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Research Article

Robust Stability and H_∞ Stabilization of Switched Systems with Time-Varying Delays Using Delta Operator Approach

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This paper considers the problems of the robust stability and robust H_∞ controller design for time-varying delay switched systems using delta operator approach. Based on the average dwell time approach and delta operator theory, a sufficient condition of the robust exponential stability is presented by choosing an appropriate Lyapunov-Krasovskii functional candidate. Then, a state feedback controller is designed such that the resulting closed-loop system is exponentially stable with a guaranteed H_∞ performance. The obtained results are formulated in the form of linear matrix inequalities (LMIs). Finally, a numerical example is provided to explicitly illustrate the feasibility and effectiveness of the proposed method.

1. Introduction

Switched systems are a kind of hybrid systems consisting of a set of discrete event dynamic subsystems or continuous variable dynamic subsystems and a switching rule which defines a particular subsystem working during a certain interval of time. Switched systems have numerous applications in network control systems [1], robot control systems [2], intelligent traffic control systems [3], chemical industry control systems [4], and many other areas [5, 6]. Many important achievements on stability and stabilization of switched systems have been developed [7–10]. It was shown in the literature that the average dwell time (ADT) method is a powerful tool to deal with the stability of switched systems.

The delta operator which is a novel approach with good finite word length performance under fast sampling rates has been investigated by many researchers due to their extensive applications [11–13], for instance, optimal filtering [14], signal processing [15], robust control [16], system identification [17], and so forth. As stated in [15], the standard shift operator was mostly adopted in the study of control theories for discrete-time systems. However, the dynamic response of a discrete system does not converge smoothly to its continuous counterpart when the sampling period tends to zero; namely, data are taken at high sampling rates. The delta operator method can solve the above problem. In addition, it was shown in [15] that delta operator requires smaller word length when implemented in fixed-point digital control processors than shift operator does. So far, some useful results on delta operator systems have been formulated in [18–21]. As is well known, time delay phenomena which often cause instability or undesirable performance in control systems are involved in a variety of real systems, such as chaotic systems, and hydraulic pressure systems [22]. In the past years, a mass of results on delta operator systems with time delay have appeared [23–27]. The delta operator is defined by

$$\delta x(t) = \begin{cases} \dfrac{dx(t)}{dt}, & T = 0, \\ \dfrac{(x(t+T) - x(t))}{T}, & T \neq 0, \end{cases} \tag{1}$$

where T is a sampling period. When $T \rightarrow 0$, the delta operator model will approach the continuous system before discretization and reflect a quasicontinuous performance [28].

It should be noted that external disturbances are generally inevitable, and the output will be subsequently affected by disturbances in the system. Some results on H_∞ control were developed by many researchers to restrain the external disturbances [29–33]. The H_∞ control problem for a class of discrete systems was solved by using delta operator approach [34]. Low order sampled data H_∞ control using the delta operator was reported in the literature [35]. Robust H_∞ control for a class of uncertain switched systems using delta operator was investigated [36]. However, few results on the issues of robust stability and H_∞ controller design for delta operator switched systems with time-varying delay are presented, which motivates the present investigations.

In this paper, we concentrate our interest on investigating the stability and H_∞ controller design problems for delta operator switched systems with time-varying delay. The main contributions of this paper can be summarized as follows: (1) by constructing a new Lyapunov-Krasovskii functional candidate and using the average dwell time approach, an exponential stability criterion for the considered system is proposed and (2) a state feedback controller design scheme is developed such that the corresponding closed-loop system is exponentially stable with a guaranteed H_∞ performance.

The remainder of the paper is organized as follows. The formulation of the considered systems and some corresponding definitions and lemmas is given in Section 2. In Section 3, the exponential stability analysis and H_∞ control for the underlying system are developed. A numerical example is given to illustrate the feasibility and effectiveness of the proposed method in Section 4. Finally, concluding remarks are presented in Section 5.

Notations. $\|\cdot\|_2$ denotes the Euclidean norm. $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and maximum eigenvalues of a matrix, respectively; A^T means the transpose of matrix A; R denotes the set of all real numbers; R^n represents the n-dimensional real vector space; $R^{m\times n}$ is the set of all $(m\times n)$-dimensional real matrices. The notation $A>0(\geq 0)$ means that the matrix A is positive (nonnegative) definite; diag$\{\cdots\}$ refers to the block-diagonal matrix; I is the identity matrix of appropriate dimension. $l_2[k_0,\infty)$ stands for the space of square summable functions on $[k_0,\infty)$.

2. Problem Formulation

Consider the following delta operator switched system with time-varying delay:

$$\begin{gathered}\delta x(k)=\widehat{A}_{\sigma(k)}x(k)+\widehat{A}_{d\sigma(k)}x(k-\tau(k))+D_{\sigma(k)}w(k),\\ z(k)=C_{\sigma(k)}x(k)+G_{\sigma(k)}w(k),\\ x(k_0+\theta)=\phi(\theta),\quad \theta=-\overline{\tau},-\overline{\tau}+1,\ldots,0,\end{gathered}\tag{2}$$

where $x(k)\in R^n$ is the state vector, $z(k)\in R^l$ denotes the controlled output, and $w(k)\in R^w$ represents the disturbance input belonging to $l_2[k_0,\infty)$. k means the time $t=kT$ and $T>0$ is the sampling period; k_0 is the initial instant. $\sigma(k):[k_0,\infty)\to\underline{N}=\{1,2,\ldots,N\}$ is the switching signal with N being the number of subsystems. $\tau(k)$ is the time-varying delay satisfying $0\leq\underline{\tau}\leq\tau(k)\leq\overline{\tau}$ for known constants $\underline{\tau}$ and $\overline{\tau}$. $\phi(\theta)$ is the discrete vector-valued initial function. C_i, D_i, and G_i are constant matrices with proper dimensions. $\widehat{A}_i$ and $\widehat{A}_{di}$ are uncertain real-valued matrices with appropriate dimensions and have the following form:

$$\left[\widehat{A}_i\ \ \widehat{A}_{di}\right]=\left[A_i\ \ A_{di}\right]+H_iF_i(k)\left[E_{ai}\ \ E_{adi}\right],\tag{3}$$

where A_i, A_{di}, H_i, E_{ai}, and E_{adi} are known real constant matrices of suitable dimensions and $F_i(k)$ is an unknown time-varying matrix which satisfies

$$F_i^T(k)F_i(k)\leq I.\tag{4}$$

To obtain the main results, we first give some definitions and lemmas which will be essential in our later development.

Definition 1 (see [36]). Consider system (2) with $w(k)=0$. It is said to be exponentially stable under a switching signal $\sigma(k)$ if, for the initial condition $x(k_0+\theta)=\phi(\theta)$, $\theta=-\overline{\tau},-\overline{\tau}+1,\ldots,0$, there exist constants $\alpha>0$ and $\beta>0$ such that the solution $x(k)$ satisfies

$$\|x(k)\|\leq\alpha\|x(k_0)\|_c e^{-\beta(k-k_0)},\quad \forall k\geq k_0,\tag{5}$$

where $\|x(k_0)\|_c=\sup_{-\overline{\tau}\leq k\leq 0}\|x(k_0+\theta)\|$.

Definition 2. For given $0<\alpha<1/T$ and $\gamma>0$, system (2) is said to have an H_∞ performance level γ if there exists a switching signal $\sigma(k)$ such that the following conditions are satisfied:

(1) system (2) is exponentially stable when $w(k)=0$;

(2) under the zero-initial condition, that is, $\phi(\theta)=0$, $\theta=-\overline{\tau},-\overline{\tau}+1,\ldots,-1,0$, system (2) satisfies

$$\sum_{k=k_0}^{\infty}(1-T\alpha)^{(k-k_0)}\|z(k)\|^2\leq\gamma^2\sum_{k=k_0}^{\infty}\|w(k)\|^2,\qquad \forall w(k)\in l_2[k_0,\infty).\tag{6}$$

Definition 3 (see [37]). For any switching signal $\sigma(k)$ and any $k_2>k_1\geq 0$, let $N_{\sigma(k)}(k_1,k_2)$ denote the number of switching of $\sigma(k)$ over the interval $[k_1,k_2)$. For given $\tau_a>0$ and $N_0\geq 0$, if the inequality

$$N_{\sigma(k)}(k_1,k_2)\leq N_0+\frac{k_2-k_1}{\tau_a}\tag{7}$$

holds, then the positive constant τ_a is called the average dwell time and N_0 is called the chattering bound. As commonly used in the literature, we choose $N_0=0$ in this paper.

Lemma 4 (see [20]). *For a given matrix* $S=\begin{bmatrix}S_{11} & S_{12}\\ S_{12}^T & S_{22}\end{bmatrix}$, *where* S_{11} *and* S_{22} *are square matrices, the following conditions are equivalent:*

(i) $S<0$;

(ii) $S_{11}<0$, $S_{22}-S_{12}^TS_{11}^{-1}S_{12}<0$;

(iii) $S_{22}<0$, $S_{11}-S_{12}S_{22}^{-1}S_{12}^T<0$.

Lemma 5 (see [20]). *Let U, V, W, and X be real matrices of appropriate dimensions with X satisfying $X = X^T$; then, for all $V^TV \leq I$,*

$$X + UVW + W^TV^TU^T < 0 \tag{8}$$

if and only if there exists a scalar ε such that

$$X + \varepsilon UU^T + \varepsilon^{-1}W^TW < 0. \tag{9}$$

Lemma 6 (see [28]). *For any time function $x(t)$ and $y(t)$, the following equation holds:*

$$\begin{aligned}\delta(x(t)y(t)) &= \delta(x(t))y(t) + x(t)\delta(y(t)) \\ &\quad + T\delta(x(t))\delta(y(t)),\end{aligned} \tag{10}$$

where T is the sampling period.

The objectives of the paper are (1) to find a class of switching signal $\sigma(k)$ such that system (2) is exponentially stable with a guaranteed H_∞ performance and (2) to determine a class of switching signal and design a state feedback controller $u(k) = K_{\sigma(k)}x(k)$ for the following delta operator switched system with time-varying delay:

$$\begin{aligned}\delta x(k) &= \widehat{A}_{\sigma(k)}x(k) + \widehat{A}_{d\sigma(k)}x(k-\tau(k)) \\ &\quad + \widehat{B}_{\sigma(k)}u(k) + D_{\sigma(k)}w(k), \\ z(k) &= C_{\sigma(k)}x(k) + G_{\sigma(k)}w(k), \\ x(k_0+\theta) &= \phi(\theta), \quad \theta = -\overline{\tau}, -\overline{\tau}+1, \dots, 0\end{aligned} \tag{11}$$

such that the corresponding closed-loop system is exponentially stable with a guaranteed H_∞ performance.

3. Main Results

3.1. Robust Stability Analysis. In this section, we will focus on the stability of system (2) with $w(k) = 0$.

Theorem 7. *For a given positive constant $0 < \alpha < 1/T$, if there exist scalars ε_i and positive definite symmetric matrices X_i and Q_i, $i \in \underline{N}$, with appropriate dimensions, such that*

$$\begin{bmatrix} E_i & A_{di}X_i & TX_iA_i^T & \varepsilon_iH_i & X_iE_{ai}^T \\ X_iA_{di}^T & -(1-T\alpha)^{(\overline{\tau}+1)}Q_i & TX_iA_{di}^T & 0 & X_iE_{adi}^T \\ TA_iX_i & TA_{di}X_i & -TX_i & \varepsilon_iTH_i & 0 \\ \varepsilon_iH_i^T & 0 & \varepsilon_iTH_i^T & -\varepsilon_iI & 0 \\ E_{ai}X_i & E_{adi}X_i & 0 & 0 & -\varepsilon_iI \end{bmatrix} < 0, \tag{12}$$

where $E_i = A_iX_i + X_iA_i^T + \alpha X_i + (1-T\alpha)(\overline{\tau} - \underline{\tau} + 1)Q_i$, then system (2) with $w(k) = 0$ is exponentially stable for any switching signal $\sigma(k)$ with the following average dwell time scheme:

$$\tau_a > \tau_a^* = -\frac{\ln\mu}{\ln(1-T\alpha)}, \tag{13}$$

where $\mu \geq 1$ satisfies

$$X_i \leq \mu X_j, \quad Q_i \leq \mu Q_j, \quad \forall i, j \in \underline{N}. \tag{14}$$

Proof. Choose the following Lyapunov-Krasovskii functional candidate for the ith subsystem

$$V_i(k) = V_{i1}(k) + V_{i2}(k) + V_{i3}(k), \quad \forall i \in \underline{N}, \tag{15}$$

where

$$\begin{aligned}V_{i1}(k) &= x^T(k)P_ix(k), \\ V_{i2}(k) &= T\sum_{s=k-\tau(k)}^{k-1}(1-T\alpha)^{(k-s)}x^T(s)S_ix(s), \\ V_{i3}(k) &= T\sum_{l=-\overline{\tau}+1}^{-\underline{\tau}}\sum_{s=k+l}^{k-1}(1-T\alpha)^{k-s}x^T(s)S_ix(s).\end{aligned} \tag{16}$$

Taking the delta operator manipulations of Lyapunov functional candidate $V_i(k)$ along the trajectory of system (2), by Lemma 6, we have

$$\begin{aligned}&\delta V_{i1}(k) \\ &= \delta\left(x^T(k)P_ix(k)\right) \\ &= \delta\left(x^T(k)P_i\right)x(k) + x^T(k)P_i\delta(x(k)) \\ &\quad + T\delta\left(x^T(k)P_i\right)\delta(x(k)) \\ &= \left(\widehat{A}_ix(k) + \widehat{A}_{di}x(k-\tau(k))\right)^TP_ix(k) \\ &\quad + x^T(k)P_i\left(\widehat{A}_ix(k) + \widehat{A}_{di}x(k-\tau(k))\right) \\ &\quad + T\left(\widehat{A}_ix(k) + \widehat{A}_{di}x(k-\tau(k))\right)^T \\ &\quad \times P_i\left(\widehat{A}_ix(k) + \widehat{A}_{di}x(k-\tau(k))\right) \\ &= x^T(k)P_i\widehat{A}_ix(k) + x^T(k)P_i\widehat{A}_{di}x(k-\tau(k)) \\ &\quad + x^T(k)\widehat{A}_i^TP_ix(k) + x^T(k-\tau(k))\widehat{A}_{di}^TP_ix(k) \\ &\quad + Tx^T(k)\widehat{A}_i^TP_i\widehat{A}_ix(k) + Tx^T(k)\widehat{A}_i^TP_i\widehat{A}_{di}x(k-\tau(k)) \\ &\quad + Tx^T(k-\tau(k))\widehat{A}_{di}^TP_i\widehat{A}_ix(k) \\ &\quad + Tx^T(k-\tau(k))\widehat{A}_{di}^TP_i\widehat{A}_{di}x(k-\tau(k)) \\ &= \begin{bmatrix} x(k) \\ x(k-\tau(k)) \end{bmatrix}^T \\ &\quad \times \begin{bmatrix} P_i\widehat{A}_i + \widehat{A}_i^TP_i + T\widehat{A}_i^TP_i\widehat{A}_i & P_i\widehat{A}_{di} + T\widehat{A}_i^TP_i\widehat{A}_{di} \\ \widehat{A}_{di}^TP_i + T\widehat{A}_{di}^TP_i\widehat{A}_i & T\widehat{A}_{di}^TP_i\widehat{A}_{di} \end{bmatrix} \\ &\quad \times \begin{bmatrix} x(k) \\ x(k-\tau(k)) \end{bmatrix},\end{aligned} \tag{17}$$

$$\begin{aligned}
&\delta V_{i2}(k)\\
&= \frac{1}{T}\left(V_{i2}(k+1) - V_{i2}(k)\right)\\
&= \frac{1}{T}\left(T\sum_{s=k+1-\tau(k+1)}^{k+1-1}(1-T\alpha)^{(k+1-s)}x^T(s)S_ix(s)\right.\\
&\qquad\left. - T\sum_{s=k-\tau(k)}^{k-1}(1-T\alpha)^{(k-s)}x^T(s)S_ix(s)\right)\\
&\le -T\alpha\sum_{s=k-\tau(k)}^{k-1}(1-T\alpha)^{(k-s)}x^T(s)S_ix(s)\\
&\qquad + (1-T\alpha)x^T(k)S_ix(k)\\
&\qquad - (1-T\alpha)^{(\overline{\tau}+1)}x^T(k-\tau(k))S_ix(k-\tau(k))\\
&\qquad + \sum_{s=k+1-\overline{\tau}}^{k-\underline{\tau}}(1-T\alpha)^{(k+1-s)}x^T(s)S_ix(s),
\end{aligned} \tag{18}$$

$$\begin{aligned}
&\delta V_{i3}(k)\\
&= \frac{1}{T}\left(V_{i3}(k+1) - V_{i3}(k)\right)\\
&= \frac{1}{T}\left(T\sum_{l=-\overline{\tau}+1}^{-\underline{\tau}}\sum_{s=k+1+l}^{k-1+1}(1-T\alpha)^{(k+1-s)}x^T(s)S_ix(s)\right.\\
&\qquad\left. -T\sum_{l=-\overline{\tau}+1}^{-\underline{\tau}}\sum_{s=k+l}^{k-1}(1-T\alpha)^{(k-s)}x^T(s)S_ix(s)\right)\\
&= -T\alpha\sum_{l=-\overline{\tau}+1}^{-\underline{\tau}}\sum_{s=k+l}^{k-1}(1-T\alpha)^{(k-s)}x^T(s)S_ix(s)\\
&\qquad + (1-T\alpha)\left(\overline{\tau}-\underline{\tau}\right)x^T(k)S_ix(k)\\
&\qquad - \sum_{s=k+1-\overline{\tau}}^{k-\underline{\tau}}(1-T\alpha)^{(k+1-s)}x^T(s)S_ix(s).
\end{aligned} \tag{19}$$

Combining (17)–(19), we have

$$\begin{aligned}
&\delta V_i(k) + \alpha V_i(k)\\
&= \begin{bmatrix} x(k) \\ x(k-\tau(k))\end{bmatrix}^T\\
&\quad\times\begin{bmatrix} P_i\widehat{A}_i + \widehat{A}_i^TP_i + T\widehat{A}_i^TP_i\widehat{A}_i & P_i\widehat{A}_{di} + T\widehat{A}_i^TP_i\widehat{A}_{di} \\ \widehat{A}_{di}^TP_i + T\widehat{A}_{di}^TP_i\widehat{A}_i & T\widehat{A}_{di}^TP_i\widehat{A}_{di}\end{bmatrix}\\
&\quad\times\begin{bmatrix} x(k) \\ x(k-\tau(k))\end{bmatrix}\\
&\quad + \alpha P_i + (1-T\alpha)\left(\overline{\tau}-\underline{\tau}+1\right)x^T(k)S_ix(k)\\
&\quad - (1-T\alpha)^{(\overline{\tau}+1)}x^T(k-\tau(k))S_ix(k-\tau(k))\\
&\le \begin{bmatrix} x(k) \\ x(k-\tau(k))\end{bmatrix}^T\Omega_i\begin{bmatrix} x(k) \\ x(k-\tau(k))\end{bmatrix},
\end{aligned} \tag{20}$$

where

$$\Omega_i = \begin{bmatrix} P_i\widehat{A}_i + \widehat{A}_i^TP_i + \alpha P_i + (1-T\alpha)\times\left(\overline{\tau}-\underline{\tau}+1\right)S_i + T\widehat{A}_i^TP_i\widehat{A}_i & P_i\widehat{A}_{di} + T\widehat{A}_i^TP_i\widehat{A}_{di} \\ \widehat{A}_{di}^TP_i + T\widehat{A}_{di}^TP_i\widehat{A}_i & T\widehat{A}_{di}^TP_i\widehat{A}_{di} - (1-T\alpha)^{(\overline{\tau}+1)}S_i\end{bmatrix}. \tag{21}$$

Applying Lemma 4, we can obtain that $\Omega_i < 0$ is equivalent to

$$\begin{bmatrix} P_i\widehat{A}_i + \widehat{A}_i^TP_i + \alpha P_i + (1-T\alpha)\left(\overline{\tau}-\underline{\tau}+1\right)S_i & P_i\widehat{A}_{di} & T\widehat{A}_i^T \\ \widehat{A}_{di}^TP_i & -(1-T\alpha)^{(\overline{\tau}+1)}S_i & T\widehat{A}_{di}^T \\ T\widehat{A}_i & T\widehat{A}_{di} & -TP_i^{-1}\end{bmatrix} < 0. \tag{22}$$

Using $\operatorname{diag}\{P_i^{-1}\ \ P_i^{-1}\ \ I\}$ to pre- and post-multiply both sides of (22), respectively, we have

$$\begin{bmatrix} \overline{E}_i & \widehat{A}_{di}P_i^{-1} & TP_i^{-1}\widehat{A}_i^T \\ P_i^{-1}\widehat{A}_{di}^T & -(1-T\alpha)^{(\overline{\tau}+1)}P_i^{-1}SP_i^{-1} & TP_i^{-1}\widehat{A}_{di}^T \\ T\widehat{A}_iP_i^{-1} & T\widehat{A}_{di}P_i^{-1} & -TP_i^{-1}\end{bmatrix} < 0, \tag{23}$$

where $\overline{E}_i = \widehat{A}_iP_i^{-1} + P_i^{-1}\widehat{A}_i^T + \alpha P_i^{-1} + (1-T\alpha)(\overline{\tau}-\underline{\tau}+1)P_i^{-1}S_iP_i^{-1}$.

Denote $Q_i = P_i^{-1}S_iP_i^{-1}$ and $X_i = P_i^{-1}$; then, substituting (3) into (23) and applying Lemmas 4 and 5, we obtain that (23) is equivalent to (12). Thus, from (12), we can easily obtain

$$\delta V_i(k) + \alpha V_i(k) \le 0. \tag{24}$$

It follows from (24) that

$$\delta V_i(k) = \frac{V_i(k+1) - V_i(k)}{T} \leq -\alpha V_i(k),$$
$$V_i(k+1) - V_i(k) \leq -\alpha_i T V_i(k), \quad (25)$$
$$V_i(k+1) \leq (1 - \alpha T) V_i(k).$$

Let $k_1 < \cdots < k_q$ denote the switching instants of $\sigma(k)$ over the interval $[k_0, k)$. Consider the following piecewise Lyapunov functional for system (2):

$$V(k) = V_{\sigma(k)}(k) = V_{\sigma(k_p)}(k),$$
$$\forall k \in [k_p, k_p + 1), \quad p = 0, 1, \ldots, q. \quad (26)$$

From (14), we obtain

$$V_{\sigma(k_p)}(k_p) \leq \mu V_{\sigma(k_p^-)}(k_p^-), \quad p = 0, 1, \ldots, q. \quad (27)$$

It can be obtained from (24), (27), and Definition 3 that

$$\begin{aligned} V_{\sigma(k)}(k) &\leq (1 - T\alpha)^{(k-k_q)} V_{\sigma(k_q)}(k_q) \\ &\leq \mu(1 - T\alpha)^{(k-k_q)} V_{\sigma(k_q^-)}(k_q^-) \\ &\leq \mu(1 - T\alpha)^{(k-k_{q-1})} V_{\sigma(k_{q-1})}(k_{q-1}) \\ &\leq \mu^2(1 - T\alpha)^{(k-k_{q-1})} V_{\sigma(k_{q-1}^-)}(k_{q-1}^-) \\ &\leq \cdots \\ &\leq \mu^{N_{\sigma(k)}(k_0,k)}(1 - T\alpha)^{(k-k_0)} V_{\sigma(k_0)}(k_0) \\ &\leq \mu^{(k-k_0)/\tau_a}(1 - T\alpha)^{(k-k_0)} V_{\sigma(k_0)}(k_0) \\ &= \left(\mu^{1/\tau_a}(1 - T\alpha)\right)^{(k-k_0)} V_{\sigma(k_0)}(k_0). \end{aligned} \quad (28)$$

Considering the definition of $V_{\sigma(k)}(k)$, it yields that

$$V_{\sigma(k)}(k) \geq a\|x(k)\|^2, \quad (29)$$

$$V_{\sigma(k)}(k_0) \leq b\|x(k_0)\|_c^2, \quad (30)$$

where

$$a = \min_{i \in \underline{N}} \lambda_{\min}(P_i),$$
$$b = \max_{i \in \underline{N}} \left\{\lambda_{\max}(P_i) + T\left(\overline{\tau}^2 - \overline{\tau}\underline{\tau} + \overline{\tau}\right)\lambda_{\max}(S_i)\right\}, \quad (31)$$
$$\|x(k_0)\|_c = \sup_{-\overline{\tau} \leq \theta \leq 0} \|x(k_0 + \theta)\|.$$

Combining (29) and (30), we have

$$\|x(k)\|^2 \leq \frac{b}{a}\left(\mu^{1/\tau_a}(1 - T\alpha)\right)^{(k-k_0)} \|x(k_0)\|^2. \quad (32)$$

Therefore, system (2) with $w(k) = 0$ is exponentially stable under the average dwell time scheme (13).

The proof is completed. □

Remark 8. When $\mu = 1$ in (14), which leads to $X_i = X_j$, $Q_i = Q_j$, $\forall i, j \in \underline{N}$, and $\tau_a^* = 0$ by (13), system (2) has a common Lyapunov-Krasovskii functional and the switching signal can be arbitrary.

When $\tau(k) = 0$, system (2) with $w(k) = 0$ becomes the following system:

$$\delta x(k) = \left(\widehat{A}_{\sigma(k)} + \widehat{A}_{d\sigma(k)}\right) x(k),$$
$$z(k) = C_{\sigma(k)} x(k). \quad (33)$$

Then we have the following corollary.

Corollary 9. *For a given positive constant $0 < \alpha < 1/T$, if there exist scalars ε_i and positive definite symmetric matrices X_i, $\forall i \in \underline{N}$, of appropriate dimensions, such that*

$$\begin{bmatrix} (A_i + A_{di}) X_i + X_i \left(A_i^T + A_{di}^T\right) + \alpha X_i & T X_i \left(A_i^T + A_{di}^T\right) & \varepsilon_i H_i & X_i \left(E_{ai}^T + E_{adi}^T\right) \\ T (A_i + A_{di}) X_i & -T X_i & \varepsilon_i T H_i & 0 \\ \varepsilon_i H_i^T & \varepsilon_i T H_i^T & -\varepsilon_i I & 0 \\ (E_{ai} + E_{adi}) X_i & 0 & 0 & -\varepsilon_i I \end{bmatrix} < 0, \quad (34)$$

then system (33) is exponentially stable for any switching signal $\sigma(k)$ with average dwell time scheme (13), where $\mu \geq 1$ satisfies

$$X_i \leq \mu X_j, \quad \forall i, j \in \underline{N}. \quad (35)$$

3.2. H_∞ Performance Analysis. The following theorem gives sufficient conditions for the existence of an H_∞ performance level for system (2).

Theorem 10. *For given positive constants γ and $0 < \alpha < 1/T$, if there exist scalars ε_i and positive definite symmetric matrices X_i and Q_i, $i \in \underline{N}$, of appropriate dimensions, such that*

$$\begin{bmatrix} \Delta_i & A_{di}X_i & D_i & TX_iA_i^T & X_iC_i^T & \varepsilon_iH_i & X_iE_{ai}^T \\ X_iA_{di}^T & -(1-T\alpha)^{\overline{\tau}+1}Q_i & 0 & TX_iA_{di}^T & 0 & 0 & X_iE_{adi}^T \\ D_i^T & 0 & -\gamma^2 I & TD_i^T & G_i^T & 0 & 0 \\ TA_iX_i & TA_{di}X_i & TD_i & -TX_i & 0 & \varepsilon_iTH_i & 0 \\ C_iX_i & 0 & G_i & 0 & -I & 0 & 0 \\ \varepsilon_iH_i^T & 0 & 0 & \varepsilon_iTH_i^T & 0 & -\varepsilon_iI & 0 \\ E_{ai}X_i & E_{adi}X_i & 0 & 0 & 0 & 0 & -\varepsilon_iI \end{bmatrix} < 0, \quad (36)$$

where $\Delta_i = A_iX_i + X_iA_i^T + \alpha X_i + (1-T\alpha)(\overline{\tau} - \underline{\tau} + 1)Q_i$, then system (2) is exponentially stable with an H_∞ performance level γ for any switching signal $\sigma(k)$ with average dwell time scheme (13), where $\mu \geq 1$ satisfies (14).

Proof. Equation (12) in Theorem 7 can be directly derived from (36). Thus, system (2) is exponentially stable. We are now in a position to show the H_∞ performance of system (2).

Choosing the Lyapunov-Krasovskii functional candidate (15) and following the proof line of Theorem 7, we get

$$\begin{aligned} &\delta V_{i1}(k) \\ &= x^T(k)P_i\delta x(k) + (\delta x(k))^T P_i x(k) \\ &\quad + T(\delta x(k))^T P_i \delta x(k) \\ &= x^T(k)P_i\left(\widehat{A}_i x(k) + \widehat{A}_{di}x(k-\tau(k)) + D_i w(k)\right) \\ &\quad + \left(\widehat{A}_i x(k) + \widehat{A}_{di}x(k-\tau(k)) + D_i w(k)\right)^T P_i x(k) \\ &\quad + T\left(\widehat{A}_i x(k) + \widehat{A}_{di}x(k-\tau(k)) + D_i w(k)\right)^T \\ &\quad \times P_i\left(\widehat{A}_i x(k) + \widehat{A}_{di}x(k-\tau(k)) + D_i w(k)\right) \\ &= \begin{bmatrix} x(k) \\ x(k-\tau(k)) \\ w(k) \end{bmatrix}^T \widetilde{\Theta}_i \begin{bmatrix} x(k) \\ x(k-\tau(k)) \\ w(k) \end{bmatrix}, \end{aligned} \quad (37)$$

where

$$\widetilde{\Theta}_i = \begin{bmatrix} P_i\widehat{A}_i + \widehat{A}_i^TP_i + T\widehat{A}_i^TP_i\widehat{A}_i & P_i\widehat{A}_{di} + T\widehat{A}_i^TP_i\widehat{A}_{di} & P_iD_i + T\widehat{A}_i^TP_iD_i \\ \widehat{A}_{di}^TP_i + T\widehat{A}_{di}^TP_i\widehat{A}_i & T\widehat{A}_{di}^TP_i\widehat{A}_{di} & T\widehat{A}_{di}^TP_iD_i \\ D_i^TP_i + TD_i^TP_i\widehat{A}_i & TD_i^TP_i\widehat{A}_{di} & TD_i^TP_iD_i \end{bmatrix},$$

$$\begin{aligned} \delta V_{i2}(k) \leq{}& -T\alpha \sum_{s=k-\tau(k)}^{k-1} (1-T\alpha)^{(k-s)} x^T(s) S_i x(s) \\ &+ (1-T\alpha) x^T(k) S_i x(k) \\ &- (1-\alpha T)^{(\overline{\tau}+1)} x^T(k-\tau(k)) S_i x(k-\tau(k)) \\ &+ \sum_{s=k+1-\overline{\tau}}^{k-\underline{\tau}} (1-T\alpha)^{(k+1-s)} x^T(s) S_i x(s), \\ \delta V_{i3}(k) ={}& -T\alpha \sum_{l=-\overline{\tau}+1}^{-\underline{\tau}} \sum_{s=k+l}^{k-1} (1-T\alpha)^{(k-s)} x^T(s) S_i x(s) \\ &+ (1-T\alpha)(\overline{\tau} - \underline{\tau}) x^T(k) S_i x(k) \\ &- \sum_{s=k+1-\overline{\tau}}^{k-\underline{\tau}} (1-T\alpha)^{(k+1-s)} x^T(s) S_i x(s). \end{aligned} \quad (38)$$

It follows from (37)-(38) that

$$\begin{aligned} &\delta V_i(k) + \alpha V_i(k) + z^T(k) z(k) - \gamma^2 w^T(k) w(k) \\ &\quad = \delta V_i(k) + \alpha V_i(k) \\ &\qquad + (Cx(k) + Gw(k))^T (Cx(k) + Gw(k)) \\ &\qquad - \gamma^2 w^T(k) w(k) \\ &\quad = \begin{bmatrix} x(k) \\ x(k-\tau(k)) \\ w(k) \end{bmatrix}^T \Theta_i \begin{bmatrix} x(k) \\ x(k-\tau(k)) \\ w(k) \end{bmatrix}, \end{aligned} \quad (39)$$

where

$$\begin{gathered} \Theta_i = \begin{bmatrix} \Pi_i & P_i\widehat{A}_{di} + T\widehat{A}_i^TP_i\widehat{A}_{di} & P_iD_i + T\widehat{A}_i^TP_iD_i + C_i^TG_i \\ \widehat{A}_{di}^TP_i + T\widehat{A}_{di}^TP_i\widehat{A}_i & T\widehat{A}_{di}^TP_i\widehat{A}_{di} - (1-T\alpha)^{\overline{\tau}+1}S_i & T\widehat{A}_{di}^TP_iD_i \\ D_i^TP_i + TD_i^TP_i\widehat{A}_i + G_i^TC_i & TD_i^TP_i\widehat{A}_{di} & TD_i^TP_iD_i + G_i^TG_i - \gamma^2 I \end{bmatrix}, \\ \Pi_i = P_i\widehat{A}_i + \widehat{A}_i^TP_i + T\widehat{A}_i^TP_i\widehat{A}_i + \alpha P_i + (1-T\alpha)(\overline{\tau} - \underline{\tau} + 1) S_i + C_i^TC_i. \end{gathered} \quad (40)$$

Applying Lemma 4, we can obtain that $\Theta_i < 0$ is equivalent to the following inequality:

$$\begin{bmatrix} P_i\widehat{A}_i + \widehat{A}_i^T P_i + \alpha P_i + (1-T\alpha)(\overline{\tau}-\underline{\tau}+1)S_i & P_i\widehat{A}_{di} & P_iD_i & T\widehat{A}_i^T & C_i^T \\ \widehat{A}_{di}^T P_i & -(1-T\alpha)^{\overline{\tau}+1}S_i & 0 & T\widehat{A}_{di}^T & 0 \\ D_i^T P_i & 0 & -\gamma^2 I & TD_i^T & G_i^T \\ T\widehat{A}_i & T\widehat{A}_{di} & TD_i & -TP_i^{-1} & 0 \\ C_i & 0 & G_i & 0 & -I \end{bmatrix} < 0. \quad (41)$$

Using diag $\{P_i^{-1}\ \ P_i^{-1}\ \ I\ \ I\ \ I\}$ to pre- and post-multiply both sides of (41), respectively, we have

$$\begin{bmatrix} M_i & \widehat{A}_{di}P_i^{-1} & D_i & TP_i^{-1}\widehat{A}_i^T & P_i^{-1}C_i^T \\ P_i^{-1}\widehat{A}_{di}^T & -(1-T\alpha)^{\overline{\tau}+1}P_i^{-1}S_iP_i^{-1} & 0 & TP_i^{-1}\widehat{A}_{di}^T & 0 \\ D_i^T & 0 & -\gamma^2 I & TD_i^T & G_i^T \\ T\widehat{A}_iP_i^{-1} & T\widehat{A}_{di}P_i^{-1} & TD_i & -TP_i^{-1} & 0 \\ C_iP_i^{-1} & 0 & G_i & 0 & -I \end{bmatrix} < 0, \quad (42)$$

where $M_i = \widehat{A}_iP_i^{-1} + P_i^{-1}\widehat{A}_i^T + \alpha P_i^{-1} + (1-T\alpha)(\overline{\tau}-\underline{\tau}+1)P_i^{-1}S_iP_i^{-1}$.

Set $Q_i = P_i^{-1}S_iP_i^{-1}$ and $X_i = P_i^{-1}$; then, substituting (3) into (42) and applying Lemmas 4 and 5, we can obtain that (42) is equivalent to (36).

Therefore, one has, for $k \in [k_p, k_{p+1})$,

$$V(k) \le (1-T\alpha)^{(k-k_p)}V_{\sigma(k_p)}(k_p) - \sum_{s=k_p}^{k-1}(1-T\alpha)^{(k-1-s)}\Lambda(s), \quad (43)$$

where $\Lambda(s) = T\|z(s)\|^2 - \gamma^2 T\|w(s)\|^2$.

Following the proof line of (28), we obtain

$$\begin{aligned} &V_{\sigma(k)}(k) \\ &\le \mu(1-T\alpha)^{(k-k_q)}V_{\sigma(k_q^-)}(k_q^-) \\ &\quad - \sum_{s=k_q}^{k-1}(1-T\alpha)^{(k-1-s)}\Lambda(s) \\ &\le \mu(1-T\alpha)^{(k-k_{q-1})}V_{\sigma(k_{q-1})}(k_{q-1}) \\ &\quad - \mu\sum_{s=k_{q-1}}^{k_q-1}(1-T\alpha)^{(k-1-s)}\Lambda(s) \\ &\quad - \sum_{s=k_q}^{k-1}(1-T\alpha)^{(k-1-s)}\Lambda(s) \\ &= \mu^{N_{\sigma(k)}(k_{q-1},k)}(1-T\alpha)^{(k-k_{q-1})}V_{\sigma(k_{q-1})}(k_{q-1}) \\ &\quad - \mu^{N_{\sigma(k)}(k_{q-1},k)}\sum_{s=k_{q-1}}^{k_q-1}(1-T\alpha)^{(k-1-s)}\Lambda(s) \\ &\quad - \sum_{s=k_q}^{k-1}(1-T\alpha)^{(k-1-s)}\Lambda(s) \\ &\le \cdots \\ &\le \mu^{N_{\sigma(k)}(k_0,k)}(1-T\alpha)^{(k-k_0)}V_{\sigma(k_0)}(k_0) \\ &\quad - \mu^{N_{\sigma(k)}(k_0,k)}\sum_{s=k_0}^{k_1-1}(1-T\alpha)^{(k-1-s)}\Lambda(s) \\ &\quad - \mu^{N_{\sigma(k)}(k_1,k)}\sum_{s=k_1}^{k_2-1}(1-T\alpha)^{(k-1-s)}\Lambda(s) \\ &\quad - \cdots - \sum_{s=k_q}^{k-1}(1-T\alpha)^{(k-1-s)}\Lambda(s) \\ &= \mu^{N_{\sigma(k)}(k_0,k)}(1-T\alpha)^{(k-k_0)}V_{\sigma(k_0)}(k_0) \\ &\quad - \sum_{s=k_0}^{k-1}\mu^{N_{\sigma(k)}(s,k)}(1-T\alpha)^{(k-1-s)}\Lambda(s). \end{aligned} \quad (44)$$

Under the zero initial condition, we get

$$0 \le -\sum_{s=k_0}^{k-1}\mu^{N_{\sigma(k)}(s,k)}(1-T\alpha)^{(k-1-s)}\Lambda(s). \quad (45)$$

Namely,

$$\sum_{s=k_0}^{k-1}\mu^{N_{\sigma(k)}(s,k)}(1-T\alpha)^{(k-s)}\|z(s)\|^2 \le \gamma^2\sum_{s=k_0}^{k-1}\mu^{N_{\sigma(k)}(s,k)}(1-T\alpha)^{(k-s)}\|w(s)\|^2. \quad (46)$$

Multiplying both sides of (46) by $\mu^{-N_{\sigma(k)}(k_0,k)}$ leads to

$$\sum_{s=k_0}^{k-1}\mu^{-N_{\sigma(k)}(k_0,s)}(1-T\alpha)^{(k-s)}\|z(s)\|^2 \le \gamma^2\sum_{s=k_0}^{k-1}\mu^{-N_{\sigma(k)}(k_0,s)}(1-T\alpha)^{(k-s)}\|w(s)\|^2. \quad (47)$$

From Definition 3 and (13), we have

$$\mu^{-N_{\sigma(k)}(k_0,s)} \le (1-T\alpha)^{s-k_0}. \quad (48)$$

Combining (47) and (48) leads to

$$\sum_{s=k_0}^{k-1}(1-T\alpha)^{(s-k_0)}(1-T\alpha)^{(k-s)}\|z(s)\|^2 \le \gamma^2\sum_{s=k_0}^{k-1}(1-T\alpha)^{(k-s)}\|w(s)\|^2. \quad (49)$$

Then, summing both sides of (49) from k_0 to ∞ leads to

$$\sum_{k=k_0}^{\infty}(1-T\alpha)^{(k-k_0)}\|z(k)\|^2 \le \gamma^2\sum_{k=k_0}^{\infty}\|w(k)\|^2. \quad (50)$$

According to Definition 2, we can conclude that the theorem is true.

The proof is completed. □

3.3. H_∞ Controller Design. In this section, a state feedback controller $u(k) = K_{\sigma(k)}x(k)$ will be designed for system (11) such that the corresponding closed-loop system (51) is exponentially stable and satisfies an H_∞ performance. Consider

$$\begin{aligned}\delta x(k) &= \left(\widehat{A}_{\sigma(k)} + \widehat{B}_{\sigma(k)}K_{\sigma(k)}\right)x(k) + \widehat{A}_{d\sigma(k)}x(k-\tau(k)) + D_{\sigma(k)}w(k),\\ z(k) &= C_{\sigma(k)}x(k) + E_{\sigma(k)}w(k),\\ x(k_0+\theta) &= \varphi(\theta), \quad \theta = -\overline{\tau}, -\overline{\tau}+1, \ldots, -1, 0,\end{aligned} \quad (51)$$

where K_i, $i \in \underline{N}$, are the controller gains to be determined. $\widehat{B}_i$ are uncertain real-valued matrices with appropriate dimensions and have the following form

$$\widehat{B}_i = B_i + H_iF_i(k)E_{bi}. \quad (52)$$

Theorem 11. *Consider system* (11). *For given positive constants γ and $0 < \alpha < 1/T$, if there exist scalars ε_i, positive definite symmetric matrices Q_i and X_i, and any matrices W_i, $i \in \underline{N}$, of appropriate dimensions, such that*

$$\begin{bmatrix} \Upsilon_i & A_{di}X_i & D_i & T(A_iX_i+B_iW_i)^T & X_iC_i^T & \varepsilon_iH_i & (E_{ai}X_i+E_{bi}W_i)^T \\ X_iA_{di}^T & -(1-T\alpha)^{\overline{\tau}+1}Q_i & 0 & TX_iA_{di}^T & 0 & 0 & X_iE_{adi}^T \\ D_i^T & 0 & -\gamma^2 I & TD_i^T & G_i^T & 0 & 0 \\ T(A_iX_i+B_iW_i) & TA_{di}X_i & TD_i & -TX_i & 0 & \varepsilon_iTH_i & 0 \\ C_iX_i & 0 & G_i & 0 & -I & 0 & 0 \\ \varepsilon_iH_i^T & 0 & 0 & \varepsilon_iTH_i^T & 0 & -\varepsilon_iI & 0 \\ (E_{ai}X_i+E_{bi}W_i) & E_{adi}X_i & 0 & 0 & 0 & 0 & -\varepsilon_iI \end{bmatrix} < 0, \quad (53)$$

where $\Upsilon_i = (A_iX_i + B_iW_i) + (A_iX_i + B_iW_i)^T + \alpha X_i + (1 - T\alpha)(\overline{\tau} - \underline{\tau} + 1)Q_i$, then under the state feedback controller

$$u(k) = K_{\sigma(k)}x(k), \qquad K_i = W_iX_i^{-1} \quad (54)$$

and the average dwell time scheme (13), *the closed-loop system* (51) *is exponentially stable with a prescribed H_∞ performance level γ, where $\mu \ge 1$ satisfies* (14).

Proof. Replacing $\widehat{A}_i$ in (36) with $\widehat{A}_i + \widehat{B}_iK_i$, we get

$$\Theta_i = \begin{bmatrix} \varphi_i & A_{di}X_i & D_i & TX_i(A_i+B_iK_i)^T & X_iC_i^T & \varepsilon_i H_i & X_i(E_{ai}+E_{bi}K_i)^T \\ X_iA_{di}^T & -(1-T\alpha)^{\overline{\tau}+1}Q_i & 0 & TX_iA_{di}^T & 0 & 0 & X_iE_{adi}^T \\ D_i^T & 0 & -\gamma^2 I & TD_i^T & G_i^T & 0 & 0 \\ T(A_i+B_iK_i)X_i & TA_{di}X_i & TD_i & -TX_i & 0 & \varepsilon_i TH_i & 0 \\ C_iX_i & 0 & G_i & 0 & -I & 0 & 0 \\ \varepsilon_i H_i^T & 0 & 0 & \varepsilon_i TH_i^T & 0 & -\varepsilon_i I & 0 \\ (E_{ai}+E_{bi}K_i)X_i & E_{adi}X_i & 0 & 0 & 0 & 0 & -\varepsilon_i I \end{bmatrix} < 0, \qquad (55)$$

where $\varphi_i = (A_i+B_iK_i)X_i + X_i(A_i+B_iK_i)^T + \alpha X_i + (1-T\alpha)(\overline{\tau} - \underline{\tau} + 1)Q_i$.

Denoting $W_i = K_iX_i$, (53) is directly obtained.

The proof is completed. □

We are now in a position to give an algorithm for determining K_i and τ_a^*.

Algorithm 12. Step 1. Input the system matrices.

Step 2. Choose the parameters $0 < \alpha < 1/T$ and $\gamma > 0$. By solving (53), one can obtain the solutions of ε_i, W_i, X_i, and Q_i.

Step 3. By (54), with the obtained W_i and X_i, one can compute the gain matrices K_i.

Step 4. Compute μ and τ_a^* by (13)-(14).

4. Numerical Example

Consider system (11) with parameters as follows:

$$A_1 = \begin{bmatrix} 0.5 & -0.7 \\ 0 & 0.4 \end{bmatrix}, \qquad A_{d1} = \begin{bmatrix} 0.2 & 0 \\ 0 & -0.4 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.12 & 0 \\ 0.4 & -1 \end{bmatrix}, \qquad C_1 = \begin{bmatrix} 0.2 & -0.18 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 0.01 \\ 0 \end{bmatrix}, \qquad G_1 = 0.01,$$

$$H_1 = \begin{bmatrix} 0.05 \\ -0.05 \end{bmatrix}, \qquad E_{a1} = \begin{bmatrix} -0.11 \\ 0.03 \end{bmatrix}^T,$$

$$E_{ad1} = \begin{bmatrix} 0.03 \\ -0.1 \end{bmatrix}^T, \qquad E_{b1} = \begin{bmatrix} 0.02 \\ -0.01 \end{bmatrix}^T,$$

$$A_2 = \begin{bmatrix} 1.2 & -1.3 \\ 1.2 & -0.8 \end{bmatrix}, \qquad A_{d2} = \begin{bmatrix} 0 & 0.3 \\ 0.1 & 0 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} -0.1 & 0 \\ 0 & 0.3 \end{bmatrix}, \qquad C_2 = \begin{bmatrix} 0.1 & -0.18 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 0.02 \\ -0.01 \end{bmatrix}, \qquad G_2 = 0.05,$$

$$H_2 = \begin{bmatrix} 0.07 \\ -0.1 \end{bmatrix}, \qquad E_{a2} = \begin{bmatrix} 0.06 \\ -0.13 \end{bmatrix}^T,$$

$$E_{ad2} = \begin{bmatrix} 0.01 \\ -0.03 \end{bmatrix}^T, \qquad E_{b2} = \begin{bmatrix} -0.24 \\ 0.01 \end{bmatrix}^T,$$

$$F_1(k) = F_2(k) = \sin(k). \qquad (56)$$

Choosing $\overline{\tau} = 1$, $\underline{\tau} = 0$, $\alpha = 0.9$, $\gamma = 2$, and $T = 0.25$ and solving (53) in Theorem 11, we obtain

$$X_1 = \begin{bmatrix} 48.9936 & 28.9063 \\ 28.9063 & 57.2489 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 28.3482 & 16.5940 \\ 16.5940 & 39.3879 \end{bmatrix},$$

$$X_2 = \begin{bmatrix} 7.7646 & 12.2103 \\ 12.2103 & 35.4768 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} 4.3523 & 9.9187 \\ 9.9187 & 35.9927 \end{bmatrix}, \qquad (57)$$

and the state feedback gain matrices are as follows:

$$K_1 = \begin{bmatrix} -25.2606 & 14.3786 \\ -11.3117 & 7.9375 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 24.9665 & -8.2606 \\ 16.0614 & -12.3235 \end{bmatrix}. \qquad (58)$$

According to (14), we have $\mu = 6.5134$. Then, from (13), we get $\tau_a > \tau_a^* = 7.3516$. Choosing $\tau_a = 7.5$, the simulation results are shown in Figures 1 and 2, where the initial conditions are $x(0) = \begin{bmatrix} -1 & 1 \end{bmatrix}^T$, $x(k) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$, and $k \in [-1, 0)$ and the exogenous disturbance input is $w(k) = 0.05e^{-0.5k}$. The switching signal with average dwell time $\tau_a = 7.5$ is shown in

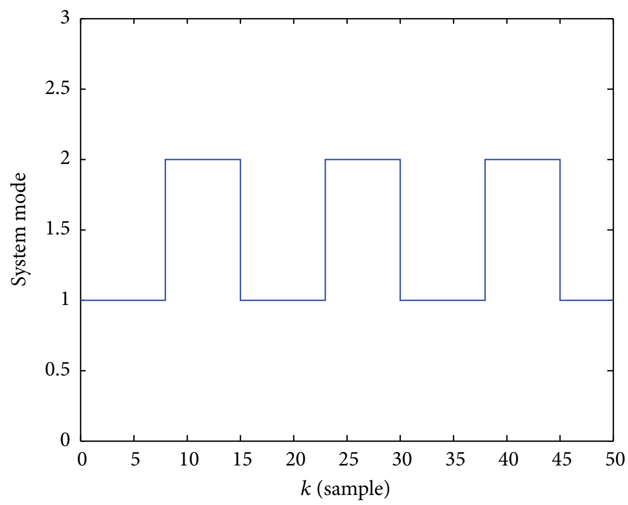

FIGURE 1: Switching signal.

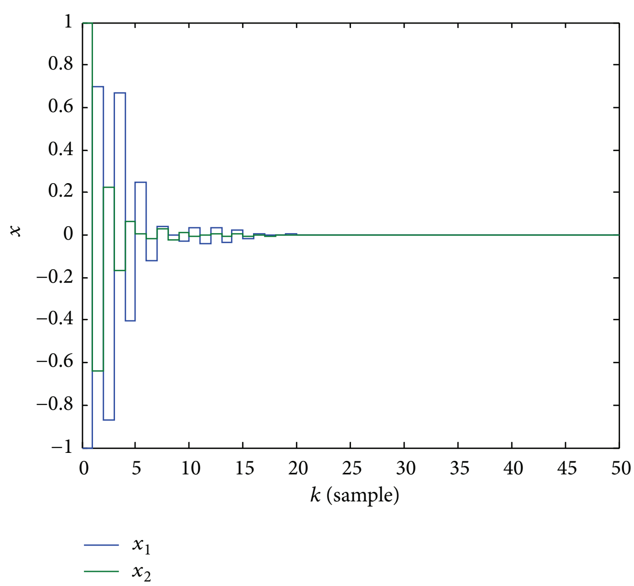

FIGURE 2: State responses of the closed-loop system.

Figure 1 and the state responses of the corresponding closed-loop system are given in Figure 2.

From Figures 1 and 2, it is easy to see that the designed controller can guarantee that the resulting closed-loop system is exponentially stable. This demonstrates the effectiveness of the proposed method.

5. Conclusions

In this paper, the robust stability and H_∞ controller design problems for time-varying delay switched system using delta operator approach have been investigated. By using the average dwell time approach and constructing a Lyapunov-Krasovskii functional candidate, sufficient conditions for the existence of a state feedback H_∞ controller are presented. Finally, a numerical example is given to illustrate the feasibility of the proposed approach. In our future work, we will study the problem of robust H_∞ filtering for delta operator switched systems with uncertainties and time-varying delays.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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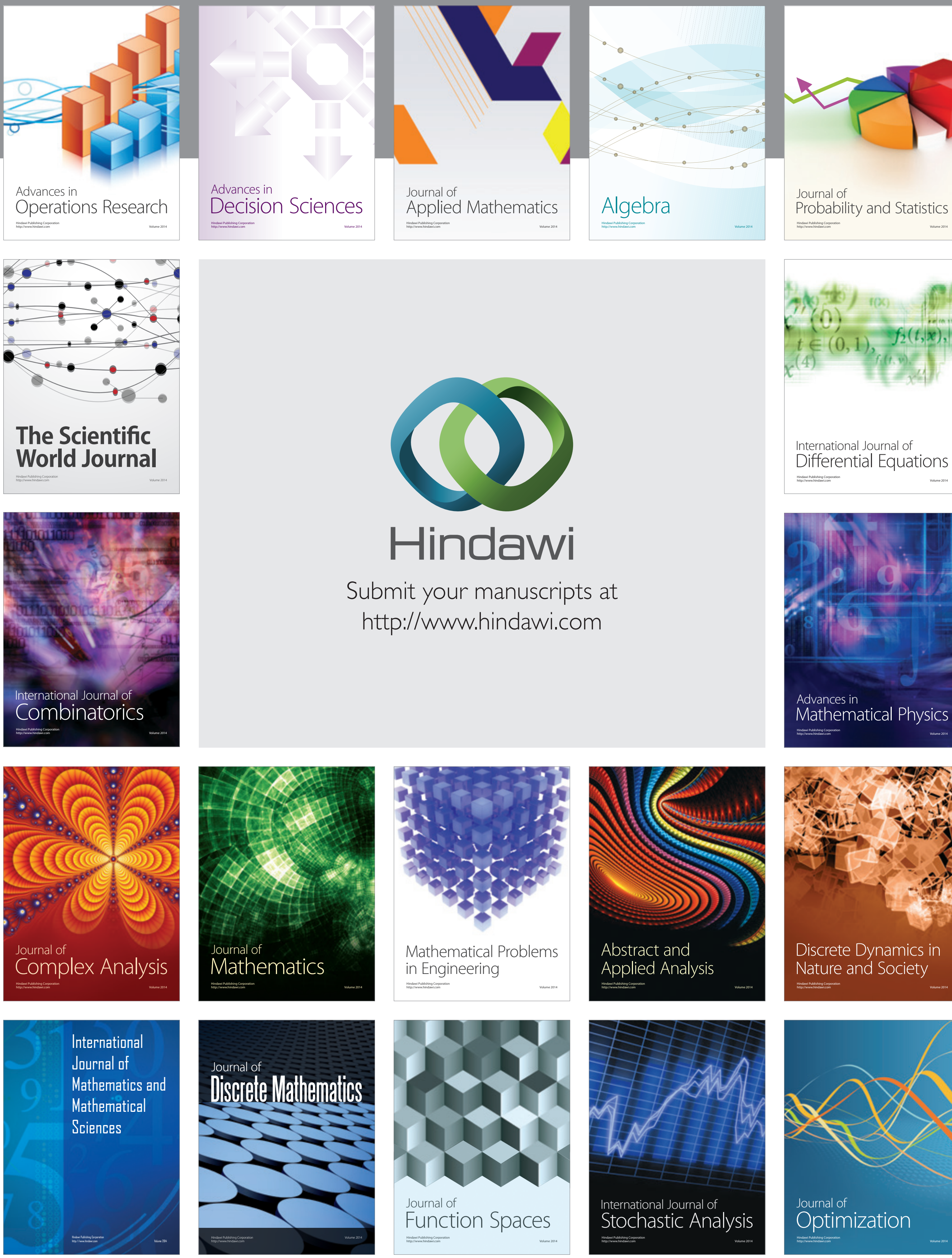
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