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Research Article

Robust H_∞ Control for Switched Nonlinear System with Multiple Delays

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This paper focuses on the problem of robust H_∞ control for a class of switched nonlinear systems with multiple time delays which are allowed to appear not only in state but also in input. We will design a controller and a switching law under which the system is "robust stable" and has performance L_2-gain γ. Besides, multiple Lyapunov-Razumikhin functions which are relatively less conservative are introduced to investigate the stability of the candidate switched system. In detail, firstly, for uncertain nonlinear switched system with time-varying delays, sufficient condition is given by inequalities which are equivalent to LMIs, so it is easy to be solved; then, as a special case, nonlinear switched system with constant time delay is also considered. Finally, by providing numerical examples, the feasibility of the proposed approach is demonstrated.

1. Introduction

Switched systems have constituted a very active field of current scientific research; many real world processes and systems can be modeled as switched systems, such as chemical processes and computer controlled systems. Besides, switched systems are extensively applied in many domains, including mechanical systems, automation, aircraft and air traffic control, and many other fields. And stability analysis and control, as the most important topics for the study of switched systems, have been studied widely [1–11]. Up to now, some methods are given to solve the stability analysis for switched system. For example, by constructing a common Lyapunov function, stability under arbitrary switching can be obtained. However, most switched systems still may be stable under certain switching laws, though they do not own a common Lyapunov function. The key idea to select such switching laws is the dwell time [12, 13]. Lately, H_∞ control theory, which is another effective tool to solve robust stabilization problems, has been well built [14].

On the other hand, time-delay phenomenon is very common in many kinds of engineering systems, for instance, long-distance transportation systems, hydraulic pressure systems, network control systems, and so on. It is regularly a source of instability and often causes undesirable performance and even makes the system out of control. So time-delay systems have also drawn more and more attention [15–23]. Based on the above two aspects, the problem of stability analysis and controller synthesis for switched system with time delay (switched delay system) has aroused growing interest. And switched delay system, as a new brand, can be found in many applications and there are quite a lot of related results [24–34]. However, in the above papers, systems with only one delay are considered. And system with multiple delays, which is widely used in practice, has gained more and more attention [35–38]. For switched systems with multiple time delays, the problem of robust stability is considered in [39]. In [39], delays are constant and appear only in state. In fact, in engineering control design for the actual system, time-varying delays are universal, and due to various reasons delays can also appear in control input. On this occasion, the system will become more complicated and then the response of state may also be affected. In [40], the problem of robust H_∞ control for nonswitched system with input delays is considered. Motivated by this, it is meaningful to investigate the following problem: if the system in [40] is switched, how is its stability and also how do we design a controller and a switching law to make the system stable? As yet, as far as we know, it has not been fully investigated despite its potential in practical applications, and the purpose of this paper is therefore to shorten such a gap by providing a rather general framework.

The contribution of this paper lies in the following aspects. First, we address the problem of H_∞ control for a more general class of switched systems with multiple time-varying delays, in which delays not only appear in the state but also appear in the input. Second, by choosing a new set of multiple Lyapunov-Razumikhin functionals (which is relatively less conservative than the CLF approach), we derive sufficient conditions of the existence of H_∞ state-feedback controller for a class of switching signals with average dwell time scheme; besides, the specific expression of controller is also given in the paper. Third, in this paper, the inequality we derived is easy to compute for it can be rewritten as LMIs.

The remainder of this paper is organized as follows. Firstly, problem formulation and preliminaries are stated in Section 2. In the following section, the main results are addressed. And a numerical example is given in Section 4. Section 5 draws the conclusion.

Notations. Throughout this paper, the notation $P > 0$ ($\geq, \leq, <$) means that P is a positive definite (positive-semidefinite, negative-semidefinite, and negative definite) matrix P. $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ denote the maximum and minimum eigenvalues of P, respectively; the superscript "T" stands for the transpose; $\|x(t)\|$ denotes the Euclidean norm; $L_2[0,\infty)$ represents the space of square integrable vector functions over $[0,\infty)$; I is an identity matrix with appropriate dimension, and if not stated, matrices are assumed to have compatible dimensions. $\left(\begin{smallmatrix} A & B \\ * & C \end{smallmatrix}\right)$ denotes a symmetric matrix.

2. Problem Formulation and Preliminaries

Consider a class of uncertain switched delay systems of the form

$$\begin{aligned}
\dot{x}(t) = {} & A_{\sigma(t)}x(t) + A_{1\sigma(t)}x(t-\tau_1(t)) \\
& + E_{\sigma(t)}f_{0\sigma(t)}(x(t),t) \\
& + E_{1\sigma(t)}f_{1\sigma(t)}(x(t-\tau_2(t)),t) \\
& + E_{2\sigma(t)}f_{2\sigma(t)}(x(t-\tau_3(t)),t) + B_{\sigma(t)}\omega(t) \\
& + B_{1\sigma(t)}u_{\sigma(t)}(t) + B_{2\sigma(t)}u_{\sigma(t)}(t-\tau_4(t)), \\
z(t) = {} & C_{\sigma(t)}x(t), \\
x(t) = {} & 0, \quad t \leq 0,
\end{aligned} \tag{1}$$

where $x(t) \in R^n$, $u(t) \in R^q$, and $z(t) \in R^p$ are the state vector, input, and output, respectively. $\omega(t) \in R^l$ which is assumed to belong to $L_2[0,\infty)$ is external disturbance input; $\sigma(t) : [0,\infty) \to M = \{1,2,\ldots,m\}$ is the switching signal; A_i, A_{1i}, B_i, E_i, and E_{1i} are given constant matrices with appropriate dimensions; $\tau_i(t)$ ($i = 1,2,3,4$) are the time-varying delays which satisfy the following assumption.

Assumption 1. Consider $0 \leq \tau_i(t) \leq d_i < \infty$, $\dot{\tau}_i(t) \leq \alpha_i < 1$, $i = 1,2,3,4$, for all $t > 0$.

Remark 2. With regard to the switching signal $\sigma(t)$, there exists a switching sequence $\{x_{t_0}; (i_0,t_0),\ldots,(i_k,t_k),\ldots, \mid i_k \in M,\ k = 0,1,\ldots\}$, which means that when $t \in [t_k, t_{k+1})$, the i_kth subsystem is activated.

Remark 3. In general, the nonlinear uncertainties $f_{0\sigma(t)}(x(t),t)$, $f_{1\sigma(t)}(x(t-\tau_2(t)))$, and $f_{2\sigma(t)}(x(t-\tau_3(t)))$ which refer to the nonlinear internal parameter uncertainties with respect to the current state and delayed states are assumed to satisfy the following inequalities.

Assumption 4. Consider

$$\begin{aligned}
\|f_{0\sigma(t)}(x(t),t)\| &\leq \|F_\sigma(t)x(t)\|, \\
\|f_{1\sigma(t)}(x(t-\tau_2(t)))\| &\leq \|F_{1\sigma(t)}x(t-\tau_2(t))\|, \\
\|f_{2\sigma(t)}(x(t-\tau_3(t)))\| &\leq \|F_{2\sigma(t)}x(t-\tau_3(t))\|,
\end{aligned} \tag{2}$$

where $F_\sigma(t)$, $F_{1\sigma(t)}$, and $F_{2\sigma(t)}$ are known constant matrices.

Remark 5. Compared with the switched system in [39] (where the control input is not contained), in this paper, delays are time varying and control input is also contained with delays, so system (1) is more general and more practical for we can choose a controller to make an unstable system stable.

In this paper, we are interested in designing a state-feedback controller which is described by $u(t) = Kx(t)$ and a switching law $\sigma(t)$ such that system (1) is "robust stable." Firstly, the relevant definitions are given below.

Definition 6. System (1) is said to be exponentially stable under controller $u(t)$ and switching law $\sigma(t)$, if, with $\omega = 0$, the solution $x(t)$ of system (1) satisfies

$$\|x(t)\| \leq k\|x_{t_0}\| e^{-\lambda(t-t_0)}, \quad \forall t \geq t_0, \tag{3}$$

where $k \geq 1$ and $\lambda > 0$.

Definition 7 (see [11]). For any $T_2 > T_1 \geq 0$, let $N_\sigma(T_1, T_2)$ represent the switching number of $\sigma(t)$ on (T_1,T_2). If $N_\sigma(T_1,T_2) \leq N_0 + (T_2 - T_1)/\tau_a$ holds for $\tau_a > 0$, $N_0 \geq 0$, then τ_a is called average dwell time.

Remark 8. The concept of average dwell time, which was an effective tool for the stability analysis of switched systems, was put forward for continuous switched systems firstly by Hespanha and Morse (see [11]). As commonly used in the literature, we choose $N_0 = 0$ in this paper.

Definition 9 (see [41]). For $\gamma > 0$, $\alpha > 0$, consider system (1) and design a switching law $i = \sigma(t)$ and controller $u(t)$, if it holds that

(1) system (1) is exponentially stable when $\omega(t) \equiv 0$;

(2) under zero initial conditions system (1) satisfies

$$\int_0^\infty e^{-\alpha s} z^T(s) z(s) ds \le \gamma^2 \int_0^\infty \omega^T(s) \omega(s) ds. \quad (4)$$

Then system (1) is said to have exponential H_∞ performance γ (weighted L_2-gain).

Now the following lemma which will be used to draw the main results in this paper is presented.

Lemma 10 (see [40]). *For any function vector $F(x)$, symmetric matrix P, and constant matrix D, it holds that*

$$2x^T PDF(x) \le \beta^2 x^T PDD^T Px + \beta^{-2} F^T(x) F(x), \quad \forall x \in R^n, \ \forall \beta > 0. \quad (5)$$

3. Main Results

In this paper, we focus on the switched delay system with time-varying delays and constant delays, respectively. And we will tackle the robust H_∞ control problem for switched delay system (1). When $t \in [t_k, t_{k+1})$, the "i_kth" subsystem of system (1) is activated. At this moment it can be seen as a "nonswitched" system corresponding to system (1). Based on this, we will first consider the nonswitched system:

$$\begin{aligned}
\dot{x}(t) &= Ax(t) + A_1 x(t - \tau_1(t)) + Ef_0(x(t), t) \\
&\quad + E_1 f_1(x(t - \tau_2(t)), t) \\
&\quad + E_2 f_2(x(t - \tau_3(t)), t) + B\omega(t) + B_1 u(t) \\
&\quad + B_2 u(t - \tau_4(t)), \\
z(t) &= Cx(t), \\
x(t) &= 0, \quad t \le 0.
\end{aligned} \quad (6)$$

Lemma 11. *Consider the nonswitched system (6). For a given positive definite matrix $R > 0$ and constants $\alpha > 0$, $\gamma > 0$, $\mu > 0$, and $\varepsilon_i > 0$, $i = 1, 2, 3, 4, 5$, define*

$$V(x,t) = x^T(t) Px(t) + \sum_{i=1}^4 \int_{t-\tau_i(t)}^t e^{\alpha(s-t)} x^T(s) Q_i x(s) ds \quad (7)$$

with

$$\begin{aligned}
Q_1 &= \frac{e^{\alpha d_1}}{\varepsilon_1^2(1-\alpha_1)} I, \\
Q_2 &= \frac{e^{\alpha d_2}}{\varepsilon_3^2(1-\alpha_2)} F_1^T F_1, \\
Q_3 &= \frac{e^{\alpha d_3}}{\varepsilon_4^2(1-\alpha_3)} F_2^T F_2, \\
Q_4 &= \frac{e^{\alpha d_4}}{\varepsilon_5^2(1-\alpha_4)} K^T K;
\end{aligned} \quad (8)$$

the linear feedback controller is designed by $u(t) = Kx(t)$, $K = -(1/2\mu)R^{-1}B_1^T P$, such that the inequality

$$A^T P + PA + \alpha P - \frac{1}{\mu} PB_1 R^{-1} B_1^T P + C^T C + \widetilde{Q} + PNP < 0 \quad (9)$$

has a positive definite solution P, where

$$\begin{aligned}
\widetilde{Q} &= \sum_{i=1}^4 Q_i + \varepsilon_2^{-2} F^T F, \\
N &= \varepsilon_1^2 A_1 A_1^T + \varepsilon_2^2 EE^T + \varepsilon_3^2 E_1 E_1^T + \varepsilon_4^2 E_2 E_2^T + \varepsilon_5^2 B_2 B_2^T \\
&\quad + \gamma^{-2} BB^T.
\end{aligned} \quad (10)$$

Then, along the trajectory of the system, when $\omega(t) = 0$,

$$V(t) \le e^{(-\alpha)(t-t_0)} V(t_0). \quad (11)$$

For general $\omega(t)$,

$$V(t) \le e^{(-\alpha)(t-t_0)} V(t_0) - \int_{t_0}^t e^{(-\alpha)(t-s)} \Gamma(s) ds, \quad (12)$$

where $\Gamma(t) = \|z(t)\|^2 - \gamma^2 \|\omega(t)\|^2$.

Proof. The following proof is motivated by the method in [40].

For the function $V(x,t)$ defined by (7),

$$\begin{aligned}
&\dot{V}(x,t) + \alpha V(x,t) + \|z\|^2 - \gamma^2 \|\omega\|^2 = x^T(t) \\
&\quad \cdot \left((A + B_1 K)^T P + P(A + B_1 K) + \alpha P + C^T C\right) x(t) + 2x^T(t) \\
&\quad \cdot PA_1 x(t - \tau_1(t)) + 2x^T(t) PB_2 Kx(t - \tau_4(t)) + 2x^T(t) PB\omega(t) + 2x^T(t) PEf_0(x(t), t) \\
&\quad + 2x^T(t) PE_1 f_1(x(t - \tau_2(t))) + 2x^T(t) \\
&\quad \cdot PE_2 f_2(x(t - \tau_3(t))) + \sum_{i=1}^4 x^T(t) Q_i x(t) \\
&\quad - \sum_{i=1}^4 (1 - \dot{\tau}_i(t)) e^{-\alpha \tau_i(t)} x^T(t - \tau_i(t)) Q_i x(t - \tau_i(t)) \\
&\quad - \gamma^2 \|\omega\|^2.
\end{aligned} \quad (13)$$

Using Assumptions 1 and 4 and Lemma 10, we get the following inequalities:

$$\begin{aligned}
&-\left(1-\dot{\tau}_i(t)\right) \le \alpha_i - 1 < 0, \\
&2x^T(t)PA_1x\left(t-\tau_1(t)\right) \\
&\quad \le \varepsilon_1^2 x^T(t)PA_1A_1^TPx(t) \\
&\qquad + \varepsilon_1^{-2}x^T\left(t-\tau_1(t)\right)x\left(t-\tau_1(t)\right), \\
&2x^T(t)PEf_0\left(x(t),t\right) \\
&\quad \le x^T(t)\left(\varepsilon_2^2 PEE^TP + \varepsilon_2^{-2}F^TF\right)x(t), \\
&2x^T(t)PE_1f_1\left(x\left(t-\tau_2(t)\right)\right) \\
&\quad \le \varepsilon_3^2 x^T(t)PE_1E_1^TPx(t) \\
&\qquad + \varepsilon_3^{-2}x^T\left(t-\tau_2(t)\right)F_1^TF_1x\left(t-\tau_2(t)\right), \\
&2x^T(t)PE_2f_2\left(x\left(t-\tau_3(t)\right)\right) \\
&\quad \le \varepsilon_4^2 x^T(t)PE_2E_2^TPx(t) \\
&\qquad + \varepsilon_4^{-2}x^T\left(t-\tau_3(t)\right)F_2^TF_2x\left(t-\tau_3(t)\right), \\
&2x^T(t)PB\omega(t) \\
&\quad \le \gamma^2\omega^T(t)\omega(t) + \gamma^{-2}x^T(t)PBB^TPx(t), \\
&2x^T(t)PB_2Kx\left(t-\tau_4(t)\right) \\
&\quad \le \varepsilon_5^2 x^T(t)PB_2B_2^TPx(t) \\
&\qquad + \varepsilon_5^{-2}x^T\left(t-\tau_4(t)\right)K^TKx\left(t-\tau_4(t)\right).
\end{aligned} \tag{14}$$

Substituting inequalities (14) into (13), it holds that

$$\begin{aligned}
&\dot{V}(x,t) + \alpha V(x,t) + \|z\|^2 - \gamma^2\|\omega\|^2 \le x^T(t)\Big[(A \\
&+ B_1K)^TP + P(A+B_1K) + \alpha P + C^TC + \sum_{i=1}^{4}Q_i \\
&+ \varepsilon_2^{-2}F^TF + P\left(\varepsilon_1^2A_1A_1^T + \varepsilon_2^2EE^T + \varepsilon_3^2E_1E_1^T\right. \\
&\left.+ \varepsilon_4^2E_2E_2^T + \varepsilon_5^2B_2B_2^T + \gamma^{-2}BB^T\right)P\Big]x(t) + \varepsilon_1^{-2}x^T(t \\
&- \tau_1(t))x\left(t-\tau_1(t)\right) + \varepsilon_3^{-2}x^T\left(t-\tau_2(t)\right)F_1^TF_1x(t \\
&- \tau_2(t)) + \varepsilon_4^{-2}x^T\left(t-\tau_3(t)\right)F_2^TF_2x\left(t-\tau_3(t)\right) \\
&+ \varepsilon_5^{-2}x^T\left(t-\tau_4(t)\right)K^TKx\left(t-\tau_4(t)\right) - \sum_{i=1}^{4}\left(1-\alpha_i\right) \\
&\cdot e^{-\alpha d_i}x^T\left(t-\tau_i(t)\right)Q_ix\left(t-\tau_i(t)\right).
\end{aligned} \tag{15}$$

Considering (8), inequality (15) can be expressed as

$$\begin{aligned}
&\dot{V}(x,t) + \alpha V(x,t) + \|z\|^2 - \gamma^2\|\omega\|^2 \le x^T(t)\Big[(A \\
&+ B_1K)^TP + P(A+B_1K) + \alpha P + C^TC + \varepsilon_2^{-2}F^TF \\
&+ \sum_{i=1}^{4}Q_i + P\left(\varepsilon_1^2A_1A_1^T + \varepsilon_2^2EE^T + \varepsilon_3^2E_1E_1^T + \varepsilon_4^2E_2E_2^T\right. \\
&\left.+ \varepsilon_5^2B_2B_2^T + \gamma^{-2}BB^T\right)P\Big]x(t).
\end{aligned} \tag{16}$$

Let $K = -(1/2\mu)R^{-1}B_1^TP$ in (16); we can get

$$\begin{aligned}
&\dot{V}(x,t) + \alpha V(x,t) + \|z\|^2 - \gamma^2\|\omega\|^2 \le x^T(t)\Big[A^TP \\
&+ PA + \alpha P - \frac{1}{\mu}PB_1R^{-1}B_1^TP + C^TC + \varepsilon_2^{-2}F^TF \\
&+ \sum_{i=1}^{4}Q_i + P\left(\varepsilon_1^2A_1A_1^T + \varepsilon_2^2EE^T + \varepsilon_3^2E_1E_1^T + \varepsilon_4^2E_2E_2^T\right. \\
&\left.+ \varepsilon_5^2B_2B_2^T + \gamma^{-2}BB^T\right)P\Big]x(t).
\end{aligned} \tag{17}$$

Hence

$$\dot{V}(x,t) + \alpha V(x,t) + \|z\|^2 - \gamma^2\|\omega\|^2 < 0. \tag{18}$$

When $\omega(t) = 0$, it yields that

$$\dot{V}(x,t) + \alpha V(x,t) < \|z\|^2 < 0. \tag{19}$$

Then, integrating inequalities (18) and (19), respectively, we can get inequalities (11) and (12) in Lemma 11. This completes the proof. □

When the delays in the nonswitched system (6) are constant, we assume that the following conditions hold:

$$\begin{aligned}
\left\|f_0\left(x(t),t\right)\right\| &\le \left\|\beta_0x(t)\right\|, \\
\left\|f_1\left(x\left(t-\tau_2\right)\right)\right\| &\le \left\|\beta_1x\left(t-\tau_2\right)\right\|, \\
\left\|f_2\left(x\left(t-\tau_3\right)\right)\right\| &\le \left\|\beta_2x\left(t-\tau_3\right)\right\|,
\end{aligned} \tag{20}$$

where β_0, β_1, and β_2 are positive real constants.

Lemma 12. *For given constants $\alpha > 0$, $\gamma > 0$, $\beta_0 > 0$, $\beta_1 > 0$, $\beta_2 > 0$, $p > 0$, and $u(t) = Kx(t)$, such that*

$$\beta_0\|E\| + \beta_1\left\|E_1\right\| + \beta_2\left\|E_2\right\| < \frac{p}{2\|P\|}, \tag{21}$$

define

$$V(x,t) = x^T(t)Px(t), \tag{22}$$

where P is a positive symmetric definite matrix of the following inequality:

$$\begin{aligned}&(A+B_1K)^T P+P(A+B_1K)+\alpha P\\&\quad+P\left(A_1A_1^T+B_2KK^TB_2\right)P+\gamma^{-2}PBB^TP+C^TC\\&<-pI.\end{aligned}\tag{23}$$

Then, along the trajectory of system (6), when $\omega(t)=0$,

$$V(t)\le e^{(-\alpha)(t-t_0)}V(t_0)\tag{24}$$

and, for general $\omega(t)$,

$$V(t)\le e^{(-\alpha)(t-t_0)}V(t_0)-\int_{t_0}^{t}e^{(-\alpha)(t-s)}\Gamma(s)\,ds,\tag{25}$$

where $\Gamma(t)=\|z(t)\|^2-\gamma^2\|\omega(t)\|^2$.

Proof. The proof is similar to that of Lemma 11.

For $V(x,t)=x^T(t)Px(t)$ and $u(t)=Kx(t)$,

$$\begin{aligned}&\dot V(x,t)+\alpha V(x,t)+\|z\|^2-\gamma^2\|\omega\|^2=x^T(t)\\&\quad\cdot\left((A+B_1K)^TP+P(A+B_1K)+\alpha P+C^TC\right)\\&\quad\cdot x(t)+2x^T(t)PA_1x(t-\tau_1)+2x^T(t)\\&\quad\cdot PEf_0(x(t),t)+2x^T(t)PE_1f_1(x(t-\tau_2))\\&\quad+2x^T(t)PE_2f_2(x(t-\tau_3))+2x^T(t)\\&\quad\cdot PB_2K(x(t-\tau_4))+2x^T(t)PB\omega(t)-\gamma^2\|\omega\|^2.\end{aligned}\tag{26}$$

Using a similar step of the proof in Lemma 11, we can obtain the following equality:

$$\begin{aligned}&\dot V(x,t)+\alpha V(x,t)+\|z\|^2-\gamma^2\|\omega\|^2\le x^T(t)\\&\quad\cdot\left[(A+B_1K)^TP\right.\\&\quad+P(A+B_1K)+\alpha P+C^TC\\&\quad\left.+P\left(A_1A_1^T+B_2KK^TB_2^T\right)P+\gamma^{-2}PBB^TP\right]x(t)\\&\quad+2\beta_0\|P\|\|E\|\|x(t)\|^2+2\beta_1\|P\|\|E_1\|\|x(t)\|\\&\quad\cdot\|\|x(t-\tau_2)\|\\&\quad+2\beta_2\|P\|\|E_2\|\|x(t)\|\|\|x(t-\tau_3)\|+\|x(t-\tau_4)\|^2\\&\quad+\|x(t-\tau_1)\|^2.\end{aligned}\tag{27}$$

If we assume

$$\|x(t-\tau_i)\|\le q_i\|x(t)\|,\quad q_i>1,\ i=1,2,3,4,\tag{28}$$

inequality (27) becomes

$$\begin{aligned}&\dot V(x,t)+\alpha V(x,t)+\|z\|^2-\gamma^2\|\omega\|^2\le x^T(t)\\&\quad\cdot\left[(A+B_1K)^TP\right.\\&\quad+P(A+B_1K)+\alpha P+C^TC\\&\quad\left.+P\left(A_1A_1^T+B_2KK^TB_2^T\right)P+\gamma^{-2}BB^TP\right]x(t)\\&\quad+\left(2\beta_0\|P\|\|E\|+2\beta_1\|P\|\|E_1\|q_2\right.\\&\quad\left.+2\beta_2\|P\|\|E_2\|q_3+q_1^2+q_4^2\right)\|x(t)\|^2.\end{aligned}\tag{29}$$

Considering that $q_i>1$, it holds that

$$\begin{aligned}&-2\left(\beta_0\|P\|\|E\|+\beta_1\|P\|\|E_1\|\mid q_2+\beta_2\|P\|\|E_2\|q_3\right)\\&\quad-q_1^2-q_4^2<-2\left(\beta_0\|P\|\|E\|+\beta_1\|P\|\|E_1\|\right.\\&\quad\left.+\beta_2\|P\|\|E_2\|\right)-q_4^2-q_1^2<-2\left(\beta_0\|P\|\|E\|\right.\\&\quad\left.+\beta_1\|P\|\|E_1\|+\beta_2\|P\|\|E_2\|\right)=-2\|P\|\left(\beta_0\|E\|\right.\\&\quad\left.+\beta_1\|E_1\|+\beta_2\|E_2\|\right)<-p.\end{aligned}\tag{30}$$

Hence from inequalities (29) and (30), we have

$$\dot V(x,t)+\alpha V(x,t)+\|z\|^2-\gamma^2\|\omega\|^2<0.\tag{31}$$

Then, when $\omega(t)=0$,

$$\dot V(x,t)+\alpha V(x,t)<0.\tag{32}$$

Similar to Lemma 11, we can get (24) and (25). □

Remark 13. Assumption (28), that is, $\|x(t-\tau_i)\|\le q_i\|x(t)\|$, $q_i>1$, $i=1,2,3,4$, in the above proof is commonly used in papers (see [42]).

Now, we will design a controller $u(t)$ and a switching law $\sigma(t)$, under which switched system (1) has exponential H_∞ performance γ. Firstly, for system (1) with time-varying delays, the following theorem is given.

Theorem 14. *For a given matrix $R_i>0$ and constants $\alpha>0$, $\gamma>0$, and $\varepsilon_{ij}>0$, $j=1,2,3,4,5$, $\nu>0$, if there exist matrices $P_i>0$, $i\in M$, and a linear state-feedback controller $u_i(t)=K_ix(t)$, $K_i=-(1/2\nu)R_i^{-1}B_{1i}^TP_i$, such that*

$$\begin{aligned}&A_i^TP_i+P_iA_i+\alpha P_i-\frac{1}{\nu}P_iB_{1i}R_i^{-1}B_{1i}^TP_i+C_i^TC_i+\widetilde{Q}_i\\&\quad+P_iN_iP_i<0\end{aligned}\tag{33}$$

has a positive definite solution P_i, where

$$\begin{aligned}\widetilde{Q}_i&=\sum_{j=1}^{4}Q_{ji}+\varepsilon_{i2}^{-2}F_i^TF_i,\\N_i&=\varepsilon_{i1}^2A_{1i}A_{1i}^T+\varepsilon_{i2}^2E_iE_i^T+\varepsilon_{i3}^2E_{1i}E_{1i}^T+\varepsilon_{i4}^2E_{2i}E_{2i}^T\\&\quad+\varepsilon_{i5}^2B_{2i}B_{2i}^T+\gamma^{-2}B_iB_i^T,\end{aligned}\tag{34}$$

then system (1) is exponentially stable and has weighted L_2-gain γ for any switching signal with average dwell time satisfying $\tau_a > \tau_a^* = (\ln\mu)/\alpha$, where $\mu \geq 1$ satisfies $P_i \leq \mu P_j$, $Q_{1i} \leq \mu Q_{1j}$, $Q_{2i} \leq \mu Q_{2j}$, $Q_{3i} \leq \mu Q_{3j}$, $Q_{4i} \leq \mu Q_{4j}$, $\forall i, j \in M$, where

$$\begin{aligned} Q_{1i} &= \frac{e^{\alpha d_1}}{\varepsilon_{i1}^2 (1-\alpha_1)} I, \\ Q_{2i} &= \frac{e^{\alpha d_2}}{\varepsilon_{i3}^2 (1-\alpha_2)} F_{1i}^T F_{1i}, \\ Q_{3i} &= \frac{e^{\alpha d_3}}{\varepsilon_{i4}^2 (1-\alpha_3)} F_{2i}^T F_{2i}, \\ Q_{4i} &= \frac{e^{\alpha d_4}}{\varepsilon_{i5}^2 (1-\alpha_4)} K_i^T K_i. \end{aligned} \tag{35}$$

Proof. Define a set of Lyapunov-Razumikhin function candidates as follows:

$$\begin{aligned} V(x(t)) &= V_\sigma(t)(x(t)) \\ &= x^T(t) P_\sigma(t) x(t) \\ &\quad + \sum_{i=1}^{4} \int_{t-\tau_i(t)}^{t} e^{\alpha(s-t)} x^T(s) Q_{i\sigma(t)} x^T(s)\, ds. \end{aligned} \tag{36}$$

Then when $t \in [t_k, t_{k+1})$ and $\omega(t) = 0$, it follows from Lemma 11 that

$$V_{\sigma(t_k)}(x(t)) \leq e^{(-\alpha)(t-t_k)} V_{\sigma(t_k)}(x_{t_k}), \tag{37}$$

and from (36) the following inequality holds at switching instant t_i:

$$V_{\sigma(t_i)}(x(t_i)) \leq \mu V_{\sigma(t_i^-)}(x(t_i^-)). \tag{38}$$

Letting $k = N_\sigma(t_0, t) \leq (t - t_0)/\tau_a$, it follows from inequalities (37) and (38) that

$$\begin{aligned} &V_{\sigma(t_k)}(x(t))(x(t)) \\ &\quad \leq e^{(-\alpha)(t-t_k)} \mu V_{\sigma(t_k^-)}(x(t_k^-)) \\ &\quad \leq \cdots e^{(-\alpha)(t-t_0)} \mu^k V_{\sigma(t_0)}(x(t_0)) \\ &\quad \leq e^{-(\alpha-(\ln\mu)/\tau_a)}(t-t_0) V_{\sigma(t_0)}(x(t_0)). \end{aligned} \tag{39}$$

Let

$$\begin{aligned} a &= \min_{\forall i \in M} \lambda_{\min}(P_i), \\ b &= \max_{\forall i \in M} \lambda_{\max}\Bigg[P_i + \frac{d_1}{\varepsilon_{i1}^2(1-\alpha_1)} I \\ &\quad + \frac{d_2}{\varepsilon_{i3}^2(1-\alpha_2)}(F_{1i}^T F_{1i}) + \frac{d_3}{\varepsilon_{i4}^2(1-\alpha_3)}(F_{2i}^T F_{2i}) \\ &\quad + \frac{d_4}{\varepsilon_{i5}^2(1-\alpha_4)} K_i^T K_i\Bigg]. \end{aligned} \tag{40}$$

Then

$$\begin{aligned} a\|x(t)\|^2 &\leq V(x(t)), \\ V_{\sigma(t_0)}(x(t_0)) &\leq b\|x(t_0)\|^2. \end{aligned} \tag{41}$$

Combining inequalities (39) and (41), we obtain

$$\begin{aligned} \|x(t)\|^2 &\leq \frac{V(x(t))}{a} \\ &\leq \frac{a}{b} e^{-(\alpha-(\ln\mu)/\tau_a)}(t-t_0)\|x(t_0)\|^2 \end{aligned} \tag{42}$$

which leads to the exponential stability of the switched system (1) with $\omega(t) = 0$.

Now we will show the weighted L_2-gain of system (1). Firstly for any $t \in [t_k, t_{k+1})$, using Lemma 10, we have

$$V(x_t) \leq e^{(-\alpha)(t-t_k)} V(t_k) - \int_{t_k}^{t} e^{(-\alpha)(t-s)} \Gamma(s)\, ds. \tag{43}$$

Then considering both inequalities (38) and (43), it holds that

$$\begin{aligned} V(x_t) &\leq \mu V(x(t_k)) e^{(-\alpha)(t-t_k)} V(t_k) \\ &\quad - \int_{t_k}^{t} e^{(-\alpha)(t-s)} \Gamma(s)\, ds \\ &\leq \mu^k V(x(t_0)) e^{-\alpha t} - \mu^k \int_0^{t_1} e^{(-\alpha)(t-s)} \Gamma(s)\, ds \\ &\quad - \mu^{k-1} \int_{t_1}^{t_2} e^{(-\alpha)(t-s)} \Gamma(s)\, ds - \cdots \\ &\quad - \int_{t_k}^{t} e^{(-\alpha)(t-s)} \Gamma(s)\, ds \\ &= e^{-\alpha t + N_\sigma(0,t)\ln\mu} V(x_0) \\ &\quad - \int_0^t e^{(-\alpha)(t-s) + N_\sigma(s,t)\ln\mu} \Gamma(s)\, ds. \end{aligned} \tag{44}$$

This gives

$$-\int_0^t e^{(-\alpha)(t-s) + N_\sigma(s,t)\ln\mu} \Gamma(s)\, ds \geq 0. \tag{45}$$

Multiplying both sides in (45) by $e^{-N_\sigma(0,t)\ln\mu}$, it yields

$$\begin{aligned} &\int_0^t e^{(-\alpha)(t-s) - N_\sigma(0,s)\ln\mu} z^T(s) z(s)\, ds \\ &\quad \leq \int_0^t e^{(-\alpha)(t-s) - N_\sigma(0,s)\ln\mu} \gamma^2 \omega^T(s) \omega(s)\, ds. \end{aligned} \tag{46}$$

Observing that $N_\sigma(0,s) \le s/\tau_a$ and $\tau_a > \tau_a^* = (\ln\mu)/\alpha$, we have $N_\sigma(0,s)\ln\mu \le \alpha s$; consequently

$$\begin{aligned}&\int_0^t e^{(-\alpha)(t-s)-\alpha s} z^T(s)\, z(s)\, ds\\ &\quad\le \int_0^t e^{(-\alpha)(t-s)} \gamma^2 \omega^T(s)\, \omega(s)\, ds.\end{aligned} \tag{47}$$

Integrating both sides of inequality (47) from $t = 0$ to ∞, we obtain

$$\int_0^\infty e^{-\alpha s} z^T(s)\, z(s)\, ds \le \int_0^\infty \gamma^2 \omega^T(s)\, \omega(s)\, ds. \tag{48}$$

This ends the proof. □

Remark 15. Inequality (33) in Theorem 14 is equivalent to LMIs as follows:

$$\begin{pmatrix} A_i^T P_i + P_i A_i + \alpha P_i + C_i^T C_i + \widetilde{Q}_i - \dfrac{\alpha d_4}{\varepsilon_{i5}^2 (1-\alpha_4)} K_i^T K_i & P\widetilde{N}_i & PB_{1i} \\ * & -\widetilde{N}_i & 0 \\ * & 0 & -\dfrac{1}{\nu} R_i \end{pmatrix} < 0, \tag{49}$$

where $\widetilde{N}_i = N_i + (\alpha d_4/4\nu^2\varepsilon_{i5}^2(1-\alpha_4))B_{1i}^T(R_i^{-1})^T R_i^{-1} B_{1i}$. So it can be calculated easily.

When the delays in system (1) are constant, we can get the following corollary.

Corollary 16. *For given constants $\alpha > 0$, $\gamma > 0$, $\beta_0 > 0$, $\beta_1 > 0$, $\beta_2 > 0$, and $p > 0$, $i \in M$, if there exists a linear state feedback controller $u_i(t) = K_i x(t)$ such that*

$$\beta_0 \|E_i\| + \beta_1 \|E_{1i}\| + \beta_2 \|E_{2i}\| < \frac{p}{2\|P_i\|}, \tag{50}$$

where P_i is a positive symmetric definite matrix of the following inequality:

$$\begin{aligned}&(A_i + B_{1i}K_i)^T P_i + P_i (A_i + B_{1i}K_i) + \alpha P_i\\ &\quad + P_i \left(A_{1i}A_{1i}^T + B_{2i}K_iK_i^T B_{2i}\right) P_i + \gamma^{-2} P_i B_i B_i^T P_i\\ &\quad + C_i^T C_i < -pI,\end{aligned} \tag{51}$$

then switched system (1) with constant delays is exponentially stable and has weighted L_2-gain γ for any switching signal with average dwell time satisfying $\tau_a > \tau_a^* = (\ln\mu)/\alpha$, where $\mu \ge 1$ satisfies $P_i \le \mu P_j$, $\forall i, j \in M$.

Proof. Define the Lyapunov-Razumikhin function candidate in Theorem 14 as follows:

$$V(x(t)) = V_\sigma(t)(x(t)) = x^T(t) P_\sigma(t) x(t); \tag{52}$$

the proof is similar to that of Theorem 14 by Lemma 12. □

Remark 17. Inequality (51) is easy to be solved for it can be rewritten as a LMI:

$$\begin{pmatrix} G & P_iA_{1i} & P_iB_{2i}K_i & \gamma^{-2}P_iB_i \\ * & -I & 0 & 0 \\ * & * & -I & 0 \\ * & * & * & -I \end{pmatrix} < 0, \tag{53}$$

where $G = (A_i + B_{1i}K_i)^T P_i + P_i(A_i + B_{1i}K_i) + \alpha P_i + C_i^T C_i + pI$.

Remark 18. In [40], conditions are given to guarantee that the nonswitched system with input delays is asymptotically stable. Compared with [40], we get the conditions when the switched system is exponentially stable.

4. Examples

To illustrate the main results, we consider the following examples.

Example 1. Consider the time-varying delay switched system

$$\begin{aligned}\dot{x}(t) &= A_{\sigma(t)}x(t) + A_{1\sigma(t)}x(t-\tau_1(t))\\ &\quad + E_{\sigma(t)} f_{0\sigma(t)}(x(t), t)\\ &\quad + E_{1\sigma(t)} f_{1\sigma(t)}(x(t-\tau_2(t)), t)\\ &\quad + E_{2\sigma(t)} f_{2\sigma(t)}(x(t-\tau_3(t)), t) + B_{\sigma(t)}\omega(t)\\ &\quad + B_{1\sigma(t)}u_{\sigma(t)}(t) + B_{2\sigma(t)}u_{\sigma(t)}(t-\tau_4(t)),\\ z(t) &= C_{\sigma(t)}x(t),\\ x(t) &= 0, \quad t \le 0,\end{aligned} \tag{54}$$

where $i = 1, 2$ and $\tau_1(t) = 0.2\sin t + 0.3$, $\tau_2(t) = 0.3\cos t + 0.4$, $\tau_3(t) = 0.4\sin t + 0.5$, $\tau_4(t) = 0.5\cos t + 0.7$; thus $0 \le \tau_1(t) \le 0.5$, $0 \le \tau_2(t) \le 0.7$, $0 \le \tau_3(t) \le 0.9$, $0 \le \tau_4(t) \le 1.2$, and, $\dot{\tau}_1(t) \le 0.2 < 1$, $\dot{\tau}_2(t) \le 0.3 < 1$, $\dot{\tau}_3(t) \le 0.4 < 1$, $\dot{\tau}_4(t) \le 0.5 < 1$. So we have $\alpha_1 = 0.2$, $\alpha_2 = 0.3$, $\alpha_3 = 0.4$, $\alpha_4 = 0.5$, $d_1 = 0.5$, $d_2 = 0.7$, $d_3 = 0.9$, $d_4 = 1.2$. And $A_1 = \begin{pmatrix} -4 & 1 \\ 2 & -4 \end{pmatrix}$, $A_{11} = \begin{pmatrix} 1 & 0 \\ -1 & 3 \end{pmatrix}$, $E_1 = \begin{pmatrix} 1 & 0 \\ 2 & -3 \end{pmatrix}$, $E_{11} = E_{12} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $E_{21} = \begin{pmatrix} 2 & -1 \\ 0 & 2 \end{pmatrix}$, $B_1 = \begin{pmatrix} 1 & 0 \\ 3 & 4 \end{pmatrix}$, $B_{11} = \begin{pmatrix} 1 & -1 \\ 0 & 3 \end{pmatrix}$, $B_{21} = \begin{pmatrix} 2 & -1 \\ -2 & 3 \end{pmatrix}$, $C_1 = \begin{pmatrix} 2 & 4 \end{pmatrix}$, $R_1 = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$, $A_2 = \begin{pmatrix} -3 & 1 \\ 1 & -2 \end{pmatrix}$, $A_{12} = \begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix}$, $E_{22} = \begin{pmatrix} 2 & -1 \\ 0 & 2 \end{pmatrix}$, $B_2 = \begin{pmatrix} 1 & 0 \\ 4 & 3 \end{pmatrix}$, $B_{12} = \begin{pmatrix} 2 & -1 \\ -1 & 3 \end{pmatrix}$, $B_{22} = \begin{pmatrix} 3 & -2 \\ -1 & 3 \end{pmatrix}$, $C_2 = \begin{pmatrix} 2 & 3 \end{pmatrix}$, $R_2 = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$, $f_{0i} = \begin{pmatrix} a(t)\sin x_1(t) \\ 0 \end{pmatrix}$, $f_{1i} = \begin{pmatrix} 0 \\ b(t)\sin x_2(t-\tau_2) \end{pmatrix}$, $f_{2i} = \begin{pmatrix} c(t)\cos x_1(t-\tau_2) \\ 0 \end{pmatrix}$, with $|a(t)| \le 1$, $|b(t)| \le 1$, $|c(t)| \le 1$. Then

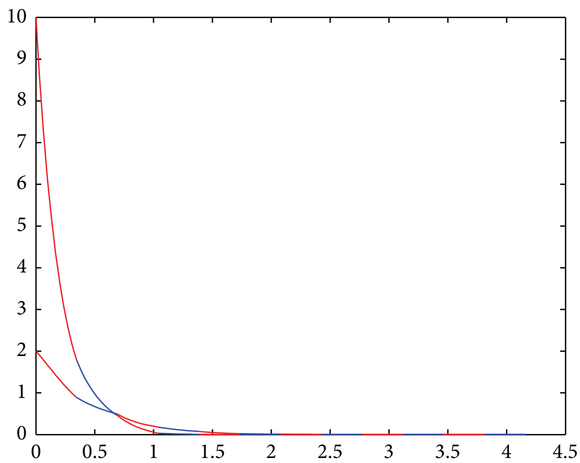

Figure 1: The state response of the closed-loop system (54) in Example 1.

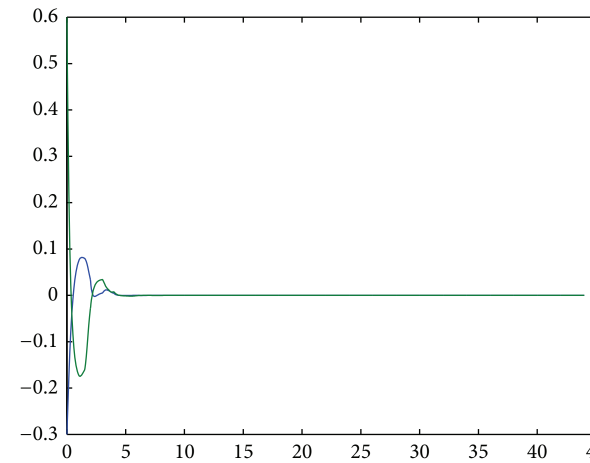

Figure 2: The state response of the closed-loop system (54) in Example 2.

$F_i = F_{1i} = F_{2i} = I_2$. Let $\varepsilon_{i1} = 1$, $\varepsilon_{i2} = \varepsilon_{i3} = \varepsilon_{i4} = \varepsilon_{i5} = 0.01$ and $\alpha = 2$, $\gamma = 1$; solving (33) using Matlab software, we can get $P_1 = (1.0e + 0.003)\begin{pmatrix} 5.6571 & 0.0619 \\ 0.0619 & 1.6519 \end{pmatrix}$ and $P_2 = (1.0e + 0.003)\begin{pmatrix} 4.6105 & 1.2150 \\ 1.2150 & 4.1704 \end{pmatrix}$; besides $Q_{11} = Q_{21} = \begin{pmatrix} 2.1746 & 0 \\ 0 & 2.1746 \end{pmatrix}$, $Q_{12} = Q_{22} = (1.0e + 0.004)\begin{pmatrix} 2.8386 & 0 \\ 0 & 2.8386 \end{pmatrix}$, $Q_{13} = Q_{23} = (1.0e+0.004)\begin{pmatrix} 3.6298 & 0 \\ 0 & 3.6298 \end{pmatrix}$, $Q_{14} = (1.0e+0.004)\begin{pmatrix} 0.15 & -0.15 \\ -0.15 & 1.5 \end{pmatrix}$, $Q_{24} = (1.0e + 0.003)\begin{pmatrix} 3.3333 & -3.3333 \\ -3.3333 & 6.6667 \end{pmatrix}$. And we can get $\mu = 20$ satisfying the condition of Theorem 14.

Therefore according to Theorem 14, system (54) is exponentially stable and has L_2-gain γ for any switching signal with average dwell time satisfying $\tau_a > \tau_a^* = (\ln \mu)/\alpha = (\ln 20)/2$. The simulation result is depicted in Figure 1.

Example 2. Consider the constant time-delay switched system with $A_1 = \begin{pmatrix} -4 & 0 \\ 0 & -4 \end{pmatrix}$, $A_{11} = \begin{pmatrix} 0 & 0.5 \\ 0.5 & 0 \end{pmatrix}$, $B_{11} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $B_{21} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$, $C_1 = (2\ \ 3)$, $B_1 = \begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}$, $A_2 = \begin{pmatrix} -4 & 1 \\ 0 & -4 \end{pmatrix}$, $A_{12} = \begin{pmatrix} -2 & 1 \\ 3 & 2 \end{pmatrix}$, $B_{12} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $B_{22} = \begin{pmatrix} -2 & 0 \\ 1 & 0 \end{pmatrix}$, $C_2 = (2\ \ 3)$, $B_2 = \begin{pmatrix} 1 & -3 \\ -1 & -2 \end{pmatrix}$, $E_1 = E_2 = E_{11} = E_{12} = E_{21} = E_{22} = I$, $\tau_1 = 1$, $\tau_2 = 1.5$, $\tau_3 = 2$, $\tau_4 = 0.5$, $f_{0i} = \begin{pmatrix} a(t)\sin x_1(t) \\ 0 \end{pmatrix}$, $f_{1i} = \begin{pmatrix} 0 \\ b(t)\sin x_2(t-\tau_2) \end{pmatrix}$, $f_{2i} = \begin{pmatrix} c(t)\sin x_1(t-\tau_3) \\ 0 \end{pmatrix}$, with $|a(t)| \le 0.01$, $|b(t)| \le 0.1$, $|c(t)| \le 0.08$. Then let $\beta_0 = 0.01$, $\beta_1 = 0.1$, and $\beta_2 = 0.08$, and let $p = 1$, $\alpha = 2$, $\gamma = 1$, $K_1 = \begin{pmatrix} -1 & -1 \\ 0 & 0 \end{pmatrix}$, $K_2 = \begin{pmatrix} -1 & 2 \\ 0 & 0 \end{pmatrix}$; solving (51) by the Matlab software, we can get $P_1 = \begin{pmatrix} 0.6896 & 0.8038 \\ 0.8038 & 1.9159 \end{pmatrix}$ and $P_2 = \begin{pmatrix} 0.2115 & 0.3434 \\ 0.3434 & 1.9159 \end{pmatrix}$ and $\|P_1\| = 2.3137$, $\|P_2\| = 1.1453$.

It is obvious that $\beta_0\|E_i\| + \beta_1\|E_{1i}\| + \beta_2\|E_{2i}\| < p/2\|P_i\|$, $i = 1, 2$, and we can get $\mu = 4$ satisfying $P_2 < \mu P_1$, $P_1 < \mu P_2$. Therefore according to Corollary 16, this system is exponentially stable and has L_2-gain γ for any switching signal with average dwell time satisfying $\tau_a > \tau_a^* = (\ln \mu)/\alpha$. The simulation result is depicted in Figure 2.

Remark 19. In the figures, horizontal axis stands for time and vertical axis stands for the states $x_1(t)$ and $x_2(t)$. The red and blue lines denote the state response of $x_1(t)$ and $x_2(t)$, respectively. We can see from the figures that the two systems are both exponentially stable under given controllers and switching law and this shows that the method proposed in this paper is feasible.

Remark 20. It is easy to verify that both subsystems are unstable. And using the method given in [40], controllers are only given to make the systems asymptotically stable. However, compared with [40], in this paper, we can see that, under given controllers, the switched systems in the examples are exponentially stable from the simulation results.

5. Summary

In this paper, we considered the problem of robust H_∞ control for a class of uncertain nonlinear switched systems with state and input delays. The controller and switching law are designed and sufficient conditions have been presented for uncertain nonlinear switched delay systems under which the system is robust stable and has H_∞ performance γ. Firstly, we considered the system with time-varying delays; then, as a special case, system with constant delays was also considered. The feasibility of the developed results has been proved using numerical examples.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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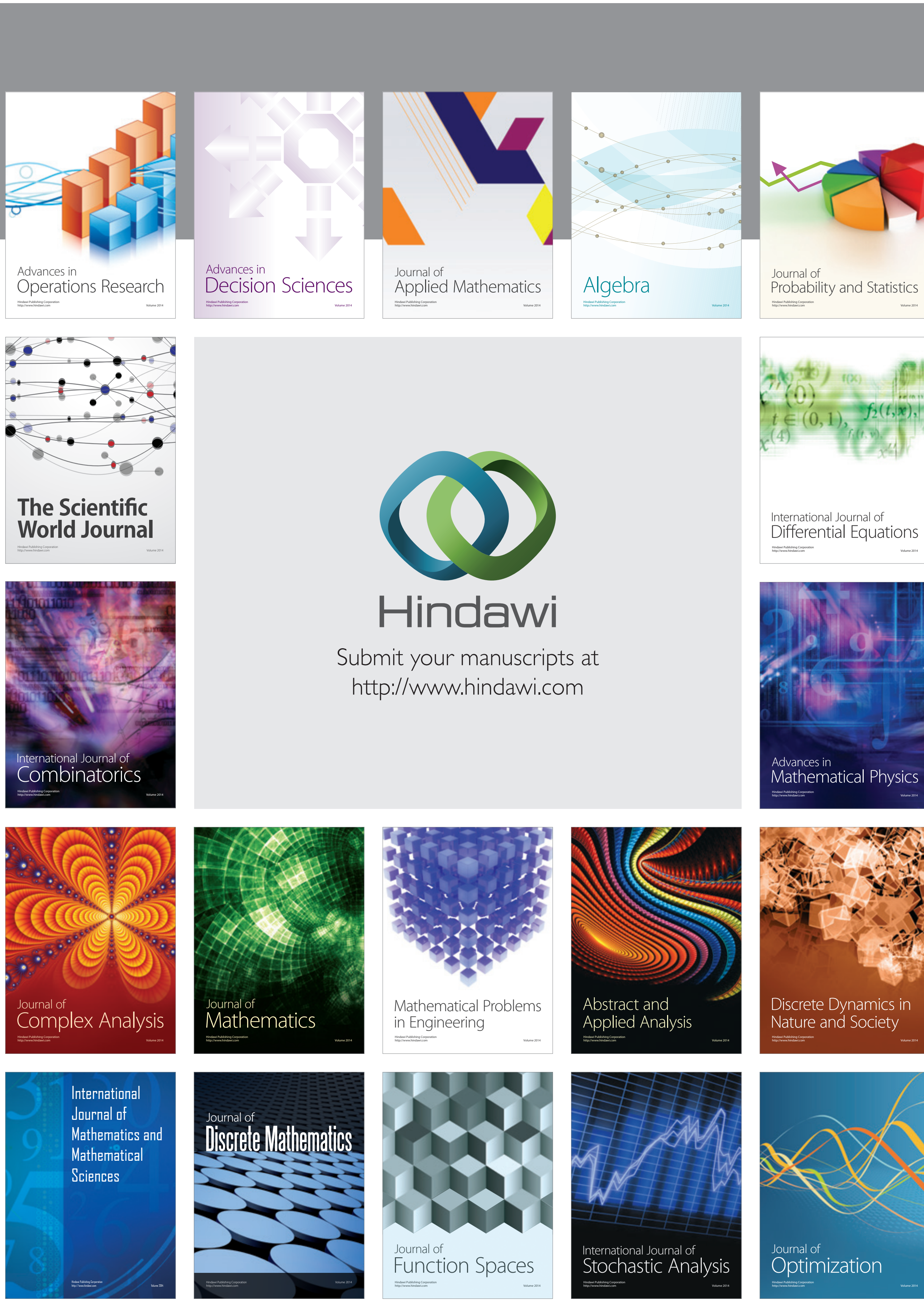
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