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Research Article

Assesment of New Analytical Method for Solving the Foam Drainage Equation

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The drainage of liquid foams involves the interplay of gravity, surface tension, and viscous forces. Foaming occurs in many distillation and absorption processes. In this study, a new reliable technique is used to handle the foam drainage equation. This new method resulted from VIM by a simple modification, that is, variational iteration method-II (VIM-II). It has been shown that the VIM-II is a powerful technique in obtaining accurate solutions that cannot be given otherwise by perturbation and other methods. The accuracy and convergence of the method are also investigated and compared with other methods. The results showed that there are good agreements between the results.

1. Introduction

Foams [1, 2] are a prime example of a multiphase "soft condensed matter" system. They have important applications in the food and chemical industries, firefighting, mineral processing, and structural material science [3], and their properties are of subject of intensive studies from both practical and scientific points of view [4]. Foams are common in personal care products such as creams and lotions, and foams often occur, even when not desired, during cleaning (clothes, dishes, scrubbing) and dispensing processes (cf. [5]). Less obviously they appear in acoustic cladding, lightweight mechanical components, and impact absorbing parts on cars, heat exchangers, and textured wallpapers (incorporated as foaming inks) and even have an analogy in cosmology. History connects foams with a number of eminent scientists, and foams continue to excite imaginations [6]. Although there are now many applications of polymeric foams [7] and more recently metallic foams, which are foams made out of metals such as aluminum [8]. In addition, industrial applications of polymeric foams and porous metals include their use for structural purposes and as heat exchange media analogous to common "finned" structures [9].

Recent research in foams and emulsions has been centered on three topics which are often treated separately but are in fact interdependent: drainage, coarsening, and rheology; see Figure 1. We focus here on a quantitative description of the coupling of drainage and coarsening.

The flow of liquid relative to the bubbles is called drainage. Drainage plays an important role in foam stability: indeed, when foam dries, its structure becomes more fragile; the liquid films between adjacent bubbles being thinner, then can break, leading to foam collapse. In the case of aqueous foams, surfactant is added into water and it adsorbs at the surface of the films, protecting them against rupture (cf. [10]).

Foam drainage is the flow of liquid through channels (Plateau borders) and nodes (intersections of four channels) between the bubbles, driven by gravity and capillarity [11–13].

The foam drainage equation models the dynamics of the liquid volume in the foam on length scales larger than the bubble size. Generally drainage is driven by gravity and/or capillary (surface tension) forces and is resisted by viscous forces (cf. [5]).

Recent theoretical studies by Verbist and Weaire describe the main features of both free drainage [14, 15], where liquid drains out of a foam due to gravity, and forced drainage [16], where liquid is introduced to the top of a column of. Forced foam drainage may well be the best prototype for certain general phenomena described by nonlinear differential equations, particularly the type of solitary wave which is most

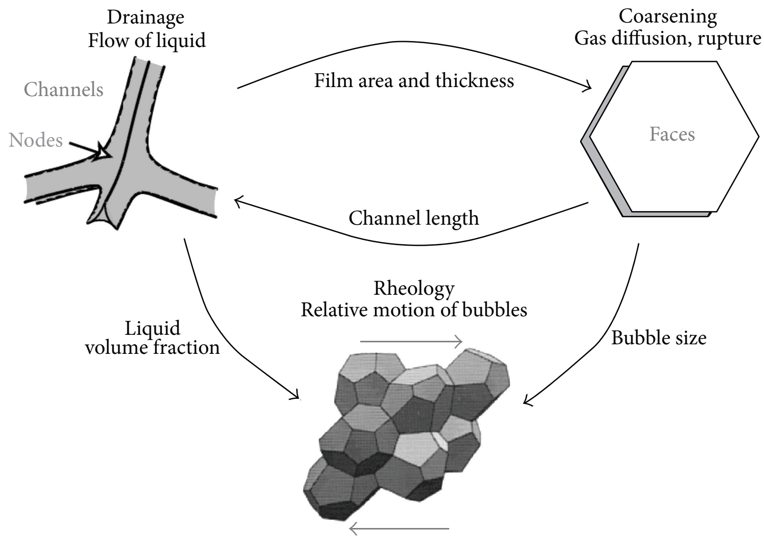

FIGURE 1: Schematic illustration of the interdependence of drainage, coarsening, and rheology of foams [3].

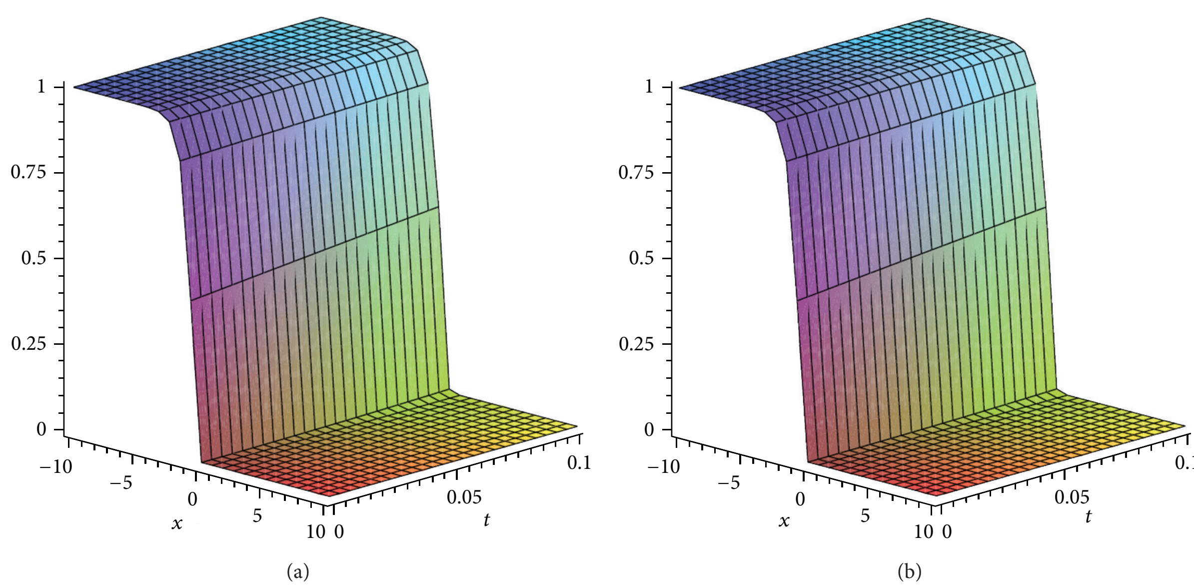

FIGURE 2: The surfaces on both columns, respectively, show the solutions, $u(x,t)$, for VIM-II on the right and exactly on the left when $c = 1$.

familiar in tidal bores. Fadravi et al. [17] employed homotopy analysis method for solving foam drainage equation with space- and time-fractional derivatives. homotopy perturbation method was used for solving the foam drainage equation by Fereidoon et al. [18].

In recent years, several such techniques have drawn special attention, such as inverse scattering method [19], Adomian decomposition method [20, 21], Hamiltonian approach [22], variational iteration method [23–25], homotopy analysis method [26, 27], variational approach [28], and homotopy perturbation method [29–38].

The aim of current study is to analytically investigate nonlinear foam drainage equation in the form of (1), using variational iteration method-II (VIM-II) [39, 40]. We will apply new algorithm that is a powerful and efficient technique in finding the approximate solutions for the following foam drainage equation:

$$\frac{\partial A}{\partial t}+\frac{\partial}{\partial x}\left(A^{2}-\frac{\sqrt{A}}{2}\frac{\partial A}{\partial x}\right)=0, \tag{1}$$

where x and t are scaled position and time coordinates, respectively.

In the case of forced drainage, the solution can be expressed as [41]

$$A(x,t)=c\tanh^{2}\left(\sqrt{c}\,(x-ct)\right), \tag{2}$$

where c is the velocity of the wave front [16].

The pursuit of analytical solutions for foam drainage equation is of intrinsic scientific interest. To the best of

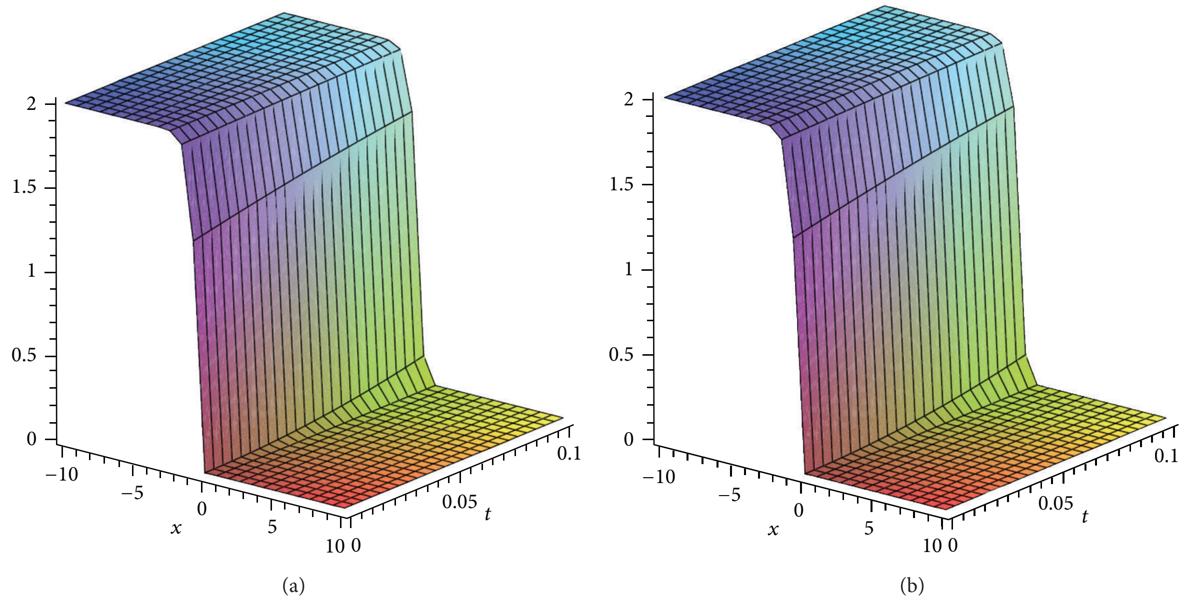

FIGURE 3: The surfaces on both columns, respectively, show the solutions, $u(x, t)$, for VIM-II on the right and exactly on the left when $c = 2$.

the authors' knowledge, there is no paper that has solved the nonlinear foam drainage equation by VIM-II. In this paper, the basic idea of VIM-II is described, and then, it is applied to study the following nonlinear foam drainage equation. Finally, the results of VIM-II and HPM as analytical solutions are then compared with those derived from Adomian decomposition method.

2. Basic Concept of VIM-II

In the following section, to clarify the idea of the proposed method for solution of the nonlinear governing equation of a cantilever beam undergoing large deformation, the basic concept of variational iteration method-II [39, 40] is firstly treated. A general nonlinear equation of kth order is considered at the following form:

$$u^{(k)} + f\left(u' + u'' + \cdots + u^{(k)}\right) = 0. \tag{3}$$

The classical variational iteration algorithm is as follows:

$$u_{n+1}(t) = u_n(t) + \int_0^t \lambda\left(u_n^{(k)} + \tilde{f}_n\right) d\varepsilon, \tag{4}$$

where λ is a general Lagrange multiplier. We apply Laplace transform to identify the Lagrange multiplier [42–44]. By using Laplace transform, we have

$$\begin{gathered} s^k U(s) + \ell\left\{f\left(u, u', \ldots, u^{(k)}\right)\right\} = 0, \\ U(s) = \frac{-\ell\left\{f\left(u, u', \ldots, u^{(k)}\right)\right\}}{s^k}. \end{gathered} \tag{5}$$

The inverse Laplace transform reads as

$$u(t) = (-1)^k \int_0^t \frac{(\varepsilon - t)^{k-1}}{(k-1)!} f\left(u_n(\varepsilon), u_n'(\varepsilon), \ldots, u_n^{(k)}(\varepsilon)\right) d\varepsilon. \tag{6}$$

Hence, after identifying the Lagrange multiplier λ, the variational iteration algorithm-II [39, 40] is constructed as follows:

$$\begin{aligned} u_{n+1}(t) = {} & u_0(t) + (-1)^k \\ & \times \int_0^t \frac{(\varepsilon - t)^{k-1}}{(k-1)!} f\left(u_n(\varepsilon), u_n'(\varepsilon), \ldots, u_n^{(k)}(\varepsilon)\right) d\varepsilon. \end{aligned} \tag{7}$$

The above equation is generally called the variational iteration algorithm-II, in which $u_0(t)$ is the initial solution. The initial values are usually used for selecting the zeroth approximation u_0. With u_0 determined, then several approximations u_n $n > 0$, follow immediately. Consequently, the exact solution could be obtained as follows:

$$u(t) = \lim_{n \to \infty} u_n. \tag{8}$$

3. Implementation of VIM-II

In this section, we will apply the VIM-II to solve foam drainage equation. Foam drainage equation (1) can be written as [41]:

$$u_t + 2uu_x - \frac{\sqrt{u}}{2} u_{xx} - \frac{1}{4\sqrt{u}}\left(u_x\right)^2 = 0, \tag{9}$$

with initial conditions

$$u(x, 0) = 3\tanh^2\left(\sqrt{3}x\right). \tag{10}$$

Table 1: Comparison between errors of ADM, HPM, and VIM-II for $t = 0.01$ and $c = 3$.

X	$u_{\text{exact}} - u_{\text{HPM}}$	$u_{\text{exact}} - u_{\text{ADM}}$	$u_{\text{exact}} - u_{\text{VIM-II}}$
−10	2.4E − 18	1.77636E − 15	0
−8	2.01600E − 15	1.86962E − 12	0
−6	2.05744E − 12	1.9087E − 9	0
−4	2.09993E − 9	1.94811E − 6	6E − 08
−2	0.000002123	0.00197296	−6.0881E − 0.5
−1	0.000050433	0.0485679	0
0	0.000014557	0.00051592	8.08544E − 3

Table 2: Comparison between errors of ADM, HPM, and VIM-II for $t = 0.001$ and $c = 3$.

X	$u_{\text{exact}} - u_{\text{HPM}}$	$u_{\text{exact}} - u_{\text{ADM}}$	$u_{\text{exact}} - u_{\text{VIM-II}}$
−10	0	4.44089E − 16	0
−8	2.4E − 18	2.24265E − 13	0
−6	2.1059E − 15	2.29754E − 10	0
−4	2.14918E − 12	2.34498E − 7	0
−2	2.17229E − 9	0.000236656	3.54069E − 15
−1	5.10302E − 8	0.00523834	1.64365E − 13
0	1.45797E − 9	5.2479E − 8	0

The exact solution for this problem is

$$u(x,t) = c\tanh^2\left(\sqrt{c}\,(x - ct)\right). \tag{11}$$

The VIM-II is implemented in (9). First, according to the method, by applying Laplace transform to identify the Lagrange multiplier, we have

$$\lambda = 1. \tag{12}$$

So, variational iteration algorithm-II is derived:

$$u_{n+1} = u_0 + \int_0^t \left(-2uu_x + \frac{\sqrt{u}}{2}u_{xx} + \frac{1}{4\sqrt{u}}(u_x)^2\right) d\varepsilon. \tag{13}$$

According to the above equation, for first order approximation, it can be written as follows:

$$u_1 = u_0 + \int_0^t \left(-2u_0u_{0x} + \frac{\sqrt{u_0}}{2}u_{0xx} + \frac{1}{4\sqrt{u_0}}(u_{0x})^2\right) d\varepsilon. \tag{14}$$

We have the following successive approximation:

$$\begin{aligned} u_1 = {} & 3\tanh\left(\sqrt{3}x\right)^2 - 36\tanh\left(\sqrt{3}x\right)^3 \\ & \times \left(1 - \tanh\left(\sqrt{3}x\right)^2\right)\sqrt{3}t \\ & + \frac{\sqrt{3}}{2}\sqrt{\tanh\left(\sqrt{3}x\right)^2} \times 18\left(1 - \tanh\left(\sqrt{3}x\right)^2\right)^2 \\ & - 36\tanh\left(\sqrt{3}x\right)^2\left(1 - \tanh\left(\sqrt{3}x\right)^2\right)t \\ & + \frac{9\tanh\left(\sqrt{3}x\right)^2\left(1 - \tanh\left(\sqrt{3}x\right)^2\right)^2\sqrt{3}t}{\sqrt{\tanh\left(\sqrt{3}x\right)^2}}. \end{aligned} \tag{15}$$

Proceeding in the same way, we can obtain the high order approximations. The VIM-II method admits the use of

$$u = \lim_{n\to\infty} u_n. \tag{16}$$

By Figures 2 and 3, we may simply compare the VIM-II solution and exact solution of (1) for $c = 1$ and $c = 2$, respectively. Tables 1 and 2 investigated comparison presented method and HPM [18] between errors of ADM [41].

4. Conclusions

In this paper, an explicit analytical solution is obtained for foam drainage equation by means of the variational iteration method-II (VIM-II), which is a powerful mathematical tool in dealing with nonlinear equations. By looking at the results of figures and tables, we can see that the agreement between VIM-II and Adomian decomposition method (ADM) is satisfactory. Clearly, VIM-II is easy to calculate with the explicit polynomial expressions. This is especially convenient for practical engineering applications with minimum requirements on calculation and computation.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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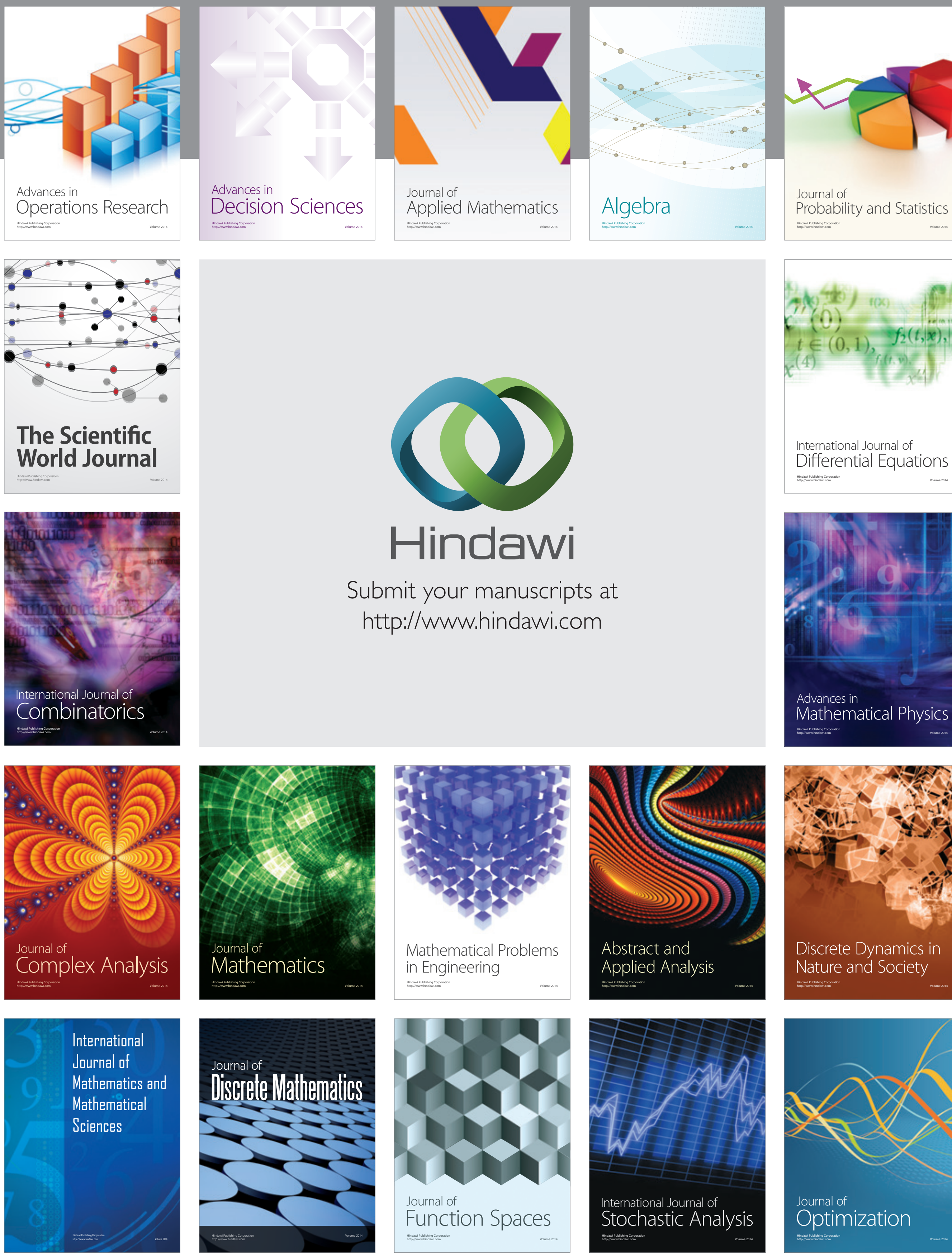
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