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Research Article

On the Study of Oscillons in Scalar Field Theories: A New Approach

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We study configurations in one-dimensional scalar field theory, which are time-dependent, localized in space, and extremely long-lived, called oscillons. How the action of changing the minimum value of the field configuration representing the oscillon affects its behavior is investigated. We find that one of the consequences of this procedure is the appearance of a pair of oscillon-like structures presenting different amplitudes and frequencies of oscillation. We also compare our analytical results to numerical ones, showing excellent agreement.

1. Introduction

The presence of topologically stable configurations is an important feature of a large number of interesting nonlinear models. Among other types of nonlinear field configurations, there is a specially important class of time-dependent stable solutions, the breathers appearing in the Sine-Gordon-like models. Another time-dependent field configuration whose stability is granted by charge conservation is the Q-balls as coined by Coleman [1] or nontopological solitons [2]. However, considering the fact that many physical systems interestingly may present a metastable behavior, a further class of nonlinear systems may present a very long-living configuration usually known as oscillon. This class of solutions was discovered in the seventies of the last century by Bogolyubsky and Makhankov [3] and rediscovered posteriorly by Gleiser [4]. Those solutions appeared in the study of the dynamics of first-order phase transitions and bubble nucleation. Since then, an increasing number of works have been dedicated to the study of these objects [5–36].

Oscillons are quite general configurations and are found in the Abelian-Higgs $U(1)$ models [5], in the standard model $SU(2) \times U(1)$ [6], in inflationary cosmological models [7], in axion models [8], in expanding universe scenarios [9, 10], and in systems involving phase transitions [11]. In a recent work by Gleiser et al. [12], the problem of the hybrid inflation characterized by two real scalar fields interacting quadratically was analyzed. In that reference, the authors have shown that a new class of oscillons arise both in excited and in ground states. Here, it is important to remark that an earlier mention of composite oscillons both in excited and in ground states was given in [13], where oscillons in the $SU(2)$ Gauged Higgs Model (GHM) have been obtained.

The usual oscillon aspect is typically that of a bell shape which oscillates sinusoidally. Recently, Amin and Shirokoff [10] have shown that, depending on the intensity of the coupling constant of the self-interacting scalar field, it is possible to observe oscillons with a kind of plateau at their top. In fact, they have shown that these new oscillons are more robust against collapse instabilities in three spatial dimensions. In a recent work, the impact of the Lorentz and CPT breaking symmetries was discussed in the context of the so-called flat-top oscillons [14].

At this point, it is interesting to remark that Segur and Kruskal [15] have shown that the asymptotic expansion does not represent in general an exact solution for the scalar field; in other words, it simply represents an asymptotic expansion of the first order in ϵ, and it is not valid at all orders of the expansion. They have also shown that in one spatial dimension they radiate [15]. In a recent work, the computation of the emitted radiation of the oscillons was extended for the case of two and three spatial dimensions [16]. Another important result was put forward by Hertzberg [17]. In that work, he was able to compute the decaying rate of quantized oscillons, showing that the quantum rate decay is very distinct from the classical one.

On the other hand, in a thermal background [18], it was shown that the presence of a thermal bath affects the existence and longevity of the oscillon. In another context, applying the principles of discrete time mechanics, Norton and Jaroszkiewicz [19] have shown that oscillons can also arise from the discrete time Dirac equation. In this case, their principal characteristic is that they oscillate in phase with a period twice that of the fundamental time.

Research involving oscillons has also been done in a stellar scenario [20], in small lattices [21], in collisions of nucleated bubbles [22, 23], in Bose-Einstein condensates confined in two- and three-dimensional optical lattices [24], in asymmetric double-well potentials [25, 26], in oscillating Higgs field in pseudovacuum state [27], in the planar Ginzburg-Landau equation [28], in the modified nonlinear Schrödinger equation [29], in the collapse of asymmetrical bubbles in $(2+1)$-dimensional φ^4 models [30], in coupled Bose-Einstein condensates [31], in D spatial dimensions [32], in the presence of quantum fluctuations [33], in chaotic attractors [34], and in the signum-Gordon model context [35].

Thus, in this work, we introduce novel configurations in one-dimensional scalar field theories, which are time-dependent, localized in space, and extremely long-lived like the oscillons. This is done through the investigation of how displacement of the oscillons' position in the field potential affects their features.

This paper is organized as follows. In Section 2, we present the symmetrical model which will be analyzed and we show the essential idea of the work. In Section 3, we find the respective oscillons-like configurations which we will call "phantom oscillons." In Section 4, we address the problem of the emitted radiation of the phantom oscillons. Discussion of some physical features of the solutions is presented in Section 5.

2. The Basics

In this work, we study a real scalar field theory in $1+1$ space-time dimensions described by the following action:

$$\mathcal{S} = \int dt\, dx \left[\frac{1}{2}\left(\partial_t \phi\right)^2 - \frac{1}{2}\left(\partial_x \phi\right)^2 - V(\phi)\right], \tag{1}$$

where $\partial_t = \partial/\partial t$, $\partial_x = \partial/\partial x$, and $V(\phi)$ is the field dependent potential. Thus, from the variation of the above action, the corresponding classical field equation of motion can be written as

$$\frac{\partial^2 \phi}{\partial t^2} - \frac{\partial^2 \phi}{\partial x^2} = -\frac{\partial V(\phi)}{\partial \phi}. \tag{2}$$

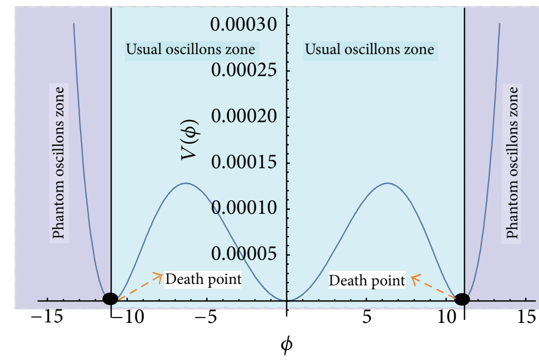

FIGURE 1: Profile of the potential of the model on analysis with $a = 1.2$, $b = 0.01$, and $\lambda = 10^{-5}$.

In order to introduce the idea we are pursuing, we will analyze the case of the symmetric ϕ^6 model, which has been used in several contexts devoted to the study of oscillons. Then, in this work, the model that we will choose is represented by the following field potential:

$$V(\phi) = \frac{\lambda}{2}\phi^2\left(a - b\phi^2\right)^2, \tag{3}$$

where λ, a, and b are real positive valued parameters.

The profile of this potential is illustrated in Figure 1. This figure shows that the potential presents three degenerate minima, localized in $\phi_v^{(0)} = 0$ and $\phi_v^{(\pm)} = \pm\sqrt{a/b}$.

This kind of potential presents kink-like configurations interpolating adjacent vacua. Here, however, we are looking for time-dependent field configurations which are localized in the space.

Therefore, since our primordial interest is to find localized and periodic solutions, it is useful, as usual in the study of oscillons, to introduce the following scale transformation in x and t:

$$\begin{aligned} y &= \epsilon x, \\ \tau &= \sqrt{1-\epsilon^2}\, t, \end{aligned} \tag{4}$$

where $0 < \epsilon \ll 1$. Under these transformations, the field equation (2) becomes

$$\left(1-\epsilon^2\right)\frac{\partial^2 \phi}{\partial \tau^2} - \epsilon^2 \frac{\partial^2 \phi}{\partial y^2} + \lambda\left(a^2\phi - 4ab\phi^3 + 3b^2\phi^5\right) = 0. \tag{5}$$

From the above equation, it is possible to find the usual oscillons which are localized in the central vacuum $\phi_v^{(0)} = 0$ of the model described by potential (3). In this case, the classical scalar field ϕ is spatially localized and periodic in time. The usual procedure to obtain oscillons configurations in $1+1$ dimensions consists in applying a small amplitude expansion of the scalar field ϕ in powers of ϵ in the following form:

$$\phi(y, \tau) = \sum_{n=1}^{\infty} \epsilon^n \phi_n(y, \tau). \tag{6}$$

However, in this work, we are interested in analyzing how some simple displacement in the above expansion may affect the configuration and stability of the oscillons. Then, we are searching for a small amplitude solution where we expand the scalar field ϕ in powers of ϵ as

$$\phi(y,\tau) = k + \sum_{n=1}^{\infty} \epsilon^n \phi_n(y,\tau). \tag{7}$$

Note that the above expansion differs from the usual treatment of oscillons, since we have an additive term, which corresponds to translation in the field. Furthermore, we can recover the usual expansion setting $k = 0$. In fact, considering that the terms in the expansion oscillate in such a way that the mean value of the field configuration is k (in fact, as we are going to see below, it will appear as an effective k_{eff}), one can see that this new oscillon can be at regions of the potential which are far from its vacua. As a consequence of this, we expect that these new field configurations should be more unstable than the usual oscillons located at the vacua of the potential. However, as we are going to see below, they are still considerably long-living configurations. Furthermore, we discovered that, after a given "death point," these oscillon type configurations are separated from their "phantoms." This is going to be done in the next section.

It is important to remark that our approach is general and can be applied to different nonlinear field theories in order to investigate oscillons configurations. A special case is that one given by choosing $b = 0$ in model (3). In this case, the model becomes linear and the solution involves both the spatial and the temporal part. The temporal part is oscillatory, but the spatial one is not localized. Nevertheless, the method still can be applied to find the solution.

3. Phantom Oscillons

In this section, we will derive the profile of the proposed oscillon type configurations using the expansion given by (7). Let us substitute this expansion of the scalar field into the field equation (5). Thus, it is not difficult to conclude that one gets

$$\begin{aligned}
&\lambda k\left(a^2 - 4abk^2 + 3b^2k^4\right) + \epsilon\left(\frac{\partial^2\phi_1}{\partial\tau^2} + \Gamma_0^2\phi_1\right) \\
&+ \epsilon^2\left(\frac{\partial^2\phi_2}{\partial\tau^2} + \Gamma_0^2\phi_2 + \Gamma_1^2\phi_1^2\right) \\
&+ \epsilon^3\left(\frac{\partial^2\phi_3}{\partial\tau^2} - \frac{\partial^2\phi_1}{\partial y^2} - \frac{\partial^2\phi_1}{\partial\tau^2} + \Gamma_0^2\phi_3 + \Gamma_2^2\phi_1^3 + \Gamma_3^2\phi_1\phi_2\right) \\
&+ \cdots = 0,
\end{aligned} \tag{8}$$

where we define

$$\begin{aligned}
\Gamma_0^2 &= \Gamma_0^2(\lambda,k,a,b) \equiv \lambda\left(a^2 - 12abk^2 + 15b^2k^4\right), \\
\Gamma_1^2 &= \Gamma_1^2(\lambda,k,a,b) \equiv 6bk\lambda\left(5bk^2 - 2a\right), \\
\Gamma_2^2 &= \Gamma_2^2(\lambda,k,a,b) \equiv 2b\lambda\left(15bk^2 - 2a\right), \\
\Gamma_3^2 &= \Gamma_3^2(\lambda,k,a,b) = 2\Gamma_1^2.
\end{aligned} \tag{9}$$

We note that the procedure of performing a small amplitude expansion shows that the scalar field solution ϕ can be obtained from a set of scalar fields which satisfy coupled nonlinear differential equations. This set of differential equations is found by taking the terms in all orders of ϵ in the above equation. However, using expansion (7) which has an additive term, one gets a constant term in order $\mathcal{O}(1)$ as output in (8). Aiming at assuring that the field configuration is still oscillating in time, we must now suppose that the first term is such that

$$\lambda k\left(a^2 - 4abk^2 + 3b^2k^4\right) = \eta\epsilon^3, \tag{10}$$

where η is a real arbitrary constant. This condition means that in order to find the values of k we need to establish the values of ϵ and η. In other words, $k = k(\epsilon,\eta)$. Note that the above supposition can be arranged through higher powers of ϵ. However, in order to obtain a correct relation between both sides of (10), we impose that the constant η is a number with magnitude of order 10^0. Thus, only for convenience do we put both sides of relation (10) in the same order of magnitude choosing the coupling constant very small (for $\eta \sim 1$, $\lambda \sim \epsilon^3$). In fact, imposition (10) guarantees that no undesirable mixing between the orders of the nonlinear differential equations appears. The motivation of our choice arises from the simple fact that we are including a residual effect in order $\mathcal{O}(\epsilon^3)$, which can be considered as an important contribution in the oscillon configuration; otherwise, we have no relevant effect in the configuration. Thus, it becomes immediately clear that up to ϵ^3 the above supposition leads to

$$\begin{aligned}
&\frac{\partial^2\phi_1}{\partial\tau^2} + \Gamma_0^2\phi_1 = 0, \\
&\frac{\partial^2\phi_2}{\partial\tau^2} + \Gamma_0^2\phi_2 = -\Gamma_1^2\phi_1^2,
\end{aligned} \tag{11}$$

$$\frac{\partial^2\phi_3}{\partial\tau^2} - \frac{\partial^2\phi_1}{\partial y^2} - \frac{\partial^2\phi_1}{\partial\tau^2} + \Gamma_0^2\phi_3 + \Gamma_2^2\phi_1^3 + \Gamma_3^2\phi_1\phi_2 = -\eta. \tag{12}$$

Here, it is necessary to impose that Γ_0 has a real value; otherwise, the solution of $\phi_1(x,t)$ will not be oscillatory in time and we will not obtain oscillon configurations. Furthermore, it can be observed that since we are limited to small coupling situations ($\lambda \sim \epsilon^3$), the frequency of oscillation Γ_0 will be small by construction. Therefore, from this condition, we must impose that $a^2 - 12abk^2 + 15b^2k^4 > 0$. Now, under this restriction, we have only a few acceptable ranges of validity for k, but we will see below that they will be enough to reproduce all the values of the effective translation in the field. Furthermore, we will see later that this restriction is not unique to get the possible values of the translation constant.

Let us now look for the solution of (11) and (12). First, it is not difficult to conclude that, for real Γ_0, the solution of (11) can be given by

$$\phi_1(y,\tau) = \varphi(y)\cos(\Gamma_0\tau). \tag{13}$$

Looking at the second equation of (11), we see that the solution of $\phi_2(y,\tau)$ can be found by using the above solution. Thus, substituting (13) into the equation of ϕ_2, one obtains

$$\phi_2(y,\tau) = -\frac{\Gamma_1^2\varphi^2(y)\left[3-\cos(2\Gamma_0\tau)\right]}{6\Gamma_0^2}. \tag{14}$$

Similarly, from solutions (13) and (14), we can obtain $\phi_3(y,\tau)$. Then, after straightforward calculations, one can verify that (12) takes the form

$$\begin{aligned}\frac{\partial^2\phi_3}{\partial\tau^2} &+ \Gamma_0^2\phi_3 \\ &= -\eta \\ &\quad + \left[\frac{d^2\varphi}{dy^2} - \Gamma_0^2\varphi + \left(\frac{5\Gamma_1^2}{6\Gamma_0^2} - \frac{3\Gamma_0^2}{4}\right)\varphi^3\right]\cos(\Gamma_0\tau) \\ &\quad - \left(\frac{\Gamma_2^2}{4} + \frac{\Gamma_1^2}{6\Gamma_0^2}\right)\varphi^3\cos(3\Gamma_0\tau).\end{aligned} \tag{15}$$

Our primordial goal is to get configurations which are periodical in time. Then, if we solve the above partial differential equation in the presented form, we will have a term linear in τ. As a consequence, the solution for ϕ_3 is neither periodical nor localized. This result comes from the contribution of the function $\cos(\Gamma_0\tau)$ in the right-hand side of the partial differential equation (15). However, we can construct solutions for ϕ_3 which are periodical in time if we impose that

$$\frac{d^2\varphi}{dy^2} = \Gamma_0^2\varphi - \Omega_0^2\varphi^3, \quad \text{with } \Omega_0^2 \equiv \frac{5\Gamma_1^4}{6\Gamma_0^2} - \frac{3\Gamma_2^2}{4}. \tag{16}$$

At this point, it is necessary to impose that $\Omega_0^2 > 0$ in order to find solutions with profile of hyperbolic secant. We recall from our studies that Γ_0 is characterized by another condition; then, we can now combine the inequalities in order to encounter the valid region of the values of k. Here, we will not be concerned with such analytic detail. In fact, such region is easily obtained in a numerical context; for instance, with $\lambda = 10^{-5}$, $a = 1.2$, $b = 0.01$, and $\epsilon = 0.01$, we find $-3.3674 < k < 3.3674$, $k < -9.20112$, and $k > 9.20112$. Despite this restriction, we will see later that these values will produce all possible translations from the vacuum located at $\phi_v^{(0)} = 0$. As a consequence, we can choose the field configuration to be at any region of potential (3).

Now, coming back to (16) and solving it, we obtain

$$\varphi(y) = \frac{\Gamma_0\sqrt{2}\,\mathrm{sech}(\Gamma_0 y)}{\Omega_0}. \tag{17}$$

Thus, using the condition that ϕ_3 should be localized, (15) becomes

$$\frac{\partial^2\phi_3}{\partial\tau^2} + \Gamma_0^2\phi_3 = -\eta - \left(\frac{\Gamma_2^2}{4} + \frac{\Gamma_1^2}{6\Gamma_0^2}\right)\varphi^3\cos(3\Gamma_0\tau), \tag{18}$$

which has the corresponding solution

$$\phi_3(y,\tau) = -\frac{\eta}{\Gamma_0^2} + \frac{\omega_0^2\varphi^3\cos(3\Gamma_0\tau)}{8\Gamma_0^2}, \qquad \text{with } \omega_0^2 \equiv \frac{\Gamma_2^2}{4} + \frac{\Gamma_1^2}{6\Gamma_0^2}. \tag{19}$$

From the above results, as one can see, up to order $\mathcal{O}(\epsilon^3)$, the corresponding solution for the classical field is given by

$$\begin{aligned}\phi(y,\tau) &= k_{\text{eff}} + \epsilon\varphi(y)\cos(\Gamma_0\tau) \\ &\quad + \epsilon^2\frac{\Gamma_1^2\varphi^2(y)\left[\cos(2\Gamma_0\tau)-3\right]}{6\Gamma_0^2} \\ &\quad + \epsilon^3\frac{\omega_0^2\varphi^3(y)\cos(3\Gamma_0\tau)}{8\Gamma_0^2} + \sum_{n=4}^{\infty}\epsilon^n\phi_n(y,\tau),\end{aligned} \tag{20}$$

with $k_{\text{eff}} \equiv 4bk^3(3bk^2-2a)/(a^2-12abk^2+15b^2k^4)$, which is the corresponding effective mean value of the field configuration as mentioned in Section 1. Since the parameter ϵ is taken as extremely small, the profile of the solution is defined up to order $\mathcal{O}(\epsilon)$, once the subsequent orders become even smaller. Thus, the field configuration written in terms of the original variables, in a good approximation, is given by

$$\begin{aligned}\phi_{\text{osc}}(x,t) &\approx k_{\text{eff}} + \epsilon\left[\frac{\sqrt{2}\Gamma_0\,\mathrm{sech}(\epsilon\Gamma_0 x)}{\Omega_0}\right]\cos(\Gamma_{\text{eff}}t) \\ &\quad + \mathcal{O}(\epsilon^2),\end{aligned} \tag{21}$$

with $\Gamma_{\text{eff}} = \Gamma_0\sqrt{1-\epsilon^2}$. At this point, it is important to remark that despite the fact that there are no constant solutions for the original differential equation we are dealing with, except for the vacua located at $0, \pm\sqrt{a/b}$, we are not dealing with exact solutions; instead, we work with solutions approximated up to order ϵ^3. In fact, even in the case of the usual oscillon, this condition holds; the only difference is that, in case of the usual oscillons, the solutions are exact at the boundaries, and here the solution is taken as an ϵ^3 approximation along the entire spatial axis, including the boundaries.

We can note that the fundamental difference of our solution, compared to the usual oscillon, is given by the presence of k_{eff}. Here, we have the advantage of moving the configuration for any region within the potential; this important mechanism opens a window to show that it is possible to find oscillons living in any region of the potential, but a natural question arises about the stability of this configuration, once the usual oscillons are highly stable configurations when they are oscillating around the vacuum of the potential. In the next section, we will address this

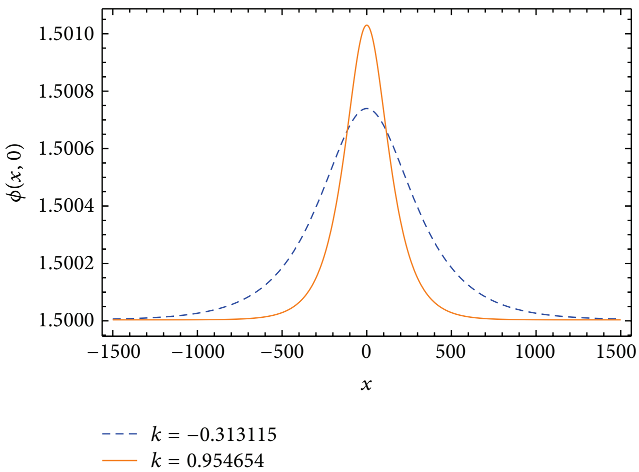

Figure 2: Oscillon (thin line) and its phantom (dashed line) for $t = 0$, $\epsilon = 0.01$, $a = 1.2$, $\lambda = 10^{-5}$, and $b = 0.01$.

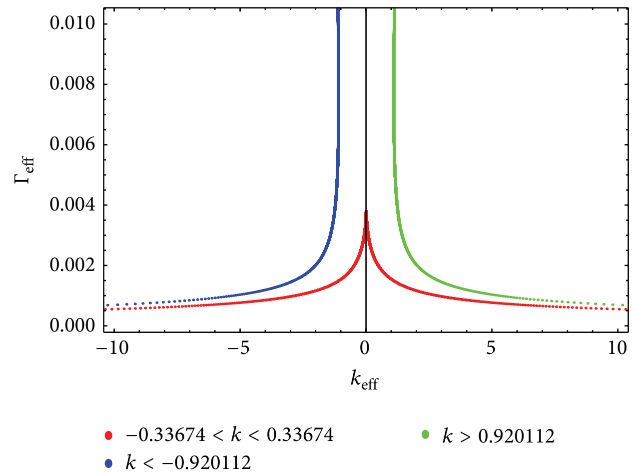

Figure 3: Effective frequency of the oscillon (bottom line) and its phantom (top line) for $t = 0$, $\lambda = 10^{-5}$, $\epsilon = 0.01$, $a = 1.2$, and $b = 0.01$.

question about the stability of the our solution and we will see that these oscillons are more unstable than the ones localized in the vacuum.

Another important and interesting result that arises from our solution is the emergence of a new "phantom oscillon" after a certain threshold value of k_{eff}. Above that value, what we called a "death point," we will have always the presence of two oscillons, in a kind of "phantom zone" (see Figure 1).

In Figure 2, we sketch the typical profile of the oscillon and its phantom. In that figure, we see that the field configurations are oscillating around the corresponding effective mean value of the field configuration. Furthermore, we can note that both the real oscillon and its phantom have different amplitudes in their structures located at the origin. Moreover, the tail of the phantom reaches a value of the effective field in a spatial region farther than the real oscillon.

We also emphasize that, in the phantom zone, the effective frequency Γ_{eff} of the oscillations in time for the oscillons decreases with k_{eff}. In Figure 3, we plot Γ_{eff} as a function of k_{eff}. There, one can see that the oscillon frequency is decreasing with k_{eff} and that the phantom oscillon has a higher frequency than that of the corresponding oscillon for a given value of k_{eff}.

4. Radiation

An important characteristic of the oscillon is its radiation emission, responsible for its unavoidable decaying. In a seminal work by Segur and Kruskal [15], it was shown that oscillons in one spatial dimension decay emitting radiation. Recently, the computation of the emitted radiation in two and three spatial dimensions was done in [16]. On the other hand, in a recent paper by Hertzberg [17], it was found that the quantum radiation is very distinct from the classic one. It is important to remark that the author has shown that the amplitude of the classical radiation emitted can be found using the amplitude of the Fourier transform of the spatial structure of the oscillon.

Thus, in this section, we compute the outgoing radiation of these oscillon type configurations. Here, we will use a method in 1 + 1 dimensional Minkowski space-time which allows computing the classical radiation and which is similar to the one presented in [17]. This approach supposes that we can write the solution of the classical equation of motion in the following form:

$$\phi_{\text{sol}}(x,t) = \phi_{\text{osc}}(x,t) + \xi(x,t), \tag{22}$$

where $\phi_{\text{osc}}(x,t)$ is the oscillon solution and $\xi(x,t)$ represents small perturbation. Let us substitute this decomposition of the scalar field into the equation of motion (2). This leads to

$$\frac{\partial^2 \xi}{\partial t^2} - \frac{\partial^2 \xi}{\partial x^2} + \Lambda(x,t)\,\xi = -J(x,t), \tag{23}$$

where $\Lambda(x,t) = \lambda(a^2 - 12ab\phi_{\text{osc}}^2 + 15b^2\phi_{\text{osc}}^4)$ and the function $J(x,t)$ acts as an external source. In this case, it is written as

$$\begin{aligned} J(x,t) = {} & \frac{\partial^2 \phi_{\text{osc}}}{\partial t^2} - \frac{\partial^2 \phi_{\text{osc}}}{\partial x^2} \\ & + \lambda\left(a^2\phi_{\text{osc}} - 4ab\phi_{\text{osc}}^3 + 3b^2\phi_{\text{osc}}^5\right). \end{aligned} \tag{24}$$

Since ξ represents a small correction, we naturally assume that the dependence of the nonlinear terms in higher powers of ξ can be neglected. On the other hand, we will look for solutions of $\xi(x,t)$ where the amplitudes of the tails of the oscillons are much smaller than those of the core. In other words, we are looking for solutions at large distances. This says that for $x \gg 1$ the field configuration is given by $\phi_{\text{osc}} \simeq k_{\text{eff}} + \epsilon\varphi(x)\cos(\Gamma_{\text{eff}} t)$. Thus, (23) can be rewritten as

$$\frac{\partial^2 \tilde{\xi}}{\partial t^2} - \frac{\partial^2 \tilde{\xi}}{\partial x^2} + \tilde{\Lambda}(x,t)\,\tilde{\xi} = -\tilde{J}(x,t), \tag{25}$$

where

$$\widetilde{\xi}(x,t) = \xi(x,t) + \mu_{\text{eff}},$$
$$\widetilde{\Lambda}(x,t) \simeq \gamma_0 \operatorname{sech}(\epsilon\Gamma_0 x)\cos(\Gamma_{\text{eff}} t),$$
$$\widetilde{J}(x,t) \simeq -\frac{\sqrt{2}\Gamma_0^3}{\Omega_0}\operatorname{sech}(\epsilon\Gamma_0 x)\cos(\Gamma_{\text{eff}} t) + \left(\epsilon\widetilde{A} + \epsilon^2\frac{2\sqrt{2}\Gamma_0^2}{\Omega_0}\right)\operatorname{sech}(\epsilon\Gamma_0 x)\cos(\Gamma_{\text{eff}} t), \quad (26)$$
$$\mu_{\text{eff}} \equiv \frac{a^2 k_{\text{eff}} - 4abk_{\text{eff}}^3 + 3b^2 k_{\text{eff}}^5}{a^2 - 12abk_{\text{eff}}^2 + 15b^2 k_{\text{eff}}^4},$$
$$\gamma_0 \equiv -12A\epsilon\lambda b k_{\text{eff}}\left(2a - 5bk_{\text{eff}}^2\right),$$
$$\widetilde{A} \equiv \frac{\sqrt{2}\Gamma_0 a\lambda\left(a - 12bk_{\text{eff}}^2\right)}{\Omega_0}.$$

At this point, we can obtain from (25) that

$$\widetilde{\xi}(x,t) = -\frac{1}{(2\pi)^2}\lim_{p\to 0^+}\int d\overline{\omega}\int d\overline{k}\frac{J(\overline{\omega},\overline{k})e^{i(\overline{k}x-\overline{\omega}t)}}{\overline{k}^2 - \overline{\omega}^2 \pm ip}, \quad (27)$$

with

$$J(\overline{\omega},\overline{k}) = \int d\overline{x}\int d\overline{t} J(\overline{x},\overline{t})e^{-i(\overline{k}x-\overline{\omega}t)}. \quad (28)$$

It is important to remark that, in order to solve (25) analytically, we will impose that b is of the order of $\mathcal{O}(\epsilon)$; as a consequence, $\gamma_0 \sim \epsilon^2$. Therefore, at far distances from the core of the oscillon, we have $\widetilde{\Lambda}(x,t) \sim 0$. Thus, after straightforward computations, one can conclude that

$$\widetilde{\xi}(x,t) \simeq \frac{\sqrt{2}J_0\sqrt{\pi}}{4\epsilon\Gamma_0\Gamma_{\text{eff}}}\operatorname{sech}\left(\frac{\Gamma_{\text{eff}}\pi}{2\epsilon\Gamma_0}\right)\sin(\Gamma_{\text{eff}} x)\cos(\Gamma_{\text{eff}} t), \quad (29)$$

where $J_0 = -\sqrt{2}\Gamma_0^3/\Omega_0 + \epsilon\widetilde{A} + 2\sqrt{2}\epsilon^2\Gamma_0^3/\Omega_0$. Using (29), we can see in Figure 4 the profile of the radiation power emitted, where the reader can observe that the stability of the dislocated oscillon diminishes for increasing values of k_{eff} and that the phantom oscillon located at the second vacuum is as stable as the oscillon located at the central vacuum.

5. Numerical Results

In this section, in order to check our analytical results, we will compute the numerical solutions for the oscillon profile. In this way, to analyze the oscillon configuration numerically, we use the initial configuration in the form

$$\phi_{\text{num}}(x,0) = k_{\text{eff}} + \epsilon\frac{\Gamma_0\sqrt{2}\operatorname{sech}(\Gamma_0\epsilon x)}{\Omega_0}. \quad (30)$$

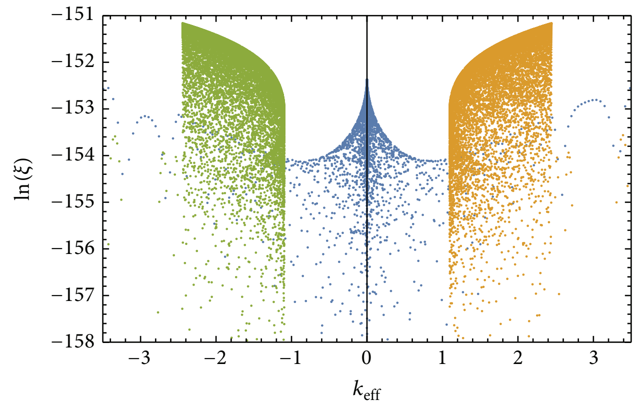

FIGURE 4: Radiation power for $t = 0$, $x = 250$, $\lambda = 10^{-5}$, $\epsilon = 0.01$, $a = 1.2$, and $b = 0.01$. The blue dots are the values in the range $-0.33674 < k < 0.33674$, the green dots correspond to the range $k < -0.920112$, and the orange dots correspond to the values in the range $k > 0.920112$.

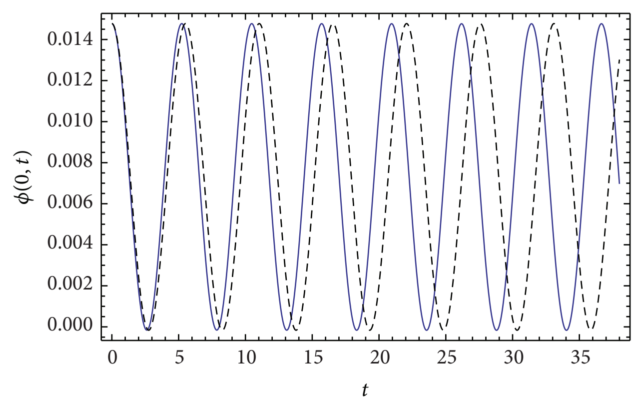

FIGURE 5: Analytical and numerical results. The figure is a comparison of the field at $x = 0$. Dashed line is analytical and the solid curve is numerical.

In general, oscillons are not an exact solution for the scalar field. Thus, it is convenient to begin with the above initial configuration for evolution of the numerical solution. Another important condition is given by $\partial\phi(x,0)/\partial t = 0$.

Now, for evolution of the numerical solution of the field equation (5), we will use $\epsilon = 0.01$, $k = -0.1$, $a = 1.2$, $b = 1$, and $\lambda = 1$. As a result of this choice, one has $k_{\text{eff}} \simeq 0.07$. To illustrate the field configurations which corresponds to the oscillons, we graph the field at $x = 0$ of the field $\phi(x,t)$. We can see this numerical field solution in Figure 5. That figure shows a comparison of analytical solution and numerical ones. We can observe that the analytical value of the field at the centre of the oscillon shows a small disparity in the horizontal position when the time increases showing a profile of phase difference. However, looking in a large range of the time, we can find that the disparity is of order ~6% showing, as we would expect, that the theoretical solutions are in good agreement with the numerical ones. Furthermore, in the vertical position, the analytical amplitude of the oscillons is correctly predicted by the numerical results.

6. Conclusions

In this work, we have presented novel oscillon-like configurations which we call phantom oscillons. We have found that displacement of the minimum value of the field configuration representing the oscillon affects its behavior. In this case, the procedure of introducing those displacement instances act as a kind of source of new oscillons; in fact, this leads us to think about the possibility of the appearance of a higher number of additional oscillons when one deals with a field potential containing a bigger number of degenerate vacua. This possibility is under analysis and hopefully will be reported in a future work. Moreover, it can be observed in Figure 4 that the original oscillon (created at $\phi_{\text{vac}} = 0$) is more stable than the ones located at $\phi_{\text{vac}} = k_{\text{eff}}$ and the stability decreases when k_{eff} increases. This happens until one reaches the position of another vacuum of the model. At this point, a second oscillon-like configuration shows up (the phantom oscillon), and it is remarkable that it presents approximately the same degree of stability as the original oscillon. Furthermore, one can also note that the frequency of the phantom oscillon is always higher than the corresponding oscillon (see Figure 3). An interesting consequence of these configurations is that one can think about the behavior of a gas of oscillons, where a statistical distribution of oscillons would appear [37], each one having a different value of k_{eff}, and, due to the relative stabilities, some of them would decay or combine to produce more stable structures. This kind of scenario could be of interest for some cosmological models [38–40].

Competing Interests

The authors declare that there are no competing interests regarding the publication of this paper.

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