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Research Article

H_∞ Stochastic Control of a Class of Networked Control Systems with Time Delays and Packet Dropouts

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This paper studies the H-infinity stochastic control problem for a class of networked control systems (NCSs) with time delays and packet dropouts. The state feedback closed-loop NCS is modeled as a Markovian jump linear system. Through using a Lyapunov function, a sufficient condition is obtained, under which the system is stochastically exponential stability with a desired H-infinity disturbance attenuation level. The designed H-infinity controller is obtained by solving a set of linear matrix inequalities with some inversion constraints. An numerical example is presented to demonstrate the effectiveness of the proposed method.

1. Introduction

In the past few years, the networked control systems (NCSs) whose control loops are connected via communication networks have received increasing attention due to their advantages, such as reduced cost, low weight, easier installation, and maintenance. Time delay and packet dropout are the two major causes of instability of system and deterioration of system performance. Therefore, the time delay and packet dropout problems have been investigated in the existing literature. In [1], time delays are time-varying in intervals. In [2, 3], the bounds were imposed on the maximum number of successive dropouts. In [4], the sufficient condition that establishes the quantitative relation between the packet-dropout rate and the stability of the NCS with a constant delay is obtained.

Considering the disturbance attenuation problem, there has been much research effort on H_∞ controller design. In [5–7], the controller dynamics is continuous, but in many NCSs, the system is controlled by a discrete-time controller with sample and hold devices. In [8–10], a discrete-time controller is designed; however, it should be pointed out that the packet dropout or the delay problem is studied separately.

In [11, 12], H_∞ control of a class of systems with random packet dropout is investigated. It is noticed that the plant is a discrete-time system and the delay is a multiple of the sampling time; therefore, the result of the papers cannot be applied to the NCSs when the plant is a continuous-time system and the delay is smaller than the sampling period. In [13], the plant studied is a continuous-time system; the delay takes values in a finite set at a fixed rate. In fact, the time delays and packet dropouts may be random and modeled as Markov chains in most cases. Unfortunately, they do not take into account the time delay and packet dropout with Markovian characterization in [13].

In this paper, we investigate the H_∞ stochastic control of a class of NCSs with time delays and packet dropouts. The random time delay and packet dropout are described by a Markov chain. By using a Lyapunov function, we obtain the system with exponential stability with a desired H_∞ disturbance attenuation level. The designed H_∞ stochastic

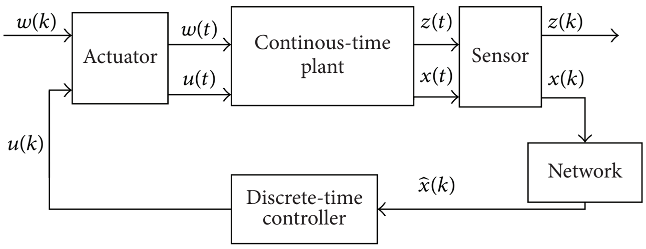

Figure 1: The structure of NCS.

controller is obtained by an iterative linear matrix inequality approach. To demonstrate the effectiveness of the method, an illustrative example is presented.

2. Model for Networked Control System

The structure of the NCS is shown in Figure 1. Consider a continuous-time linear system described by

$$\dot{x} = A_p x(t) + B_p u(t) + E_p w(t), \quad z(t) = Cx(t), \tag{1}$$

where $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the plant input, $w(t) \in R^q$ is the disturbance input, and $z(t) \in R^l$ is the plant output. A_p, B_p, E_p, and C are constant matrices of appropriate dimensions.

The following assumptions are made for the considered NCS throughout the paper [13].

The controller is event-driven; both the sensor and the actuator are time-driven. The sampling period of the sensor is T. The actuator has a receiving buffer which contains the most recently updated packet from the controller. The actuator reads the buffer periodically at a smaller period than T, say $T_0 = T/N$ for some integer N large enough. The sensor and the actuator are time synchronized. Upon reading a new value, the actuator with a zero-order-hold device will update the output value. The network-induced delay $\tau(k)$ satisfies $0 \le \tau(k) < T$.

Based on the above assumptions, the discrete-time state feedback H_∞ controller can be expressed as follows:

$$u(k) = K\hat{x}(k), \tag{2}$$

where

$$\hat{x}(k) = \begin{cases} x(k), & \text{if } x(k) \text{ is successfully transmitted,} \\ \hat{x}(k-1), & \text{if } x(k) \text{ is lost during transmission,} \end{cases} \tag{3}$$

where $x(k)$ is the value of $x(t)$ at the sampling time kT.

Consider

$$z(k) = Cx(k), \tag{4}$$

where $z(k)$ is the value of the $z(t)$ at the sampling time kT.

During each sampling period, several different cases may arise, which leads to the following discrete-time switched system model [13]:

$$\tilde{x}(k+1) = A_{\sigma(k)}\tilde{x}(k) + \tilde{E}w(k), \quad z(k) = \tilde{C}\tilde{x}(k), \tag{5}$$

where

$$\tilde{x}(k+1) = \begin{bmatrix} x(k+1) \\ \hat{x}(k) \end{bmatrix}, \qquad \tilde{E} = \begin{bmatrix} E \\ 0 \end{bmatrix}, \qquad \tilde{C} = \begin{bmatrix} C & 0 \end{bmatrix}, \tag{6}$$

$$A_{\sigma(k)} = \begin{bmatrix} A + A_{1\sigma(k)}K & A_{0\sigma(k)}K \\ I & 0 \end{bmatrix}, \quad \text{for } \sigma(k) = 0, 1, 2 \ldots, N-1 \tag{7}$$

$$A_{\sigma(k)} = \begin{bmatrix} A & BK \\ 0 & I \end{bmatrix}, \quad \text{for } \sigma(k) = N \tag{8}$$

$$\begin{gathered} A = \exp\left\{A_p T\right\}, \qquad A_{0\sigma(k)} = \int_{T-\sigma(k)T_0}^{T} \exp\left\{A_p \tau\right\} B_p \mathrm{d}\tau, \\ B = \int_0^T \exp\left\{A_p \tau\right\} B_p \mathrm{d}\tau, \\ A_{1\sigma(k)} = \int_0^{T-\sigma(k)T_0} \exp\left\{A_p \tau\right\} B_p \mathrm{d}\tau, \\ E = \int_0^T \exp\left\{A_p \tau\right\} E_p \mathrm{d}\tau. \end{gathered} \tag{9}$$

The $\sigma(k)$ is called a switching signal. Note that $\sigma(k) = i$, $i = 0, 1, \ldots, N-1$, implies $\tau(k) = iT_0$, while $\sigma(k) = N$ implies packet dropout.

$\sigma(k)$ is modeled as Markov chain that takes values in $\{0, 1, \ldots, N-1, N\}$. The transition probability matrices of $\sigma(k)$ are $\Pi = [\pi_{ij}]$. That means that $\sigma(k)$ jump from mode i to mode j, from mode with probabilities π_{ij}:

$$\pi_{ij} = \Pr\left(\sigma(k+1) = j \mid \sigma(k) = i\right), \tag{10}$$

where $\pi_{ij} \ge 0$ and $\sum_{j=0}^{N} \pi_{ij} = 1$.

3. H_∞ Disturbance Attenuation Analysis

Definition 1. System (5) is said to be stochastically and exponentially stable, if there exist constants $C > 0$ and $0 < \lambda < 1$, such that $\mathscr{E}(\|\tilde{x}(k)\|^2) \le C\lambda^k \mathscr{E}(\|\tilde{x}(0)\|^2)$ for $w(t) \equiv 0$.

Definition 2. System (5) is said to be stochastically and exponentially stable with an H_∞ disturbance attenuation level γ, if system (5) is stochastically and exponentially stable and for the zero initial condition, $\sum_{k=0}^{\infty} \mathscr{E}\{z^T(k)z(k)\} \le \gamma^2 \sum_{k=0}^{\infty} \mathscr{E}\{w^T(k)w(k)\}$.

Lemma 3 (see [14]). *Define $V_{\sigma(k)}(k) = \tilde{x}^T(k)P_{\sigma(k)}\tilde{x}(k)$, where $P_{\sigma(k)}$ is a positive definite matrix; then there exist constant scalars $\beta_1, \beta_2 > 0$ such that*

$$\beta_1\|\tilde{x}(k)\|^2 \le V_{\sigma(k)}(k) \le \beta_2\|\tilde{x}(k)\|^2, \quad \sigma(k) = 0, 1, 2, \ldots, N. \tag{11}$$

Theorem 4. *For given positive scalars π_{ij} $(i, j = 0, 1, 2, \ldots, N)$, λ, and γ, if there exist matrices $P_i > 0$, $Q_i > 0$, such that*

$$\Omega_i = \begin{bmatrix} \mathscr{C}_{i11} & * \\ \mathscr{C}_{i21} & \mathscr{C}_{i22} \end{bmatrix} < 0, \quad i = 0, 1, \ldots, N-1, \tag{12}$$

$$\overline{\Omega} = \begin{bmatrix} \overline{\mathscr{C}}_{11} & * \\ \overline{\mathscr{C}}_{21} & \overline{\mathscr{C}}_{22} \end{bmatrix} < 0, \tag{13}$$

where

$$\mathscr{C}_{i11} = \begin{bmatrix} \sum_{j=0}^{N}\pi_{ij}Q_j - \lambda P_i + C^TC & 0 & 0 \\ 0 & -\lambda Q_i & 0 \\ 0 & 0 & -\gamma^2 I \end{bmatrix},$$

$$\mathscr{C}_{i21} = \begin{bmatrix} \sqrt{\pi_{i0}}(A + A_{1i}K) & \sqrt{\pi_{i0}}A_{0i}K & \sqrt{\pi_{i0}}E \\ \sqrt{\pi_{i1}}(A + A_{1i}K) & \sqrt{\pi_{i1}}A_{0i}K & \sqrt{\pi_{i1}}E \\ \vdots & \vdots & \vdots \\ \sqrt{\pi_{iN}}(A + A_{1i}K) & \sqrt{\pi_{iN}}A_{0i}K & \sqrt{\pi_{iN}}E \end{bmatrix},$$

$$\mathscr{C}_{i22} = \operatorname{diag}\{-P_0^{-1} \;\; -P_1^{-1} \;\; \cdots \;\; -P_N^{-1}\},$$

$$\overline{\mathscr{C}}_{11} = \begin{bmatrix} -\lambda P_N + C^TC & 0 & \sum_{j=0}^{N}\pi_{Nj}P_jE \\ * & \sum_{j=0}^{N}\pi_{Nj}Q_j - \lambda Q_N & 0 \\ * & * & \sum_{j=0}^{N}\pi_{Nj}E^TP_jE - \gamma^2 I \end{bmatrix},$$

$$\overline{\mathscr{C}}_{21} = \begin{bmatrix} \sqrt{\pi_{N0}}A & \sqrt{\pi_{N0}}BK & 0 \\ \sqrt{\pi_{N1}}A & \sqrt{\pi_{N1}}BK & 0 \\ \vdots & \vdots & \vdots \\ \sqrt{\pi_{NN}}A & \sqrt{\pi_{NN}}BK & 0 \end{bmatrix},$$

$$\overline{\mathscr{C}}_{22} = \operatorname{diag}\{-P_0^{-1} \;\; -P_1^{-1} \;\; \cdots \;\; -P_N^{-1}\} \tag{14}$$

then system (5) *is stochastically and exponentially stable with an H_∞ disturbance attenuation level γ.*

Proof. Let the Lyapunov function

$$\begin{aligned} V_i(k) &= x^T(k)P_ix(k) + \tilde{x}^T(k-1)Q_i\tilde{x}(k-1) \\ &= \tilde{x}^T(k)\tilde{P}_i\tilde{x}(k) \end{aligned} \tag{15}$$

correspond to the subsystem as follows:

$$\begin{aligned} \tilde{x}(k+1) &= A_i\tilde{x}(k) + \tilde{E}w(k), \\ z(k) &= \tilde{C}\tilde{x}(k), \end{aligned} \tag{16}$$

where

$$\tilde{P}_i = \begin{bmatrix} P_i & 0 \\ 0 & Q_i \end{bmatrix}. \tag{17}$$

When $\sigma(k) = i$ $(i = 0, 1, 2, \ldots, N-1)$, we obtain

$$\begin{aligned} &\mathscr{E}\left(V_{\sigma(k+1)}(k+1) - \lambda V_{\sigma(k)}(k) + z^T(k)z(k) - \gamma^2 w^T(k)w(k)\right) \\ &= \mathscr{E}\left(V_{\sigma(k+1)}(k+1) \mid \sigma(k) = i\right) - \lambda V_i(k) \\ &\quad + \tilde{x}^T(k)\tilde{C}^T\tilde{C}\tilde{x}(k) - \gamma^2 w^T(k)w(k) \\ &= \sum_{j=0}^{N}\pi_{ij}\left(\tilde{x}^T(k)A_i^T + w^T(k)\tilde{E}^T\right)\tilde{P}_j\left(A_i\tilde{x}(k) + \tilde{E}w(k)\right) \\ &\quad - \lambda\tilde{x}^T(k)\tilde{P}_i\tilde{x}(k) + x^T(k)\tilde{C}^T\tilde{C}x(k) - \gamma^2 w^T(k)w(k) \\ &= \tilde{x}^T(k)\left(\sum_{j=0}^{N}\pi_{ij}A_i^T\tilde{P}_jA_i\right)\tilde{x}(k) + \tilde{x}^T(k) \\ &\quad \times\left(\sum_{j=0}^{N}\pi_{ij}A_i^T\tilde{P}_j\tilde{E}\right)w(k) + w^T(k)\left(\sum_{j=0}^{N}\pi_{ij}\tilde{E}^T\tilde{P}_jA_i\right)\tilde{x}(k) \\ &\quad + w^T(k)\left(\sum_{j=0}^{N}\pi_{ij}\tilde{E}^T\tilde{P}_j\tilde{E}\right)w(k) - \lambda\tilde{x}^T(k)\tilde{P}_i\tilde{x}(k) \\ &\quad + x^T(k)\tilde{C}^T\tilde{C}x(k) - \gamma^2 w^T(k)w(k) \\ &= \begin{bmatrix}\tilde{x}^T(k) & w^T(k)\end{bmatrix}\Theta_i\begin{bmatrix}\tilde{x}(k) \\ w(k)\end{bmatrix}, \end{aligned} \tag{18}$$

where

$$\Theta_i = \begin{bmatrix} \sum_{j=0}^{N}\pi_{ij}A_i^T\tilde{P}_jA_i - \lambda\tilde{P}_i + \tilde{C}^T\tilde{C} & \sum_{j=0}^{N}\pi_{ij}A_i^T\tilde{P}_j\tilde{E} \\ \sum_{j=0}^{N}\pi_{ij}\tilde{E}^T\tilde{P}_jA_i & \sum_{j=0}^{N}\pi_{ij}\tilde{E}^T\tilde{P}_j\tilde{E} - \gamma^2 I \end{bmatrix}. \tag{19}$$

From (6) and (7), it can be obtained that

$$\Theta_i = \begin{bmatrix} \mathbb{A}_{i11} & \mathbb{A}_{i12} & \mathbb{A}_{i13} \\ * & \mathbb{A}_{i22} & \mathbb{A}_{i23} \\ * & * & \mathbb{A}_{i33} \end{bmatrix}.$$

$$\begin{aligned} \mathbb{A}_{i11} &= \sum_{j=0}^{N}\pi_{ij}\left[\left(A^T + K^TA_{1i}^T\right)P_j\left(A + A_{1i}K\right) + Q_j\right] \\ &\quad - \lambda P_i + C^TC, \end{aligned}$$

$$\begin{aligned}
\mathbb{A}_{i12} &= \sum_{j=0}^{N}\pi_{ij}\left(A^T + K^T A_{1i}^T\right)P_j A_{0i}K,\\
\mathbb{A}_{i13} &= \sum_{j=0}^{N}\pi_{ij}(A + A_{1i}K)^T P_j E,\\
\mathbb{A}_{i22} &= \sum_{j=0}^{N}\pi_{ij}K^T A_{0i}^T P_j A_{0i}K - \lambda Q_i,\\
\mathbb{A}_{i23} &= \sum_{j=0}^{N}\pi_{ij}(A_{0i}K)^T P_j E,\\
\mathbb{A}_{i33} &= \sum_{j=0}^{N}\pi_{ij}E^T P_j E - \gamma^2 I.
\end{aligned} \tag{20}$$

$\Theta_i < 0$ can be rewritten as follows:

$$\Phi_i + \begin{bmatrix} \sqrt{\pi_{i0}}\left(A^T + K^T A_{1i}^T\right) \\ \sqrt{\pi_{i0}}K^T A_{0i}^T \\ \sqrt{\pi_{i0}}E^T \end{bmatrix} \times P_0\left[\sqrt{\pi_{i0}}\left(A + A_{1i}K\right) \quad \sqrt{\pi_{i0}}A_{0i}K \quad \sqrt{\pi_{i0}}E\right] < 0, \tag{21}$$

where

$$\Phi_i = \begin{bmatrix} \mathbb{B}_{i11} & \mathbb{B}_{i12} & \mathbb{B}_{i13} \\ * & \mathbb{B}_{i22} & \mathbb{B}_{i23} \\ * & * & \mathbb{B}_{i33} \end{bmatrix}.$$

$$\begin{aligned}
\mathbb{B}_{i11} &= \sum_{j=1}^{N}\pi_{ij}\left(A^T + K^T A_{1i}^T\right)P_j\left(A + A_{1i}K\right) + \sum_{j=0}^{N}\pi_{ij}Q_j - \lambda P_i + C^T C,\\
\mathbb{B}_{i12} &= \sum_{j=1}^{N}\pi_{ij}\left(A^T + K^T A_{1i}^T\right)P_j A_{0i}K,\\
\mathbb{B}_{i13} &= \sum_{j=1}^{N}\pi_{ij}(A + A_{1i}K)^T P_j E,\\
\mathbb{B}_{i22} &= \sum_{j=1}^{N}\pi_{ij}K^T A_{0i}^T P_j A_{0i}K - \lambda Q_i,\\
\mathbb{B}_{i23} &= \sum_{j=1}^{N}\pi_{ij}(A_{0i}K)^T P_j E,\\
\mathbb{B}_{i33} &= \sum_{j=1}^{N}\pi_{ij}E^T P_j E - \gamma^2 I.
\end{aligned} \tag{22}$$

From the Schur complement, we have that (21) is equivalent to

$$\Psi_i = \begin{bmatrix} \mathbb{B}_{i11} & \mathbb{B}_{i12} & \mathbb{B}_{i13} & \sqrt{\pi_{i0}}\left(A^T + K^T A_{10}^T\right) \\ * & \mathbb{B}_{i22} & \mathbb{B}_{i23} & \sqrt{\pi_{i0}}K^T A_{0i}^T \\ * & * & \mathbb{B}_{i33} & \sqrt{\pi_{i0}}E^T \\ * & * & * & -P_0^{-1} \end{bmatrix} < 0. \tag{23}$$

Similarly, we can see that $\Psi_i < 0$ is equivalent to

$$\Omega_i = \begin{bmatrix} \mathscr{C}_{i11} & * \\ \mathscr{C}_{i21} & \mathscr{C}_{i22} \end{bmatrix} < 0. \tag{24}$$

It can be seen that if (12) holds, $\Theta_i < 0$ is true, which means

$$\mathscr{E}\left(V_{\sigma(k+1)}(k+1) - \lambda V_{\sigma(k)}(k) + z^T(k)\,z(k) - \gamma^2 w^T(k)\,w(k)\right) < 0. \tag{25}$$

When $\sigma(k) = N$,

$$\begin{aligned}
&\mathscr{E}\left(V_{\sigma(k+1)}(k+1) - \lambda V_{\sigma(k)}(k) + z^T(k)\,z(k) - \gamma^2 w^T(k)\,w(k)\right)\\
&= \left[\tilde{x}^T(k) \quad w^T(k)\right]\overline{\Theta}\begin{bmatrix}\tilde{x}(k)\\ w(k)\end{bmatrix},
\end{aligned} \tag{26}$$

where

$$\overline{\Theta} = \begin{bmatrix} \sum_{j=0}^{N}\pi_{Nj}A_N^T\widetilde{P}_j A_N - \lambda\widetilde{P}_N + \widetilde{C}^T\widetilde{C} & \sum_{j=0}^{N}\pi_{Nj}A_N^T\widetilde{P}_j\widetilde{E} \\ \sum_{j=0}^{N}\pi_{Nj}\widetilde{E}^T\widetilde{P}_j A_N & \sum_{j=0}^{N}\pi_{Nj}\widetilde{E}^T\widetilde{P}_j\widetilde{E} - \gamma^2 I \end{bmatrix}. \tag{27}$$

From (6) and (8), it can be seen that

$$\begin{aligned}
\overline{\Theta} &= \begin{bmatrix} \overline{\mathbb{A}}_{11} & \overline{\mathbb{A}}_{12} & \overline{\mathbb{A}}_{13} \\ * & \overline{\mathbb{A}}_{22} & \overline{\mathbb{A}}_{23} \\ * & * & \overline{\mathbb{A}}_{33} \end{bmatrix},\\
\overline{\mathbb{A}}_{11} &= \sum_{j=0}^{N}\pi_{Nj}A^T P_j A - \lambda P_N + C^T C,\\
\overline{\mathbb{A}}_{12} &= \sum_{j=0}^{N}\pi_{Nj}A^T P_j BK,\\
\overline{\mathbb{A}}_{13} &= \sum_{j=0}^{N}\pi_{Nj}P_j E,\\
\overline{\mathbb{A}}_{22} &= \sum_{j=0}^{N}\pi_{Nj}K^T B^T P_j BK + \sum_{j=0}^{N}\pi_{Nj}Q_j - \lambda Q_N,\\
\overline{\mathbb{A}}_{23} &= 0,\\
\overline{\mathbb{A}}_{33} &= \sum_{j=0}^{N}\pi_{Nj}E^T P_j E - \gamma^2 I.
\end{aligned} \tag{28}$$

$\overline{\Theta} < 0$ can be rewritten as follows:

$$\overline{\Phi} + \begin{bmatrix} \sqrt{\pi_{N0}} A^T \\ \sqrt{\pi_{N0}} K^T B^T \\ 0 \end{bmatrix} P_0 \left[\sqrt{\pi_{N0}} A \quad \sqrt{\pi_{N0}} BK \quad 0\right] < 0, \quad (29)$$

where

$$\overline{\Phi} = \begin{bmatrix} \overline{\overline{\mathbb{B}}}_{11} & \overline{\mathbb{B}}_{12} & \overline{\mathbb{B}}_{13} \\ * & \overline{\mathbb{B}}_{22} & \overline{\mathbb{B}}_{23} \\ * & * & \overline{\mathbb{B}}_{33} \end{bmatrix},$$
$$\overline{\mathbb{B}}_{11} = \sum_{j=1}^{N} \pi_{Nj} A^T P_j A - \lambda P_N + C^T C,$$
$$\overline{\mathbb{B}}_{12} = \sum_{j=1}^{N} \pi_{Nj} A^T P_j BK,$$
$$\widetilde{\mathbb{B}}_{13} = \sum_{j=0}^{N} \pi_{Nj} P_j E, \quad (30)$$
$$\overline{\mathbb{B}}_{22} = \sum_{j=1}^{N} \pi_{Nj} K^T B^T P_j BK + \sum_{j=0}^{N} \pi_{Nj} Q_j - \lambda Q_N,$$
$$\overline{\mathbb{B}}_{23} = 0,$$
$$\overline{\mathbb{B}}_{33} = \sum_{j=0}^{N} \pi_{Nj} E^T P_j E - \gamma^2 I.$$

From the Schur complement, we have that (29) is equivalent to

$$\overline{\Psi} = \begin{bmatrix} \overline{\mathbb{B}}_{11} & \overline{\mathbb{B}}_{12} & \overline{\mathbb{B}}_{13} & \sqrt{\pi_{N0}} A^T \\ * & \overline{\mathbb{B}}_{22} & \overline{\mathbb{B}}_{23} & \sqrt{\pi_{N0}} K^T B^T \\ * & * & \overline{\mathbb{B}}_{33} & 0 \\ * & * & * & -P_0^{-1} \end{bmatrix} < 0. \quad (31)$$

Similarly, it is easy to see that $\overline{\Psi} < 0$ is equivalent to

$$\overline{\Omega} = \begin{bmatrix} \overline{\mathscr{C}}_{11} & * \\ \overline{\mathscr{C}}_{21} & \overline{\mathscr{C}}_{22} \end{bmatrix} < 0. \quad (32)$$

It can be seen that if (13) holds, $\overline{\Theta} < 0$ is true, which means

$$\mathscr{E}\left(V_{\sigma(k+1)}(k+1) - \lambda V_N(k) + z^T(k) z(k) - \gamma^2 w^T(k) w(k)\right) < 0. \quad (33)$$

It follows from (25) and (33) that

$$\mathscr{E}\left(V_{\sigma(k+1)}(k+1) - \lambda V_{\sigma(k)}(k) + z^T(k) z(k) - \gamma^2 w^T(k) w(k)\right) < 0 \quad (34)$$

which means

$$\begin{aligned} &\mathscr{E}\left(V_{\sigma(k+1)}(k+1)\right) \\ &\quad < \mathscr{E}\left(V_{\sigma(k)}(k)\right) - \mathscr{E}\left(z^T(k) z(k)\right) + \gamma^2 \mathscr{E}\left(w^T(k) w(k)\right), \\ &\mathscr{E}\left(V_{\sigma(\infty)}(k+1)\right) \\ &\quad < \mathscr{E}\left(V_{\sigma(0)}(0)\right) - \sum_{k=0}^{\infty} \mathscr{E}\left(z^T(k) z(k)\right) \\ &\qquad + \gamma^2 \sum_{k=0}^{\infty} \mathscr{E}\left(w^T(k) w(k)\right), \\ &\qquad \sum_{k=0}^{\infty} \mathscr{E}\left(z^T(k) z(k)\right) \le \gamma^2 \sum_{k=0}^{\infty} \mathscr{E}\left(w^T(k) w(k)\right). \end{aligned} \quad (35)$$

Next, we prove the stochastically and exponentially stable system (5). The perturbation $w(t)$ is assumed to be zero.

When $\sigma(k) = i$, $(i = 0, 1, 2, \ldots, N-1)$, we obtain

$$\begin{aligned} &\mathscr{E}\left(V_{\sigma(k+1)}(k+1) - \lambda V_{\sigma(k)}(k)\right) \\ &\quad = \mathscr{E}\left(V_{\sigma(k+1)}(k+1) \mid \sigma(k) = i\right) - \lambda V_i(k) \\ &\quad = \widetilde{x}^T(k) \left(\sum_{j=0}^{N} \pi_{ij} A_i^T \widetilde{P}_j A_i - \lambda \widetilde{P}_i\right) \widetilde{x}. \end{aligned} \quad (36)$$

From (21), (23), and (24), it can be seen that $\Theta < 0$ is equivalent to $\Omega < 0$.

Then, it can be seen from (19) that if (12) holds, we have

$$\sum_{j=1}^{N} \pi_{ij} A_i^T \widetilde{P}_j A_i - \lambda \widetilde{P}_i + \widetilde{C}^T \widetilde{C} < 0 \quad (37)$$

and then

$$\sum_{j=1}^{N} \pi_{ij} A_i^T \widetilde{P}_j A_i - \lambda \widetilde{P}_i < 0 \quad (38)$$

which means

$$\mathscr{E}\left(V_{\sigma(k+1)}(k+1) - \lambda V_{\sigma(k)}(k)\right) < 0. \quad (39)$$

When $\sigma(k) = N$,

$$\begin{aligned} &\mathscr{E}\left(V_{\sigma(k+1)}(k+1) - \lambda V_N(k)\right) \\ &\quad = \widetilde{x}^T(k) \left(\sum_{j=0}^{N} \pi_{Nj} A_N^T \widetilde{P}_j A_N - \lambda \widetilde{P}_N\right) \widetilde{x}(k). \end{aligned} \quad (40)$$

From (27), (29), (31), and (32), it can be seen that $\overline{\Theta} < 0$ is equivalent to $\overline{\Omega} < 0$.

Then, it can be seen from (27) that if (13) holds, we have

$$\sum_{j=0}^{N} \pi_{Nj} A_N^T \widetilde{P}_j A_N - \lambda \widetilde{P}_N + \widetilde{C}^T \widetilde{C} < 0 \quad (41)$$

and then

$$\sum_{j=0}^{N}\pi_{Nj}A_N^T\widetilde{P}_jA_N-\lambda\widetilde{P}_N<0 \quad (42)$$

which means

$$\mathscr{E}\left(V_{\sigma(k+1)}(k+1)-\lambda V_N(k)\right)<0. \quad (43)$$

From (36) and (43), we have

$$\begin{gathered}\mathscr{E}\left(V_{\sigma(k+1)}(k+1)\right)<\mathscr{E}\left(\lambda V_{\sigma(k)}(k)\right),\\ \mathscr{E}\left(V_{\sigma(k)}(k)\right)<\lambda^k\mathscr{E}\left(V_{\sigma(0)}(0)\right).\end{gathered} \quad (44)$$

From Lemma 3, we get

$$\mathscr{E}\left(\|\widetilde{x}(k)\|^2\right)\le C\lambda^k\mathscr{E}\left(\|\widetilde{x}(0)\|^2\right). \quad (45)$$

Then, the result is established. □

The conditions in Theorem 4 are a set of LMIs with some inversion constraints. K can be solved by an iterative LMI approach which is called the cone complementarity linearization algorithm [15, 16].

4. Numerical Example

Consider the following system [13]. Suppose $\gamma = 0.91$, $\lambda = 0.9760$. The transition probability matrices of $\sigma(k)$ are taken as follow:

$$\begin{bmatrix}0.1 & 0.8 & 0 & 0.1\\ 0.2 & 0.7 & 0 & 0.1\\ 0.4 & 0.5 & 0 & 0.1\\ 0.6 & 0.3 & 0 & 0.1\end{bmatrix} \quad (46)$$

which means

$$\begin{gathered}\pi_{01}=0.1,\quad \pi_{02}=0.8,\quad \pi_{03}=0,\quad \pi_{04}=0.1,\\ \pi_{11}=0.3,\quad \pi_{12}=0.7,\quad \pi_{13}=0,\quad \pi_{14}=0,\\ \pi_{21}=0.3,\quad \pi_{22}=0.7,\quad \pi_{23}=0,\quad \pi_{24}=0,\\ \pi_{31}=0.6,\quad \pi_{32}=0.3,\quad \pi_{33}=0,\quad \pi_{34}=0.1.\end{gathered} \quad (47)$$

Using Theorem 4 and the cone complementarity linearization algorithm, we obtain

$$K=\left[-1.4252\quad -5.7880\right]. \quad (48)$$

Figure 2 is the possible realizations of the mode $\sigma(k)$. Under this mode sequence, the corresponding state trajectories of the closed-loop system are shown in Figure 3. It is shown that the closed-loop system is stochastically and exponentially stable.

5. Conclusions

In this paper, by modeling the random delays and packet dropouts as a Markov chain, a new Markovian jump system model is presented to describe the networked control system with disturbance attenuation. The criteria for the system are stochastically and exponentially stable with an H_∞ disturbance attenuation level which is derived by an iterative LMI approach.

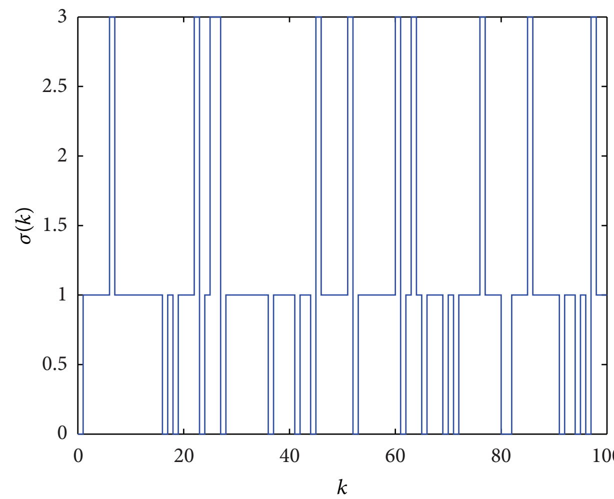

Figure 2: Random mode $\sigma(k)$.

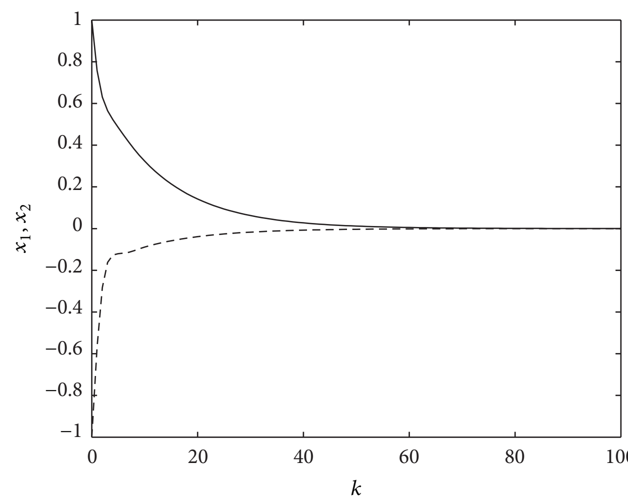

Figure 3: State trajectories of the NCS.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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