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Research Article

Global Attractivity and Periodic Solution of a Discrete Multispecies Cooperation and Competition Predator-Prey System

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We propose a discrete multispecies cooperation and competition predator-prey systems. For general nonautonomous case, sufficient conditions which ensure the permanence and the global stability of the system are obtained; for periodic case, sufficient conditions which ensure the existence of a globally stable positive periodic solution of the system are obtained.

1. Introduction

In this paper, we consider the dynamic behavior of the following non-autonomous discrete $n+m$-species cooperation and competition predator-prey systems

$$\begin{aligned} x_i(k+1) &= x_i(k)\exp\left[r_{1i}(k)\left(1-\frac{x_i(k)}{a_i(k)+\sum_{l=1,l\neq i}^{n} b_{il}(k)x_l(k)}-c_i(k)x_i(k)\right)\right. \\ &\qquad\left.-\sum_{l=1}^{m} d_{il}(k)y_l(k)\right], \\ y_j(k+1) &= y_j(k)\exp\left[r_{2j}(k)+\sum_{l=1}^{n} e_{jl}(k)x_l(k)-\sum_{l=1}^{m} p_{jl}(k)y_l(k)\right], \end{aligned} \tag{1.1}$$

where $i=1,2,\ldots n;\ j=1,2,\ldots,m;\ x_i(k)$ is the density of prey species i at kth generation. $y_j(k)$ is the density of predator species j at kth generation.

Dynamic behaviors of population models governed by difference equations had been studied by a number of papers, see [1–4] and the references cited therein. It has been found that the autonomous discrete systems can demonstrate quite rich and complicated dynamics, see [5, 6]. Recently, more and more scholars paid attention to the non-autonomous discrete population models, since such kind of model could be more appropriate.

May [7] suggested the following set of equations to describe a pair of mutualists:

$$\begin{aligned} \dot{u} &= r_1 u\left(1 - \frac{u}{a_1 + b_1 v} - c_1 u\right), \\ \dot{v} &= r_2 v\left(1 - \frac{v}{a_2 + b_2 u} - c_2 v\right), \end{aligned} \tag{1.2}$$

where u, v are the densities of the species U, V at time t, respectively. $r_i,\ a_i,\ b_i,\ c_i,\ i = 1,2$ are positive constants. He showed that system (1.2) has a globally asymptotically stable equilibrium point in the region $u > 0, v > 0$.

Bai et al. [8] argued that the discrete case of cooperative system is more appropriate, and they proposed the following system:

$$\begin{aligned} x_1(k+1) &= x_1(k)\exp\left\{r_1(k)\left[1 - \frac{x_1(k)}{a_1(k) + b_1(k)x_2(k)} - c_1(k)x_1(k)\right]\right\}, \\ x_2(k+1) &= x_2(k)\exp\left\{r_2(k)\left[1 - \frac{x_2(k)}{a_2(k) + b_2(k)x_2(k)} - c_2(k)x_1(k)\right]\right\}. \end{aligned} \tag{1.3}$$

Chen [9] investigated the dynamic behavior of the following discrete $n+m$-species Lotka-Volterra competition predator-prey systems

$$\begin{aligned} x_i(k+1) &= x_i(k)\exp\left[b_i(k) - \sum_{l=1}^{n} a_{il}(k)x_l(k) - \sum_{l=1}^{m} c_{il}(k)y_l(k)\right], \\ y_j(k+1) &= y_j(k)\exp\left[r_j(k) + \sum_{l=1}^{n} d_{jl}(k)x_l(k) - \sum_{l=1}^{m} e_{jl}(k)y_l(k)\right], \end{aligned} \tag{1.4}$$

he investigated the dynamic behavior of the system (1.4).

The aim of this paper is, by further developing the analysis technique of Huo and Li [10] and Chen [9], to obtain a set of sufficient condition which ensure the permanence and the global stability of the system (1.1); for periodic case, sufficient conditions which ensure the existence of a globally stable positive periodic solution of the system (1.1) are obtained.

We say that system (1.1) is permanent if there are positive constants M and m such that for each positive solution $(x_1(k), \ldots, x_n(k), y_1(k), \ldots, y_m(k))$ of system (1.1) satisfies

$$\begin{aligned} m &\le \lim_{k\to+\infty}\inf x_i(k) \le \lim_{k\to+\infty}\sup x_i(k) \le M, \\ m &\le \lim_{k\to+\infty}\inf y_i(k) \le \lim_{k\to+\infty}\sup y_i(k) \le M, \end{aligned} \tag{1.5}$$

for all $i = 1, 2, \ldots, n;\ j = 1, 2, \ldots, m$.

Throughout this paper, we assume that $r_{1i}(k)$, $b_{il}(k)$, $a_i(k)$, $c_i(k)$, $r_{2j}(k)$, $d_{il}(k)$, $e_{jl}(k)$, $p_{jl}(k)$ are all bounded nonnegative sequence, and use the following notations for any bounded sequence $\{a(k)\}$

$$a^u = \sup_{k \in N} a(k), \qquad a^l = \inf_{k \in N} a(k). \tag{1.6}$$

For biological reasons, we only consider solution $(x_1(k), \ldots, x_n(k), y_1(k), \ldots, y_m(k))$ with

$$x_i(0) > 0; \quad i = 1, 2, \ldots, n, \qquad y_j(0) > 0, \quad j = 1, 2, \ldots, m. \tag{1.7}$$

Then system (1.1) has a positive solution $(x_1(k), \ldots, x_n(k), y_1(k), \ldots, y_m(k))_{k=0}^{\infty}$ passing through $(x_1(0), \ldots, x_n(0), y_1(0), \ldots, y_m(0))$.

The organization of this paper is as follows. In Section 2, we obtain sufficient conditions which guarantee the permanence of the system (1.1). In Section 3, we obtain sufficient conditions which guarantee the global stability of the positive solution of system (1.1). As a consequence, for periodic case, we obtain sufficient conditions which ensure the existence of a globally stable positive solution of system (1.1).

2. Permanence

In this section, we establish permanence results for system (1.1).

Lemma 2.1 (see [11]). *Let $k \in N_{k_0}^+ = \{k_0, k_0 + 1, \ldots, k_0 + l, \ldots\}$, $r \geq 0$. For any fixed k, $g(k, r)$ is a non-decreasing function with respect to r, and for $k \geq k_0$, the following inequalities hold:*

$$\begin{aligned} y(k+1) &\leq g(k, y(k)), \\ u(k+1) &\geq g(k, u(k)). \end{aligned} \tag{2.1}$$

If $y(k_0) \leq u(k_0)$, then $y(k) \leq u(k)$ for all $k \geq k_0$.

Now let one consider the following single species discrete model:

$$N(k+1) = N(k) \exp\{a(k) - b(k)N(k)\}, \tag{2.2}$$

where $\{a(k)\}$ and $\{b(k)\}$ are strictly positive sequences of real numbers defined for $k \in N = \{0, 1, 2, \ldots\}$ and $0 < a^l \leq a^u$, $0 < b^l \leq b^u$.

Lemma 2.2 (see [12]). *Any solution of system* (2.5) *with initial condition $N(0) > 0$ satisfies*

$$m \leq \lim_{k \to +\infty} \inf N(k) \leq \lim_{k \to +\infty} \sup N(k) \leq M, \tag{2.3}$$

where

$$M = \frac{1}{b^l}\exp\{a^u - 1\}, \qquad m = \frac{a^l}{b^u}\exp\left\{a^l - b^u M\right\}. \tag{2.4}$$

Proposition 2.3. *Assume that*

$$-r_{2j}^u + \sum_{l=1}^{n} e_{jl}^l M_l > 0, \quad j = 1, 2, \ldots, m \tag{2.5}$$

holds, then

$$\begin{aligned} &\lim_{k\to+\infty}\sup x_i(k) \le M_i, \quad i = 1, 2, \ldots, n, \\ &\lim_{k\to+\infty}\sup y_i(k) \le Q_i, \quad i = 1, 2, \ldots, m, \end{aligned} \tag{2.6}$$

where

$$\begin{aligned} M_i &= \frac{1}{c_i^l r_{1i}^l}\exp\{r_{1i}^u - 1\}, \\ Q_i &= \frac{1}{p_{ii}^l}\exp\left\{\sum_{l=1}^{n} e_{il}^u M_l - r_{2i}^l - 1\right\}. \end{aligned} \tag{2.7}$$

Proof. Let $u(k) = (x_1(k), \ldots, x_n(k), y_1(k), \ldots, y_m(k))$ be any positive solution of system (1.1), from the ith equation of (1.1) we have

$$\begin{aligned} x_i(k+1) &\le x_i(k)\exp\{r_{1i}(k)[1 - c_i(k)x_i(k)]\} \\ &= x_i(k)\exp\{r_{1i}(k) - r_{1i}(k)c_i(k)x_i(k)\}. \end{aligned} \tag{2.8}$$

By applying Lemmas 2.1 and 2.2, it immediately follows that

$$\lim_{k\to+\infty}\sup x_i(k) \le \frac{1}{c_i^l r_{1i}^l}\exp\{r_{1i}^u - 1\} := M_i. \tag{2.9}$$

For any positive constant ε small enough, it follows from (2.9) that there exists enough large K_0 such that

$$x_i(k) \le M_i + \varepsilon, \quad i = 1, 2, \ldots, n, \quad \forall k \ge K_0. \tag{2.10}$$

From the $n + j$th equation of the system (1.1) and (2.10), we can obtain

$$y_j(k+1) \le y_j(k)\exp\left\{-r_{2j}(k) + \sum_{l=1}^{n} e_{jl}(k)(M_l + \varepsilon) - p_{jj}(k)y_j(k)\right\}. \tag{2.11}$$

Condition (2.5) shows that Lemmas 2.1 and 2.2 could be applied to (2.11), and so, by applying Lemmas 2.1 and 2.2, it immediately follows that

$$\lim_{k\to+\infty} \sup y_j(k) \le \frac{1}{p_{jj}^l} \exp\left\{\sum_{l=1}^{n} e_{jl}^u (M_l+\varepsilon) - r_{2j}^l - 1\right\}, \quad 1 \le j \le m. \tag{2.12}$$

Setting $\varepsilon \to 0$ in the above inequality leads to

$$\lim_{k\to+\infty} \sup y_j(k) \le \frac{1}{p_{jj}^l} \exp\left\{\sum_{l=1}^{n} e_{jl}^u M_l - r_{2j}^l - 1\right\} := Q_j, \quad 1 \le j \le m. \tag{2.13}$$

This completes the proof of Proposition 2.3. □

Now we are in the position of stating the permanence of the system (1.1).

Theorem 2.4. *In addition to* (2.5)*, assume further that*

$$\begin{gathered} r_{1i}^l - \sum_{l=1}^{m} d_{il}^u Q_l > 0, \quad i = 1,2,\ldots,n, \\ -r_{2j}^u + \sum_{l=1}^{n} e_{jl}^l m_l - \sum_{l=1, l\ne j}^{m} p_{jl}^u Q_l > 0, \quad j = 1,2,\ldots m, \end{gathered} \tag{2.14}$$

then system (1.1) *is permanent, where*

$$\begin{gathered} m_i = \frac{r_{1i}^l - \sum_{l=1}^{m} d_{il}^u Q_l}{r_{1i}^u\left(1/a_i^l + c_i^u\right)} \exp\left\{ r_{1i}^l - \sum_{l=1}^{m} d_{il}^u Q_l - r_{1i}^u\left(\frac{1}{a_i^l} + c_i^u\right) M_i \right\}, \\ Q_i = \frac{1}{p_{ii}^l} \exp\left\{\sum_{l=1}^{n} e_{il}^u M_l - r_{2i}^l - 1\right\}. \end{gathered} \tag{2.15}$$

Proof. By applying Proposition 2.3, we see that to end the proof of Theorem 2.4, it is enough to show that under the conditions of Theorem 2.4,

$$\begin{gathered} \lim_{k\to+\infty} \inf x_j(k) \ge m_i, \quad 1 \le i \le n, \\ \lim_{k\to+\infty} \inf y_j(k) \ge q_j, \quad 1 \le j \le m. \end{gathered} \tag{2.16}$$

From Proposition 2.3, $\forall \varepsilon > 0$, there exists a $K_1 > 0, K_1 \in N, \forall k > K_1$,

$$x_i(k) \le M_i + \varepsilon, \quad i = 1,2,\ldots,n, \qquad y_j(k) \le Q_j + \varepsilon, \quad j = 1,2,\ldots,m. \tag{2.17}$$

From the ith equation of system (1.1) and (2.17), we have

$$
\begin{aligned}
x_i(k+1) &\geq x_i(k)\exp\left\{r_{1i}(k) - r_{1i}(k)\left(\frac{1}{a_i^l} + c_i^u\right)x_i(k) - \sum_{l=1}^{m} d_{il}(k)(Q_l+\varepsilon)\right\} \\
&= x_i(k)\exp\left\{r_{1i}(k) - \sum_{l=1}^{m} d_{il}(k)(Q_l+\varepsilon) - r_{1i}(k)\left(\frac{1}{a_i^l} + c_i^u\right)x_i(k)\right\},
\end{aligned}
\tag{2.18}
$$

for all $k > K_1$.

By applying Lemmas 2.1 and 2.2 to (2.18), it immediately follows that

$$
\begin{aligned}
\liminf_{k\to+\infty} x_i(k) \geq{}& \frac{r_{1i}^l - \sum_{l=1}^{m} d_{il}^u(Q_l+\varepsilon)}{r_{1i}^u\left(1/a_i^l + c_i^u\right)} \\
&\times \exp\left\{r_{1i}^l - \sum_{l=1}^{m} d_{il}^u(Q_l+\varepsilon) - r_{1i}^u\left(\frac{1}{a_i^l} + c_i^u\right)M_i\right\}.
\end{aligned}
\tag{2.19}
$$

Setting $\varepsilon \to 0$ in (2.19) leads to

$$
\begin{aligned}
\liminf_{k\to+\infty} x_i(k) \geq{}& \frac{r_{1i}^l - \sum_{l=1}^{m} d_{il}^u Q_l}{r_{1i}^u\left(1/a_i^l + c_i^u\right)} \\
&\times \exp\left\{r_{1i}^l - \sum_{l=1}^{m} d_{il}^u Q_l - r_{1i}^u\left(\frac{1}{a_i^l} + c_i^u\right)M_i\right\} := m_i.
\end{aligned}
\tag{2.20}
$$

Then, for any positive constant ε small enough, from (2.20) we know that there exists an enough large $K_2 > K_1$ such that

$$
x_i(k) \geq m_i - \varepsilon, \ \forall\ k \geq k_2. \tag{2.21}
$$

Equations (2.17), (2.21) combining with the $n+j$th equation of the system (1.1) leads to

$$
\begin{aligned}
y_j(k+1) \geq y_j(k)\exp\Bigg\{&-r_{2j}(k) + \sum_{l=1}^{n} e_{jl}(k)(m_l-\varepsilon) \\
&- \sum_{l=1, l\neq j}^{m} p_{jl}(k)(Q_l+\varepsilon) - p_{jj}(k)y_j(k)\Bigg\},
\end{aligned}
\tag{2.22}
$$

under the condition (2.14), by applying Lemmas 2.1 and 2.2 to (2.22), it immediately follows that

$$\liminf_{k\to+\infty} y_j(k) \geq \frac{-r_{2j}^u + \sum_{l=1}^n e_{jl}^l (m_l - \varepsilon) - \sum_{l=1, l\neq j}^m p_{jl}^u (Q_l + \varepsilon)}{p_{jj}^u} \times \exp\left\{ -r_{2j}^u + \sum_{l=1}^n e_{jl}^l (m_l - \varepsilon) - \sum_{l=1, l\neq j}^m p_{jl}^u (Q_l + \varepsilon) - p_{jj}^u Q_j \right\}. \tag{2.23}$$

Setting $\varepsilon \to 0$ in (2.23) leads to

$$\liminf_{k\to+\infty} y_j(k) \geq \frac{-r_{2j}^u + \sum_{l=1}^n e_{jl}^l m_l - \sum_{l=1, l\neq j}^m p_{jl}^u Q_l}{p_{jj}^u} \times \exp\left\{ -r_{2j}^u + \sum_{l=1}^n e_{jl}^l m_l - \sum_{l=1, l\neq j}^m p_{jl}^u Q_l - p_{jj}^u Q_j \right\} := q_j. \tag{2.24}$$

This ends the proof of Theorem 2.4. □

It should be noticed that, under the assumption of Theorem 2.4, the set

$$[m_1, M_1] \times \cdots [m_n, M_n] \times [q_1, Q_1] \times \cdots [q_m, Q_m] \tag{2.25}$$

is an invariant set of system (1.1).

3. Global Stability

Now we study the stability property of the positive solution of system (1.1).

Theorem 3.1. *Assume that*

$$\lambda_i = \max\left\{ \left| 1 - \left(c_i^u + \frac{1}{a_i^l} \right) r_{1i}^u M_i \right|, \left| 1 - \left(c_i^l + \frac{1}{a_i^u} \right) r_{1i}^l m_i \right| \right\} + \frac{r_{1i}^u M_i}{\left(a_i^l\right)^2} \sum_{l=1, l\neq i}^n b_{il}^u M_l + \sum_{l=1}^m d_{il}^u Q_l < 1, \tag{3.1}$$

$$\delta_j = \max\left\{ \left| 1 - p_{jj}^u Q_j \right|, \left| 1 - p_{jj}^l q_j \right| \right\} + \sum_{l=1}^n e_{jl}^u M_l + \sum_{l=1}^n p_{jl}^u Q_l < 1.$$

Then for any two positive solution $(x_1(k), \ldots, x_n(k), y_1(k), \ldots, y_m(k))$ *and* $(\widetilde{x}_1(k), \ldots, \widetilde{x}_n(k), \widetilde{y}_1(k), \ldots, \widetilde{y}_m(k))$ *of system* (1.1)*, one has*

$$\lim_{k\to+\infty} (\widetilde{x}_i(k) - x_i(k)) = 0, \qquad \lim_{k\to+\infty} \left(\widetilde{y}_j(k) - y_j(k)\right) = 0. \tag{3.2}$$

Proof. Let

$$x_i(k) = \tilde{x}_i(k)\exp\{u_i(k)\}, \qquad y_j(k) = \tilde{y}_j(k)\exp\{v_j(k)\}. \tag{3.3}$$

Then system (1.1) is equivalent to

$$\begin{aligned} u_i(k+1) &= u_i(k) + \frac{r_{1i}(k)\tilde{x}_i(k)}{a_i(k) + \sum_{l=1,l\neq i}^{n} b_{il}(k)\tilde{x}_l(k)} - \frac{r_{1i}(k)\tilde{x}_i(k)\exp\{u_i(k)\}}{a_i(k) + \sum_{l=1,l\neq i}^{n} b_{il}(k)\tilde{x}_l(k)\exp\{u_l(k)\}} \\ &\quad - r_{1i}(k)c_i(k)\tilde{x}_i(k)\left(\exp\{u_i(k)\} - 1\right) - \sum_{l=1}^{m} d_{il}(k)\tilde{y}_l(k)\left(\exp\{v_l(k)\} - 1\right), \\ v_j(k+1) &= v_j(k) - \sum_{l=1}^{n} e_{jl}(k)\tilde{x}_l(k)\left(\exp\{u_l(k)\} - 1\right) \\ &\quad - \sum_{l=1}^{m} p_{jl}(k)\tilde{y}_l(k)\left(\exp\{v_l(k)\} - 1\right). \end{aligned} \tag{3.4}$$

So,

$$\begin{aligned} |u_i(k+1)| &\leq \left|1 - \left(c_i(k) + \frac{1}{a_i(k)}\right)r_{1i}(k)\tilde{x}_i(k)\exp\{\theta_i(k)u_i(k)\}\right||u_i(k)| \\ &\quad + \frac{r_{1i}(k)\tilde{x}_i(k)}{a_i^2(k)}\sum_{l=1,l\neq i}^{n} b_{il}(k)\tilde{x}_l(k)\exp\{\theta_l(k)u_l(k)\}|u_l(k)| \\ &\quad + \sum_{l=1}^{m} d_{il}(k)\tilde{y}_l(k)\exp\{\xi_l(k)v_l(k)\}|v_l(k)| \\ |v_i(k+1)| &\leq \left|1 - p_{jj}(k)\tilde{y}_j(k)\exp\{\xi_j(k)v_j(k)\}\right|\left|v_j(k)\right| \\ &\quad + \sum_{l=1}^{n} e_{jl}(k)\tilde{x}_l(k)\exp\{\theta_l(k)u_l(k)\}|u_l(k)| \\ &\quad + \sum_{l=1}^{m} p_{il}(k)\tilde{y}_l(k)\exp\{\xi_l(k)v_l(k)\}|v_l(k)|, \end{aligned} \tag{3.5}$$

where $\theta_l(k), \xi_l(k) \in [0,1]$, to complete the proof, it suffices to show that

$$\lim_{k\to+\infty} u_i(k) = 0, \qquad \lim_{k\to+\infty} v_j(k) = 0. \tag{3.6}$$

In view of (3.1), we can choose $\varepsilon > 0$ small enough such that

$$\lambda_i^{\varepsilon} = \max\left\{\left|1-\left(c_i^u+\frac{1}{a_i^l}\right)r_{1i}^u(M_i+\varepsilon)\right|,\left|1-\left(c_i^l+\frac{1}{a_i^u}\right)r_{1i}^l(m_i-\varepsilon)\right|\right\} + \frac{r_{1i}^u(M_i+\varepsilon)}{\left(a_i^l\right)^2}\sum_{l=1,l\neq i}^{n} b_{il}^u(M_l+\varepsilon) + \sum_{l=1}^{m} d_{il}^u(Q_l+\varepsilon) < 1,$$
$$\delta_j^{\varepsilon} = \max\left\{\left|1-p_{jj}^u(Q_j+\varepsilon)\right|,\left|1-p_{jj}^l(q_j-\varepsilon)\right|\right\} + \sum_{l=1}^{n} e_{jl}^u(M_l+\varepsilon) + \sum_{l=1}^{n} p_{jl}^u(Q_l+\varepsilon) < 1. \tag{3.7}$$

For the above $\varepsilon > 0$, according to Theorem 2.4 in Section 2, there exists a $k^* \in N$ such that

$$m_i - \varepsilon \le x_i(k), \quad \tilde{x}_i(k) \le M_i + \varepsilon,$$
$$q_j - \varepsilon \le y_j(k), \quad \tilde{y}_j(k) \le Q_j + \varepsilon, \tag{3.8}$$

for all $k \ge k^*$.

Noticing that $\theta_l(k), \xi_l(k) \in [0,1]$ implies that $\tilde{x}_l(k)\exp\{\theta_l(k)u_l(k)\}$ lies between $\tilde{x}_l(k)$ and $x_l(k)$, $\tilde{y}_l(k)\exp\{\xi_l(k)v_l(k)\}$ lies between $\tilde{y}_l(k)$ and $y_l(k)$. From (3.5), we get

$$|u_i(k+1)| \le \max\left\{\left|1-\left(c_i^u+\frac{1}{a_i^l}\right)r_{1i}^u(M_i+\varepsilon)\right|,\left|1-\left(c_i^l+\frac{1}{a_i^u}\right)r_{1i}^l(m_i-\varepsilon)\right|\right\}|u_i(k)| + \frac{r_{1i}^u(M_i+\varepsilon)}{\left(a_i^l\right)^2}\sum_{l=1,l\neq i}^{n} b_{il}^u(M_l+\varepsilon)|u_l(k)| + \sum_{l=1}^{m} d_{il}^u(Q_l+\varepsilon)|v_l(k)|,$$
$$\left|v_j(k+1)\right| \le \max\left\{\left|1-p_{jj}^u(Q_j+\varepsilon)\right|,\left|1-p_{jj}^l(q_j-\varepsilon)\right|\right\}\left|v_j(k)\right| + \sum_{l=1}^{n} e_{jl}^u(M_l+\varepsilon)|u_l(k)| + \sum_{l=1}^{n} p_{jl}^u(Q_l+\varepsilon)|v_l(k)|. \tag{3.9}$$

Let $\gamma = \max\{\lambda_i^{\varepsilon}, \delta_j^{\varepsilon}\}$, then $\gamma < 1$. In view of (3.9), for $k \ge k^*$, we get

$$\max\left\{|u_i(k+1)|,\left|v_j(k+1)\right|\right\} \le \gamma \max\left\{|u_i(k)|,\left|v_j(k)\right|\right\}. \tag{3.10}$$

This implies

$$\max\left\{|u_i(k)|,\left|v_j(k)\right|\right\} \le \gamma^{k-k^*}\max\left\{|u_i(k^*)|,\left|v_j(k^*)\right|\right\}. \tag{3.11}$$

From (3.11), we have

$$\lim_{k\to+\infty} u_i(k) = 0, \qquad \lim_{k\to+\infty} v_j(k) = 0. \tag{3.12}$$

This ends the proof of Theorem 3.1. □

4. Existence and Stability of Periodic Solution

In this section, we further assume that the coefficients of the system (1.1) satisfies (4.1).

There exists a positive integer ω such that for $k \in N$,

$$\begin{aligned} &0 < r_{1i}(k+\omega) = r_{1i}(k), \qquad 0 < b_{il}(k+\omega) = b_{1i}(k), \\ &0 < a_i(k+\omega) = a_i(k), \qquad 0 < c_i(k+\omega) = c_i(k), \\ &0 < r_{2j}(k+\omega) = r_{2j}(k), \qquad 0 < d_{il}(k+\omega) = d_{il}(k) \\ &0 < e_{jl}(k+\omega) = e_{jl}(k), \qquad 0 < p_{jl}(k+\omega) = p_{jl}(k). \end{aligned} \tag{4.1}$$

Our first result concerned with the existence of a positive periodic solution of system (1.1).

Theorem 4.1. *Assume that* (2.5) *and* (2.14) *hold, then system* (1.1) *admits at least one positive ω-periodic solution which ones denotes by* $(\tilde{x}_1(k), \dots, \tilde{x}_n(k), \tilde{y}_1(k), \dots, \tilde{y}_m(k))$.

Proof. As noted at the end of Section 2,

$$D^{n+m} := [m_1, M_1] \times \cdots [m_n, M_n] \times [q_1, Q_1] \times \cdots [q_m, Q_m] \tag{4.2}$$

is an invariant set of system (1.1). Thus, we can define a mapping F on D^{n+m} by

$$F(x_1(0), \dots, x_n(0), y_1(0), \dots, y_m(0)) = (x_1(\omega), \dots, x_n(\omega), y_1(\omega), \dots, y_m(\omega)), \tag{4.3}$$

for $(x_1(0), \dots, x_n(0), y_1(0), \dots, y_m(0)) \in D^{n+m}$. Obviously, F depends continuously on $(x_1(0), \dots, x_n(0), y_1(0), \dots, y_m(0))$. Thus, F is continuous and maps the compact set D^{n+m} into itself. Therefore, F has a fixed point. It is easy to see that the solution $(\tilde{x}_1(k), \dots, \tilde{x}_n(k), \tilde{y}_1(k), \dots, \tilde{y}_m(k))$ passing through this fixed point is an ω-periodic solution of the system (1.1). This completes the proof of Theorem 4.1. □

Theorem 4.2. *Assume that* (2.5), (2.14), *and* (3.1) *hold, then system* (1.1) *has a global stable positive ω-periodic solution.*

Proof. Under the assumption of Theorem 4.2, it follows from Theorem 4.1 that system (1.1) admits at least one positive ω-periodic solution. Also, Theorem 3.1 ensures the positive solution to be globally stable. This ends the proof of Theorem 4.2. □

References

[1] P. Cull, "Global stability of population models," *Bulletin of Mathematical Biology*, vol. 43, no. 1, pp. 47–58, 1981.

[2] J. E. Franke and A.-A. Yakubu, "Geometry of exclusion principles in discrete systems," *Journal of Mathematical Analysis and Applications*, vol. 168, no. 2, pp. 385–400, 1992.

[3] Y. Huang, "A note on global stability for discrete one-dimensional population models," *Mathematical Biosciences*, vol. 102, no. 1, pp. 121–124, 1990.

[4] W. D. Wang and Z. Y. Lu, "Global stability of discrete models of Lotka-Volterra type," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 35, no. 8, pp. 1019–1030, 1999.

[5] Y. Saito, W. Ma, and T. Hara, "A necessary and sufficient condition for permanence of a Lotka-Volterra discrete system with delays," *Journal of Mathematical Analysis and Applications*, vol. 256, no. 1, pp. 162–174, 2001.

[6] M. P. Hassell and H. N. Comins, "Discrete time models for two-species competition," *Theoretical Population Biology*, vol. 9, no. 2, pp. 202–221, 1976.

[7] R. M. May, *Theoretical Ecology*, Principles and Applications, Sounders, Philadelphia, Pa, USA, 1976.

[8] L. Bai, M. Fan, and K. Wang, "Existence of positive periodic solution for difference equations of a cooperative system," *Journal of Biomathematics*, vol. 19, no. 3, pp. 271–279, 2004 (Chinese).

[9] F. Chen, "Permanence and global attractivity of a discrete multispecies Lotka-Volterra competition predator-prey systems," *Applied Mathematics and Computation*, vol. 182, no. 1, pp. 3–12, 2006.

[10] H.-F. Huo and W.-T. Li, "Permanence and global stability for nonautonomous discrete model of plankton allelopathy," *Applied Mathematics Letters*, vol. 17, no. 9, pp. 1007–1013, 2004.

[11] L. Wang and M. Q. Wang, *Ordinary Difference Equation*, Xin-jiang University Press, Xin-jiang, China, 1991.

[12] F. Chen, "Permanence of a discrete n-species food-chain system with time delays," *Applied Mathematics and Computation*, vol. 185, no. 1, pp. 719–726, 2007.