Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2013, Article ID 836314, 9 pages
http://dx.doi.org/10.1155/2013/836314

Research Article

H_∞ Control of Network-Based Systems with Packet Disordering and Packet Loss Compensation

Jinna Li,[1,2] Lifeng Wei,[3] Chao Liu,[4] and Haibin Yu[2]

[1] *Lab of Operation and Control, Shenyang, University of Chemical Technology, Liaoning 110142, China*
[2] *Lab of Industrial Control Networks and Systems, Shenyang Institute of Automation, Chinese Academy of Sciences, Liaoning 110016, China*
[3] *College of Information Engineering, Shenyang University of Chemical Technology, Liaoning 110142, China*
[4] *Institute of Systems Science, Northeastern University, Shenyang, Liaoning 110004, China*

Correspondence should be addressed to Jinna Li; lijinna_721@126.com

Received 3 April 2013; Accepted 14 May 2013

Academic Editor: Guanghui Wen

Compensation scheme-based H_∞ control is investigated for networked control systems with packet disordering and packet loss. Since the existence of packet disordering and packet loss inevitably degrades the control performance of networked control systems, it is worth studying a control scheme to compensate for them, such that the control performance can be improved. Thus, a compensation control strategy is first proposed following this direction. Next, a mathematical model of networked control systems with Markovian property is constructed due to the signals executed by the plant subject to Markovian chain. Based on it, a sufficient condition for stochastic stability of networked control systems with uncertain parameters as well as compensation strategy is presented, and an adaptive controller is designed based on linear matrix inequality (LMI) technique. Finally, a numerical example and simulations are given to illustrate the effectiveness of the proposed method.

1. Introduction

Networked control systems (NCSs), in which nodes communicate over communication networks, have attracted lots of researchers' attention [1–4]. Since networks-based control gives rise to many advantages including low cost, easy maintenance, and flexible system structure, the successful application of NCSs can be found in a wide range of areas such as industrial automation, intelligent transportation system, and smart grid. However, packet disordering and packet dropout inevitably exist in the transmission of signals. They are recognized to be two main causes for performance deterioration or even instability of NCSs, hence, considerable research has been done (see, e.g. [5–21] and the references therein).

So far, the majority of NCSs research has focused on controller design to provide sufficient stability conditions for NCSs with packet loss. A lot of effort has also been taken for modeling NCSs in presence of packet losses as asynchronous dynamic systems or Markovian jumping systems [5, 6]. With further study on packet loss, some effective compensation strategies for packet loss that occurred during communication are proposed to improve control performance of NCSs. Predictive control is a typical method with wich the control prediction generator provides a set of future control predictions to enable the closed-loop system to achieve the desired control performance leading to removing the effects of data dropout [7–11]. Another typical compensation methodology for packet loss is observer-based state estimation [12]. In addition, [13] proposed a packet dropout-based compensation scheme, namely, the latest control signal is used for compensation if the ideal control input is missing. However, note that, in all of the aforementioned literature, packet disordering is not considered, but packet disordering and packet loss coexist in packets delivered network communication.

Packet disordering means that a packet sent earlier may arrive at the destination node later or vice versa. Packet disordering of NCSs has drawn an increasing attention [10, 14–21]. In [14], packets that arrived late at control nodes were discarded, and stability and H_∞ compensation control were investigated. However, the packet disordering is not described clearly. In [15, 16], the sampling instants of received signals were compared to describe packet disordering, and stability analysis and synthesis were studied. Some literature using the similar method can be found in the existing reported results (e.g., [17]). Recently, [10, 18] proposed an active compensation for packet disordering; that is, the latest control actions applied to the plant are available by comparing the time stamps of packets. The so called compensation method has been also presented in [19–21], where the latest signals are executed by the plant by defining an operator constructing a mapping between the newest signals and packet displacement values. However, note that a situation where no new control actions arrive at the actuator may occur due to packet disordering and packet loss during a sampling interval. In this case, it is critical how to control the plant. To the best of the authors' knowledge, this problem has been not fully investigated to date, which motivates this work for proposing a new compensation scheme.

The specific problem addressed in this paper is the compensation control when the newest signal is not available for NCSs due to the packet disordering and packet loss. The highlighted method is that control inputs are determined by defining some operators associated with packet displacements; that is, the latest control input is chosen when no new signals arrive at the actuator during the sampling interval $((k-1)T, kT]$; otherwise, the newest signal controls the plant. After that, a Markovian jumping model of NCSs is put forward. Stability analysis and the controller synthesis are thoroughly investigated and the adaptive controller design is obtained in terms of linear matrix inequalities (LMI). A numerical example is provided to demonstrate the effectiveness of the proposed approach.

The rest of the paper is organized as follows. Section 2 is concerned with problem statement. In this section, a compensation control scheme is proposed, and a model of networked control systems is constructed. Section 3 investigates the stability and H_∞ control for NCSs with packet disordering and packet losses. The results of numerical simulation are presented in Section 4. Conclusions are stated in Section 5.

Notation. R^n denotes the n dimensional Euclidean space. $\mathbf{P} > \mathbf{0}$ means that matrix $\mathbf{P}$ is real symmetric and positive definite, and $\mathbf{I}$ is the identity matrix of appropriate dimensions. The subscript "T" denotes the matrix transpose. In symmetric block matrices, we use "$*$" to represent a term that is induced by symmetry, and $\mathrm{diag}(\cdots)$ stands for a block-diagonal matrix. $\|\mathbf{x}\|$ stands for the standard l_2 norm of vector $\mathbf{x}$; that is, $\|\mathbf{x}\| = (\mathbf{x}^{\mathrm{T}}\mathbf{x})^{1/2}$.

2. Problem Statement

The following system is considered:

$$\begin{aligned} \mathbf{x}_{k+1} &= \overline{\mathbf{A}}\mathbf{x}_k + \overline{\mathbf{B}}\mathbf{u}_k + \mathbf{D}_1\mathbf{w}_k \\ \mathbf{z}_k &= \mathbf{C}\mathbf{x}_k + \mathbf{D}_2\mathbf{w}_k, \end{aligned} \tag{1}$$

where $\mathbf{x}_k = \mathbf{x}(kT) \in \Re^n$, $\mathbf{u}_k = \mathbf{u}(kT) \in \Re^M$, and $\mathbf{z}_k = \mathbf{z}(kT) \in \Re^P$ are the state vector, control input vector, and controlled output vector, respectively. T is the sampling period. $\overline{\mathbf{A}} = \mathbf{A} + \Delta\mathbf{A}$, $\overline{\mathbf{B}} = \mathbf{B} + \Delta\mathbf{B}$, $(\Delta\mathbf{A}\ \ \Delta\mathbf{B}) = \mathbf{D}\mathbf{F}(k)(\mathbf{E}_1\ \ \mathbf{E}_2)$, $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{D}_1, \mathbf{D}_2, \mathbf{E}_1$, and $\mathbf{E}_2$ are some constant matrices of appropriate dimensions, and $\mathbf{F}^T(k)\mathbf{F}(k) \le \mathbf{I}$. $\mathbf{w}_k = \mathbf{w}(kT) \in \iota_2[0, \infty]$ denotes the exogenous disturbance signal.

The state feedback controller can be expressed as

$$\mathbf{u}_k = \mathbf{K}\mathbf{x}_k, \tag{2}$$

where $\mathbf{K}$ is some constant matrix of appropriate dimensions.

Here, we assume that sensor and actuator are time-driven synchronously, the period is identical and equal to T, and the controller is event-driven. Since the states of systems and control signals are transmitted over the communication networks with limited bandwidth, the packet disordering and intermittent packet dropouts are inevitable in the communication channels. To describe the phenomenon of packet disordering and design compensation scheme, we define the displacement values of packets and some operators determining the control actions. The details are as follows.

Without loss of generalization, we consider a sequence of packets $\mathbf{x}_{k-h}, \mathbf{x}_{k-h+1}, \ldots, \mathbf{x}_k$ transmitted over the network from the sensor, where h is a given integer. The maximum delay bound is an alternative solution to h. For $\mathbf{x}_{k-h}, \mathbf{x}_{k-h+1}, \ldots, \mathbf{x}_k$, it is well known that the corresponding expected arrival sequence numbers are $1, 2, \ldots, h+1$. Then, the expected arrival sequence number of packet $\mathbf{x}_{k-i}$ is $h + 1 - i$ $(i = 0, 1, \ldots, h)$ is easily obtained. A receive_index l $(l = 1, 2, \ldots, h+1)$ is assigned to each nonduplicate packet as it arrives at the point of measurement, which we refer to as the destination (actuator) since the control is event-driven. To describe the newest signal executed by the plant, we assume that packets which have not appeared or lost during the sampling interval $((k-1)T, kT]$ arrive at the actuator in order after the kT time instant, and their receive_index values are 1 more than the real values. Moreover, if the sampled packets behind p (p is some positive integer) lost packets arrive at the plant before the kT time instant (including kT time instant), their receive index values are p more than the real values. $R_k(i)$ and $d_k(h+1-i)$ denote the receive_index and displacement value of packet $\mathbf{x}_{k-i}$, respectively. For packet $\mathbf{x}_{k-i}$ arriving at the actuator before the kT instant (including kT instant), if $d_k(h+1-i) \neq 0$, then a "disordering event" has occurred in communication. Packet $\mathbf{x}_{k-i}$ is late if $d_k(h+1-i) > 0$, early if $d_k(h+1-i) < 0$, and in order if $d_k(h+1-i) = 0$ (see [19–21]). To guarantee the newest signals being executed by the plant, the packets that arrive at the actuator late are discarded. Define the following operators:

$$\delta\left(d_k(h+1-i)\right) = \begin{cases} 1 & d_k(h+1-i) \le 0 \\ 0 & d_k(h+1-i) > 0, \end{cases} \tag{3}$$

$$\theta_k(i) = \prod_{j=0}^{i-1}\left(1 - \delta\left(d_k(h+1-j)\right)\right)\delta\left(d_k(h+1-i)\right), \tag{4}$$

Table 1: An example in Figure 1.

	x_{k-2}	x_{k-1}	x_k
Exception values	1	2	3
Receive_index	2	3	4
Displacement values	1	1	1
Parameters		$\theta_k(-1) = 1$ $\theta_k(0) = 0$ $\theta_k(1) = 0$ $\theta_k(2) = 0$	

Table 2: An example in Figure 2.

	x_{k-1}	x_{k-1}	x_k
Exception values	1	2	3
Receive_index	1	3	4
Displacement values	0	1	1
Parameters		$\theta_k(-1) = 0$ $\theta_k(0) = 0$ $\theta_k(1) = 0$ $\theta_k(2) = 1$	

where $\prod_{j=0}^{-1}(1 - \delta(d_k(h + 1 - j))) = 1$ $(i = 0, 1, \ldots, h)$. The function of $\theta_k(i)$ $(i = 0, 1, \ldots, h)$ is to guarantee that the newest signal is executed if it has arrived at the actuator during the interval $((k-1)T, kT]$. Moreover, note that it may happen not to receive new signal due to late coming packet or packet loss; thus we design the following controller:

$$\mathbf{u}_k = \sum_{i=0}^{h} \theta_k(i)\,\mathbf{K}\mathbf{x}_{k-i} + \theta_k(-1)\,\mathbf{u}_{k-1}, \tag{5}$$

where

$$\theta_k(-1) = \begin{cases} 1 & d_k(h+1-i) > 0, \\ & \forall i\,(i = 0, 1, \ldots, h) \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

Remark 1. As a matter of fact, the operators $\theta_k(i)$ $(i = -1, 0, \ldots, h)$ are defined for the purpose of selecting control input. Note that $\theta_k(i) = 1$ or 0, and $\sum_{i=-1}^{h}\theta_k(i) = 1$. $\theta_k(i) = 1$ or $\theta_k(i) = 0$ $(i = 0, 1, \ldots, h)$, which is determined in terms of the displacement values of packets. More detailed explanations can be found in the examples in Figures 1 and 2, Tables 1 and 2. Since we choose event-triggered controller, once $\mathbf{x}_{k-i}$ arrives at the controller, control action $\mathbf{K}\mathbf{x}_{k-i}$ is sent to the actuator. When the displacement values of all packets $d_k(h + 1 - i) > 0$ $(i = 0, 1, \ldots, h)$; that is, there are no new signals arriving the actuator during sampling interval $((k-1)T, kT]$, $\theta_k(-1)$ is set to be 1 by (6) and other $\theta_k(i) = 0$ $(i = 0, 1, \ldots, h)$ hold due to (4), which indicates the control input $\mathbf{u}_{k-1}$ to act on the plant (see Figure 1 and Table 1). Otherwise, $\theta_k(-1) = 0$ and there exists one $\theta_k(i) = 1$ $(i = 0, 1, \ldots, h)$. In this context, the newest control signal $\mathbf{K}\mathbf{x}_{k-i}$ is regarded as the control action $\mathbf{u}_k$. It is worth pointing out that the packet disordering, random time-varying transmission delay, and packet loss are taken into account in the suggested compensation control scheme (5), simultaneously. From this point of view, it is readily seen that the proposed control scheme (5) is quite general.

For further understanding compensation control scheme (5), we study two examples shown in Figures 1 and 2 which illustrate the arriving timing of signals transmitted. Expected arrival sequence numbers, assigned receive_index values, and displacement values for a sequence transmitted,

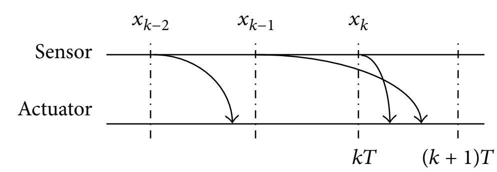


Figure 1: Timing diagram of signals transmitting in the example with packet disordering.

corresponding to Figures 1 and 2, are given in Tables 1 and 2, respectively. In the two examples, we choose $h = 2$, which means that a group of 3 packets is used as the studying object. From Table 1, we find that, packets $\mathbf{x}_{k-2}$, $\mathbf{x}_{k-1}$ and $\mathbf{x}_k$ are displaced by one unit from their positions, then all of displacement values are equal to 1. In this context, the actuator does not receive new signal during the sampling interval $((k-1)T, kT]$, which can be seen in Figure 1. By (3), (4), and (6), we obtain $\theta_k(-1) = 1$ and $\theta_k(0) = \theta_k(1) = \theta_k(2) = 0$. And by (5), the control input $\mathbf{u}_{k-1}$ acts on the plant, which is entirely consistent with the proposed compensation control scheme that the latest control action is utilized to control the plant when no new signal is available during the sampling interval $((k-1)T, kT]$. In Figure 2, note that packet $\mathbf{x}_{k-1}$ has lost. Similarly, by calculating, we obtain $\theta_k(2) = 1$ and $\theta_k(0) = \theta_k(1) = \theta_k(-1) = 0$. Then, $\mathbf{K}\mathbf{x}_{k-2}$ is used as the newest control input by virtue of (5). Obviously, this result accords with the actual situation shown in Figure 2.

Remark 2. Similar to [13], we execute the compensation control scheme. However, there are two distinct differences from [13]. The first one is that the stability analysis and compensation strategy are investigated in the presence of both packet loss and packet disordering simultaneously. In this sense, the theory method presented in this paper extends the results presented in [13], since packet disordering is not taken into account in [13]. The second difference is that an operator deciding how to choose control actions is clearly defined based on the displacement values of packets, while [13] control strategy is proposed in terms of delay information.

Remark 3. Compared with the existing studies on NCSs with packet disordering [10, 14–21], a key difference is

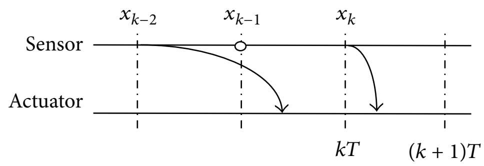


FIGURE 2: Timing diagram of signals transmitting in the example with packet loss.

that a compensation control scheme is proposed when no new signal arrives at the actuator.

The closed-loop system can be obtained by substituting (5) into (1):

$$\mathbf{x}_{k+1} = \overline{\mathbf{A}}x_k + \overline{\mathbf{B}}\left(\sum_{i=0}^{h}\theta_k(i)\,\mathbf{K}x_{k-i} + \theta_k(-1)\,\mathbf{u}_{k-1}\right) + \mathbf{D}_1\mathbf{w}_k$$
$$\mathbf{z}_k = \mathbf{C}\mathbf{x}_k + \mathbf{D}_2\mathbf{w}_k. \tag{7}$$

Letting $\xi_k^{\mathrm{T}} = [\mathbf{x}_k^{\mathrm{T}}\ \ \mathbf{x}_{k-1}^{\mathrm{T}}\cdots\mathbf{x}_{k-h}^{\mathrm{T}}]$ and $\eta_k^{\mathrm{T}} = [\xi_k^{\mathrm{T}}\ \ \mathbf{u}_{k-1}^{\mathrm{T}}]$, (7) is expressed as

$$\xi_{k+1} = \mathbf{A}_{11,k}\xi_k + \mathbf{A}_{12,k}\mathbf{u}_{k-1} + \widetilde{\mathbf{D}}_1\mathbf{w}_k$$
$$\mathbf{u}_k = \mathbf{A}_{21,k}\xi_k + \theta_k(-1)\,\mathbf{u}_{k-1}, \tag{8}$$

where

$$\mathbf{A}_{11,k} = \begin{bmatrix} \overline{\mathbf{A}}+\mathbf{M}_0 & \mathbf{M}_1 & \cdots & \mathbf{M}_{h-1} & \mathbf{M}_h \\ \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{I} & \mathbf{0} \end{bmatrix},$$
$$\mathbf{A}_{12,k} = \begin{bmatrix} \overline{\mathbf{B}}\theta_k(-1) \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix}, \qquad \widetilde{\mathbf{D}}_1 = \begin{bmatrix} \mathbf{D}_1 \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix}, \tag{9}$$
$$\mathbf{A}_{21,k} = [\theta_k(0)\,\mathbf{K}\ \ \theta_k(1)\,\mathbf{K}\ \ \cdots\ \ \theta_k(h)\,\mathbf{K}],$$
$$\mathbf{M}_i = \overline{\mathbf{B}}i_k(i)\,\mathbf{K} \quad (i = 0,1,\ldots,h).$$

Further, we have

$$\eta_{k+1} = \overline{\mathbf{A}}_k\eta_k + \overline{\mathbf{D}}_1\mathbf{w}_k$$
$$\mathbf{z}_k = \widehat{\mathbf{C}}\eta_k + \mathbf{D}_2\mathbf{w}_k, \tag{10}$$

where

$$\overline{\mathbf{A}}_k = \begin{bmatrix} \Gamma & \mathbf{R} \\ \Lambda(\theta_k(0),\ldots,\theta_k(h))\,\overline{\mathbf{K}} & \theta_k(-1)\,\mathbf{I} \end{bmatrix},$$
$$\mathbf{R} = \widehat{\mathbf{D}}\mathbf{F}(k)\,\mathbf{E}_2\theta_k(-1) + \widehat{\mathbf{B}}\theta_k(-1),$$
$$\Gamma = \widehat{\mathbf{A}} + \widehat{\mathbf{B}}\Lambda(\theta_k(0),\ldots,\theta_k(h))\,\overline{\mathbf{K}} + \widehat{\mathbf{D}}\mathbf{F}(k)\left(\widehat{\mathbf{E}}_1 + \mathbf{E}_2\Lambda(\theta_k(0),\ldots,\theta_k(h))\,\overline{\mathbf{K}}\right),$$
$$\Lambda(\theta_k(0),\ldots,\theta_k(h)) = [\theta_k(0)\,\mathbf{I}\ \ \cdots\ \ \theta_k(h)\,\mathbf{I}],$$
$$\overline{\mathbf{K}} = \operatorname{diag}\{\underbrace{\mathbf{K},\mathbf{K},\ldots,\mathbf{K}}_{(h+1)}\}, \qquad \widehat{\mathbf{C}} = [\mathbf{C}\ \ \mathbf{0}\ \ \cdots\ \ \mathbf{0}],$$
$$\widehat{\mathbf{A}} = \begin{bmatrix} \mathbf{A} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{I} & \mathbf{0} \end{bmatrix}, \qquad \widehat{\mathbf{B}} = \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix},$$
$$\widehat{\mathbf{E}}_1 = [\mathbf{E}\ \ \mathbf{0}\ \ \cdots\ \ \mathbf{0}],$$
$$\widehat{\mathbf{D}} = \begin{bmatrix} \mathbf{D} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix}, \qquad \overline{\mathbf{D}}_1 = \begin{bmatrix} \widetilde{\mathbf{D}}_1 \\ \mathbf{0} \end{bmatrix}. \tag{11}$$

It is well known that the newest signals executed by plant may be subject to some probability distribution [6]; here we will assume that the newest signals transmitted over communication network are subject to Markovian chain. Define $\mathbf{d}_k = [\theta_k(-1),\theta_k(0),\ldots,\theta_k(h)]^{\mathrm{T}}$; since $\theta_k(i) = 1$ or 0 $(i = -1,0,\ldots,h)$ and $\sum_{i=-1}^{h}\theta_k(i) = 1$, then, there are $h+2$ possible values for $\mathbf{d}_k$, obviously. Similar to our prior effort [21], for ease of notation, we define a vector-valued function $f : \mathbf{d}_k \rightarrow \sigma(k)$ to map the vector $\mathbf{d}_k$ into a scalar number $\sigma(k) \in \mathfrak{I} = \{1,2,\ldots,r\}$, where $r = h+2$. $\sigma(k) = i$ also denotes the No. i $(i \in \mathfrak{I})$ subsystem of NCS (10). Moreover, transition probability associated with the newest signals executed is defined as $\pi_{ij} = \operatorname{Prob}(\sigma(k+1) = j \mid \sigma(k) = i)$, where $\sigma(k) = i$ denotes $\mathbf{d}_k = [0,\ldots,1,\ldots,0]^{\mathrm{T}}$, namely, $\theta_k(i-2) = 1$. Obviously, $\sum_{j\in\mathfrak{I}}\pi_{ij} = 1$. Thus, (10) is a Markovian jumping system.

Remark 4. In this paper, the stability analysis and H_∞ controller design are investigated on the premise that transition probability matrix is fully known. Actually, the analysis and control methods for NCSs with uncertain transition probabilities have been developed, and we can refer to [22, 23].

3. Stability Analysis and H_∞ Controller Design

In this section, we will present a sufficient condition for H_∞ control and the design of the controller gains $\mathbf{K}_i$ $(i = 1,2,\ldots,r)$ adapting to No. i switched subsystem.

Lemma 5 (see [24]). *For any matrices* $\mathbf{W}$, $\mathbf{M}$, $\mathbf{N}$, $\mathbf{F}(k)$ *with* $\mathbf{F}^T(k)\mathbf{F}(k) < \mathbf{I}$, *and any scalar* $\varepsilon > 0$, *the following inequality holds:*

$$\mathbf{W} + \mathbf{MF}(k)\mathbf{N} + \mathbf{N}^{\mathrm{T}}\mathbf{F}^{\mathrm{T}}(k)\mathbf{M}^{\mathrm{T}} \leq \mathbf{W} + \varepsilon\mathbf{MM}^{\mathrm{T}} + \varepsilon^{-1}\mathbf{N}^{\mathrm{T}}\mathbf{N}. \quad (12)$$

Theorem 6. *For given scalars* h *and* $\gamma > 0$, *matrices* $\mathbf{K}_i$, *if there exist matrices* $\mathbf{P}_i > \mathbf{0}$, $\mathbf{Q}_i > \mathbf{0}$ $(i \in \mathfrak{I})$, *such that*

$$\begin{bmatrix} \Upsilon_1 & \Upsilon_2 \\ * & \Upsilon_3 \end{bmatrix} < 0, \quad (13)$$

then the closed-loop system (10) *is stochastic stable with an* H_∞ *norm bound* γ, *where* $\Upsilon_1 = \sum_{j\in\mathfrak{I}} \pi_{ij}\overline{\mathbf{A}}_k^{\mathrm{T}}\mathbf{W}_j\overline{\mathbf{A}}_k + \gamma^{-1}\widehat{\mathbf{C}}^{\mathrm{T}}\widehat{\mathbf{C}} - \mathbf{W}_i$, $\Upsilon_2 = \sum_{j\in\mathfrak{I}} \pi_{ij}\overline{\mathbf{A}}_k^{\mathrm{T}}\mathbf{W}_j\overline{\mathbf{D}}_1 + \gamma^{-1}\widehat{\mathbf{C}}^{\mathrm{T}}\mathbf{D}_2$, $\Upsilon_3 = \sum_{j\in\mathfrak{I}} \pi_{ij}\overline{\mathbf{D}}_1^{\mathrm{T}}\mathbf{W}_j\overline{\mathbf{D}}_1 + \gamma^{-1}\mathbf{D}_2^{\mathrm{T}}\mathbf{D}_2 - \gamma\mathbf{I}$.

Proof. Without loss of generalization, we set $\sigma(k)$ to be i. Choosing a Lyapunov-Krasovskii functional candidate which is given by

$$V_k = \eta_k^{\mathrm{T}}\mathbf{W}_i\eta_k, \quad (14)$$

where

$$\mathbf{W}_i = \begin{bmatrix} \mathbf{P}_i & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_i \end{bmatrix}, \quad (15)$$

we can obtain

$$EV_{k+1} - V_k = \sum_{j\in\mathfrak{I}} \pi_{ij}\left(\overline{\mathbf{A}}_k\eta_k + \overline{\mathbf{D}}_1\mathbf{w}_k\right)^{\mathrm{T}}\mathbf{W}_j \times \left(\overline{\mathbf{A}}_k\eta_k + \overline{\mathbf{D}}_1\mathbf{w}_k\right) - \eta_k^{\mathrm{T}}\mathbf{W}_i\eta_k. \quad (16)$$

Let $\mathbf{e}_k = \begin{bmatrix}\eta_k^{\mathrm{T}} & \mathbf{w}_k^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}$; we can obtain $EV_{k+1} - V_k = \mathbf{e}_k^{\mathrm{T}}\Theta_i\mathbf{e}_k$, where

$$\Theta_i = \begin{bmatrix} -\mathbf{W}_i + \sum_{j\in\mathfrak{I}} \pi_{ij}\overline{\mathbf{A}}_k^{\mathrm{T}}\mathbf{W}_j\overline{\mathbf{A}}_k & \sum_{j\in\mathfrak{I}} \pi_{ij}\overline{\mathbf{A}}_k^{\mathrm{T}}\mathbf{W}_j\overline{\mathbf{D}}_1 \\ * & \sum_{j\in\mathfrak{I}} \pi_{ij}\overline{\mathbf{D}}_1^{\mathrm{T}}\mathbf{W}_j\overline{\mathbf{D}}_1 \end{bmatrix}. \quad (17)$$

And

$$\gamma^{-1}\mathbf{z}_k^{\mathrm{T}}\mathbf{z}_k - \gamma\mathbf{w}_k^{\mathrm{T}}\mathbf{w}_k + EV_{k+1} - V_k = \eta_k^{\mathrm{T}}\left(\Theta_i + \Lambda\right)\eta_k, \quad (18)$$

where

$$\Lambda = \begin{bmatrix} \gamma^{-1}\widehat{\mathbf{C}}^{\mathrm{T}}\widehat{\mathbf{C}} & \gamma^{-1}\widehat{\mathbf{C}}^{\mathrm{T}}\mathbf{D}_2 \\ * & \gamma^{-1}\mathbf{D}_2^{\mathrm{T}}\mathbf{D}_2 - \gamma\mathbf{I} \end{bmatrix}. \quad (19)$$

If (13) is satisfied, we have $(\Theta_i + \Lambda) < \mathbf{0}$. Further, we can obtain

$$\gamma^{-1}\mathbf{z}_k^{\mathrm{T}}\mathbf{y}_k - \gamma\mathbf{w}_k^{\mathrm{T}}\mathbf{w}_k + EV_{k+1} - V_k < \mathbf{0}. \quad (20)$$

Then, $\gamma^{-1}\mathbf{z}_k^{\mathrm{T}}\mathbf{y}_k - \gamma\mathbf{w}_k^{\mathrm{T}}\mathbf{w}_k < -(EV_{k+1} - V_k)$. Summing up both sides of the above inequality from $k = 0$ to $k = n$, using the zero initial condition, we have $\sum_{k=0}^{n}\|\mathbf{z}_k\|^2 < \gamma^2\sum_{k=0}^{n}\|\mathbf{w}_k\|^2 - \gamma EV_{n+1}$ for all n. Letting $n \to \infty$, we have

$$\|\mathbf{z}_k\|_2^2 < \gamma^2\|\mathbf{w}_k\|_2^2. \quad (21)$$

If $\mathbf{w}_k \equiv \mathbf{0}$ and (13) holds, it is clear that $\Gamma_i = -\mathbf{W}_i + \sum_{j\in\mathfrak{I}} \pi_{ij}\overline{\mathbf{A}}_k^{\mathrm{T}}\mathbf{W}_j\overline{\mathbf{A}}_k < 0$ can be implied by (13), then

$$EV_{k+1} - V_k \leq -\beta\mathbf{x}_k^{\mathrm{T}}\mathbf{x}_k, \quad (22)$$

where $\beta = \min\{\lambda_{\min}(-\Gamma_i), i \in \mathfrak{I}\}$. Summing up both sides of the above inequality from $k = 0$ to $k = \hbar$ and Letting $\hbar \to \infty$, we can see that for any $\hbar > 1$

$$\lim_{\hbar\to\infty} EV(\hbar + 1) - V(\varphi_0, s_0) \leq -\beta\lim_{\hbar\to\infty}\sum_{k=0}^{\hbar}E\left(\mathbf{x}_k^{\mathrm{T}}\mathbf{x}_k\right) \quad (23)$$

or

$$\lim_{\hbar\to\infty}\sum_{k=0}^{\hbar}E\left(\mathbf{x}_k^{\mathrm{T}}\mathbf{x}_k\right) \leq \lim_{\hbar\to\infty}\frac{1}{\beta}V(\varphi_0, s_0) < \infty, \quad (24)$$

where φ_0 and s_0 are the initial condition of the system. The stochastic stability is obtained. This completes the proof. □

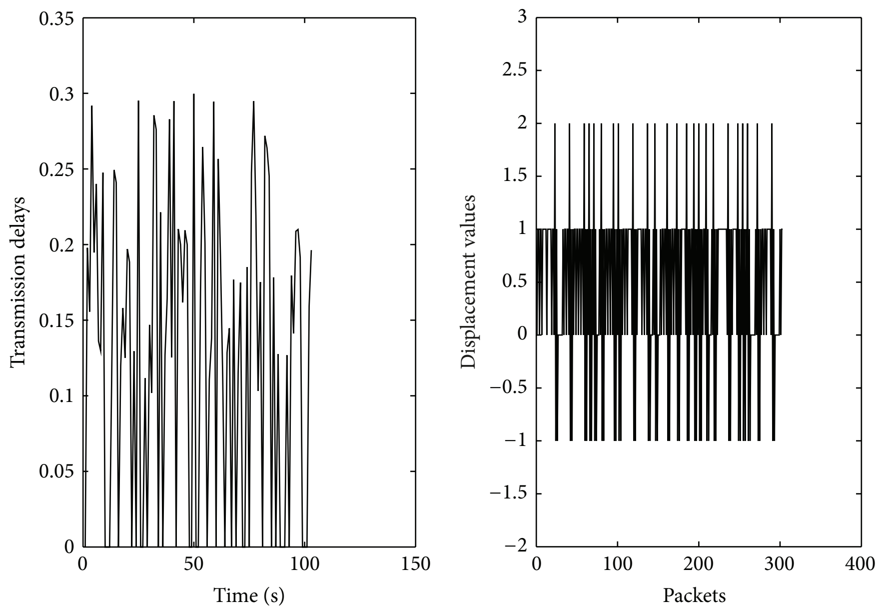
For the purpose of controller design, we give Theorem 7.

Theorem 7. *For a given scalar* h_1, *if there exist matrices* $\mathbf{X}_i > \mathbf{0}$, $\mathbf{Y}_i$, *and* $\mathbf{F}_i$ $(i \in \mathfrak{I})$, *such that*

$$\begin{bmatrix} -\mathbf{S}_i & \mathbf{0} & \Xi_{i1} & \mathbf{S}_i\widehat{\mathbf{C}}^T & \widehat{\mathbf{H}}_i^T \\ * & -\gamma\mathbf{I} & \Xi_{i2} & \mathbf{D}_2^T & \mathbf{0} \\ * & * & \Xi_{i3} & \mathbf{0} & \mathbf{0} \\ * & * & * & -\gamma\mathbf{I} & \mathbf{0} \\ * & * & * & * & -\varepsilon\mathbf{I} \end{bmatrix} < \mathbf{0}, \quad (25)$$

then $\mathbf{K}_i$ $(\overline{\mathbf{K}}_i = \mathbf{F}_i\mathbf{X}_i^{-1}, \overline{\mathbf{K}}_i = \operatorname{diag}(\mathbf{K}_i, \mathbf{K}_i, \ldots, \mathbf{K}_i))$ *are adaptive controller gains with an* H_∞ *norm bound* γ, *where*

$$\begin{gathered} \Xi_{i1} = \left[\rho_{i1}\mathbf{N}_i^T \;\; \rho_{i2}\mathbf{N}_i^T \;\; \cdots \;\; \rho_{ir}\mathbf{N}_i^T\right], \\ \Xi_{i2} = \left[\rho_{i1}\overline{\mathbf{D}}_1^T \;\; \rho_{i2}\overline{\mathbf{D}}_1^T \;\; \cdots \;\; \rho_{ir}\overline{\mathbf{D}}_1^T\right], \\ \Psi_i = \begin{bmatrix} \Psi_{i,11} & \Psi_{i,12} & \cdots & \Psi_{i,1r} \\ * & \Psi_{i,22} & \cdots & \Psi_{i,2r} \\ * & * & \ddots & \vdots \\ * & * & * & \Psi_{i,rr} \end{bmatrix}, \\ \Xi_{i3} = \operatorname{diag}\left(-\mathbf{S}_1, -\mathbf{S}_2, \ldots, -\mathbf{S}_r\right) + \Psi_i, \\ \rho_{ij} = \sqrt{\pi_{ij}} \quad (j = 1, 2, \ldots, r), \\ \mathbf{N}_i = \begin{bmatrix} \widehat{\mathbf{A}}_k\mathbf{X}_i + \widehat{\mathbf{B}}\Lambda_i\mathbf{F}_i & \widehat{\mathbf{B}}\theta_k(-1)\mathbf{Y}_i \\ \Lambda_i\mathbf{F}_i & \theta_k(-1)\mathbf{Y}_i \end{bmatrix}, \\ \widehat{\mathbf{H}}_i = \left[\widehat{\mathbf{E}}_1\mathbf{X}_i + \mathbf{E}_2\Lambda_i\mathbf{F}_i \;\; \mathbf{E}_2\theta_k(-1)\mathbf{Y}_i\right], \\ \Psi_{i,lj} = \rho_{il} * \rho_{ij}\varepsilon\widetilde{\mathbf{D}}\widetilde{\mathbf{D}}^T \quad (l, j = 1, 2, \ldots, r). \end{gathered} \quad (26)$$



(a) (b)

FIGURE 3: Transmission delay (a) and displacement values (b).

Proof. By Schur complement and Lemma 5, (13) is equivalent to

$$\begin{bmatrix} -\mathbf{W}_i & \mathbf{0} & \Lambda_{i1} & \widehat{\mathbf{C}}^{\mathrm{T}} & \mathbf{H}_i^{\mathrm{T}} \\ * & -\gamma\mathbf{I} & \Xi_{i2} & \mathbf{D}_2^{\mathrm{T}} & \mathbf{0} \\ * & * & \Lambda_{i3} & \mathbf{0} & \mathbf{0} \\ * & * & * & -\gamma\mathbf{I} & \mathbf{0} \\ * & * & * & * & -\varepsilon\mathbf{I} \end{bmatrix} < \mathbf{0}, \tag{27}$$

where

$$\Lambda_{i1} = \left[\rho_{i1}\widehat{\mathbf{A}}_k^{\mathrm{T}} \ \ \rho_{i2}\widehat{\mathbf{A}}_k^{\mathrm{T}} \ \ \cdots \ \ \rho_{ir}\widehat{\mathbf{A}}_k^{\mathrm{T}}\right],$$
$$\Lambda_{i3} = \operatorname{diag}\left(-\mathbf{W}_1, -\mathbf{W}_2, \ldots, -\mathbf{W}_r\right) + \Psi_i, \tag{28}$$
$$\mathbf{H}_i = \left[\widehat{\mathbf{E}}_1 + \mathbf{E}_2\Lambda_i\overline{\mathbf{K}}_i \ \ \ \mathbf{E}_2\theta_k(-1)\right].$$

Pre- and postmultiplying both sides of (27) with $\operatorname{diag}(\mathbf{W}_i^{-1}, \underbrace{\mathbf{I}, \ldots, \mathbf{I}}_{r+3})$ and its transpose, letting $\mathbf{X}_i = \mathbf{P}_i^{-1}$, $\mathbf{Y}_i = \mathbf{Q}_i^{-1}$, $\mathbf{S}_i = \mathbf{W}_i^{-1}$, $\overline{\mathbf{K}}_i\mathbf{X}_i = \mathbf{F}_i$ $(i = 1, 2, \ldots r)$, (25) can be derived. Theorem 7 is completed. □

4. Numerical Examples

In this section, we verify the effectiveness of the control strategy proposed for NCSs with packet disordering and packet loss. First, we show the control results for NCSs with packet disordering and packet loss under two cases. One is in the absence of structural uncertainty, and the other is with uncertain structure. One can clearly see that the method proposed has good robustness. Second, comparative studies are performed to demonstrate clearly the advantages enjoyed by the suggested compensation scheme described in this paper.

Example 1. Consider the following unstable system:

$$\begin{aligned} \mathbf{x}_{k+1} = {} & \left(\begin{bmatrix} 0.0885 & -0.0659 \\ -0.1538 & 0.2977 \end{bmatrix} + \Delta\mathbf{A}\right)\mathbf{x}_k \\ & + \left(\begin{bmatrix} 0.5234 \\ -0.0990 \end{bmatrix} + \Delta\mathbf{B}\right)\mathbf{u}_k \\ & + \begin{bmatrix} 1.2544 \\ 0.5317 \end{bmatrix}\mathbf{w}_k \end{aligned} \tag{29}$$
$$\mathbf{z}_{k+1} = \left[0.0690 \ \ -0.3554\right]\mathbf{x}_k + 0.3304\mathbf{w}_k,$$

where

$$\mathbf{D} = \begin{bmatrix} 0.3885 \\ 0.3112 \end{bmatrix}, \qquad \mathbf{E}_1 = \left[0.3237 \ \ -0.2128\right] \tag{30}$$
$$\mathbf{E}_2 = 0.5243.$$

The transmission delays and displacement values of packets delivered over network are shown in Figure 3. It should be explained that delay determines the arrival orders of packets, based on which displacement values of packets are calculated. Clearly, the bound of transmission delay $h = 2$ under assuming sampling period $0.15s$. Based on the displacement values of packets, thus the jumping process taking values in a finite set $\mathfrak{I} = \{1, 2, 3, 4\}$, standing for $\mathfrak{I} = \{[1, 0, 0, 0], [0, 1, 0, 0], [0, 0, 1, 0], [0, 0, 0, 1]\}$, governs the switching among the different system modes, then $r = 4$. $[1, 0, 0, 0] \rightarrow 1$ means no new signal arrives at the actuator, $u(k-1)$ acts on the plant; $[0, 1, 0, 0] \rightarrow 2$ means the newest signal $\mathbf{K}_2\mathbf{x}_k$ transmitted over network is executed by the plant at kT time instant, the rest may be deduced by analogy; $[0, 0, 0, 1] \rightarrow 4$ means the newest signal $\mathbf{K}_4\mathbf{x}_{k-2}$ transmitted over network is executed by the plant at kT time instant.

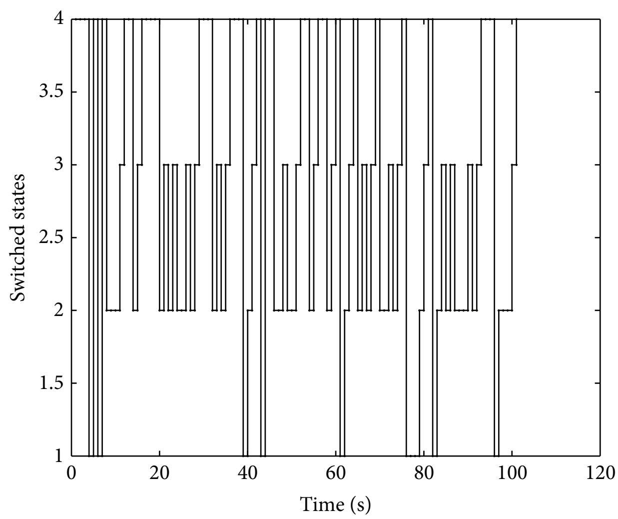


Figure 4: Markovian process of NCS with packet disordering and dropouts competition.

Thus, the newest signals executed by the plant are subject to Markovian process, whose switched states are shown in Figure 4 and transition probability matrix is given as follows:

$$\begin{bmatrix} 0.2000 & 0.5000 & 0 & 0.3000 \\ 0 & 0.2941 & 0.7059 & 0 \\ 0 & 0.4167 & 0 & 0.5833 \\ 0.2500 & 0.2813 & 0 & 0.4688 \end{bmatrix}. \tag{31}$$

4.1. Verification of Compensation Scheme. First, we consider the systems without structural uncertainty in NCS. By Theorem 7, we obtain the following adaptive controller gains

$$\begin{aligned} \mathbf{K}_2 &= \begin{bmatrix} -0.3271 & 0.5110 \end{bmatrix} \\ \mathbf{K}_3 &= 1.0e-03 * \begin{bmatrix} 0.1137 & -0.1898 \end{bmatrix} \\ \mathbf{K}_4 &= 1.0e-03 * \begin{bmatrix} -0.0474 & 0.1103 \end{bmatrix}. \end{aligned} \tag{32}$$

Second, if the uncertainty exists in the NCSs, by Theorem 7, we also obtain the corresponding adaptive controller gains

$$\begin{aligned} \mathbf{K}_2 &= \begin{bmatrix} -0.3903 & 0.5512 \end{bmatrix} \\ \mathbf{K}_3 &= \begin{bmatrix} -0.2038 & 0.3679 \end{bmatrix} \\ \mathbf{K}_4 &= \begin{bmatrix} -0.2869 & 0.3902 \end{bmatrix}. \end{aligned} \tag{33}$$

We choose the uncertain parameter $F(k) = \sin(k)$. At the initial state value $\mathbf{x}_0 = [-1 \ -3]^T$, the states and output response of the NCS, without uncertainty and with uncertainty under the network environment in the presence of packet disordering and packet loss competition, are shown in Figures 5 and 6, respectively. Compared with the result given in Figure 5, the NCS with uncertainty can be also stabilized quickly using the competition controller designed in this paper though there exist packet disordering and packet dropout in communication network. This makes it clear that the proposed control scheme has a good robustness.

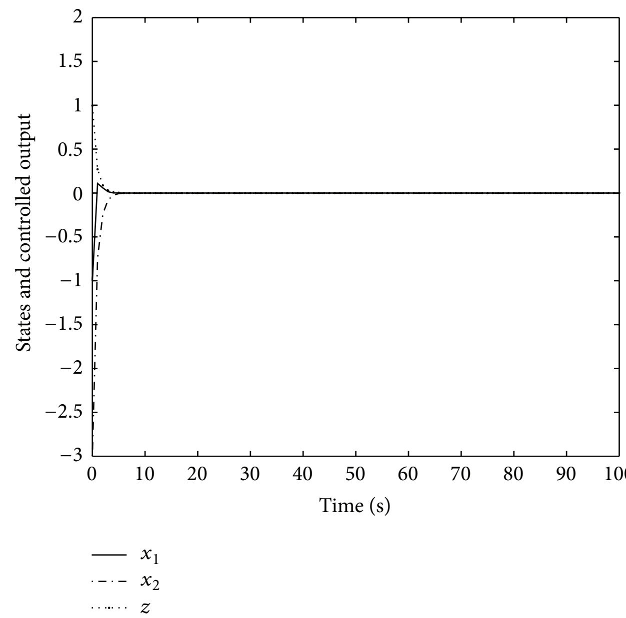


Figure 5: State and output response of the system.

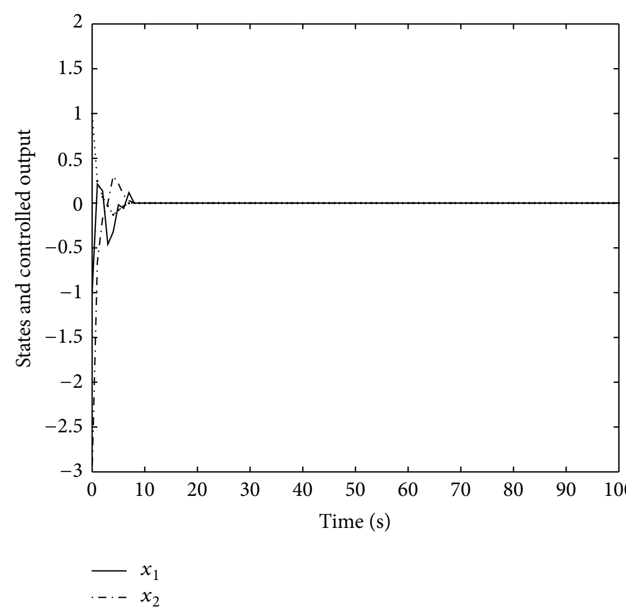


Figure 6: State and output response of the system.

4.2. Control Performance Comparison. It should be pointed out that the discrete-time system in this example can be inverted into a continuous-time system in [14, 20] if sampling period $T = 0.1s$ is given. If $\mathbf{w}_k \neq \mathbf{0}$, the H_∞ norm bounds and corresponding controller gains are shown in Table 3 ("—" denotes that the conditions are infeasible). Obviously, a more optimal H_∞ norm bound is obtained in this paper than those in [14, 20] since the competition scheme is performed when no signal is available by the plant due to packet disordering

Table 3: Comparison of convergence time.

	H_∞ norm bound	Controller gain
[14]	—	—
[20]	1.5	$\mathbf{K} = 1.0 * 10^{-7}$ $\begin{bmatrix} 0.1278 & -0.0714 \end{bmatrix}$
This paper	0.6977	$\mathbf{K}_2 = [-0.4867 \quad 0.7869]$ $\mathbf{K}_3 = [0 \quad 0]$ $\mathbf{K}_4 = [0 \quad 0]$

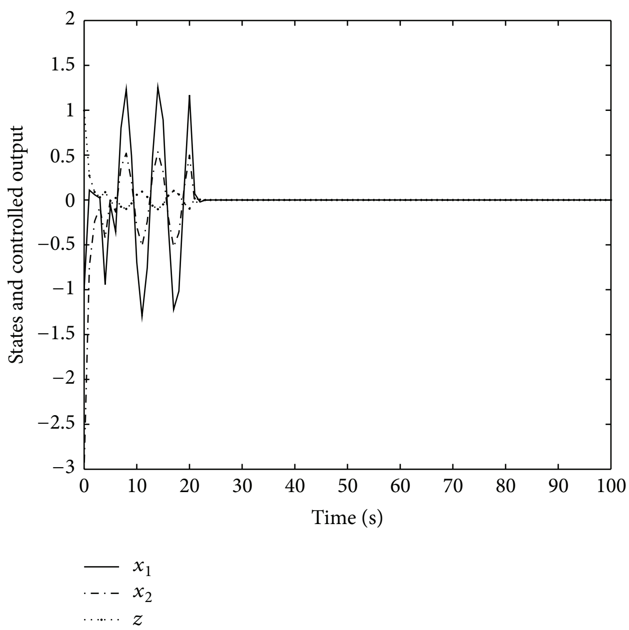


Figure 7: Curves of state and output response of the system subject to exogenous disturbance.

and packet loss. At the initial state value $\mathbf{x}_0 = [-1 \quad -3]^{\mathrm{T}}$, the disturbance input $\mathbf{w}(t)$ is as follows:

$$\mathbf{w}(t) = \begin{cases} \sin(t) & 5s \le t < 20s \\ 0 & \text{otherwise}, \end{cases} \tag{34}$$

the state and output response of the NCS in the presence of packet disordering and packet loss are shown in Figure 7.

5. Conclusions

In this paper, we are concerned with H_∞ control of NCSs with compensation scheme. The aim of devised control scheme is that the effect of packet disordering and packet loss on control performance is eliminated. The main idea is that we first describe the packet disordering and give the compensation scheme when there are no new signals executed by the plant during the sampling interval $((k-1)T, kT]$. Second, a model of NCSs with Markovian jumping property is presented. Furthermore, the stochastic stability and controller design are discussed. Finally, a numerical example and simulations are given to illustrate the advantages and the effectiveness of the developed theory.

Acknowledgments

The authors would like to acknowledge the National Natural Science Foundation of China under Grants 61104093, 61174119, 61034006, and 60774070, the Special Program for Key Basic Research Founded by MOST under Grant 2010CB334705, the National High Technology Research and Development Program of China (863 Program) under Grant 2011AA040101, and Scientific Research Project of Liaoning Province of China under Grants L2012141, L2011064.

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