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Research Article

New Results on Passivity Analysis of Delayed Discrete-Time Stochastic Neural Networks

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The problem of passivity analysis for a class of discrete-time stochastic neural networks (DSNNs) with time-varying interval delay is investigated. The delay-dependent sufficient criteria are derived in terms of linear matrix inequalities (LMIs). The results are shown to be generalization of some previous results and are less conservative than the existing works. Meanwhile, the computational complexity of the obtained stability conditions is reduced because less variables are involved. Two numerical examples are given to show the effectiveness and the benefits of the proposed method.

1. Introduction

Over the past decades, Neural Networks (NNs) have attracted considerable attention because of their extensive applications in pattern recognition, optimization solvers, model identification, signal processing, and other engineering areas [1]. Meanwhile, time delays are frequently encountered in various engineering, biological, and economic systems due to the finite switching speed of amplifiers and the inherent communication time of neurons. It has been revealed that time delay may cause instability and oscillation of the neural networks [2, 3]. For these reasons, the stability problem of NNs with delays has been extensively studied, for example, see [2–10]. It is well known that delay-dependent stability conditions are generally less conservative than delay-independent conditions, especially when the size of the delay is small. Therefore, considerable attention has been focused on the derivation of delay-dependent stability results and many effective approaches have been provided to reduce the conservatism of stability results for further improving the quality of delay-dependent stability criteria.

Most NNs studied are assumed to act in a continuous-time manner; however, in implementing and applications of neural networks, discrete-time neural networks become more and more important than their continuous-time counterparts [11]. So, the stability analysis problems for discrete-time neural networks have received more and more interests, and some stability criteria have been proposed in literature; for example, see [11–15] and the references cited therein. Moreover, stochastic disturbance usually appears in the electrical circuit design of neural networks; a neural network could be stabilized or destabilized by certain stochastic inputs. The delay-dependent stability problems of stochastic neural networks are studied in some works, such as, [16, 17] and the reference cited therein.

On the other hand, the passivity theory plays an important role in the analysis and design of linear and nonlinear delayed systems. Recently, the passivity of linear systems with delays [18–20] and the passivity of neural networks with delays [21–23] have been studied. Based on the Lyapunov-Krasovskii method and LMI framework, the passivity properties for delayed NNs were firstly studied in [21]. In [23], the problem of passivity and robust passivity for a class of discrete-time stochastic neural networks with time-varying delays was studied.

In this paper, by constructing a new Lyapunov-Krasovskii functional, the improved delay-dependent passivity and robust passivity criteria of DSNNs are obtained in the form of linear matrix inequalities (LMIs). It is shown that the obtained conditions are less conservative and more efficiency than those in [23]. Two numerical examples are also provided to show the effectiveness of the proposed stability criteria.

Notation. Throughout this paper, $\mathbb{N}^+$ stands for the set of nonnegative integers; $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n dimensional Euclidean space and the set of all $n \times m$ real matrices, respectively. Real symmetric matrix $P > 0$ (≥ 0) denotes P being a positive definite (positive semi-definite) matrix. The notation $X \geq Y$ (resp., $X > Y$) means that X and Y are symmetric matrices, and that $X - Y$ is positive semi-definite (resp., positive definite). I is used to denote an identity matrix with proper dimension. Matrices, if not explicitly stated, are assumed to have compatible dimensions. The symmetric terms in a symmetric matrix are denoted by $*$. The superscript "T" represents the transpose. We use $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ to denote the minimum and maximum eigenvalue of a real symmetric matrix, respectively. Let $(\Omega, \mathcal{F}, P)$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., it is right continuous and $\{\mathcal{F}\}_0$ contains all P-pull sets); let $L^p_{\mathcal{F}_0}([-\tau, 0]; \mathbb{R}^n)$ be the family of all $\mathcal{F}_0$-measurable $\{C([-\tau, 0]; \mathbb{R}^n)\}$-valued random variables $\{\xi = \xi(\theta) : -\tau \leq \theta \leq 0\}$ such that $\sup_{-\tau\leq\vartheta\leq 0}\mathbb{E}\{|\xi(\theta)|\}^p < \infty$, where $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure P.

2. Problem Formulation and Preliminaries

Consider the uncertain DSNNs with time-varying interval delay described by

$$\begin{aligned} x(k+1) &= C(k)x(k) + A(k)G(x(k)) + B(k)G(x(k-\tau(k))) + u(k) \\ &\quad + \sigma(x(k), x(k-\tau(k)), k)\omega(k), \quad k = 1, 2, \ldots, \\ y(k) &= G(x(k)), \end{aligned} \tag{2.1}$$

where $x(k) = [x_1(k), \ldots, x_n(k)]^T \in \mathbb{R}^n$ is the neural state vector; $G(x(k)) = [g_1(x_1(k)), \ldots, g_n(x_n(k))]^T$, and $G(x(k - \tau(k))) = [g_1(x_1(k-\tau(k))), \ldots, g_n(x_n(k-\tau(k)))]^T$ denote the neuron activation functions; $y(k) = G(x(k))$ is the output of the neural network; $u(k) = (u_1(k), u_2(k), \ldots, u_n(k))^T$ is the input vector; $\tau(k)$ denotes the time-varying delay satisfying

$$0 \leq \tau_m \leq \tau(k) \leq \tau_M, \tag{2.2}$$

and τ_m, τ_M are known positive integers. $C(k) = C + \Delta C(k)$, $A(k) = A + \Delta A(k)$, and $B(k) = B + \Delta B(k)$, where $C = \operatorname{diag}(c_1, c_2, \ldots, c_n)$ with $|c_i| < 1$ describing the rate with which the ith neuron will reset its potential to the resting state in isolation when disconnected from the networks and external inputs, $A = (a_{ij})_{n\times n}$ is the connection weight matrix, and $B = (b_{ij})_{n\times n}$ is the delayed connection weight matrix. $\Delta C(k), \Delta A(k)$, and $\Delta B(k)$ represent the time-varying parameter uncertainties and are assumed to satisfy the following admissible condition:

$$\begin{bmatrix}\Delta C & \Delta A & \Delta B\end{bmatrix} = EF(k)\begin{bmatrix}G_C & G_A & G_B\end{bmatrix}, \tag{2.3}$$

where E, G_C, G_A, and G_B are known real constant matrices of appropriate dimensions; $F(k)$ is the unknown time-varying matrix-valued function satisfying

$$F(k)^T F(k) \leq I, \quad \forall k \in \mathbb{N}^+. \tag{2.4}$$

In system (2.1), $\omega(k)$ is a scalar Wiener process (Brownian Motion) on $(\Omega, \mathcal{F}, P)$ with

$$\mathbb{E}[\omega(k)] = 0, \qquad \mathbb{E}\left[\omega^2(k)\right] = 1, \qquad \mathbb{E}[\omega(i)\omega(j)] = 0(i \neq j), \tag{2.5}$$

and there exist two positive constants that ρ_1 and ρ_2 such that

$$\begin{aligned}&\sigma^T(x(k), x(k-\tau(k)), k)\sigma(x(k), x(k-\tau(k)), k)\\&\quad \leq \rho_1 x(k)^T x(k) + \rho_2 x(k-\tau(k))^T x(k-\tau(k)),\end{aligned} \tag{2.6}$$

The initial condition of system (2.1) is given by

$$x(s) = \varphi(s), \quad s \in [-\tau_M, 0]. \tag{2.7}$$

The activation functions in (2.1) satisfy the following assumption.

Assumption 2.1. Activation functions $g_i(\cdot)$ in (2.1) are bounded and satisfy $g_i(0) = 0$:

$$\sigma_i^- \leq \frac{g_i(x) - g_i(y)}{x - y} \leq \sigma_i^+, \quad \forall x, y \in \mathbb{R}, x \neq y, i = 1, 2, \ldots, n, \tag{2.8}$$

where σ_i^+, σ_i^- are constants. Denote $\Gamma = \text{diag}(\sigma_1^-, \dots, \sigma_n^-)$, and

$$\Sigma_1 = \text{diag}(\sigma_1^+\sigma_1^-, \dots, \sigma_n^+\sigma_n^-), \qquad \Sigma_2 = \text{diag}\left(\frac{\sigma_1^+ + \sigma_1^-}{2}, \dots, \frac{\sigma_n^+ + \sigma_n^-}{2}\right). \tag{2.9}$$

Remark 2.2. As pointed out in [11], the constants σ_i^+, σ_i^- in Assumption 2.1 are allowed to be positive, negative, or zero. So the assumption is weaker in comparison with those made in [2, 13], and so forth.

Definition 2.3. The delayed DSNNs (2.1) are said passive if there exists a scalar $\gamma > 0$ such that

$$2\sum_{j=0}^{k_0}\mathbb{E}\left\{y^T(j)u(j)\right\} \geq -\gamma\sum_{j=0}^{k_0}\mathbb{E}\left\{u^T(j)u(j)\right\}, \quad \forall k_0 \in \mathbb{N}. \tag{2.10}$$

The purpose of this paper is to find the maximum allowed delay bound τ_M for the given lower bound τ_m such that the system described by (2.1) with uncertainties (2.3) is robust passivity.

In obtaining the main results of this paper, the following lemma will be useful for the proofs.

Lemma 2.4 (see [3]). *Given constant matrices $P, Q,$ and R, where $P^T = P, Q^T = Q$, then the LMI $\begin{bmatrix} P & R \\ R^T & -Q \end{bmatrix} < 0$ is equivalent to the following condition: $Q > 0$, $P + RQ^{-1}R^T < 0$.*

Lemma 2.5 (see [11]). *Given matrices $P, Q,$ and R with $P = P^T$, then*

$$P + QF(k)R + (QF(k)R)^T < 0 \tag{2.11}$$

holds for all $F(k)$ satisfying $F^T(k)F(k) \leq I$ if and only if there exists a scalar $\epsilon > 0$ such that

$$P + \epsilon^{-1}QQ^T + \epsilon RR^T < 0. \tag{2.12}$$

3. Main Results

In this section, we present a delay-dependent criterion guaranteeing the passivity of DSNNs with time-varying delay:

$$x(k+1) = Cx(k) + AG(x(k)) + BG(x(k-\tau(k))) + u(k) + \sigma(x(k), x(k-\tau(k)), k)\omega(k). \tag{3.1}$$

Theorem 3.1. *Under Assumption 2.1 given two scalars $\tau_m > 0$ and $\tau_M > 0$, for any delay $\tau(k)$ satisfying* (2.2)*, system* (3.1) *is passive in the sense of Definition 2.3 if there exist two scalars $\gamma > 0, \lambda^* > 0$, five positive definite matrices $P, Q_1, Q_2, Q_3, Z > 0$, three diagonal matrices $K > 0, U_1 > 0, U_2 > 0$, and six matrices $L_1, L_2, M_1, M_2, N_1, N_2$ with appropriate dimensions, such that the following LMIs hold:*

$$\tau_M Z + P < \lambda^* I, \tag{3.2}$$

$$\Psi = \begin{bmatrix} \Psi_1 & \Psi_2 \\ * & \Psi_3 \end{bmatrix} < 0, \tag{3.3}$$

where

$$\begin{gathered}\Psi_1 = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} & \Psi_{15} & M_1 - N_1 & -L_1 \\ * & \Psi_{22} & \Psi_{23} & \Psi_{24} & 0 & 0 & 0 \\ * & * & \Psi_{33} & \Psi_{34} & \Psi_{35} & 0 & 0 \\ * & * & * & \Psi_{44} & 0 & 0 & 0 \\ * & * & * & * & \Psi_{55} & M_2 - N_2 & -L_2 \\ * & * & * & * & * & -Q_1 & 0 \\ * & * & * & * & * & * & -Q_3 \end{bmatrix}, \\ \Psi_2 = \left[\sqrt{\tau_M - \tau_m} L \quad \sqrt{\tau_M - \tau_m} M \quad \sqrt{\tau_M} N\right], \\ \Psi_3 = \operatorname{diag}\{-Z, -Z, -Z\},\end{gathered} \tag{3.4}$$

with

$$\begin{aligned}
\Psi_{11} &= -P + Q_1 + \tau Q_2 + Q_3 - \tau\left(\Gamma^T K + K^T \Gamma\right) + N_1 + N_1^T \\
&\quad + C^T P C - U_1 \Sigma_1 + \rho_1 \lambda^* + \tau_M (C - I)^T Z (C - I), \\
\Psi_{12} &= \tau K + \Sigma_2 U_1 + C^T P A + \tau_M (C - I)^T Z A, \\
\Psi_{13} &= \tau_M C^T Z B + C^T P B, \\
\Psi_{14} &= \tau_M (C - I)^T Z + C^T P, \\
\Psi_{15} &= L_1 + N_2^T - M_1, \\
\Psi_{22} &= -U_1 + \tau_M A^T Z A + A^T P A, \\
\Psi_{23} &= \tau_M A^T Z B + A^T P B, \\
\Psi_{24} &= \tau_M A^T Z + A^T P - I,
\end{aligned}$$

$$
\begin{aligned}
\Psi_{33} &= -U_2 + \tau_M B^T Z B + B^T P B, \\
\Psi_{34} &= \tau_M B^T Z + B^T P, \\
\Psi_{35} &= -K + U_2 \Sigma_2, \\
\Psi_{44} &= \tau_M Z + P - \gamma I, \\
\Psi_{55} &= -Q_2 - U_2 \Sigma_1 + L_2 + L_2^T - M_2 - M_2^T + \rho_2 \lambda^* + \Gamma^T K + K^T \Gamma, \\
\tau &= \tau_M - \tau_m + 1, \\
L &= \begin{bmatrix} L_1 \\ 0 \\ 0 \\ 0 \\ L_2 \\ 0 \\ 0 \\ \ \end{bmatrix}, \quad M = \begin{bmatrix} M_1 \\ 0 \\ 0 \\ 0 \\ M_2 \\ 0 \\ 0 \\ \ \end{bmatrix}, \quad N = \begin{bmatrix} N_1 \\ 0 \\ 0 \\ 0 \\ N_2 \\ 0 \\ 0 \\ \ \end{bmatrix}.
\end{aligned}
\tag{3.5}
$$

Proof. Choose a new Lyapunov-Krasovskii functional candidate as follows:

$$
V(k) = V_1(k) + V_2(k) + V_3(k) + V_4(k) + V_5(k), \tag{3.6}
$$

where

$$
\begin{aligned}
V_1(k) &= x^T(k) P x(k), \\
V_2(k) &= \sum_{i=k-\tau_m}^{k-1} x^T(i) Q_1 x(i) + \sum_{i=k-\tau_M}^{k-1} x^T(i) Q_3 x(i), \\
V_3(k) &= \sum_{i=k-\tau(k)}^{k-1} x^T(i) Q_2 x(i) + \sum_{j=k-\tau_M+1}^{k-\tau_m} \sum_{i=j}^{k-1} x^T(i) Q_2 x(i), \\
V_4(k) &= \sum_{j=k-\tau_M}^{k-1} \sum_{i=j}^{k-1} \eta^T(i) Z \eta(i), \\
V_5(k) &= 2 \sum_{i=k-\tau(k)}^{k-1} [G(x(i)) - \Gamma x(i)]^T K x(i) + 2 \sum_{i=k-\tau_M+1}^{k-\tau_m} \sum_{j=i}^{k-1} [G(x(j)) - \Gamma x(j)]^T K x(j), \\
\eta(i) &= x(i+1) - x(i).
\end{aligned}
\tag{3.7}
$$

Defining $\Delta V(k) = V(k+1) - V(k)$, calculating the difference of $V(k)$ along the system (3.1), and taking the mathematical expectation, we have

$$\mathbb{E}\{\Delta V(k)\} = \mathbb{E}\{\Delta V_1(k)\} + \mathbb{E}\{\Delta V_2(k)\} + \mathbb{E}\{\Delta V_3(k)\} + \mathbb{E}\{\Delta V_4(k)\} + \mathbb{E}\{\Delta V_5(k)\}, \tag{3.8}$$

where,

$$\begin{aligned}
\mathbb{E}\{\Delta V_1(k)\} &= \mathbb{E}\Big\{[Cx(k) + AG(x(k)) + BG(x(k-\tau(k))) + u(k)]^T P \\
&\qquad \times [Cx(k) + AG(x(k)) + BG(x(k-\tau(k))) + u(k)] \\
&\qquad + \sigma^T(x(k), x(k-\tau(k)), k) P \sigma(x(k), x(k-\tau(k)), k) - x^T(k) P x(k)\Big\}, \\
\mathbb{E}\{\Delta V_2(k)\} &= \mathbb{E}\Big\{x^T(k)(Q_1 + Q_3)x(k) - x^T(k-\tau_m)Q_1 x(k-\tau_m) - x^T(k-\tau_M)Q_3 x(k-\tau_M)\Big\}, \\
\mathbb{E}\{\Delta V_3(k)\} &= \mathbb{E}\Bigg\{x^T(k)Q_2 x(k) - x^T(k-\tau(k))Q_2 x(k-\tau(k)) \\
&\qquad + \sum_{i=k+1-\tau_m}^{k-1} x^T(i)Q_2 x(i) + \sum_{i=k+1-\tau(k+1)}^{k-\tau_m} x^T(i)Q_2 x(i) - \sum_{i=k+1-\tau(k)}^{k-1} x^T(i)Q_2 x(i) \\
&\qquad + (\tau_M - \tau_m)x^T(k)Q_2 x(k) - \sum_{i=k+1-\tau_M}^{k-\tau_m} x^T(i)Q_2 x(i)\Bigg\} \\
&\le \mathbb{E}\Big\{x^T(k)Q_2 x(k) - x^T(k-\tau(k))Q_2 x(k-\tau(k)) + (\tau_M - \tau_m)x^T(k)Q_2 x(k)\Big\}, \\
\mathbb{E}\{\Delta V_4(k)\} &= \mathbb{E}\Bigg\{\tau_M \eta^T(k) Z \eta(k) - \sum_{i=k-\tau_M}^{k-1} \eta^T(i) Z \eta(i)\Bigg\} \\
&= \mathbb{E}\Bigg\{\tau_M x^T(k)(C-I)^T Z (C-I)x(k) + 2\tau_M x^T(k)(C-I)^T Z A G(x(k)) \\
&\qquad + 2\tau_M x^T(k)(C-I)^T Z B G(x(k-\tau(k))) + 2\tau_M x^T(k)(C-I)^T Z u(k) \\
&\qquad + \tau_M G^T(x(k)) A^T Z A G(x(k)) + 2\tau_M G^T(x(k)) A^T Z B G(x(k-\tau(k))) \\
&\qquad + 2\tau_M G^T(x(k)) A^T Z u(k) + \tau_M G^T(x(k-\tau(k))) B^T Z B G(x(k-\tau(k))) \\
&\qquad + 2\tau_M G^T(x(k-\tau(k))) B^T Z u(k) + \tau_M u^T(k) Z u(k) \\
&\qquad + \tau_M \sigma^T(x(k), x(k-\tau(k)), k) Z \sigma(x(k), x(k-\tau(k)), k) \\
&\qquad - \sum_{i=k-\tau_M}^{k-\tau(k)-1} \eta^T(i) Z \eta(i) - \sum_{i=k-\tau(k)}^{k-\tau_m-1} \eta^T(i) Z \eta(i) - \sum_{i=k-\tau_m}^{k-1} \eta^T(i) Z \eta(i)\Bigg\},
\end{aligned}$$

$$\begin{aligned}\mathbb{E}\{\Delta V_5(k)\} &= \mathbb{E}\Bigg\{2\sum_{i=k+1-\tau(k+1)}^{k-1}[G(x(i))-\Gamma x(i)]^T Kx(i)+2[G(x(k))-\Gamma x(k)]^T Kx(k)\\ &\quad +2\sum_{i=k-\tau_M+2}^{k+1-\tau_m}\sum_{j=i}^{k}[G(x(j))-\Gamma x(j)]^T Kx(j)-2\sum_{i=k+1-\tau(k)}^{k-1}[G(x(i))-\Gamma x(i)]^T\\ &\quad \times Kx(i)-2[G(x(k-\tau(k)))-\Gamma x(k-\tau(k))]^T Kx(k-\tau(k))\\ &\quad -2\sum_{i=k-\tau_M+1}^{k-\tau_m}\sum_{j=i}^{k-1}[G(x(j))-\Gamma x(j)]^T Kx(j)\Bigg\}\\ &\leq \mathbb{E}\Big\{2(\tau_M-\tau_m+1)(G(x(k))-\Gamma x(k))^T Kx(k)-2G^T(x(k-\tau(k)))Kx(k-\tau(k))\\ &\quad +2x^T(k-\tau(k))\Gamma^T Kx(k-\tau(k))\Big\}.\end{aligned} \tag{3.9}$$

Define $\xi^T(k) = [x^T(k)\ G^T(x(k))\ G^T(x(k-\tau(k)))\ u^T(k)x^T(k-\tau(k))\ x^T(k-\tau_m)x^T(k-\tau_M)]$, and it is easy to see

$$\begin{aligned}2\xi^T(k)L\left[x(k-\tau(k))-x(k-\tau_M)-\sum_{i=k-\tau_M}^{k-\tau(k)-1}\eta(i)\right]&\equiv 0,\\ 2\xi^T(k)M\left[x(k-\tau_m)-x(k-\tau(k))-\sum_{i=k-\tau(k)}^{k-\tau_m-1}\eta(i)\right]&\equiv 0,\\ 2\xi^T(k)N\left[x(k)-x(k-\tau_m)-\sum_{i=k-\tau_m}^{k-1}\eta(i)\right]&\equiv 0.\end{aligned} \tag{3.10}$$

From Assumption 2.1, if follows that for $i = 1, 2, \ldots, n$

$$\begin{aligned}\left(g_i(x_i(k))-\sigma_i^+ x_i(k)\right)\left(g_i(x_i(k))-\sigma_i^- x_i(k)\right)&\leq 0,\\ \left(g_i(x_i(k-\tau(k)))-\sigma_i^+ x_i(k-\tau(k))\right)\times\left(g_i(x_i(k-\tau(k)))-\sigma_i^- x_i(k-\tau(k))\right)&\leq 0.\end{aligned} \tag{3.11}$$

For any diagonal matrices $U_1 \geq 0, U_2 \geq 0$, and Σ_i $(i = 1, 2)$ in Assumption 2.1, it is easy to get

$$\begin{aligned}0 \leq \Bigg\{&\begin{bmatrix}x(k)\\ G(x(k))\end{bmatrix}^T\begin{bmatrix}-U_1\Sigma_1 & U_1\Sigma_2\\ * & -U_1\end{bmatrix}\begin{bmatrix}x(k)\\ G(x(k))\end{bmatrix}+\begin{bmatrix}x(k-\tau(k))\\ G(x(k-\tau(k)))\end{bmatrix}^T\\ &\times\begin{bmatrix}-U_2\Sigma_1 & U_2\Sigma_2\\ * & -U_2\end{bmatrix}\begin{bmatrix}x(k-\tau(k))\\ G(x(k-\tau(k)))\end{bmatrix}\Bigg\}.\end{aligned} \tag{3.12}$$

On the other hand, using (3.2)

$$\begin{aligned}&\mathbb{E}\Big\{\sigma^T(x(k),x(k-\tau(k)),k)[P+\tau_M Z]\sigma(x(k),x(k-\tau(k)),k)\Big\}\\&\quad\leq \mathbb{E}\Big\{\rho_1\lambda^* x^T(k)G_1^T x(k)+\rho_2\lambda^* x^T(k-\tau(k))x(k-\tau(k))\Big\}.\end{aligned}\tag{3.13}$$

Since $y(k)=G(x(k))$, combining (3.9)–(3.13), we get

$$\begin{aligned}&\mathbb{E}\Big\{\Delta V(k)-2G^T(x(k))u(k)-\gamma u^T(k)u(k)\Big\}\\&\quad\leq \mathbb{E}\Bigg\{\xi^T(k)\Big[\Psi_1+(\tau_M-\tau_m)LZ^{-1}L^T+(\tau_M-\tau_m)MZ^{-1}M^T+\tau_M NZ^{-1}N^T\Big]\xi(k)\\&\qquad-\sum_{l=k-\tau_M}^{k-\tau(k)-1}\Big[L^T\xi(k)+Z\eta(l)\Big]^T Z^{-1}\Big[L^T\xi(k)+Z\eta(l)\Big]\\&\qquad-\sum_{l=k-\tau(k)}^{k-\tau_m-1}\Big[M^T\xi(k)+Z\eta(l)\Big]^T Z^{-1}\Big[M^T\xi(k)+Z\eta(l)\Big]\\&\qquad-\sum_{l=k-\tau(k)}^{k-1}\Big[N^T\xi(k)+Z\eta(l)\Big]^T Z^{-1}\Big[N^T\xi(k)+Z\eta(l)\Big]\Bigg\}\\&\quad\leq \mathbb{E}\Big\{\xi^T(k)\Big[\Psi_1+(\tau_M-\tau_m)LZ^{-1}L^T+(\tau_M-\tau_m)MZ^{-1}M^T+\tau_M NZ^{-1}N^T\Big]\xi(k)\Big\}.\end{aligned}\tag{3.14}$$

Applying Lemma 2.4 to (3.3) yields

$$\mathbb{E}\Big\{\Psi_1+(\tau_M-\tau_m)LZ^{-1}L^T+(\tau_M-\tau_m)MZ^{-1}M^T+\tau_M NZ^{-1}N^T\Big\}<0.\tag{3.15}$$

From (3.15), we obtain

$$2\sum_{j=0}^{k}\mathbb{E}\Big\{y^T(j)u(j)\Big\}\geq\sum_{j=0}^{k}\mathbb{E}\{\Delta V(k)\}-\gamma\sum_{j=0}^{k}\mathbb{E}\Big\{u^T(j)u(j)\Big\}\tag{3.16}$$

for all $k_0\in\mathbb{N}$. By the definition of $V(k)$, we know that

$$\sum_{j=0}^{k}\mathbb{E}\{\Delta V(k)\}=\mathbb{E}\{V(k_0+1)-V(0)\}=\mathbb{E}\{V(k_0+1)\}\geq 0\tag{3.17}$$

for all $k_0\in\mathbb{N}$. Thus,

$$2\sum_{j=0}^{k}\mathbb{E}\Big\{y^T(j)u(j)\Big\}\geq-\gamma\sum_{j=0}^{k}\mathbb{E}\Big\{u^T(j)u(j)\Big\}.\tag{3.18}$$

then by Definition 2.3, it completes the proof of Theorem 3.1. □

Remark 3.2. In Theorem 3.1, the free-weighing matrices $L_1, L_2, M_1, M_2, N_1,$ and N_2 are introduced so as to reduce the conservatism of the delay-dependent results.

Remark 3.3. Since the activation functions satisfy Assumption 2.1, we know that

$$\sigma_i^- \leq \frac{g_i(x(k))}{x(k)} \leq \sigma_i^+, \quad \forall x(k) \in \mathbb{R},\ i = 1, 2, \ldots, n. \tag{3.19}$$

So,

$$0 \leq \frac{g_i(x(k))}{x(k)} - \sigma_i^- \leq \sigma_i^+ - \sigma_i^-, \tag{3.20}$$

then

$$\left(g_i(x(k)) - \sigma_i^- x(k)\right) x(k) \geq 0, \quad \forall x(k) \in \mathbb{R}. \tag{3.21}$$

So, $V_5(k)$ is nonnegative definite. Including the term $V_5(k)$ in the Lyapunov functional is to reduce the conservatism further, and it is illustrated by the later example. If $V_5(k)$ is not included in (3.6), we can get Corollary 3.4.

Corollary 3.4. *Given two scalars $\tau_m > 0$ and $\tau_M > 0$. Then, for any delay $\tau(k)$ satisfying* (2.2), *system* (3.1) *is passive in the sense of Definition 2.3 if there exist two scalars $\gamma > 0$, $\lambda^* > 0$, five positive definite matrices $P, Q_1, Q_2, Q_3, Z > 0$, two diagonal matrices $U_1 > 0$, $U_2 > 0$, and six matrices $L_1, L_2, M_1, M_2, N_1, N_2$ with appropriate dimensions, such that* (3.2) *and the following LMI hold:*

$$\Psi = \begin{bmatrix} \widehat{\Psi}_1 & \Psi_2 \\ * & \Psi_3 \end{bmatrix} < 0, \tag{3.22}$$

where

$$\widehat{\Psi}_1 = \begin{bmatrix} \widehat{\Psi}_{11} & \widehat{\Psi}_{12} & \Psi_{13} & \Psi_{14} & \Psi_{15} & M_1 - N_1 & -L_1 \\ * & \Psi_{22} & \Psi_{23} & \Psi_{24} & 0 & 0 & 0 \\ * & * & \Psi_{33} & \Psi_{34} & U_2\Sigma_2 & 0 & 0 \\ * & * & * & \Psi_{44} & 0 & 0 & 0 \\ * & * & * & * & \widehat{\Psi}_{55} & M_2 - N_2 & -L_2 \\ * & * & * & * & * & -Q_1 & 0 \\ * & * & * & * & * & * & -Q_3 \end{bmatrix}, \tag{3.23}$$

with

$$\begin{aligned} \widehat{\Psi}_{11} &= -P + Q_1 + \tau Q_2 + Q_3 + N_1 + N_1^T + C^T P C - U_1\Sigma_1 + \rho_1\lambda^* + \tau_M (C - I)^T Z (C - I), \\ \widehat{\Psi}_{12} &= \tau K + \Sigma_2 U_1 + C^T P A + \tau_M (C - I)^T Z A, \\ \widehat{\Psi}_{55} &= -Q_2 - U_2\Sigma_1 + L_2 + L_2^T - M_2 - M_2^T + \rho_2\lambda^*. \end{aligned} \tag{3.24}$$

Next, we provide the delay-dependent robust passivity analysis for uncertain DSNN (2.1).

Theorem 3.5. *Given two scalars $\tau_m > 0$ and $\tau_M > 0$. Then, under Assumption 2.1, for any delay $\tau(k)$ satisfying* (2.2), *system* (3.1) *is robust passive if there exist two scalars $\gamma > 0, \epsilon > 0$, five positive definite matrices $P, Q_1, Q_2, Q_3, Z > 0$, three diagonal matrices $K > 0, U_1 > 0, U_2 > 0$, and six matrices $L_1, L_2, M_1, M_2, N_1, N_2$ with appropriate dimensions, such that* (3.3) *and the following LMI hold:*

$$\Gamma = \begin{bmatrix} \Gamma_1 & \Gamma_2 & \Gamma_3 & \Gamma_4 & \Gamma_5 & \Gamma_6 & 0 & \epsilon\Gamma_7 \\ * & -Z & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -Z & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -Z & 0 & 0 & 0 & 0 \\ * & * & * & * & -P & 0 & PE & 0 \\ * & * & * & * & * & -Z & \sqrt{\tau_M}ZE & 0 \\ * & * & * & * & * & * & -\epsilon I & 0 \\ * & * & * & * & * & * & * & -\epsilon I \end{bmatrix} < 0, \tag{3.25}$$

where

$$\Gamma_1 = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & 0 & 0 & \Gamma_{15} & M_1 - N_1 & -L_1 \\ * & -U_1 & 0 & -I & 0 & 0 & 0 \\ * & * & -U_2 & 0 & \Gamma_{35} & 0 & 0 \\ * & * & * & -\gamma I & 0 & 0 & 0 \\ * & * & * & * & \Gamma_{55} & M_2 - N_2 & -L_2 \\ * & * & * & * & * & -Q_1 & 0 \\ * & * & * & * & * & * & -Q_3 \end{bmatrix},$$

$$\Gamma_2 = \begin{bmatrix} \sqrt{\tau_M - \tau_m}L_1 \\ 0 \\ 0 \\ 0 \\ \sqrt{\tau_M - \tau_m}L_2 \\ 0 \\ 0 \end{bmatrix}, \qquad \Gamma_3 = \begin{bmatrix} \sqrt{\tau_M - \tau_m}M_1 \\ 0 \\ 0 \\ 0 \\ \sqrt{\tau_M - \tau_m}M_2 \\ 0 \\ 0 \end{bmatrix}, \qquad \Gamma_4 = \begin{bmatrix} \sqrt{\tau_M}N_1 \\ 0 \\ 0 \\ 0 \\ \sqrt{\tau_M}N_2 \\ 0 \\ 0 \end{bmatrix},$$

$$\Gamma_5 = \begin{bmatrix} C^T P \\ A^T P \\ B^T P \\ P \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \Gamma_6 = \begin{bmatrix} \sqrt{\tau_M}(C-I)^T Z \\ \sqrt{\tau_M} A^T Z \\ \sqrt{\tau_M} B^T Z \\ \sqrt{\tau_M} Z \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \Gamma_7 = \begin{bmatrix} G_C^T \\ G_A^T \\ G_B^T \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \tag{3.26}$$

with

$$\begin{aligned} \Gamma_{11} &= -P + Q_1 + \tau Q_2 + Q_3 - \tau\left(\Gamma^T K + K^T \Gamma\right) + N_1 + N_1^T - U_1 \Sigma_1 + \rho_1 \lambda^*, \\ \Gamma_{12} &= \tau K + \Sigma_2 U_1, \\ \Gamma_{15} &= L_1 + N_2^T - M_1, \\ \Gamma_{35} &= -K + U_2 \Sigma_2, \\ \Gamma_{55} &= -Q_2 - U_2 \Sigma_1 + L_2 + L_2^T - M_2 - M_2^T + \rho_2 \lambda^* + \Gamma^T K + K^T \Gamma. \end{aligned} \tag{3.27}$$

Proof. Assume that inequality (3.25) holds, according to Lemma 2.4, we have

$$\begin{aligned} &\begin{bmatrix} \Gamma_1 & \Gamma_2 & \Gamma_3 & \Gamma_4 & \Gamma_5 & \Gamma_6 \\ * & -Z & 0 & 0 & 0 & 0 \\ * & * & -Z & 0 & 0 & 0 \\ * & * & * & -Z & 0 & 0 \\ * & * & * & * & -P & 0 \\ * & * & * & * & * & -Z \end{bmatrix} \\ &+ \epsilon^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ PE \\ \sqrt{\tau_M} ZE \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & E^T P & \sqrt{\tau_M} E^T Z \end{bmatrix} + \epsilon \begin{bmatrix} \Gamma_7 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \Gamma_7^T & 0 & 0 & 0 & 0 & 0 \end{bmatrix} < 0. \end{aligned} \tag{3.28}$$

Then, from Lemma 2.5, we know that LMI (3.28) is equivalent to the following inequality:

$$\begin{bmatrix} \Gamma_1 & \Gamma_2 & \Gamma_3 & \Gamma_4 & \Gamma_5 & \Gamma_6 \\ * & -Z & 0 & 0 & 0 & 0 \\ * & * & -Z & 0 & 0 & 0 \\ * & * & * & -Z & 0 & 0 \\ * & * & * & * & -P & 0 \\ * & * & * & * & * & -Z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ PE \\ \sqrt{\tau_M} ZE \end{bmatrix} F(k) \begin{bmatrix} \Gamma_7^T & 0 & 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} \Gamma_7 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} F^T(k) \begin{bmatrix} 0 & 0 & 0 & 0 & E^T P & \sqrt{\tau_M} E^T Z \end{bmatrix} < 0. \tag{3.29}$$

It can be verified that the above inequality is exactly the left-hand side of (3.3) when C, A, and B are replaced with $C + EF(k)G_C$, $A + EF(k)G_A$, and $B + EF(k)G_B$, respectively. The result then follows from Theorem 3.1.

We now consider the DSNNs without stochastic term. In this case, the system (2.1) reduces to

$$x(k+1) = C(k)x(k) + A(k)G(x(k)) + B(k)G(x(k-\tau(k))) + u(k). \tag{3.30}$$

From Theorem 3.5, we can easily obtain the following corollary. □

Corollary 3.6. *Given two scalars $\tau_m > 0$ and $\tau_M > 0$. Then, for any delay $\tau(k)$ satisfying* (2.2)*, system* (3.30) *is robust passive if there exist three scalars $\gamma > 0, \lambda^* > 0, \epsilon > 0$, five positive definite matrices $P, Q_1, Q_2, Q_3, Z > 0$, three diagonal matrices $K > 0, U_1 > 0, U_2 > 0$, and six matrices $L_1, L_2, M_1, M_2, N_1, N_2$ with appropriate dimensions, such that the following LMI holds:s*

$$\Gamma = \begin{bmatrix} \widehat{\Gamma}_1 & \Gamma_2 & \Gamma_3 & \Gamma_4 & \Gamma_5 & \Gamma_6 & 0 & \epsilon\Gamma_7 \\ * & -Z & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -Z & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -Z & 0 & 0 & 0 & 0 \\ * & * & * & * & -P & 0 & PE & 0 \\ * & * & * & * & * & -Z & \sqrt{\tau_M} ZE & 0 \\ * & * & * & * & * & * & -\epsilon I & 0 \\ * & * & * & * & * & * & * & -\epsilon I \end{bmatrix} < 0, \tag{3.31}$$

where

$$\widehat{\Gamma}_1 = \begin{bmatrix} \widehat{\Gamma}_{11} & \Gamma_{12} & 0 & 0 & \Gamma_{15} & M_1 - N_1 & -L_1 \\ * & -U_1 & 0 & -I & 0 & 0 & 0 \\ * & * & -U_2 & 0 & \Gamma_{35} & 0 & 0 \\ * & * & * & -\gamma I & 0 & 0 & 0 \\ * & * & * & * & \widehat{\Gamma}_{55} & M_2 - N_2 & -L_2 \\ * & * & * & * & * & -Q_1 & 0 \\ * & * & * & * & * & * & -Q_3 \end{bmatrix}, \quad (3.32)$$

with

$$\begin{aligned} \widehat{\Gamma}_{11} &= -P + Q_1 + \tau Q_2 + Q_3 - \tau\left(\Gamma^T K + K^T \Gamma\right) + N_1 + N_1^T - U_1\Sigma_1, \\ \widehat{\Gamma}_{55} &= -Q_2 - U_2\Sigma_1 + L_2 + L_2^T - M_2 - M_2^T + \Gamma^T K + K^T \Gamma. \end{aligned} \quad (3.33)$$

4. Numerical Examples

This section presents two numerical examples that demonstrate the validity of the method described above.

Example 4.1. Consider a delayed DSNNs (3.1) with the following parameters:

$$C = \begin{bmatrix} 0.04 & 0 \\ 0 & 0.01 \end{bmatrix}, \qquad A = \begin{bmatrix} 0.01 & 0.08 \\ -0.05 & 0.02 \end{bmatrix}, \qquad B = \begin{bmatrix} -0.05 & 0.01 \\ 0.02 & 0.07 \end{bmatrix}, \quad (4.1)$$

$\rho_1 = 0.01$, $\rho_2 = 0.02$, and the activation functions satisfy Assumption 2.1 with $\sigma_1^- = -0.1$, $\sigma_2^- = -0.2$, $\sigma_1^+ = 0.1$, and $\sigma_2^+ = 0.2$.

The activation functions satisfy Assumption 2.1 with $\sigma_1^- = 1$, $\sigma_2^- = 2$, $\sigma_1^+ = 2$, and $\sigma_2^+ = 4$.

From Tables 1 and 2, it is easy to see that the results in this paper are superior to those in [23].

Example 4.2. Consider a delayed uncertain DSNNs (3.1) with the following parameters:

$$\begin{gathered} C = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.4 \end{bmatrix}, \qquad A = \begin{bmatrix} 0.2 & 0.6 \\ 0.5 & 0.5 \end{bmatrix}, \quad B = \begin{bmatrix} -0.25 & 0.1 \\ 0.2 & 0.5 \end{bmatrix}, \\ E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad G_C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad G_A = G_B = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, \end{gathered} \quad (4.2)$$

$\rho_1 = \rho_2 = 0.01$, and the activation functions satisfy Assumption 2.1 with $\sigma_1^- = -0.1$, $\sigma_2^- = 0.1$, $\sigma_1^+ = 0.1$, and $\sigma_2^+ = 0.2$.

Table 1: Comparisons of allowable upper bound (τ_M) for given τ_m.

Methods	$\tau_m = 1$	$\tau_m = 3$	$\tau_m = 10$	$\tau_m = 20$	Number of variables
By [23]	48	50	57	67	**81**
By Corollary 3.4	48	50	57	67	**45**

Table 2: Comparisons of allowable upper bound (τ_M) for given τ_m.

Methods	$\tau_m = 1$	$\tau_m = 3$	$\tau_m = 10$	$\tau_m = 20$	Number of variables
By [23]	7	9	16	26	**81**
By Corollary 3.4	7	9	16	26	**45**
By Theorem 3.1	8	10	17	27	**47**

If the time-varying delays satisfy $2 \leq \tau(k) \leq 15$, by the Matlab LMI Toolbox, we can find a solution to the LMI (3.25) as follows:

$$\begin{aligned}
P &= \begin{bmatrix} 558.5625 & -139.9896 \\ -139.9896 & 113.5136 \end{bmatrix}, \qquad Q_1 = \begin{bmatrix} 101.6690 & -27.6867 \\ -27.6867 & 7.8060 \end{bmatrix}, \\
Q_2 &= \begin{bmatrix} 12.4051 \\ -2.9092 \end{bmatrix}, \qquad Q_3 = \begin{bmatrix} 101.7661 & -27.7596 \\ -27.7596 & 7.7780 \end{bmatrix}, \\
Z &= \begin{bmatrix} 9.7355 & -2.4709 \\ -2.4709 & 1.0820 \end{bmatrix}, \qquad K = \begin{bmatrix} 0.3724 & 0 \\ 0 & 0.0937 \end{bmatrix}, \\
U_1 &= \begin{bmatrix} 930.7099 & 0 \\ 0 & 621.0133 \end{bmatrix}, \qquad U_2 = \begin{bmatrix} 135.3270 & 0 \\ 0 & 190.4640 \end{bmatrix}, \\
L_1 &= \begin{bmatrix} 0.0002 & 0.0012 \\ 0.0002 & -0.0004 \end{bmatrix}, \qquad L_2 = \begin{bmatrix} -0.7398 & 0.1904 \\ 0.1898 & -0.0785 \end{bmatrix}, \\
M_1 &= \begin{bmatrix} -0.0349 & 0.0028 \\ 0.0030 & -0.0140 \end{bmatrix}, \qquad M_2 = \begin{bmatrix} 0.7406 & -0.1891 \\ -0.1888 & 0.0806 \end{bmatrix}, \\
N_1 &= \begin{bmatrix} -4.4873 & 1.1774 \\ 1.1714 & -0.4350 \end{bmatrix}, \qquad N_2 = \begin{bmatrix} 0.0358 & -0.0020 \\ -0.0029 & 0.0135 \end{bmatrix},
\end{aligned} \tag{4.3}$$

and $\gamma = 8845.9$, and $\epsilon = 11.9457$. Therefore, by Theorem 3.5, we know that system (3.1) with the above given parameters is robust passive.

5. Conclusions

In this paper, we have considered the problem of passivity and robust passivity analysis for a class of DSNNs with time-varying delay. By choosing a new Lyapunov-Krasovskii functional, the improved delay-dependent criteria have been proposed. Finally, two numerical examples have been provided to illustrate the effectiveness of the obtained results.

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