Hindawi
Journal of Function Spaces
Volume 2017, Article ID 6979571, 11 pages
https://doi.org/10.1155/2017/6979571

Research Article

Existence of Mild Solutions and Controllability of Fractional Impulsive Integrodifferential Systems with Nonlocal Conditions

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Received 11 May 2017; Accepted 24 July 2017; Published 20 September 2017

Academic Editor: Xinguang Zhang

This paper is concerned with the existence results of nonlocal problems for a class of fractional impulsive integrodifferential equations in Banach spaces. We define a piecewise continuous control function to obtain the results on controllability of the corresponding fractional impulsive integrodifferential control systems. The results are obtained by means of fixed point methods. An example to illustrate the applications of our main results is given.

1. Introduction

In recent decades, existence of mild solutions of nonlocal Cauchy problems has been investigated extensively by many researchers (see [1–15] and the references cited therein). The study of abstract nonlocal semilinear initial value problems was initiated by Byszewski and Lakshmikantham [11] and Byszewski [12]. Byszewski [12] considered the existence and uniqueness of mild, strong, and classical solutions of nonlocal Cauchy problems. Lin and Liu [8] studied the existence and uniqueness of mild and classical solutions of semilinear integrodifferential equations with nonlocal Cauchy problems. Using Krasnoselskii's fixed point theorem, Schauder's fixed point theorem, and Banach contraction principle, Zhou and Jiao [13] obtained several criteria on the existence and uniqueness of mild solutions of nonlocal Cauchy problems for fractional evolution equations without impulse.

Such analysis on nonlocal Cauchy problems is important from an applied viewpoint, since the nonlocal condition has a better effect in applications than a classical initial one. For instance, the diffusion phenomenon of a small amount of gas in a transparent tube can be given a better description than using the usual local Cauchy problem. On the other hand, controllability of nonlocal problems in Banach spaces has become an active area of investigation; we refer the reader to, for example, the papers [16–29]. The most common method is to transform the controllability problem into a fixed-point problem of solutions for an appropriate operator in a function space, that is, the existence problem of differential and integrodifferential equations. Unfortunately, by [16], we know that the concept of mild solutions used in [14, 15, 17] was not suitable for fractional evolution systems.

Chang et al. [18] investigated the controllability of a class of first-order semilinear differential systems with nonlocal initial conditions in a Banach space:

$$x'(t) = Ax(t) + f(t, x(t)) + Bu(t), \quad t \in J = [0, b], \tag{1}$$
$$x(0) + g(x) = x_0 \in \mathbb{X},$$

where A generates a strongly continuous, not necessarily compact, semigroup $(T(t))_{t\geq 0}$ in the Banach space $\mathbb{X}$. Sufficient conditions for the controllability of the first-order semilinear differential system with nonlocal initial conditions were established. The approach used is Sadovskii's fixed point theorem.

Balachandran et al. [19] discussed the controllability of a class of fractional integrodifferential systems with nonlocal conditions in a Banach space:

$$\frac{d^q x(t)}{dt^q} = Ax(t) + f(t, x(t), (Hx)(t)) + Bu(t), \quad t \in J = [0, b], \quad x(0) + g(x) = x_0 \in \mathbb{X}. \tag{2}$$

Motivated by the work of the above papers and wide applications of nonlocal Cauchy problems in various fields of natural sciences and engineering, in this paper, we study the existence of nonlocal problems for a class of fractional impulsive integrodifferential systems in a Banach space of the following type:

$$\begin{aligned} D_t^q x(t) &= Ax(t) + f(t, x(t), (Hx)(t)), \quad t \in I = [0, b], \ t \neq t_k, \\ \Delta x|_{t=t_k} &= I_k(x(t_k^-)), \quad k = 1, 2, \ldots, m, \\ x(0) + g(x) &= x_0 \in \mathbb{X}, \end{aligned} \tag{3}$$

where $(Hx)(t) = \int_0^t h(t, s, x(s))ds$ and D_t^q is the Caputo fractional derivative $(0 < q < 1)$; the state $x(\cdot)$ takes values in the Banach space $\mathbb{X}$. $A : D(A) \subseteq \mathbb{X} \to \mathbb{X}$ is the infinitesimal generator of a strongly continuous semigroup $(T(t))_{t\geq 0}$ of uniformly bounded operators in $\mathbb{X}$, and A is a bounded linear operator. $f : I \times \mathbb{X} \times \mathbb{X} \to \mathbb{X}$ is a given $\mathbb{X}$-value function; $h : \Delta \times \mathbb{X} \to \mathbb{X}$ is continuous; here $\Delta = \{(t, s) : 0 \leq s \leq t \leq b\}$, $I_k : \mathbb{X} \to \mathbb{X}$, $0 = t_0 < t_1 < \cdots < t_m < t_{m+1} = b$, $\Delta x|_{t=t_k} = x(t_k^+) - x(t_k^-)$, $x(t_k^+) = \lim_{h\to 0^+} x(t_k + h)$, and $x(t_k^-) = \lim_{h\to 0^-} x(t_k + h)$ represent the right and left limits of $x(t)$ at $t = t_k$, respectively. Using the similar method and a piecewise continuous control function, we consider the controllability of a class of fractional impulsive integrodifferential systems with nonlocal initial conditions:

$$\begin{aligned} D_t^q x(t) &= Ax(t) + f(t, x(t), (Hx)(t)) + Bu(t), \quad t \in I = [0, b], \ t \neq t_k, \\ \Delta x|_{t=t_k} &= I_k(x(t_k^-)), \quad k = 1, 2, \ldots, m, \\ x(0) + g(x) &= x_0 \in \mathbb{X}, \end{aligned} \tag{4}$$

where B is a bounded linear operator from U to $\mathbb{X}$ and the control function $u(\cdot)$ is given in $L^2[I, U]$, with U as a Banach space.

We study the nonlocal initial problem (3) that describes a more general form than the previous ones reported in [18, 19]. We introduce a suitable concept of PC-mild solutions for nonlocal initial problem (3). We not only study the existence and uniqueness of a mild solution for impulsive fractional semilinear integrodifferential equation (3) but also define a piecewise continuous control function and present the results on the controllability of the corresponding fractional impulsive integrodifferential system (4) which include some known results obtained in [14, 17]. Assumptions in our results are less restrictive.

2. Preliminaries and Lemmas

Throughout this paper, let us consider the set of functions $\mathrm{PC}[I, \mathbb{X}] = \{x : I \to \mathbb{X} \mid x \in C[(t_k, t_{k+1}), \mathbb{X}]$ and there exist $x(t_k^-)$ and $x(t_k^+)$, $k = 0, 1, 2, \ldots, m$, with $x(t_k^-) = x(t_k)\}$. Endowed with the norm $\|x\|_{\mathrm{PC}} = \sup_{t\in I}\|x(t)\|$, it is easy to verify that $(\mathrm{PC}[I, \mathbb{X}], \|\cdot\|_{\mathrm{PC}})$ is a Banach space. Let $L_B(\mathbb{X})$ be the Banach space of all linear and bounded operators on $\mathbb{X}$. For a C_0-semigroup $(T(t))_{t\geq 0}$, we set $M_1 = \sup_{t\in I}\|T(t)\|_{L_B(\mathbb{X})}$. For each positive constant r, we set $B_r = \{x \in \mathrm{PC}[I, \mathbb{X}] : \|x\| \leq r\}$. Obviously, B_r is a bounded closed and convex subset.

Definition 1. The fractional integral of order γ with the lower limit zero for a function f is defined as

$$I^\gamma f(t) = \frac{1}{\Gamma(\gamma)} \int_0^t \frac{f(s)}{(t-s)^{1-\gamma}} ds, \quad t > 0, \ \gamma > 0, \tag{5}$$

provided that the right side is point-wise defined on $[0, \infty)$, where $\Gamma(\cdot)$ is the gamma function.

Definition 2. The Riemann-Liouville derivative of order γ with the lower limit zero for a function $f : [0, \infty) \to \mathbb{R}$ can be written as

$${}^L D^\gamma f(t) = \frac{1}{\Gamma(n-\gamma)} \frac{d^n}{dt^n} \int_0^t \frac{f(s)}{(t-s)^{1-n+\gamma}} ds, \quad t > 0, \ n-1 < \gamma < n. \tag{6}$$

Definition 3. The Caputo derivative of order γ for a function $f : [0, \infty) \to \mathbb{R}$ can be written as

$$D_t^\gamma f(t) = {}^L D^\gamma \left(f(t) - \sum_{k=0}^{n-1} \frac{t^k}{k!} f^{(k)}(0) \right), \quad t > 0, \ n-1 < \gamma < n. \tag{7}$$

Remark 4. If f is an abstract function with values in $\mathbb{X}$, then integrals that appear in Definitions 1–3 are taken in Bochner's sense.

Definition 5 (see [20]). Let $\mathbb{X}$ be a Banach space; a one-parameter family $T(t)$, $0 \leq t < \infty$, of bounded linear operators from $\mathbb{X}$ to $\mathbb{X}$ is a semigroup of bounded linear operators on $\mathbb{X}$ if

(1) $T(0) = I$; I is the identity operator on $\mathbb{X}$;

(2) $T(t + s) = T(t)T(s)$ for every $t, s \geq 0$ (the semigroup property).

A semigroup of bounded linear operators, $T(t)$, is uniformly continuous if $\lim_{t\downarrow 0}\|T(t) - I\| = 0$.

Definition 6 (see [21]). By a PC-mild solution of system (3), we mean a function $x \in \mathrm{PC}[I, \mathbb{X}]$ that satisfies the following integral equation:

$$x(t)=\begin{cases}\mathcal{T}(t)\left[x_0-g(x)\right]+\int_0^t(t-s)^{q-1}\mathcal{S}(t-s)f(s,x(s),(Hx)(s))\,ds, & t\in[0,t_1],\\ \mathcal{T}(t)\left[x_0-g(x)\right]+\int_0^t(t-s)^{q-1}\mathcal{S}(t-s)f(s,x(s),(Hx)(s))\,ds+\mathcal{T}(t-t_1)I_1(x(t_1^-)), & t\in(t_1,t_2],\\ \vdots \\ \mathcal{T}(t)\left[x_0-g(x)\right]+\int_0^t(t-s)^{q-1}\mathcal{S}(t-s)f(s,x(s),(Hx)(s))\,ds+\sum_{k=1}^{m}\mathcal{T}(t-t_k)I_k(x(t_k^-)), & t\in(t_m,b],\end{cases}\tag{8}$$

where $\mathcal{T}(\cdot)$ and $\mathcal{S}(\cdot)$ are called characteristic solution operators and are given by

$$\begin{aligned}\mathcal{T}(t)&=\int_0^\infty\xi_q(\theta)T(t^q\theta)\,d\theta,\\ \mathcal{S}(t)&=q\int_0^\infty\theta\xi_q(\theta)T(t^q\theta)\,d\theta,\end{aligned}\tag{9}$$

and, for $\theta\in(0,\infty)$,

$$\xi_q(\theta)=\frac{1}{q}\theta^{-1-1/q}\varpi_q\left(\theta^{-1/q}\right)\geq 0,$$

$$\varpi_q(\theta)=\frac{1}{\pi}\sum_{n=1}^{\infty}(-1)^{n-1}\theta^{-qn-1}\frac{\Gamma(nq+1)}{n!}\sin(qn\pi),\tag{10}$$

where ξ_q is a probability density function defined on $(0,\infty)$; that is,

$$\begin{gathered}\xi_q(\theta)\geq 0,\quad \theta\in(0,\infty),\\ \int_0^\infty\xi_q(\theta)\,d\theta=1.\end{gathered}\tag{11}$$

Definition 7 (see [21]). By a PC-mild solution of system (4), we mean a function $x\in PC[I,\mathbb{X}]$ that satisfies the following integral equation:

$$x(t)=\begin{cases}\mathcal{T}(t)\left[x_0-g(x)\right]+\int_0^t(t-s)^{q-1}\mathcal{S}(t-s)\left[f(s,x(s),(Hx)(s))+Bu(s)\right]ds, & t\in[0,t_1],\\ \mathcal{T}(t)\left[x_0-g(x)\right]+\int_0^t(t-s)^{q-1}\mathcal{S}(t-s)\left[f(s,x(s),(Hx)(s))+Bu(s)\right]ds+\mathcal{T}(t-t_1)I_1(x(t_1^-)), & t\in(t_1,t_2],\\ \vdots \\ \mathcal{T}(t)\left[x_0-g(x)\right]+\int_0^t(t-s)^{q-1}\mathcal{S}(t-s)\left[f(s,x(s),(Hx)(s))+Bu(s)\right]ds+\sum_{k=1}^{m}\mathcal{T}(t-t_k)I_k(x(t_k^-)), & t\in(t_m,b].\end{cases}\tag{12}$$

Definition 8. System (4) is said to be controllable on the interval I if, for every $x_0,x_1\in\mathbb{X}$, there exists a control $u\in L^2[I,U]$ such that a mild solution x of (4) satisfies $x(b)+g(x)=x_1$.

Lemma 9 (see [20]). *Linear operator A is the infinitesimal generator of a uniformly continuous semigroup if and only if A is a bounded linear operator.*

Lemma 10 (see [13] Krasnoselskii's fixed point theorem). *Let $\mathbb{X}$ be a Banach space, let B be a bounded closed and convex subset of $\mathbb{X}$, and let F_1,F_2 be maps of B into $\mathbb{X}$ such that $F_1x+F_2y\in B$ for every pair $x,y\in B$. If F_1 is a contraction and F_2 is completely continuous, then the equation $F_1x+F_2x=x$ has a solution in B.*

Lemma 11 (see [22, 23]). *The operators $\mathcal{T}(t)$ and $\mathcal{S}(t)$ defined by (9) have the following properties:*

(i) *For any fixed $t\geq 0$, $\mathcal{T}(t)$ and $\mathcal{S}(t)$ are linear and bounded operators; that is, for any $x\in\mathbb{X}$,*

$$\begin{aligned}\|\mathcal{T}(t)x\|&\leq M_1\|x\|,\\ \|\mathcal{S}(t)x\|&\leq\frac{qM_1}{\Gamma(1+q)}\|x\|.\end{aligned}\tag{13}$$

(ii) *$\{\mathcal{T}(t),t\geq 0\}$ and $\{\mathcal{S}(t),t\geq 0\}$ are strongly continuous.*

(iii) *$\{\mathcal{T}(t),t\geq 0\}$ and $\{\mathcal{S}(t),t\geq 0\}$ are uniformly continuous.*

Remark 12. Since the infinitesimal generator A is a linear bounded operator and thanks to Definition 5 and Lemma 9, we can get that (iii) is satisfied.

Lemma 13 (see [21]). *For $\sigma\in(0,1]$ and $0<a\leq b$, $|a^\sigma-b^\sigma|\leq(b-a)^\sigma$.*

3. Existence and Uniqueness of PC-Mild Solutions

In order to prove the existence and uniqueness of mild solutions of (3), we have the following assumptions:

($\mathbf{H}_1$) $f : I \times \mathbb{X} \times \mathbb{X} \to \mathbb{X}$ is continuous and there exist two functions $\mu_1, \mu_2 \in L[I, \mathbb{R}^+]$ such that

$$\|f(t, x_1, y_1) - f(t, x_2, y_2)\| \le \mu_1(t)\|x_1 - x_2\| + \mu_2(t)\|y_1 - y_2\|, \quad x_1, x_2, y_1, y_2 \in \mathbb{X}. \tag{14}$$

($\mathbf{H}_2$) $h : \triangle \times \mathbb{X} \to \mathbb{X}$ is continuous and there exists a function $\nu_1 \in C[I, \mathbb{R}^+]$ such that

$$\|h(t, s, x_1) - h(t, s, x_2)\| \le \nu_1(t)\|x_1 - x_2\|, \quad x_1, x_2 \in \mathbb{X}. \tag{15}$$

($\mathbf{H}_3$) $I_k : \mathbb{X} \to \mathbb{X}$ are continuous and there exist $\omega_k \in C[I, \mathbb{R}^+]$ such that

$$\|I_k(x_1) - I_k(x_2)\| \le \omega_k(t)\|x_1 - x_2\|, \quad x_1, x_2 \in \mathbb{X},\ k = 1, 2, \ldots, m. \tag{16}$$

($\mathbf{H}_4$) g is continuous and there exists a function $\phi \in C[I, \mathbb{R}^+]$ such that

$$\|g(x_1) - g(x_2)\| \le \phi(t)\|x_1 - x_2\|. \tag{17}$$

($\mathbf{H}_5$) The function $\Omega_m(t) : I \to \mathbb{R}^+$ is defined by

$$\Omega_m(t) = m\omega_0 M_1 + M_1\phi(t) + \frac{qM_1}{\Gamma(1+q)} \times \int_0^t (t-s)^{q-1}\left(\mu_1(s) + \nu_1^0 b\mu_2(s)\right) ds, \tag{18}$$

where $\nu_1^0 = \max\{\nu_1(t) \mid t \in I\}$, $\omega_0 = \max\{\omega_k(t) \mid t \in I, k = 1, 2, \ldots, m\}$, and $0 < \Omega_m(t) < 1$, $t \in I$.

($\mathbf{H}_5'$) The constant Ω_u and function $\Omega_m'(t) : I \to \mathbb{R}^+$ are defined by

$$\begin{aligned}\Omega_u &= \omega_0 m M_1 + \phi_0 M_1 + \frac{qKM_1}{\Gamma(1+q)} \times \int_0^b (b-s)^{q-1}\left(\mu_1(s) + \nu_1^0 b\mu_2(s)\right) ds,\\ \Omega_m'(t) &= \omega_0 m M_1 + \phi_0 M_1 + \frac{qM_1}{\Gamma(1+q)} \times \int_0^t (t-s)^{q-1}\left(\mu_1(t) + \nu_1^0 b\mu_2(t)\right) ds + \frac{qM_1\Omega_u}{\Gamma(1+q)} \int_0^t (t-s)^{q-1} ds,\end{aligned} \tag{19}$$

where $\phi_0 = \max\{\phi(t) \mid t \in I\}$ and $0 < \Omega_m'(t) < 1$, $t \in I$.

Theorem 14. *If hypotheses* ($\mathbf{H}_1$)–($\mathbf{H}_5$) *are satisfied, then (3) has a unique PC-mild solution.*

Proof. Define the operator Q on $PC[I, \mathbb{X}]$ by

$$(Qx)(t) = \begin{cases} \mathcal{T}(t)[x_0 - g(x)] + \int_0^t (t-s)^{q-1}\mathcal{S}(t-s) f(s, x(s), (Hx)(s)) ds, & t \in [0, t_1], \\ \mathcal{T}(t)[x_0 - g(x)] + \int_0^t (t-s)^{q-1}\mathcal{S}(t-s) f(s, x(s), (Hx)(s)) ds + \mathcal{T}(t-t_1) I_1(x(t_1^-)), & t \in (t_1, t_2], \\ \vdots & \\ \mathcal{T}(t)[x_0 - g(x)] + \int_0^t (t-s)^{q-1}\mathcal{S}(t-s) f(s, x(s), (Hx)(s)) ds + \sum_{k=1}^m \mathcal{T}(t-t_k) I_k(x(t_k^-)), & t \in (t_m, b]. \end{cases} \tag{20}$$

For $0 \le \tau < t \le t_1$, by virtue of (20), we conclude that

$$\begin{aligned}\|(Qx)(t) - (Qx)(\tau)\| &\le \|\mathcal{T}(t) - \mathcal{T}(\tau)\| \|x_0 - g(x)\| + \left\|\int_\tau^t (t-s)^{q-1}\mathcal{S}(t-s) f(s, x(s), (Hx)(s)) ds\right\| \\ &+ \left\|\int_0^\tau (t-s)^{q-1}[\mathcal{S}(t-s) - \mathcal{S}(\tau-s)] \times f(s, x(s), (Hx)(s)) ds\right\| \\ &+ \left\|\int_0^\tau \left[(t-s)^{q-1} - (\tau-s)^{q-1}\right]\mathcal{S}(\tau-s) \times f(s, x(s), (Hx)(s)) ds\right\|.\end{aligned} \tag{21}$$

It follows from Lemma 11, part (iii) and Lemma 13 that

$$\|(Qx)(t) - (Qx)(\tau)\| \longrightarrow 0 \quad \text{as } t \longrightarrow \tau. \tag{22}$$

Thus, we deduce that $Qx \in C[[0,t_1],\mathbb{X}]$. For $t_1 < \tau < t \le t_2$, we have

$$\begin{aligned}
&\|(Qx)(t)-(Qx)(\tau)\| \le \|\mathcal{T}(t)-\mathcal{T}(\tau)\|\,\|x_0-g(x)\| \\
&\quad + \|\mathcal{T}(t-t_1)-\mathcal{T}(\tau-t_1)\|\,\|I_1(x(t_1^-))\| \\
&\quad + \left\|\int_\tau^t (t-s)^{q-1}\mathcal{S}(t-s) \times f(s,x(s),(Hx)(s))\,ds\right\| \\
&\quad + \left\|\int_0^\tau (t-s)^{q-1}[\mathcal{S}(t-s)-\mathcal{S}(\tau-s)] \times f(s,x(s),(Hx)(s))\,ds\right\| \\
&\quad + \left\|\int_0^\tau \left[(t-s)^{q-1}-(\tau-s)^{q-1}\right]\mathcal{S}(\tau-s) \times f(s,x(s),(Hx)(s))\,ds\right\|.
\end{aligned} \tag{23}$$

From (23), we know that $Qx \in C[(t_1,t_2],\mathbb{X}]$. Using the same method, we obtain $Qx \in C[(t_2,t_3],\mathbb{X}],\ldots, Qx \in C[(t_m,b],\mathbb{X}]$, and therefore $Qx \in \mathrm{PC}[I,\mathbb{X}]$. For each $t \in (t_i,t_{i+1}]$, $1\le i\le m$, $x,y\in \mathrm{PC}[I,\mathbb{X}]$,

$$\begin{aligned}
&\|(Qx)(t)-(Qy)(t)\| \le M_1\phi(t)+\frac{qM_1}{\Gamma(1+q)}\int_0^t (t-s)^{q-1} \times \left(\mu_1(s)+\nu_1^0 b\mu_2(s)\right)ds\,\|x-y\|_{\mathrm{PC}} \\
&\quad + \left\|\sum_{k=1}^{i}\mathcal{T}(t-t_i)I_k(x(t_k^-)) - \sum_{k=1}^{i}\mathcal{T}(t-t_i)I_k(y(t_k^-))\right\| \le \Omega_i(t)\,\|x-y\|_{\mathrm{PC}}.
\end{aligned} \tag{24}$$

When $i=m$, we get

$$\|(Qx)(t)-(Qy)(t)\| \le \Omega_m(t)\,\|x-y\|_{\mathrm{PC}}. \tag{25}$$

It follows now from $\Omega_i(t)\le\Omega_m(t)$, $(\mathbf{H}_5)$, and the contraction mapping principle that Q has a unique fixed point $x\in \mathrm{PC}[I,\mathbb{X}]$; that is,

$$x(t)=\begin{cases}
\mathcal{T}(t)[x_0-g(x)]+\int_0^t (t-s)^{q-1}\mathcal{S}(t-s)f(s,x(s),(Hx)(s))\,ds, & t\in[0,t_1],\\
\mathcal{T}(t)[x_0-g(x)]+\int_0^t (t-s)^{q-1}\mathcal{S}(t-s)f(s,x(s),(Hx)(s))\,ds+\mathcal{T}(t-t_1)I_1(x(t_1^-)), & t\in(t_1,t_2],\\
\vdots & \\
\mathcal{T}(t)[x_0-g(x)]+\int_0^t (t-s)^{q-1}\mathcal{S}(t-s)f(s,x(s),(Hx)(s))\,ds+\sum_{k=1}^{m}\mathcal{T}(t-t_k)I_k(x(t_k^-)), & t\in(t_m,b],
\end{cases} \tag{26}$$

is a unique PC-mild solution of (3). The proof is complete. □

In order to obtain more existence results, we have the following assumptions:

$(\mathbf{H}_6)$ $f: I\times\mathbb{X}\times\mathbb{X}\to\mathbb{X}$ is continuous and there exist three functions $\mu_3,\mu_4,\mu_5\in L[I,\mathbb{R}^+]$ such that

$$\|f(t,x,y)\| \le \mu_3(t)+\mu_4(t)\|x\|+\mu_5(t)\|y\|, \quad t\in I,\ x,y\in\mathbb{X}. \tag{27}$$

$(\mathbf{H}_7)$ $h:\triangle\times\mathbb{X}\to\mathbb{X}$ is continuous and there exist two functions $\nu_2,\nu_3\in C[I,\mathbb{R}^+]$ such that

$$\|h(t,s,x)\| \le \nu_2(s)+\nu_3(s)\|x\|, \quad x\in\mathbb{X}. \tag{28}$$

$(\mathbf{H}_8)$ $I_k:\mathbb{X}\to\mathbb{X}$ are continuous and there exist $\psi_k\in C[I,\mathbb{R}^+]$ such that

$$\|I_k(x)\| \le \psi_k(t)\|x\|, \quad x\in\mathbb{X}. \tag{29}$$

Define $\psi_0=\max\{\psi_k(t)\mid t\in I,\ k=1,2,\ldots,m\}$.

$(\mathbf{H}_9)$ There exists a function $\kappa\in C[I,\mathbb{R}^+]$ such that

$$\|g(x)\| \le \kappa(t)\|x\|, \quad x\in\mathbb{X}. \tag{30}$$

Define $\kappa_0=\max\{\kappa(t)\mid t\in I\}$.

$(\mathbf{H}_{10})$ For all bounded subsets B_r, the set

$$\Pi_{m,h,\delta}(t)=\left\{\int_0^{t-h}(t-s)^{q-1}\mathcal{S}_\delta(t-s)F(s)\,ds + \sum_{k=1}^{m}\mathcal{T}_\delta(t-t_k)I_k(x(t_k^-)) : x\in B_r\right\} \tag{31}$$

is relatively compact in $\mathbb{X}$ for arbitrary $h\in(0,t)$ and $\delta>0$, where $\mathcal{T}_\delta(t)$ and $\mathcal{S}_\delta(t)$ are defined by

$$\begin{aligned}
\mathcal{T}_\delta(t)&=\int_\delta^\infty \xi_q(\theta)T(t^q\theta)\,d\theta,\\
\mathcal{S}_\delta(t)&=q\int_\delta^\infty \theta\xi_q(\theta)T(t^q\theta)\,d\theta.
\end{aligned} \tag{32}$$

($\mathbf{H}_{10}'$) For all bounded subsets B_r, the set

$$\Pi'_{m,h,\delta}(t) = \left\{\int_0^{t-h}(t-s)^{q-1}\mathcal{S}_\delta(t-s)[F(s)+Bu(s)]\,ds + \sum_{k=1}^{m}\mathcal{T}_\delta(t-t_k)I_k(x(t_k^-)) : x\in B_r\right\} \tag{33}$$

is relatively compact in $\mathbb{X}$ for arbitrary $h\in(0,t)$ and $\delta>0$.

Theorem 15. *Let hypotheses* ($\mathbf{H}_4$) *and* ($\mathbf{H}_6$)–($\mathbf{H}_{10}$) *be satisfied. If the inequalities*

$$\frac{qb^qM_1}{\Gamma(1+q)}\int_0^b\varphi_2(s)\,ds + mM_1\psi_0 + M_1\kappa_0 < 1, \qquad \phi_0 M_1 < 1 \tag{34}$$

hold, where $\varphi_2(s) = \mu_4(s)+\mu_5(s)\int_0^s\nu_3(\theta)\,d\theta$ *and* ϕ_0 *is as in* ($\mathbf{H}_5'$), *then (3) has at least one PC-mild solution.*

Proof. We shall present the results in six steps.

Step 1 (Continuity of Q defined by (20) *on* $(t_i,t_{i+1}]$ $(i=0,1,2,\dots,m)$). Let $x_n, x\in \mathrm{PC}[I,\mathbb{X}]$ and $\|x_n-x^*\|_{\mathrm{PC}}\to 0$ $(n\to\infty)$. Then $r=\sup_n\|x_n\|_{\mathrm{PC}}<\infty$ and $\|x^*\|_{\mathrm{PC}}<r$. For $t\in(t_i,t_{i+1}]$ $(i=0,1,2,\dots,m)$, we have

$$\begin{aligned}\|Qx_n(t)-Qx(t)\| \le{}& \frac{qM_1}{\Gamma(1+q)}\int_0^t(t-s)^{q-1}\\ &\times\|f(s,x_n(s),(Hx_n)(s))\\ &-f(s,x(s),(Hx)(s))\|\,ds\\ &+\psi_0M_1\sum_{k=1}^m\|I_k(x_n(t_k^-))-I_k(x(t_k^-))\|\\ &+M_1\|g(x_n)-g(x)\|.\end{aligned} \tag{35}$$

Since the functions f, I_k, and g are continuous, we conclude that

$$\begin{gathered} f(s,x_n(s),(Hx_n)(s)) \longrightarrow f(s,x(s),(Hx)(s)),\\ g(x_n)\longrightarrow g(x),\\ I_k(x_n(t_k^-))\longrightarrow I_k(x(t_k^-)),\quad n\longrightarrow\infty.\end{gathered} \tag{36}$$

Applications of ($\mathbf{H}_6$) and ($\mathbf{H}_7$) yield

$$\begin{aligned}&\|f(s,x_n(s),(Hx_n)(s))-f(s,x(s),(Hx)(s))\|\\ &\le 2\mu_3(s)+2\mu_5(s)\int_0^s\nu_2(\theta)\,d\theta\\ &\quad+\left(2\mu_4(s)+2\mu_5(s)\int_0^s\nu_3(\theta)\,d\theta\right)r,\end{aligned} \tag{37}$$

which implies that

$$(t-s)^{q-1}\cdot\|f(s,x_n(s),(Hx_n)(s))-f(s,x(s),(Hx)(s))\| \in L^1[I,\mathbb{R}^+]. \tag{38}$$

By Lebesgue's dominated convergence theorem, we get

$$\int_0^t(t-s)^{q-1}\times\|f(s,x_n(s),Hx_n(s)) - f(s,x(s),Hx(s))\|\,ds\longrightarrow 0, \tag{39}$$

and so

$$\lim_{n\to\infty}\|Qx_n(t)-Qx(t)\|_{\mathrm{PC}}=0. \tag{40}$$

Step 2 (Q maps bounded sets into bounded sets in $\mathrm{PC}[I,\mathbb{X}]$). From (20), we get

$$\begin{aligned}\|(Qx)(t)\| ={}& \|\mathcal{T}(t)[x_0-g(x)]\|\\ &+\int_0^t(t-s)^{q-1}\|\mathcal{S}(t-s)f(s,x(s),(Hx)(s))\|\,ds\\ &+\sum_{k=1}^m\|\mathcal{T}(t-t_k)I_k(x(t_k^-))\|,\end{aligned} \tag{41}$$

where

$$\begin{aligned}&\|f(s,x(s),(Hx)(s))\|\\ &\le\mu_3(s)+\mu_5(s)\int_0^s\nu_2(\theta)\,d\theta\\ &\quad+\left(\mu_4(s)+\mu_5(s)\int_0^s\nu_3(\theta)\,d\theta\right)\|x\|\\ &\le\varphi_1(s)+\varphi_2(s)\|x\|.\end{aligned} \tag{42}$$

By Lemma 11 and (42), we obtain

$$\begin{aligned}\|(Qx)(t)\| \le{}& \frac{qb^qM_1}{\Gamma(1+q)}\int_0^t(\varphi_1(s)+\varphi_2(s)\|x\|)\,ds\\ &+M_1\|x_0\|+M_1\kappa_0\|x\|+mM_1\psi_0\|x\|.\end{aligned} \tag{43}$$

Thus, for any $x\in B_r=\{x\in\mathrm{PC}[I,\mathbb{X}]:\|x\|_{\mathrm{PC}}\le r\}$, we have

$$\begin{aligned}&\|(Qx)(t)\|\\ &\le M_1\|x_0\|+\frac{qb^qM_1}{\Gamma(1+q)}\int_0^b\varphi_1(s)\,ds\\ &\quad+\left(\frac{qb^qM_1}{\Gamma(1+q)}\int_0^b\varphi_2(s)\,ds+mM_1\psi_0+M_1\kappa_0\right)r\\ &=\gamma_1.\end{aligned} \tag{44}$$

Hence, we deduce that $\|(Qx)(t)\| \leq \gamma_1$, that is, Q maps bounded sets into bounded sets in $PC[I, \mathbb{X}]$.

Step 3 ($Q(B_r)$ is equicontinuous with B_r on $(t_i, t_{i+1}]$ ($i = 0, 1, 2, \ldots, m$)). For any $x \in B_r$, t', $t'' \in (t_i, t_{i+1}]$ ($i = 0, 1, 2, \ldots, m$), we obtain

$$\begin{aligned}\left\|(Qx)(t'') - (Qx)(t')\right\| &\leq \left\|\mathcal{T}(t'')x_0 - \mathcal{T}(t')x_0\right\| \\ &+ \left\|\mathcal{T}(t'')g(x) - \mathcal{T}(t')g(x)\right\| \\ &+ \left\|\int_0^{t''}(t''-s)^{q-1}\mathcal{S}(t''-s)F(s)\,ds \right.\\ &\left.- \int_0^{t'}(t'-s)^{q-1}\mathcal{S}(t'-s)F(s)\,ds\right\| \\ &+ \left\|\sum_{k=1}^{m}\mathcal{T}(t''-t_k)I_k(x(t_k^-)) \right.\\ &\left.- \sum_{k=1}^{m}\mathcal{T}(t'-t_k)I_k(x(t_k^-))\right\|.\end{aligned} \tag{45}$$

Based on a straightforward computation, we have

$$\begin{aligned}\left\|(Qx)(t'') - (Qx)(t')\right\| &\leq \left\|\mathcal{T}(t'') - \mathcal{T}(t')\right\|\left\|x_0\right\| \\ &+ \left\|\mathcal{T}(t'')g(x) - \mathcal{T}(t')g(x)\right\| \\ &+ \left\|\int_{t'}^{t''}(t''-s)^{q-1}\mathcal{S}(t''-s)F(s)\,ds\right\| \\ &+ \left\|\int_0^{t'}\left[(t''-s)^{q-1} - (t'-s)^{q-1}\right]\mathcal{S}(t''-s)\right.\\ &\left.\cdot F(s)\,ds\right\| + \left\|\int_0^{t'}(t'-s)^{q-1}\right.\\ &\left.\cdot\left[\mathcal{S}(t''-s) - \mathcal{S}(t'-s)\right]F(s)\,ds\right\| \\ &+ mM_1\left\|\mathcal{T}(t''-t') - I\right\|\left\|I_k(x(t_k^-))\right\|.\end{aligned} \tag{46}$$

It follows from Lemma 11, part (iii) and Lemma 13 that $\lim_{t''\to t'}\|(Qx)(t'') - (Qx)(t')\| = 0$. Thus, $Q(B_r)$ is equicontinuous with B_r on $(t_i, t_{i+1}]$ ($i = 0, 1, 2, \ldots, m$).

Step 4 (P_i map B_r into a precompact set in $\mathbb{X}$ ($i = 1, \ldots, m$)). We define the operator

$$(Qx)(t) = (P_i x)(t) + (Lx)(t), \tag{47}$$

where

$$\begin{aligned}(Lx)(t) &= \mathcal{T}(t)\left[x_0 - g(x)\right], \\ (P_i x)(t) &\\ &= \int_0^t (t-s)^{q-1}\mathcal{S}(t-s)f(s, x(s), (Hx)(s))\,ds \\ &+ \sum_{k=1}^{i}\mathcal{T}(t-t_k)I_k(x(t_k^-)), \quad i = 1, \ldots, m.\end{aligned} \tag{48}$$

Define $\Pi = P_i B_r$ and $\Pi(t) = \{(P_i x)(t) : x \in B_r\}$ for $t \in I$. Set

$$\Pi_{i,h,\delta}(t) = \left\{(P_{i,h,\delta}x)(t) : x \in B_r\right\}, \tag{49}$$

where

$$\begin{aligned}\Pi_{i,h,\delta}(t) = \left\{\int_0^{t-h}(t-s)^{q-1}\mathcal{S}_\delta(t-s)F(s)\,ds\right.\\ \left.+ \sum_{k=1}^{i}\mathcal{T}_\delta(t-t_k)I_k(x(t_k^-)) : x \in B_r\right\}.\end{aligned} \tag{50}$$

From hypotheses we imposed and the same method used in [16, Theorem 3.2], it is not difficult to verify that the set $\Pi(t)$ can be arbitrary approximated by the relatively compact set $\Pi_{i,h,\delta}(t)$. Thus, $P_i(B_r)(t)$ are relatively compact in $\mathbb{X}$.

Step 5 ($Lx + P_i y \in B_r$ for $x, y \in B_r$ ($i = 1, \ldots, m$)). Note that

$$\frac{qb^q M_1}{\Gamma(1+q)}\int_0^b \varphi_2(s)\,ds + mM_1\psi_0 + M_1\kappa_0 < 1. \tag{51}$$

Choose

$$\begin{aligned}&\frac{M_1\left\|x_0\right\| + \left(qb^q M_1/\Gamma(1+q)\right)\int_0^b \varphi_1(s)\,ds}{1 - \left(qb^q M_1/\Gamma(1+q)\right)\int_0^b \varphi_2(s)\,ds - mM_1\psi_0 - M_1\kappa_0} \\ &\quad \leq r\end{aligned} \tag{52}$$

and define operators L and P_i on B_r by

$$\begin{aligned}(Lx)(t) &= \mathcal{T}(t)\left[x_0 - g(x)\right], \\ (P_i x)(t) &\\ &= \int_0^t (t-s)^{q-1}\mathcal{S}(t-s)f(s, x(s), (Hx)(s))\,ds \\ &+ \sum_{k=1}^{i}\mathcal{T}(t-t_k)I_k(x(t_k^-)), \quad i = 1, \ldots, m.\end{aligned} \tag{53}$$

It is sufficient to proceed exactly as in step 1 to step 4 of the proof to deduce that P_i are continuous and compact. Thus, to complete this proof, it suffices to show that L is a contraction mapping and that $Lx + P_i y \in B_r$ for $x, y \in B_r$. Indeed, for any $x \in B_r$, by virtue of (43) and (51), we have

$$\begin{aligned} \|(Qx)(t)\| &\le M_1\|x_0\| + \frac{qb^qM_1}{\Gamma(1+q)}\int_0^b \varphi_1(s)\,ds \\ &\quad + \left(\frac{qb^qM_1}{\Gamma(1+q)}\int_0^b \varphi_2(s)\,ds + mM_1\psi_0 + M_1\kappa_0\right) r \\ &\le r. \end{aligned} \tag{54}$$

Consequently, if $x, y \in B_r$, then $Lx + P_i y \in B_r$.

Step 6 (L is a contraction mapping). For any $t', t'' \in (t_i, t_{i+1}]$ $(i = 0, 1, 2, \ldots, m)$ and $x, y \in \mathrm{PC}[I, \mathbb{X}]$, we have

$$\begin{aligned} \|(Lx)(t) - (Ly)(t)\| &\le \|\mathcal{T}(t)(g(x) - g(y))\| \\ &\le \|\mathcal{T}(t)\|\,\|g(x) - g(y)\| \\ &\le \phi_0 M_1 \|x - y\|_{\mathrm{PC}}. \end{aligned} \tag{55}$$

Since $\phi_0 M_1 < 1$, L is a contraction mapping. Hence, by Lemma 10, we conclude that (3) has at least one PC-mild solution on I. This completes the proof. □

4. Controllability Results

In this section, we impose the following conditions to prove the results.

($\mathbf{H}_{11}$) Define $I_i = (t_{i-1}, t_i]$ $(i = 1, 2, \ldots, m+1)$. The linear operator W_i from $L^2[I_i, U]$ into $\mathbb{X}$ defined by

$$W_i u = \int_0^{t_i} (t_i - s)^{q-1}\mathcal{S}(t_i - s)Bu(s)\,ds \tag{56}$$

induces an invertible operator $\widetilde{W}_i^-$ defined on $L^2[I_i, U]/\mathrm{Ker}W_i$ and there exists a positive constant $K > 0$ such that $\|B\widetilde{W}_i^-\| \le K$.

Theorem 16. *If hypotheses* ($\mathbf{H}_1$)–($\mathbf{H}_4$), ($\mathbf{H}_5'$), *and* ($\mathbf{H}_{11}$) *are satisfied, then system (4) is controllable on* I.

Proof. Using ($\mathbf{H}_{11}$), for an arbitrary function $x(\cdot)$, we define the piecewise continuous control u by

$$u(t) = \begin{cases} \widetilde{W}_1^-\left[x_0 + \dfrac{x_1 - x_0}{m+1} - \mathcal{T}(t_1)[x_0 - g(x)] - \int_0^{t_1}(t_1 - s)^{q-1}\mathcal{S}(t_1 - s)f(s, x(s), (Hx)(s))\,ds\right](t), & t \in [0, t_1], \\ \widetilde{W}_2^-\left[x_0 + \dfrac{2(x_1 - x_0)}{m+1} - \mathcal{T}(t_2)[x_0 - g(x)] - \int_0^{t_2}(t_2 - s)^{q-1}\mathcal{S}(t_2 - s)f(s, x(s), (Hx)(s))\,ds - \mathcal{T}(t_2 - t_1)I_1(x(t_1^-))\right](t), & t \in (t_1, t_2], \\ \vdots \\ \widetilde{W}_{m+1}^-\left[x_1 - \mathcal{T}(b)[x_0 - g(x)] - \int_0^{b}(b - s)^{q-1}\mathcal{S}(b - s)f(s, x(s), (Hx)(s))\,ds - \sum_{k=1}^{m}\mathcal{T}(b - t_k)I_k(x(t_k^-))\right](t), & t \in (t_m, b]. \end{cases} \tag{57}$$

On the basis of this control, with a similar proof to Theorem 14, we can conclude that the operator Q defined by

$$(Qx)(t) = \begin{cases} \mathcal{T}(t)[x_0 - g(x)] + \int_0^t (t - s)^{q-1}\mathcal{S}(t - s)[f(s, x(s), (Hx)(s)) + Bu(s)]\,ds, & t \in [0, t_1], \\ \mathcal{T}(t)[x_0 - g(x)] + \int_0^t (t - s)^{q-1}\mathcal{S}(t - s)[f(s, x(s), (Hx)(s)) + Bu(s)]\,ds + \mathcal{T}(t - t_1)I_1(x(t_1^-)), & t \in (t_1, t_2], \\ \vdots \\ \mathcal{T}(t)[x_0 - g(x)] + \int_0^t (t - s)^{q-1}\mathcal{S}(t - s)[f(s, x(s), (Hx)(s)) + Bu(s)]\,ds + \sum_{k=1}^{m}\mathcal{T}(t - t_k)I_k(x(t_k^-)), & t \in (t_m, b], \end{cases} \tag{58}$$

has a fixed point $x(\cdot)$. This fixed point is a PC-mild solution of system (4), which implies that the system is controllable on I. The proof is complete. □

Theorem 17. *Assume that hypotheses* $(\mathbf{H}_4)$, $(\mathbf{H}_6)$–$(\mathbf{H}_9)$, $(\mathbf{H}'_{10})$, *and* $(\mathbf{H}_{11})$ *are satisfied. If the inequalities*

$$\frac{qb^q M_1}{\Gamma(1+q)} \int_0^b (\varphi_2(s) + N_1)\, ds + mM_1\psi_0 + M_1\kappa_0 < 1, \qquad \phi_0 M_1 < 1 \tag{59}$$

hold, where $N_1 = qKM_1 \int_0^b (b-s)^{q-1}\varphi_2(s)\, ds/\Gamma(1+q) + mKM_1\psi_0$ *and* $\varphi_2(s)$ *and* ϕ_0 *are as in Theorem 15, then system (4) is controllable on* I.

Proof. The proof is similar to that of Theorem 15 and so is omitted. □

5. Example

Consider the following nonlinear partial integrodifferential equation of the form

$$\begin{aligned}
&\frac{\partial^{1/3}}{\partial t^{1/3}} z(t,y) \\
&\quad = \int_0^1 (y-s)\, z(s,y)\, ds + f(t, z(t,y), Hz(t,y)) \\
&\qquad + \mu(t,y), \\
&z(t,0) = z(t,1) = 0, \quad t \in J = [0,1], \\
&z(0,y) + \phi(t)\, z(t,y) = z_0(y), \quad 0 \le y \le 1, \\
&\Delta z|_{t=1/2} = I_1\left(x\left(\frac{1}{2}^-\right)\right),
\end{aligned} \tag{60}$$

where $\mu : J \times (0,1) \to (0,1)$ is continuous. Let us take $\mathbb{X} = C([0,1])$. Consider the operator $A : D(A) \subseteq \mathbb{X} \to \mathbb{X}$ defined by

$$(Aw)(t) = \int_0^1 (y-s)\, w(s)\, ds. \tag{61}$$

It is not difficult to get

$$\begin{aligned}
\|Aw\| &= \|w\| \int_0^1 |y-s|\, ds = \left(\frac{1}{2} - y(1-y)\right)\|w\| \\
&\le \frac{1}{2}\|w\|
\end{aligned} \tag{62}$$

and, clearly, A is the infinitesimal generator of a uniformly continuous semigroup $(T(t))_{t\ge 0}$ on $\mathbb{X}$. Put $x(t)(y) = z(t,y)$ and $u(t)(y) = \mu(t,y)$, and take

$$\begin{aligned}
f(t,x,Hx) &= k_0 x + Hx, \\
(Hx)(t) &= \int_0^t h(t,s,x(s))\, ds, \\
h(t,s,x) &= k_1 x, \\
I_1(x) &= \omega(t)\, x, \\
g(x) &= \phi(t)\, x,
\end{aligned} \tag{63}$$

where k_0 and k_1 are positive constants and $\omega(t)$ and $\phi(t)$ are continuous functions. Then $f : [0,1] \times \mathbb{X} \times \mathbb{X} \to \mathbb{X}$ and $I_1 : \mathbb{X} \to \mathbb{X}$ are continuous functions; f, g, I_1, and h satisfy $(\mathbf{H}_6)$–$(\mathbf{H}_9)$, respectively.

For $y \in (0,1]$, we define

$$\begin{aligned}
W_1 u &= \int_0^{1/2} \left(\frac{1}{2} - s\right)^{-2/3} \mathcal{S}\left(\frac{1}{2} - s\right) Bu(s)\, ds, \\
W_2 u &= \int_0^1 (1-s)^{-2/3}\, \mathcal{S}(1-s)\, Bu(s)\, ds,
\end{aligned} \tag{64}$$

where

$$\begin{aligned}
\mathcal{T}(t)\, w(s) &= \int_0^\infty \xi_{1/3}(\theta)\, w\left(t^{1/3}\theta + s\right) d\theta, \\
\mathcal{S}(t)\, w(s) &= \frac{1}{3}\int_0^\infty \theta \xi_{1/3}(\theta)\, w\left(t^{1/3}\theta + s\right) d\theta,
\end{aligned} \tag{65}$$

and, for $\theta \in (0,\infty)$,

$$\begin{aligned}
&\xi_{1/3}(\theta) = 3\theta^{-4}\varpi_{1/3}\left(\theta^{-3}\right), \\
&\varpi_{1/3}(\theta) \\
&\quad = \frac{1}{\pi}\sum_{n=1}^\infty (-1)^{n-1}\theta^{-(n+3)/3}\frac{\Gamma((n+3)/3)}{n!}\sin\left(\frac{n\pi}{3}\right).
\end{aligned} \tag{66}$$

Moreover, the linear operator W_i from $L^2[I_i, U]$ $(i = 1,2)$ into $\mathbb{X}$ induces an invertible operator $\widetilde{W}_i^-$ defined on $L^2[I_i, U]/\operatorname{Ker} W_i$ and there exists a positive constant $K > 0$ such that $\|B\widetilde{W}_i^-\| \le K$; that is, $(\mathbf{H}_{11})$ is satisfied. With the choices of A, f, g, H, and $B = I$ (the identity operator), we see that (60) is an abstract formulation of (4). All conditions of Theorem 17 are able to be fulfilled, so we deduce that (60) is controllable on I. On the other hand, we have

$$\begin{aligned}
&\|f(t,x,Hx) - f(t,y,Hy)\| \\
&\quad \le k_0\|x-y\| + k_1\|x-y\|, \\
&\|h(t,s,x) - h(t,s,y)\| \le k_1\|x-y\|, \\
&\|I_1(x) - I_1(y)\| \le \omega(t)\|x-y\|, \\
&\|g(x) - g(y)\| \le \phi(t)\|x-y\|.
\end{aligned} \tag{67}$$

It is easy to see that all assumptions of Theorem 16 are satisfied when using the suitable choices of k_0, k_1, ω, ϕ. Hence, Theorem 16 can also yield controllability of (60) on I.

6. Conclusions

In this paper, we studied the existence and uniqueness results for a class of impulsive fractional semilinear integrodifferential equations with nonlocal initial conditions in a Banach space. Introducing the concept of PC-mild solutions and using the piecewise continuous control functions and uniformly continuous semigroup, we obtained the controllability results for the corresponding fractional impulsive integrodifferential system. Assuming that the semigroup is compact and utilizing some additional conditions, Hernández and O'Regan [30] showed that some known results on exact controllability (see the references cited therein) are valid if and only if the Banach space is finite dimensional. Recently, Hernández et al. [31] pointed out that some recent results on exact controllability of abstract differential systems with an unbounded linear operator dominated by a sectorial operator were not applicable. Contrary to those results, we do not need in our results conflicting conditions, which, in a certain sense, is a significant improvement compared to the results in the cited papers. An illustrative example is given to demonstrate the effectiveness of the results obtained. Our future work will focus on constrained controllability, nonlocal problems, and their applications in nonlinear dynamical systems (see [32–36]).

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This research is supported by Shandong Provincial Natural Science Foundation (Grants nos. ZR2016AB04 and ZR2016JL021), a Project of Shandong Province Higher Educational Science and Technology Program (Grant no. J17KB121), Major International (Regional) Joint Research Project of National Natural Science Foundation of China (Grant no. 61320106011), National Natural Science Foundation of China (Grants nos. 61503171 and 61527809), China Postdoctoral Science Foundation (Grant no. 2015M582091), Foundation for Young Teachers of Qilu Normal University (Grants nos. 2016L0605, 2017JX2311, and 2017JX2312), Doctoral Scientific Research Foundation of Linyi University (Grant no. LYDX2015BS001), and Scientific Research Foundation for University Students of Qilu Normal University (Grant no. XS2017L05).

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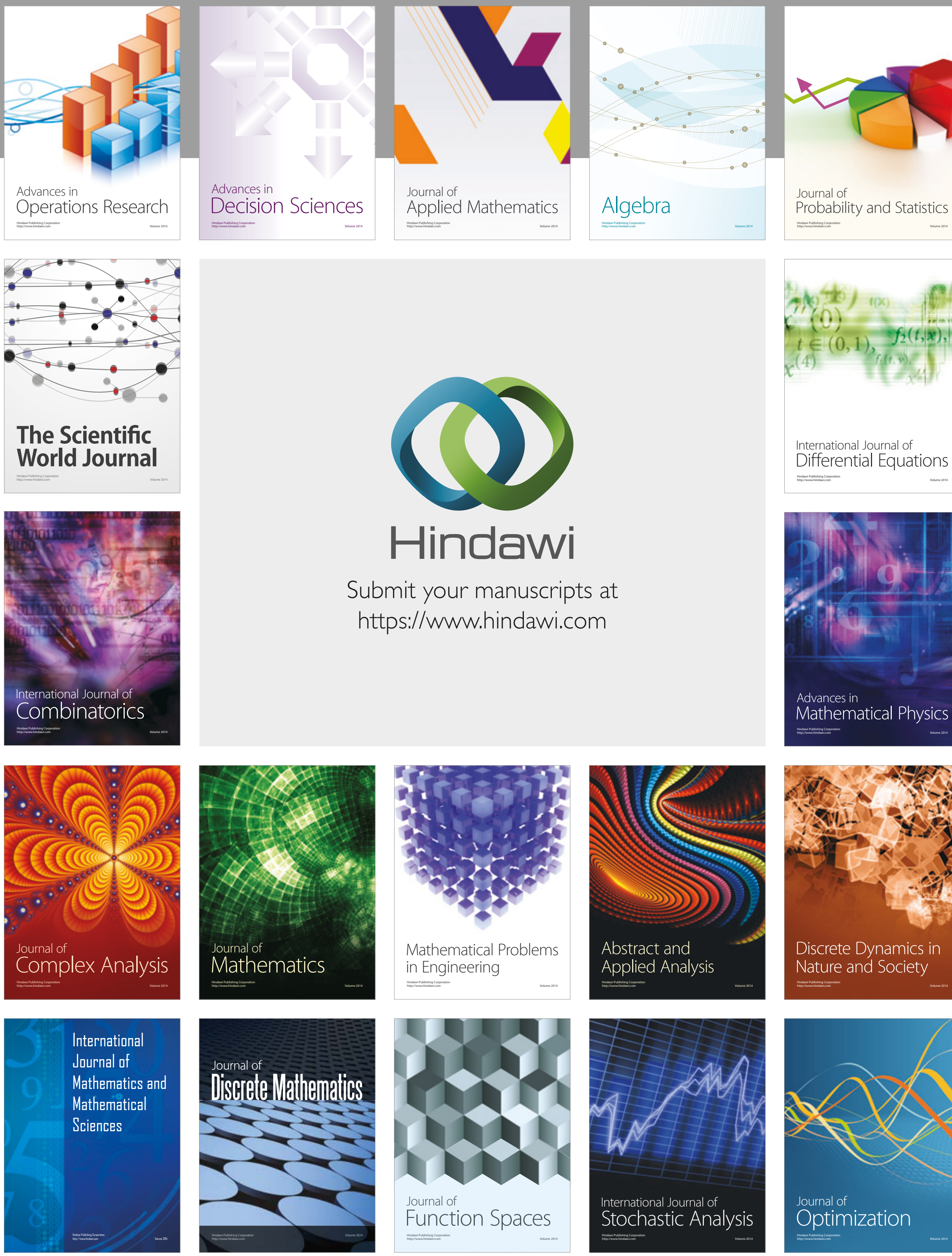
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