

Research Article

Global Behavior of a Discrete Survival Model with Several Delays

Meirong Xu^{1,2} and Yuzhen Wang¹

¹ School of Control Science and Engineering, Shandong University, Jinan 250061, China

² School of Mathematical Sciences, University of Jinan, Jinan 250022, China

Correspondence should be addressed to Meirong Xu; ss_xumr@ujn.edu.cn

Received 21 September 2013; Revised 28 October 2013; Accepted 11 November 2013

Academic Editor: Mohamad Alwash

Copyright © 2013 M. Xu and Y. Wang. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

The difference equation $y_{n+1} - y_n = -\alpha y_n + \sum_{j=1}^m \beta_j e^{-\gamma_j y_{n-k_j}}$ is studied and some sufficient conditions which guarantee that all solutions of the equation are oscillatory, or that the positive equilibrium of the equation is globally asymptotically stable, are obtained.

1. Introduction

The delay differential equation

$$\frac{dN(t)}{dt} = -\alpha N(t) + \beta e^{-\gamma N(t-\tau)}, \quad \text{for } t \geq 0, \quad (1)$$

was first proposed by Wazewska-Czyzewska and Lasota [1] as a model for the survival of red blood cell in an animal. Here, $N(t)$ denotes the number of red blood cells at time t , α is the probability of death of red blood cells, β and γ are positive constants which are related to the production of red blood cells, and τ is the time which is required to produce a red blood cell. The oscillation and global attractivity of (1) were studied by Gyóri and Ladas [2] and Li and Cheng [3], while the bifurcation and the direction of the stability were investigated by Song et al. [4]. Xu and Li [5] and Liu [6] considered its generalization with several delays and obtained sufficient conditions for the global stability of survival blood cells model with several delays and piecewise constant argument.

Research on the oscillation and global stability of the discrete analogue of (1), that is, for the equation

$$x_{n+1} - x_n = -\alpha x_n + \beta e^{-\gamma x_{n-k}}, \quad n = 0, 1, 2, \dots, \quad (2)$$

where

$$\alpha \in (0, 1), \quad \beta, \gamma \in (0, \infty), \quad k \in \{1, 2, \dots\}, \quad (3)$$

was proposed by Kocić and Ladas [7] as an open problem.

Kubiacyk and Saker [8] investigated the oscillation of (2) about its positive equilibrium point \bar{x} , where \bar{x} is the unique solution of the equation

$$\alpha \bar{x} = \beta e^{-\gamma \bar{x}}, \quad (4)$$

and showed that every solution of (2) oscillates about \bar{x} if

$$\beta \gamma e^{-\gamma \bar{x}} > \left(\frac{k}{k+1} \right)^{k+1} (1 - \alpha)^{k+1}. \quad (5)$$

Meng and Yan [9] investigated the global attractivity of the positive equilibrium point \bar{x} and showed that \bar{x} is a global attractor of all positive solutions of (2) if

$$\frac{\beta^2 \gamma^2}{\alpha^2} e^{\gamma(Q_1 + \bar{x})} < 1, \quad (6)$$

where $Q_1 = (\beta/\alpha)e^{-\beta\gamma/\alpha}$.

Zeng and Shi [10] established another condition for global attractivity of \bar{x} and showed that \bar{x} is a global attractor of all positive solutions of (2) if

$$\frac{\beta \gamma}{\alpha} \leq e. \quad (7)$$

Obviously, the condition (7) improves (6).

Kubiacyk and Saker [8] also considered (2) when $k = 1$ and proved that \bar{x} is a global attractor of all positive solutions of (2) provided that

$$\beta \gamma e^{-\gamma \bar{x}} < \alpha. \quad (7')$$

Ma and Yu [11] proved that \bar{x} is a global attractor of all solutions of (2) if

$$\gamma\bar{x}(1 - (1 - \alpha)^{k+1}) \leq 1. \tag{8}$$

By (2), we have

$$\bar{x}e^{\gamma\bar{x}} = \frac{\beta}{\alpha}. \tag{9}$$

So, if (7) holds, then we have $\gamma\bar{x}e^{\gamma\bar{x}} = \beta\gamma/\alpha \leq e$, which implies that $\gamma\bar{x} \leq 1$. Hence, (8) is satisfied. But, the converse is not true. So, the condition (8) improves (7).

In addition, we can also easily see that the conditions (7) and (7') are equivalent to the condition $\gamma\bar{x} \leq 1$.

For the system with delay, many authors deemed that arbitrary finite number of discrete delays is more appropriate than the single discrete delay; see [12–14] and the references cited therein.

Stemming from the above discussion, the difference equation in the following form will be studied in this paper:

$$y_{n+1} - y_n = -\alpha y_n + \sum_{j=1}^m \beta_j e^{-\gamma_j y_{n-k_j}}, \tag{10}$$

where

$$\alpha \in (0, 1), \quad \beta_j, \gamma_j \in (0, \infty), \quad k_j \in \{1, 2, \dots\},$$

$$j = 1, 2, \dots, m; \quad \sum_{j=1}^m \beta_j = \beta. \tag{11}$$

Besides, we denote that $k = \max_{1 \leq j \leq m} \{k_j\}$, $l = \min_{1 \leq j \leq m} \{k_j\}$, $\gamma^* = \min_{1 \leq j \leq m} \{\gamma_j\}$.

Obviously, the case $m = 1$ is the form of (2). Besides, if $y_{-k}, y_{-k+1}, \dots, y_{-1}, y_0 \in [0, \infty)$, then, the corresponding solution of (10) is positive, and (10) has a unique positive equilibrium point \bar{y} , which satisfies

$$\alpha\bar{y} = \sum_{j=1}^m \beta_j e^{-\gamma_j \bar{y}}. \tag{12}$$

The aim of this paper is to investigate the oscillation and the global asymptotic stability of (10).

2. Some Lemmas

Lemma 1 (see [7, page 6]). Assume that $p_i \in (0, \infty)$ and $k_i \in \{0, 1, \dots\}$ with $\sum_{i=1}^m (p_i + k_i) \neq 1$, $i = 1, 2, \dots, m$. Let $\{p_i(n)\}$ be sequences of positive numbers such that

$$\liminf_{n \rightarrow \infty} p_i(n) \geq p_i \quad \text{for } i = 1, 2, \dots, m. \tag{13}$$

Suppose that the linear difference inequality

$$z_{n+1} - z_n + \sum_{i=1}^m p_i(n) z_{n-k_i} \leq 0 \quad \text{for } n = 0, 1, \dots \tag{14}$$

has an eventually positive solution. Then, the difference equation

$$x_{n+1} - x_n + \sum_{i=1}^m p_i x_{n-k_i} = 0 \tag{15}$$

has a positive solution.

Lemma 2 (see [7, page 5]). Consider the linear homogeneous difference equation

$$x_{n+k} + \sum_{i=1}^k q_i x_{n+k-i} = 0 \quad \text{for } n = 0, 1, \dots, \tag{16}$$

where k is a nonnegative integer and $q_i \in R$, $i = 1, 2, \dots, k$. Then, the following statements are equivalent:

- (a) every solution of (16) oscillates;
- (b) the characteristic equation of (16)

$$\lambda^k + \sum_{i=1}^k q_i \lambda^{k-i} = 0 \tag{17}$$

has no positive roots.

Lemma 3 (see [7, page 12]). Assume that $p_1, p_2, \dots, p_k \in R$ and k is a nonnegative integer. Then, $\sum_{i=1}^k |p_i| < 1$ is a sufficient condition for the asymptotic stability of the difference equation

$$x_{n+k} + p_1 x_{n+k-1} + \dots + p_k x_n = 0 \quad \text{for } n = 0, 1, \dots \tag{18}$$

Lemma 4. Assume that (11) holds, and $\{y_n\}$ is a solution of (10) with positive initial conditions y_{-k}, \dots, y_0 . Then,

$$\limsup_{n \rightarrow \infty} y_n \leq \frac{\beta}{\alpha}. \tag{19}$$

Proof. Clearly, we have $y_n > 0$, for $n = -k, -k + 1, \dots, 0, 1, 2, \dots$. So by (10), we can find that

$$y_{n+1} = (1 - \alpha) y_n + \sum_{j=1}^m \beta_j e^{-\gamma_j y_{n-k_j}} \leq (1 - \alpha) y_n + \beta. \tag{20}$$

Define a sequence $\{\omega_n\}$ by

$$\omega_{n+1} = (1 - \alpha) \omega_n + \beta, \quad \omega_0 = y_0. \tag{21}$$

Obviously,

$$y_n \leq \omega_n = (1 - \alpha)^n \omega_0 + \frac{\beta}{\alpha} [1 - (1 - \alpha)^n]. \tag{22}$$

So, we have

$$\limsup_{n \rightarrow \infty} y_n \leq \frac{\beta}{\alpha}. \tag{23}$$

□

Lemma 5. Assume that (11) holds, and

$$\sum_{j=1}^m \beta_j \gamma_j e^{-r_j \bar{y}} < \alpha. \tag{24}$$

Then, the positive equilibrium \bar{y} of (10) is locally asymptotically stable.

Proof. To prove that the positive equilibrium \bar{y} is locally asymptotically stable, it suffices to prove that the zero solution of the linear equation of (10) is locally asymptotically stable. The linearized equation associated with (10) about positive equilibrium \bar{y} is

$$y_{n+1} = (1 - \alpha) y_n - \sum_{j=1}^m \beta_j \gamma_j e^{-\gamma_j \bar{y}} y_{n-k_j}, \tag{25}$$

which satisfies

$$|1 - \alpha| + \sum_{j=1}^m |\beta_j \gamma_j e^{-\gamma_j \bar{y}}| \leq 1. \tag{26}$$

Then, by Lemma 3, the positive equilibrium solution \bar{y} of (10) is locally asymptotically stable. \square

Lemma 6 (see [15]). The following system of inequalities,

$$\mu \leq e^{-\lambda} - 1, \quad \lambda \geq e^{-\mu} - 1, \tag{27}$$

with λ, μ being real numbers, have exactly one solution $\lambda = \mu = 0$.

3. Main Results

Theorem 7. Assume that (11) holds, and

$$\sum_{j=1}^m \beta_j \gamma_j e^{-\gamma_j \bar{y}} (1 - \alpha)^{-l-1} > 1. \tag{28}$$

Then, every positive solution of (10) oscillates about the positive equilibrium \bar{y} .

Proof. Assume for the sake of contradiction that (10) has a positive solution $\{y_n\}$ which does not oscillate about \bar{y} . We assume that $y_n > \bar{y}$ eventually. If $y_n < \bar{y}$ eventually, the proof is similar and will be omitted. So, there exists an $n_0 \geq 0$ such that $y_n > \bar{y}$ for $n \geq n_0$, and consequently $y_{n-k} > \bar{y}$ for $n \geq n_1$, where $n_1 = n_0 + k$.

From Lemma 4, we have $\{y_n\}$ as a bounded sequence. In the following, we will claim that

$$\lim_{n \rightarrow \infty} y_n = \bar{y}. \tag{29}$$

Otherwise, let

$$\mu = \limsup_{n \rightarrow \infty} y_n. \tag{30}$$

Then, $\mu > \bar{y}$ and there exists a subsequence $\{y_{n_i}\}$ such that

$$\lim_{i \rightarrow \infty} y_{n_i+1} = \mu, \quad y_{n_i+1} - y_{n_i} > 0 \tag{31}$$

for $n_i \geq n_1, i = 1, 2, \dots$

Equation (10) can be reformulated in the form

$$y_{n+1} = (1 - \alpha) y_n + \sum_{j=1}^m \beta_j e^{-\gamma_j y_{n-k_j}}. \tag{32}$$

Then, from (31) and (32), we find that

$$\alpha y_{n_i+1} \leq \sum_{j=1}^m \beta_j e^{-\gamma_j y_{n_i-k_j}}. \tag{33}$$

So, we obtain

$$\alpha \mu \leq \limsup_{i \rightarrow \infty} \sum_{j=1}^m \beta_j e^{-\gamma_j y_{n_i-k_j}} \leq \sum_{j=1}^m \beta_j e^{-\gamma_j \bar{y}} = \alpha \bar{y}, \tag{34}$$

which is a contradiction. Accordingly, (29) holds.

Set

$$y_n = \bar{y} + x_n \quad \text{for } n = -k, -k + 1, \dots \tag{35}$$

By the assumption $y_n > \bar{y}$, we have that x_n is an eventually positive solution of the difference equation

$$x_{n+1} - (1 - \alpha) x_n + \alpha \bar{y} - \sum_{j=1}^m \beta_j e^{-\gamma_j (\bar{y} + x_{n-k_j})} = 0, \tag{36}$$

$$n = 0, 1, \dots,$$

which can also be rewritten in the form

$$x_{n+1} - (1 - \alpha) x_n + \sum_{j=1}^m p(n - k_j) x_{n-k_j} = 0, \tag{37}$$

$$n = 0, 1, \dots,$$

where $p(n - k_j) = (\beta_j e^{-\gamma_j \bar{y}} - \beta_j e^{-\gamma_j (\bar{y} + x_{n-k_j})}) / (x_{n-k_j})$, $j = 0, 1, \dots, m$.

By some simple calculations and (29), we get

$$\lim_{n \rightarrow \infty} p(n - k_j) = \beta_j \gamma_j e^{-\gamma_j \bar{y}} > 0. \tag{38}$$

One can easily see that the hypothesis of Lemma 1 is satisfied and so the linear equation

$$x_{n+1} - (1 - \alpha) x_n + \sum_{j=1}^m \beta_j \gamma_j e^{-\gamma_j \bar{y}} x_{n-k_j} = 0 \tag{39}$$

has an eventually positive solution.

Let $\{x_n\}$ be an eventually positive solution of (39); then $z_n = (1 - \alpha)^{-n} x_n$ is an eventually positive solution of

$$z_{n+1} - z_n + \sum_{j=1}^m \beta_j \gamma_j e^{-\gamma_j \bar{y}} (1 - \alpha)^{-k_j-1} z_{n-k_j} = 0, \tag{40}$$

$$n = 0, 1, \dots$$

Let

$$F(\lambda) = \lambda^{n+1} - \lambda^n + \sum_{j=1}^m \beta_j \gamma_j e^{-\gamma_j \bar{y}} (1 - \alpha)^{-k_j-1} \lambda^{n-k_j} \tag{41}$$

be the characteristic polynomial of (40). Now, we prove that $F(\lambda) > 0$, for $\lambda > 0$.

If $\lambda \geq 1$, then obviously $F(\lambda) > 0$. Else if $0 < \lambda < 1$, we have

$$\begin{aligned} F(\lambda) &= \lambda^n \left(\lambda - 1 + \sum_{j=1}^m \beta_j \gamma_j e^{-\gamma_j \bar{y}} (1 - \alpha)^{-k_j - 1} \lambda^{-k_j} \right) \\ &\geq \lambda^n \left(\lambda - 1 + \sum_{j=1}^m \beta_j \gamma_j e^{-\gamma_j \bar{y}} (1 - \alpha)^{-l-1} \lambda^{-k_j} \right) \\ &\geq \lambda^n \left(\lambda - 1 + \sum_{j=1}^m \beta_j \gamma_j e^{-\gamma_j \bar{y}} (1 - \alpha)^{-l-1} \right) \\ &\geq \lambda^{n+1} > 0. \end{aligned} \tag{42}$$

Therefore, the characteristic equation of (40)

$$\lambda^{n+1} - \lambda^n + \sum_{j=1}^m \beta_j \gamma_j e^{-\gamma_j \bar{y}} (1 - \alpha)^{-k_j - 1} \lambda^{n-k_j} = 0 \tag{43}$$

has no positive roots.

According to Lemma 2, (40) has no nonoscillatory solution.

This is a contradiction. The proof is completed. \square

Theorem 8. Assume that (11) holds, and

$$\bar{y} [1 - (1 - \alpha)^{k+1}] \leq 1. \tag{44}$$

Then, the positive equilibrium \bar{y} of (10) is a global attractor of all positive solutions of (10).

Proof. To prove that the positive equilibrium \bar{y} is a global attractor of all positive solutions of (10), it suffices to show that (29) holds.

We will prove that (29) holds in each of the following two cases.

Case 1 ($\{y_n\}$ is nonoscillatory). Let $\{y_n\}$ be eventually positive. The case that $\{y_n\}$ is eventually positive is similar and will be omitted. So, there exists an $n_0 \geq 0$ such that $y_n > \bar{y}$ for $n \geq n_0$, and consequently $y_{n-k} > \bar{y}$ for $n \geq n_1$, where $n_1 = n_0 + k$.

From Lemma 4, we have $\{y_n\}$ as a bounded sequence. Assume for the sake of contradiction that (29) is not satisfied. Let

$$\mu = \limsup_{n \rightarrow \infty} y_n. \tag{45}$$

Then, $\mu > \bar{y}$ and there exists a subsequence $\{y_{n_i}\}$ such that

$$\lim_{i \rightarrow \infty} y_{n_i+1} = \mu, \quad y_{n_i+1} - y_{n_i} > 0 \tag{46}$$

$$\text{for } n_i \geq n_1, \quad i = 1, 2, \dots$$

It follows from (10) that

$$\alpha y_{n_i+1} \leq \sum_{j=1}^m \beta_j e^{-\gamma_j y_{n_i-k_j}}. \tag{47}$$

So, we obtain

$$\alpha \mu \leq \limsup_{i \rightarrow \infty} \sum_{j=1}^m \beta_j e^{-\gamma_j y_{n_i-k_j}} \leq \sum_{j=1}^m \beta_j e^{-\gamma_j \bar{y}} = \alpha \bar{y}, \tag{48}$$

which is a contradiction. Accordingly, (29) holds.

Case 2 ($\{y_n\}$ is strictly oscillatory). To show that (29) holds, it suffices to prove that $\lim_{n \rightarrow \infty} x_n = 0$ holds, when $\{x_n\}$ is a strictly oscillatory solution of (36).

To this end, let

$$\{x_{p_i+1}, x_{p_i+2}, \dots, x_{q_i}\} \tag{49}$$

be the i th positive semicycle of $\{x_n\}$ followed by the j th negative semicycle

$$\{x_{q_i+1}, x_{q_i+2}, \dots, x_s\}. \tag{50}$$

Let x_{M_i}, x_{m_i} be the extreme values in these two semicycles with the smallest possible indices M_i and m_i . Then, we claim that

$$M_i - p_i \leq k + 1, \quad m_i - q_i \leq k + 1. \tag{51}$$

In the following, we will prove that (51) holds for positive semicycles, while for negative semicycles, the proof is similar and will be omitted. Assume for the sake of contradiction that the first inequality in (51) is not true. Then, $M_i - p_i > k + 1$ and the terms $x_{M_i-k-1}, x_{M_i-k}, \dots, x_{M_i-1}$ are in a positive semicycle. Because of $x_{M_i} > x_{M_i-1}$, (36) renders

$$\alpha x_{M_i} + \alpha \bar{y} \leq \sum_{j=1}^m \beta_j e^{-\gamma_j (\bar{y} + x_{M_i-k_j-1})}. \tag{52}$$

So, we have

$$\begin{aligned} x_{M_i} &\leq \frac{1}{\alpha} \sum_{j=1}^m \beta_j e^{-\gamma_j (\bar{y} + x_{M_i-k_j-1})} - \bar{y} \\ &= \frac{1}{\alpha} \left[\sum_{j=1}^m \beta_j e^{-\gamma_j (\bar{y} + x_{M_i-k_j-1})} - \sum_{j=1}^m \beta_j e^{-\gamma_j \bar{y}} \right] \\ &= \frac{1}{\alpha} \left[\sum_{j=1}^m \beta_j e^{-\gamma_j \bar{y}} (e^{-\gamma_j x_{M_i-k_j-1}} - 1) \right]. \end{aligned} \tag{53}$$

So there exists at least a j s.t. $x_{M_i-k_j-1} < 0$, which contradicts that $x_{M_i-k_j-1}$ is in the positive semicycle. So, (51) is true. Noting that $\{y_n\}$ is bounded from Lemma 4, we can let

$$\lambda = \liminf_{n \rightarrow \infty} x_n = \liminf_{i \rightarrow \infty} x_{m_i}, \tag{54}$$

$$\mu = \limsup_{n \rightarrow \infty} x_n = \limsup_{i \rightarrow \infty} x_{M_i}.$$

To prove that $\lim_{n \rightarrow \infty} x_n = 0$ holds, it is sufficient to show that $\lambda = \mu = 0$.

From (54), it follows that if $\epsilon \in (0, \lambda)$ is given, then there exists $n_2 \geq 0$ such that

$$\lambda - \epsilon \leq x_n \leq \mu + \epsilon \quad \text{for } n \geq n_2 + k. \quad (55)$$

Equation (36) can be reformulated in the form

$$x_{n+1} - (1 - \alpha)x_n = -\alpha\bar{y} + \sum_{j=1}^m \beta_j e^{-\gamma_j(\bar{y} + x_{n-k_j})}. \quad (56)$$

Multiplying (56) by $(1 - \alpha)^{-n-1}$ and then summing up from $n = p_i$ to $n = M_i - 1$ for i being sufficiently large, we get

$$\begin{aligned} & (1 - \alpha)^{-M_i} x_{M_i} - (1 - \alpha)^{-p_i} x_{p_i} \\ &= \sum_{n=p_i}^{M_i-1} (-\alpha\bar{y})(1 - \alpha)^{-n-1} \\ &+ \sum_{n=p_i}^{M_i-1} \sum_{j=1}^m \beta_j e^{-\gamma_j(\bar{y} + x_{n-k_j})} (1 - \alpha)^{-n-1}. \end{aligned} \quad (57)$$

From (55) and $x_{p_i} < 0$, we have

$$\begin{aligned} (1 - \alpha)^{-M_i} x_{M_i} &\leq (-\alpha\bar{y}) \sum_{n=p_i}^{M_i-1} (1 - \alpha)^{-n-1} \\ &+ \sum_{n=p_i}^{M_i-1} \sum_{j=1}^m \beta_j e^{-\gamma_j(\bar{y} + x_{n-k_j})} (1 - \alpha)^{-n-1} \\ &\leq (-\alpha\bar{y}) \sum_{n=p_i}^{M_i-1} (1 - \alpha)^{-n-1} \\ &+ \sum_{n=p_i}^{M_i-1} \sum_{j=1}^m \beta_j e^{-\gamma_j\bar{y}} e^{-\gamma^*(\lambda - \epsilon)} (1 - \alpha)^{-n-1} \\ &= (-\alpha\bar{y}) \sum_{n=p_i}^{M_i-1} (1 - \alpha)^{-n-1} \\ &+ (\alpha\bar{y}) e^{-\gamma^*(\lambda - \epsilon)} \sum_{n=p_i}^{M_i-1} (1 - \alpha)^{-n-1} \\ &= (-\alpha\bar{y}) \frac{(1 - \alpha)^{-M_i} - (1 - \alpha)^{-p_i}}{\alpha} \\ &+ \alpha\bar{y} e^{-\gamma^*(\lambda - \epsilon)} \frac{(1 - \alpha)^{-M_i} - (1 - \alpha)^{-p_i}}{\alpha} \\ &= \bar{y} \left[(1 - \alpha)^{-M_i} - (1 - \alpha)^{-p_i} \right] \left[e^{-\gamma^*(\lambda - \epsilon)} - 1 \right]. \end{aligned} \quad (58)$$

So,

$$x_{M_i} \leq \bar{y} \left[e^{-\gamma^*(\lambda - \epsilon)} - 1 \right] \left[1 - (1 - \alpha)^{M_i - p_i} \right]. \quad (59)$$

By using (54), ϵ is arbitrary and $M_i - p_i \leq k + 1$; we get

$$\mu \leq \bar{y} \left[e^{-\gamma^*\lambda} - 1 \right] \left[1 - (1 - \alpha)^{k+1} \right]. \quad (60)$$

From the assumption of the theorem, we have

$$\mu \leq e^{-\gamma^*\lambda} - 1. \quad (61)$$

By the same trick as in proving (61), we can prove that

$$\lambda \geq e^{-\gamma^*\mu} - 1. \quad (62)$$

Therefore, by Lemma 6, we can get $\lambda = \mu = 0$; that is, $\lim_{n \rightarrow \infty} x_n = 0$, which implies that \bar{y} is a global attractor of all positive solutions of (10).

By Lemma 3 and Theorem 8, we can get the following result. \square

Theorem 9. Suppose that (11) holds and that

$$\sum_{j=1}^m \beta_j \gamma_j e^{-r_j \bar{y}} < \alpha, \quad \bar{y} \left[1 - (1 - \alpha)^{k+1} \right] < 1. \quad (63)$$

Then, the positive equilibrium \bar{y} is globally asymptotically stable.

Remark 10. From Theorem 7, it is clear that if the condition (28) holds, then the oscillation condition for $m = 1$ as established by Kubiacyk and Saker [8] is already satisfied.

Remark 11. When $m = 1$, the condition of Theorem 8 is independent from the argument γ .

Acknowledgments

This work is supported by the National Natural Science Foundation of China (G61074068, G61034007, G61174036, G61374065, and G61374002), the Fund for the Taishan Scholar Project of Shandong Province, the Natural Science Foundation of Shandong Province (ZR2010FM013), and the Scientific Research and Development Project of Shandong Provincial Education Department (J11LA01).

References

- [1] M. Wazewska-Czyzewska and A. Lasota, "Mathematical problems of the dynamics of the red blood cells system," *Annals of the Polish Mathematical Society Series III*, vol. 17, pp. 23–40, 1976.
- [2] I. Györi and G. Ladas, *Oscillation Theory of Delay Differential Equations with Applications*, Clarendon Press, Oxford, UK, 1991.
- [3] J.-W. Li and S. S. Cheng, "Global attractivity in an RBC survival model of Wazewska and Lasota," *Quarterly of Applied Mathematics*, vol. 60, no. 3, pp. 477–483, 2002.
- [4] Y. Song, J. Wei, and Y. Yuan, "Bifurcation analysis on a survival red blood cells model," *Journal of Mathematical Analysis and Applications*, vol. 316, no. 2, pp. 459–471, 2006.
- [5] W. Xu and J. Li, "Global attractivity of the model for the survival of red blood cells with several delays," *Annals of Differential Equations*, vol. 14, no. 2, pp. 357–363, 1998.
- [6] Y.-j. Liu, "Global stability in a differential equation with piecewisely constant arguments," *Journal of Mathematical Research and Exposition*, vol. 21, no. 1, pp. 31–36, 2001.

- [7] V. L. Kocić and G. Ladas, *Global Behavior of Nonlinear Difference Equations of Higher Order with Applications*, vol. 256, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1993.
- [8] I. Kubiacyk and S. H. Saker, "Oscillation and global attractivity of discrete survival red blood cells model," *Applications Mathematicae*, vol. 30, pp. 441–449, 2003.
- [9] Q. Meng and J. R. Yan, "Global attractivity of delay difference equations," *Indian Journal of Pure and Applied Mathematics*, vol. 30, no. 3, pp. 233–242, 1999.
- [10] X. Y. Zeng and B. Shi, "Oscillation and global attractivity on a rational recursive sequence," in *Proceedings of the 7th International Conference on Difference Equations and Applications*, Changsha, Hunan, China, 2002.
- [11] M. J. Ma and J. S. Yu, "Global attractivity of $x_{n+1} = (1 - \alpha)x_n + \beta e^{-\gamma x_{n-k}}$," *Computers & Mathematics with Applications*, vol. 49, pp. 1397–1402, 2005.
- [12] M. Xu and S. Xu, "Global behavior of a discrete haematopoiesis model with several delays," *Applied Mathematics and Computation*, vol. 217, no. 1, pp. 441–449, 2010.
- [13] L. N. Dang and R. F. Cheng, "Periodicity in a Lotka-Volterra facultative mutualism system with several delays," in *Proceedings of the 4th International Conference on Information and Computing*, pp. 377–380.
- [14] F.-D. Chen, J.-I. Shi, and X.-X. Chen, "Periodicity in a Lotka-Volterra facultative mutualism system with several delays," *Chinese Journal of Engineering Mathematics*, vol. 21, no. 3, pp. 403–409, 2004.
- [15] J. W.-H. So and J. S. Yu, "Global attractivity and uniform persistence in Nicholson's blowflies," *Differential Equations and Dynamical Systems*, vol. 2, no. 1, pp. 11–18, 1994.



Hindawi

Submit your manuscripts at
<http://www.hindawi.com>

