11-2013

What Is the Maximum Predictability Permitted by Asset Pricing Models?

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March 6, 2013

Abstract

This paper investigates whether return predictability can be explained by existing asset pricing models. Using different assumptions, I develop two theoretical upper bounds on the R-square of the regression of stock returns on predictive variables. Empirically, I find that the predictive R-square is significantly larger than the upper bounds, implying that extant asset pricing models are incapable of explaining the degree of return predictability. The reason for this inconsistency is the low correlation between the excess returns and the state variables used in the discount factor. The finding of this paper suggests the development of new asset pricing models with new state variables that are highly correlated with stock returns.

JEL Classification: C22, C53, C58, G10, G12, G14, G17

Key words: Return predictability, predictive regression, stochastic discount factor

*I am grateful to my dissertation committee, Ohad Kadan, Hong Liu, Ngoc-Khanh Tran, and Guofu Zhou (chair), for constant support and valuable comments. I thank Andy Chen, Felipe Cortes, Phil Dybvig, Frank Fabozzi, Wayne Ferson, Hung-Gay Fung, Feng Gao (CICF discussant), Fang Liu, Fernando Lopez, Cesare Robotti (FMA discussant), Anjan Thakor, Jun Tu, and seminar participants at 2012 China International Conference in Finance (CICF), 2012 FMA Doctoral Student Consortium and PhD Student Paper Presentation Session, City University of Hong Kong, Singapore Management University, University of Cincinnati, University of Missouri–St. Louis, and Washington University for helpful comments.

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1 Introduction

In the past three decades, financial economists and investors have found numerous economic variables that can be identified as predictors of stock returns.[1] The evidence on return predictability has led to the development of new asset pricing models, such as the habit formation model (Campbell and Cochrane 1999), the long-run risk model (Bansal and Yaron, 2004), and the rare disaster model (Barro, 2006; Gabaix, 2012; Gourio, 2012; Wachter, 2012). While many asset pricing models can generate time-varying expected returns, it is unclear whether they allow the same degree of predictability as observed in the data.

This paper asks whether predictability can be *fully* explained by a general asset pricing model, of which the above three models are special cases. To answer this question, I develop two theoretical upper bounds on the R^2 of the regression of stock returns on any predictive variable. If the predictive R^2 is less than the bounds, return predictability is consistent with asset pricing models. Otherwise, the models can be rejected. In this sense, the proposed bounds provide a new way to diagnose asset pricing models.

With the assumptions that the stochastic discount factor (SDF) is a function of a set of known state variables and investors' risk aversions have an upper bound (maximum risk aversion), the first bound in this paper depends on three key parameters: the multiple correlation between the excess return and the state variables of the SDF, the maximum risk aversion, and the volatility of the marginal investor's optimal wealth. The rationale of the maximum risk aversion is from Ross (2005) who shows that the volatility of the SDF is positively related to risk aversion and that any upper bound on the SDF volatility is directly related to the upper bound on the marginal investor's risk aversion.

Instead of the maximum risk aversion, the second bound assumes that the volatility of the SDF is bounded above by the market Sharpe ratio and also depends on three important

[1] Examples include the short-term interest rate (Fama and Schwert, 1977; Breen, Glosten, and Jagannathan, 1989; Ang and Bekaert, 2007), the dividend yield (Fama and French, 1988; Campbell and Yogo, 2006; Ang and Bekaert, 2007), the earnings-price ratio (Campbell and Shiller, 1988), term spreads (Campbell, 1987; Fama and French, 1988), the book-to-market ratio (Kothari and Shanken, 1997), inflation (Campbell and Vuolteenaho, 2004), corporate issuing activity (Baker and Wurgler, 2000), the consumption-wealth ratio (Lettau and Ludvigson, 2001), stock volatility (French, Schwert, and Stambaugh, 1987; Guo, 2006), output (Rangvid, 2006), oil price (Driesprong, Jacobsen, and Maat, 2008), output gap (Cooper and Prestley, 2009), and open interest (Hong and Yogo, 2012).

parameters: the multiple correlation (as used in the bound with maximum risk aversion), the market Sharpe ratio, and a parameter chosen by end-users that excludes arbitrage opportunities or "good-deals" in the sense of Cochrane and Saá-Requejo (2000). This bound is in the spirit of Ross (1976) and Cochrane and Saá-Requejo (2000) who advocate using the market Sharpe ratio to restrict the SDF volatility. The intuition is that extremely high Sharpe ratios cannot persistently exist in the market and the volatility of the SDF is intimately linked to the market Sharpe ratio. Hence, excluding extremely high Sharpe ratios is equivalent to imposing an upper bound on the SDF volatility.

In the applications, I consider ten widely explored variables utilized by Goyal and Welch (2008) to predict the excess returns of the market portfolio and cross-sectional portfolios, such as portfolios formed based on size, book-to-market ratio, momentum, and industry. For the state variables in the SDF, I first consider the consumption growth rate and the three factors used by Fama and French (1993). The results show that the predictive R^2s are almost always larger than the proposed upper bounds. When the consumption growth rate is used as the state variable in the SDF, the two proposed bounds are approximately zero regardless of any of the ten predictors is used. When the state variables are the Fama-French three factors, out of ten predictors, six predictors generate larger R^2s than the bounds with the maximum risk aversion and seven are larger than the bounds with the market Sharpe ratio. Cross-sectionally, when any one of the ten variables is used as a predictor, with several exceptions, all the predictive R^2s violate the upper bounds, no matter whether the state variables of the SDF are the consumption growth rate or the Fama-French three factors.[2].

I then consider the market portfolio forecast in the case when the state variables are those used in the habit formation model, the long-run risk model, or the rare disaster model. The state variables in the habit formation model are the consumption growth rate and the surplus consumption ratio. All the ten predictors generate larger R^2s than the two bounds. For example, when the dividend-price ratio is the predictor, the predictive R^2 is 0.27% while the upper bound is 0.03% with the maximum risk aversion and 0.02% with the market Sharpe ratio. Constantinides and Ghosh (2011) show that the state variables in the SDF of the long-run risk model can be the consumption growth rate, the risk-free rate, and the

[2]The results are robust when the momentum factor is included.

dividend-price ratio. Nine predictive R^2s violate the two bounds. With respect to the rare disaster model, Wachter (2012) shows that the state variables can be the consumption growth rate and the dividend-price ratio. In this case, both bounds are similar to that in the habit formation and long-run risk models, and all ten predictive R^2s exceed the two proposed bounds significantly. In summary, one can conclude with a high degree of confidence that the above three models explain only a fraction of predictability.

What happens when market frictions are introduced into the bounds? It may be the case that the profits documented in the literature are not attainable for investors due to the presence of market frictions. I follow Nagel (2012) by augmenting the SDF with a factor that captures different notions of transaction cost, such as the marginal value of liquidity services of tradeable assets in Holmström and Tirole (2001), the transaction costs in Acharya and Pedersen (2005), or the funding liquidity in Brunnermeier and Petersen (2009). When the liquidity factor in Pátor and Stambaugh (2003) is used as a proxy of transaction cost, the proposed bounds are improved but still less than the predictive R^2s significantly. In this sense, transaction cost or market friction is not a key source to explain return predictability.

Since the bounds are robust to any specification of investors' preference, the incapability of extant asset pricing models in explaining return predictability is mainly due to the low contemporaneous correlation between the excess return and the state variables. This explanation is supported by the fact that the upper bounds are higher when the state variables are the Fama-French three factors than the consumption growth rate, because the Fama-French three factors have a higher contemporaneous correlation with the excess return. Therefore, the finding of this paper suggests the development of new asset pricing models with new state variables that are highly correlated with stock returns. This is consistent with Cochrane and Hansen (1992) and Campbell and Cochrane (1999) who find that the low correlation exacerbates a lot of asset pricing puzzles. More recently, Albuquerque, Eichenbaum, and Rebelo (2012) introduce a demand shock to a representative agent's rate of time preference to account for the equity premium, bond term premia, and the correlation puzzle.

In the literature, most studies focus on the qualitative property of predictability, and only a few studies explicitly explore the quantitative magnitude allowed by asset pricing models. Hansen and Singleton (1983) seem to be the first to consider this problem exclusively and find that the predictability of stock returns are proportional to the predictability of the consumption growth rate. The weak predictability of the consumption growth rate implies that stock returns are almost unpredictable. Ferson and Harvey (1991) and Ferson and Korajczyk (1995) find that the multi-beta model explain a large fraction of return predictability. Kirby (1998) develops a formal test and finds that none of the recognized models can deliver sufficient predictability to accommodate the empirical pattern. Bansal, Kiku, and Yaron (2012) show that the dividend-price ratio can only generate a marginal degree of predictability with the long-run risk model. de Roon and Szymanowska (2012) show that transaction costs rather than short sale constraint can reconcile Kirby (1998). All these papers assume specific utility functions and so the results vary with different models and parameter specifications.

Ross (2005) proposes an upper bound on the predictive R^2 and finds that predictability is consistent with asset pricing models. Zhou (2010) proposes a tighter bound and shows that most predictors generate larger predictive R^2s than his bound if the SDF is driven by the consumption growth rate. This paper is closely related to Ross (2005) and Zhou (2010) but departs from them in four aspects. First, I propose a new bound with the market Sharpe ratio rather than the maximum risk aversion, giving a new choice to those who are uncertain about risk aversion. Second, Ross (2005) implicitly assumes that the correlation between the forecasted excess return and the state variables is 1, making it a special case of my bounds. Third, Zhou (2010) uses the correlation between the state variables and the default SDF,[3] while my bounds use the correlation between the excess return and the state variables, thereby providing some insights on cross-sectional predictability as to why some assets are more predictable than others. Fourth and more important, my bounds use conditional information explicitly and are much tighter than Ross (2005) and Zhou (2010). When the market portfolio is included in the state variables of the SDF, the bounds in Ross (2005) and Zhou (2010) lose the power to bind the predictive R^2 while my bounds still work well.

The rest of the paper is organized as follows. Section 2 shows how the predictive R^2 can

[3]See equation (2) in Section 2.

be bounded above by a specific SDF. Section 3 presents two semi-parametric bounds when the SDF are bounded above by the maximum risk aversion or by the market Sharpe ratio. The results of applying these two bounds to return predictability are reported in Section 4. Finally, Section 5 summarizes and concludes.

2 Model

In this section, I show how to connect the predictive regression with asset pricing models and then derive an upper bound on the predictive R^2 with the variance of the stochastic discount factor (SDF).

2.1 Asset pricing model

The central idea of finance theory is that the price of any asset is uniquely determined by a Euler equation that satisfies

$$\mathrm{E}[m_{t+1} r_{j,t+1} | I_t] = 0, \; j = 0, 1, \cdots, N, \tag{1}$$

where m_{t+1} is the SDF, $r_{j,t+1}$ is the return of asset j in excess of the risk-free rate $R_{f,t}$.[4] Equation (1) says that the risk-adjusted return process defined by the product of the excess return $r_{j,t+1}$ and the SDF m_{t+1} is a martingale and is unpredictable using any information contained in I_t. This equation is so general that it can accommodate the case when the return itself is predictable, which does not necessarily conflict with the market efficiency hypothesis. The only case of $r_{j,t+1}$ being unpredictable is when m_{t+1} is constant over time.

According to Cochrane (2005), any asset pricing model is a particular specification of m_{t+1}. One default SDF, which satisfies (1) and prices the $N+1$ assets, is given by

$$m_{0,t+1} = R_{f,t}^{-1} + (1_N - R_{f,t}^{-1}\mu)'\Sigma^{-1}(R_{t+1} - \mu), \tag{2}$$

where R_{t+1} is the $N \times 1$ vector of gross returns on the N risky assets with mean μ and

[4] I use $R_{f,t}$ rather than $R_{f,t+1}$ since it is known at the beginning of the return period.

covariance Σ, and 1_N is an N-dimensional vector of ones. I assume that μ is not proportional to 1_N and the N risky assets are not redundant.

In what follows, when it is not necessary to be explicit about the difference between assets, I will suppress the subscripts and just write r_{t+1} rather than $r_{j,t+1}$.

2.2 Predictive regression

Predictive regression is widely used in the study of return predictability, and is expressed as

$$r_{t+1} = \alpha + \beta z_t + \varepsilon_{t+1}, \tag{3}$$

where z_t is a predictive variable known at the end of period t. The degree of predictability is measured by the predictive R^2,

$$R^2 = \frac{\mathrm{Var}(\alpha + \beta z_t)}{\mathrm{Var}(r_{t+1})}. \tag{4}$$

When $R^2 > 0$, r_{t+1} can be forecasted by z_t. Otherwise, it cannot be forecasted. Following this idea, numerous variables have been identified as predictors. Ludvigson and Ng (2007) and Goyal and Welch (2008) provide a comprehensive list of predictors.

2.3 Bound on R^2

Whether return predictability can be explained by asset pricing models is equivalent to ask whether the predictive R^2 in (4) can be derived from (1). For easy of exposition, I follow Balduzzi and Kallai (1997) and normalize the SDF

$$\tilde{m}_{t+1} = \frac{m_{t+1}}{\mathrm{E}(m_{t+1})}$$

such that $E(\tilde{m}_{t+1}) = 1$ and $\mathrm{E}(\tilde{m}_{t+1} r_{t+1}) = 0$. With a little abuse of notation, I still call this normalized SDF as the SDF in the sequel.

I assume that the predictor z_t in (3) has a mean zero and variance one throughout the paper. Following Kirby (1998) and Ferson and Siegel (2003), I multiply the pricing equation

(1) by z_t in both sizes and apply the law of iterated expectations to obtain

$$\mathrm{E}(\tilde{m}_{t+1} r_{t+1} z_t) = 0, \tag{5}$$

which can be rewritten as

$$\mathrm{Cov}(r_{t+1}, z_t) = -\mathrm{Cov}(\tilde{m}_{t+1}, r_{t+1} z_t). \tag{6}$$

Since $\mathrm{Cov}(r_{t+1}, z_t) = \mathrm{E}(r_{t+1} z_t)$, equality (6) says that the expected excess return with z_t units of investment in the asset r_{t+1} is equal to the negative covariance between the SDF and the realized excess return of the investment. In other words, any dynamic trading strategy that exploits the predictability of r_{t+1} must be priced by the SDF.

Recall that $\mathrm{Var}(z_t) = 1$ and $\beta = \mathrm{Cov}(r_{t+1}, z_t)$. Combining (4) and (6) gives

$$R^2 = \frac{\mathrm{Var}(\alpha + \beta z_t)}{\mathrm{Var}(r_{t+1})} = \frac{\beta^2}{\mathrm{Var}(r_{t+1})} = \frac{\mathrm{Cov}^2(r_{t+1}, z_t)}{\mathrm{Var}(r_{t+1})} = \frac{\mathrm{Cov}^2(\tilde{m}_{t+1}, r_{t+1} z_t)}{\mathrm{Var}(r_{t+1})}. \tag{7}$$

If an asset pricing model is true, i.e., the model can match the empirical evidence, the last equality of (7) should always hold. To test this hypothesis, Kirby (1998) uses the generalized method of moments (GMM) and finds that the R^2 calculated from the last equality of (7) is much smaller than that in (4) for established consumption- and factor-based asset pricing models at that moment. Therefore, he concludes that return predictability is inconsistent with what is expected. Kirby's method is parametric and depends on the specification of $\tilde{m}_{t+1}$. Since Kirby (1998), new asset pricing models, such as the habit formation model, the long-run risk model and the rare disaster model, have been developed. This implies that we need retest the conclusion of Kirby (1998) when a new model is proposed.

I solve Kirby's problem from another perspective by developing an upper bound on (7) which can serve as a benchmark for evaluating forecasts. Statistically, the larger the predictive R^2, the higher the degree of predictability. Both financial economists and investment practitioners have paid a lot attention in the past four decades in searching for variables that can produce a better R^2. This raises two issues. First, without theoretical guidance on the R^2 permitted by asset pricing models, an investor will never know whether the used predictor is the best one. Second, given hundreds of predictors that have been identified, how does an investor use them in investment decision making? Should an investor utilize all the possible predictors or just choose a subset of them?[5] An investor cannot run two million regressions and then decides which one is the best. However, if an investor knows the maximum predictability, he can stop searching when a predictor generates an R^2 that achieves or is close to the theoretical upper bound. Moreover, an investor can directly exclude those variables with R^2s much less than the bound.

Following Kan and Zhou (2006), I impose one structure on the SDF: $\tilde{m}_{t+1} = \tilde{m}(x_{t+1})$ is a function of a set of observable state variables x_{t+1}. This structure remains general enough to accommodate many asset pricing models. For example, factor-based models, such as the capital asset pricing model (CAPM) and the Fama-French three-factor model, specify $\tilde{m}_{t+1}$ as a linear function of factors. In consumption-based models, the state variables are the surplus consumption ratio and the consumption growth rate in the habit formation model (Campbell and Cochrane, 1999; Kan and Zhou, 2006), are the risk-free rate, the dividend-price ratio, and the consumption growth rate in the long-run risk model (Constantinides and Ghosh, 2011), and are the consumption growth rate and the dividend-price ratio in the rare disaster model (Wachter, 2012). In addition, Bansal and Viswanathan (1993) specify the SDF as a nonlinear function of the market portfolio, the Treasury bill yield, and the term spread. Dittmar (2002) specifies the SDF as a cubic function of aggregate wealth. Aït-Sahalia and Lo (2000) project the SDF onto stock returns to obtain an observable kernel, thereby avoiding the use of the consumption growth rate.

Now I am in a position to present the following proposition to explain that the predictive R^2 can be bounded above.

Proposition 1 *Suppose that the SDF $\tilde{m}_{t+1} = \tilde{m}(x_{t+1})$ is a function of K-dimensional state variable x_{t+1} and $\mathrm{E}(\varepsilon_{t+1}|x_{t+1}) = 0$ in the regression $r_{t+1}z_t = a + b'x_{t+1} + \varepsilon_{t+1}$. Then,*

$$R^2 \leq \phi^2_{x,rz}\mathrm{Var}(\tilde{m}_{t+1}), \tag{8}$$

[5] This echoes Cochrane (2011) who asks how multivariate information affects the understanding of price movements.

where

$$\phi^2_{x,rz} = \frac{\rho^2_{x,rz}\mathrm{Var}(r_{t+1}z_t)}{\mathrm{Var}(r_{t+1})}, \tag{9}$$

and

$$\rho^2_{x,rz} = \frac{\mathrm{Cov}(x_{t+1}, r_{t+1}z_t)'\mathrm{Var}^{-1}(x_{t+1})\mathrm{Cov}(x_{t+1}, r_{t+1}z_t)}{\mathrm{Var}(r_{t+1}z_t)}. \tag{10}$$

The formal proof is provided in the paper's Appendix. Here I give a simplified proof showing how the predictive R^2 can be bounded by the variance of the SDF. This is the key to restrict the regression analysis of return predictability by asset pricing models. Suppose x_{t+1} and $r_{t+1}z_t$ are jointly normally distributed conditional on time t. From (7), I have

$$\begin{aligned}
R^2 &= \frac{\mathrm{Cov}^2(\tilde{m}_{t+1}, r_{t+1}z_t)}{\mathrm{Var}(r_{t+1})} = \frac{\left[\mathrm{Cov}(x_{t+1}, r_{t+1}z_t)'\mathrm{Var}^{-1}(x_{t+1})\mathrm{Cov}(\tilde{m}_{t+1}, x_{t+1})\right]^2}{\mathrm{Var}(r_{t+1})} && (11)\\
&\leq \left[\mathrm{Cov}(x_{t+1}, r_{t+1}z_t)'\mathrm{Var}^{-1}(x_{t+1})\mathrm{Cov}(x_{t+1}, r_{t+1}z_t)\right] \\
&\quad \times \frac{\left(\mathrm{Cov}(\tilde{m}_{t+1}, x_{t+1})'\mathrm{Var}^{-1}(x_{t+1})\mathrm{Cov}(\tilde{m}_{t+1}, x_{t+1})\right)}{\mathrm{Var}(r_{t+1})} && (12)\\
&= \frac{\rho^2_{x,rz}\mathrm{Var}(r_{t+1}z_t)\mathrm{Cov}(\tilde{m}_{t+1}, x_{t+1})'\mathrm{Var}^{-1}(x)\mathrm{Cov}(\tilde{m}_{t+1}, x_{t+1})}{\mathrm{Var}(r_{t+1})} && (13)\\
&\leq \frac{\rho^2_{x,rz}\mathrm{Var}(r_{t+1}z_t)\mathrm{Var}(\tilde{m}_{t+1})}{\mathrm{Var}(r_{t+1})} = \phi^2_{x,rz}\mathrm{Var}(\tilde{m}_{t+1}), && (14)
\end{aligned}$$

where (11) uses Stein's Lemma, which separates the underlying stochastic structure between r_{t+1} and x_{t+1} from the distortion of $\tilde{m}(\cdot)$ (Furman and Zitikis, 2008). Inequalities (12) and (14) use the Cauchy-Schwarz inequality. This completes the proof of (8).

Equality (11) shows that the covariance $\mathrm{Cov}(\tilde{m}_{t+1}, r_{t+1}z_t) = \mathrm{E}(r_{t+1}z_t)$ is mainly dependent on two parts: one is covariance between the excess return with z_t units of investment in r_{t+1} and the state variable x_{t+1}, $\mathrm{Cov}(x_{t+1}, r_{t+1}z_t)$, and the other is the covariance between the SDF and the state variable, $\mathrm{Cov}(\tilde{m}_{t+1}, x_{t+1})$. In the asset pricing literature, expected returns are expressed by the covariance of the returns and the SDF. The failure of asset pricing models in explaining return puzzles or anomalies is usually attributed to the inability of preferences in capturing investor's behaviors. For this reason, many different preferences have been proposed over the past three decades. With a "moment-matching" approach (calibrating parameters with real data and investigating if the estimated parameters make sense or if what the model implies with given parameters is consistent with return moments), one specific utility is usually successful in explaining one or several puzzles, but not all of them. Proposition 1, however, shows that the failure of asset pricing models may be due to the insufficient state variables x_{t+1} rather than the utility functions $\tilde{m}(\cdot)$. The covariance between the return and the SDF may blur the main reason of the inability of asset pricing models.

Proposition 1 imposes a slightly stronger assumption

$$\mathrm{E}_t(u_{t+1}|x_{t+1}) = 0, \tag{15}$$

rather than the typical $\mathrm{E}_t(u_{t+1}) = 0$ and $\mathrm{Cov}_t(u_{t+1}, x_{t+1}) = 0$. One extreme case is $u_{t+1} = 0$ when $r_{t+1}z_t$ is the same as x_{t+1} and can be fully projected on x_{t+1}. Actually, the two assumptions are equivalent if the excess return $r_{t+1}z_t$ and the state variable x_{t+1} are jointly elliptically, conditionally distributed (Muirhead, 1982).

The bound in (8) is an improvement over the bound of Ross (2005) who finds

$$R^2 \leq \mathrm{Var}(\tilde{m}_{t+1}). \tag{16}$$

This improvement is due to the fact that I use the information of x_{t+1} in $\tilde{m}_{t+1}$. Comparing (8) and (16), Ross (2005) takes the extreme possibility that the state variable and the excess return are perfectly correlated. This is obviously not the case in the real equity market. Suppose that the correlation between the consumption growth rate and the market portfolio is 0.2 and that the SDF is driven by the consumption growth rate, bound (8) will be at least 25 times tighter than that derived by Ross (2005). Cochrane (2005) notes the fact that the low correlation between the consumption growth rate and stock returns exacerbates the risk premium puzzle, but does not develop this point with respect to return predictability. In summary, Ross' bound imposes almost no structure on the SDF other than the law of one price. The consequence is that it can deliver an R^2 bound that is applicable for all SDFs. However, the cost is that the bound is too loose to be meaningful in practice. Over my sample period, Ross' bound is as large as 4.78%, but the predictive R^2 in the existing literature is less than 1% in general. To the best of my knowledge, no single predictor can produce an R^2 of 4.78%.

The bound in (16) holds with respect to the default SDF, i.e.,

$$R^2 \leq \mathrm{Var}(\tilde{m}_{0,t+1}), \tag{17}$$

which can be tightened by Kan and Zhou (2007) who show that

$$\mathrm{Var}(\tilde{m}_{0,t+1}) \leq \rho^2_{x,\tilde{m}_0} \mathrm{Var}(\tilde{m}(x_{t+1})), \tag{18}$$

where $\rho_{x,\tilde{m}_0}$ is the multiple correlation between the state variable x_{t+1} and the default SDF. Combining these two inequalities, Zhou (2010) gives the following upper bound

$$R^2 \leq \rho^2_{x,\tilde{m}_0} \mathrm{Var}(\tilde{m}_{t+1}), \tag{19}$$

which is apparently tighter than Ross (2005) bound.

An interesting question at this point is whether the bound in (8) is tighter than (19). This is equivalent to exploring whether $\phi^2_{x,rz} < \rho^2_{x,m_0}$. While there is no analytical relation between them, empirical applications will show that $\phi^2_{x,rz}$ is always smaller than ρ^2_{x,m_0}.

It is important to highlight the implication of the proposed bound of the predictive R^2 on cross-sectional return predictability. In the literature, a large number of papers find that return predictability exists and varies across cross-sectional portfolios sorted by market capitalization (Ferson and Harvey, 1991; Kirby, 1998), book-to-market ratio (Ferson and Harvey, 1991), industry (Ferson and Harvey, 1991), and volatility (Han, Yang and Zhou, 2012). Proposition 1 says that the maximum predictability of any asset is directly determined by the parameter, $\phi^2_{x,r\tilde{z}}$, in the upper bound of R^2. An asset is allowed to be more predictable if it has a higher correlation with the state variables of the SDF, regardless of the specification of the SDF.

3 Upper Bound on $\mathrm{Var}(\tilde{m}_{t+1})$

Inequality (8) provides an upper bound on the predictive R^2. However, the SDF is model-specific and unobservable. The goal of this section is to develop an upper bound on $\mathrm{Var}(\tilde{m}_{t+1})$ that is observable and model-free.

There are two approaches for the SDF specification proffered in the literature, the *absolute* approach and the *relative* approach (Cochrane and Saá-Requejo, 2000). The absolute approach makes explicit assumptions about the representative investor's preference and endowment. Under these assumptions, the SDF is uniquely, endogenously determined by the form of preferences. Although this approach is precise, it is sensitive to model and parameter misspecification errors. The relative approach assumes the existence of a set of basis assets and the absence of arbitrage opportunities, restricting the set of the SDF to those that can correctly price the basis assets in the economy and assigning positive values to payoffs in every state. Without resorting to preferences or endowments, this approach is exogenously specified and robust to model specification. The drawback is that there are usually infinite SDFs that can price the basis assets. This implies that it is difficult to choose an correct asset pricing model when all the SDFs produce the same price.

To tackle this challenge, Cochrane and Saá-Requejo (2000) and Ross (2005) propose to integrate the absolute and the relative approaches by restricting the SDF to an economically meaningful set. In contrast to Hansen and Jagannathan (1991) who restrict the SDF with a lower bound, I assume an upper bound on the SDF volatility to exclude the opportunities that may generate arbitrages.

3.1 Bound $\mathrm{Var}(\tilde{m}_{t+1})$ with relative risk aversion

Ross (2005) shows that, in an incomplete market, if all investors are bounded above by a maximum risk aversion, the set of the SDFs can be restricted by the marginal investor's SDF.

Lemma 1 (Ross, 2005) *If a utility function, $U(w)$, is bounded above in the relative risk aversion by a utility function $V(w)$, i.e., the risk aversion of $U(w)$ is less than that of*

$V(w)$, then

$$\mathrm{Var}(\tilde{m}_U) \leq \mathrm{Var}(\tilde{m}_V),$$

where $\tilde{m}_U$ and $\tilde{m}_V$ are the corresponding SDFs. Moreover, if $V(w)$ is a constant relative risk aversion utility function with risk aversion γ ($\gamma \neq 1$) and the optimal wealth is lognormally distributed such as $\log w \sim N(\mu_w, \sigma_w^2)$, then

$$\mathrm{Var}(\tilde{m}_U) \leq \gamma^2 \sigma_w^2.$$

This lemma says that the variance of any SDF can be bounded above by a maximum risk aversion.

Applying Lemma 1, I present the first semi-parametric bound in this paper as follows.

Proposition 2 *Under conditions of Propositions 1 and Lemma 1, if investors are bounded above by the maximum risk aversion γ, the upper bound of the predictive R^2 is*

$$R^2 \leq \bar{R}_{RA}^2 = \phi_{x,rz}^2 \gamma^2 \sigma_w^2. \tag{20}$$

3.2 Bound $\mathrm{Var}(\tilde{m}_{t+1})$ with market Sharpe ratio

Instead of maximum risk aversion, Ross (1976) advocates using the market Sharpe ratio to restrict the variability of the SDF. The intuition is that a high Sharpe ratio is not an arbitrage opportunity or a violation of the law of one price, but extremely high Sharpe ratios are unlikely to persist. In particular, Ross (1976) bounds the asset pricing theory residuals by assuming that no portfolio can have more than twice the market Sharpe ratio. With this idea, Cochrane and Saá-Requejo (2000) use the market Sharpe ratio to bound option prices when either market frictions or non-market risks violate simple arbitrage pricing. That is,

$$\mathrm{Std}(\tilde{m}_{t+1}) \leq h \cdot \mathrm{SR}(r_{S\&P500}), \tag{21}$$

where h is a parameter chosen by the marginal investor. Cochrane and Saá-Requejo (2000) choose $h = 2$ as the threshold for "good deals".

Proposition 3 *Under conditions of Propositions 1, if the volatility of the SDF is bounded by the market Sharpe ratio as in (21), the upper bound of the predictive R^2 is*

$$R^2 \leq \bar{R}^2_{SR} = \phi^2_{x,rz} \cdot h^2 \cdot \mathrm{SR}^2(r_{S\&P500}). \tag{22}$$

It is important to point out that the maximum risk aversion γ or h are the central parameters that a user must input to the calculation. When the upper bound of the SDF's volatility is violated, Shanken (1992) calls there have some "approximate arbitrage" opportunities. Ledoit (1995) calls a high Sharpe ratio a "δ arbitrage" that should be ruled out. Also, there are other ways to bound the volatility of $\tilde{m}_{t+1}$. For example, Bernardo and Ledoit (2000) bound the SDF as $a \leq \tilde{m}_{t+1} \leq b$, where a and b are two positive and finite real parameters. By applying the Grüss' inequality, one immediately has $\mathrm{Var}(\tilde{m}_{t+1}) \leq \frac{(b-a)^2}{4}$ for any distribution of $\tilde{m}_{t+1}$.

4 Empirical Results

This section explores empirically whether the predictive R^2s of predicting excess returns on the market portfolio and cross-sectional portfolios are smaller than the upper bounds derived from asset pricing models.

4.1 Data

The main data set used in this paper is from Goyal and Welch (2008) and the Ken French data library, spanning 1959:01-2010:12,[6] where the sources are described in detail. The excess return of the market portfolio is the gross return on the S&P 500 (including dividends) minus the gross return on a risk-free treasury bill. As discussed by Ferson and Korajczyk (1995), in the context of this paper, it is not appropriate to use continuously compounded returns, which are commonly used in the literature of return predictability. The basic pricing equation says that the expected returns are equal to the conditional covariances of returns with the marginal utility for wealth, which depends on the simple arithmetic return of

[6] I thank Amit Goyal and Ken French for making the data available.

the optimal portfolio. Moreover, continuously compounded portfolio returns are not the portfolio-weighted average of the compounded returns of the component securities. For these reasons, I use simple arithmetic returns.[7]

Ten popular economic variables in Goyal and Welch (2008) are used as predictors:

1. *Dividend-price ratio* (d/p): a 12-month moving sum of dividends paid on the S&P 500 index divided by the S&P 500 index;
2. *Earning-price ratio* (e/p): a 12-month moving sum of earnings on the S&P 500 index divided by the S&P 500 index;
3. *Dividend yield* (dy): a 12-month moving sum of dividends divided by the lagged S&P 500 index;
4. *Treasury bill rate* (tbl): the 3-month Treasury bill (secondary market) rate;
5. *Default yield spread* (dfy): the difference between BAA and AAA-rated corporate bond yields;
6. *Term spread* (tms): the difference between the long-term yield on government bonds the Treasury bill rate;
7. *Net equity expansion* (ntis): ratio of a twelve-month moving sum of net equity issues by NYSE-listed stocks to the total end-of-year market capitalization of NYSE stocks;
8. *Inflation* (infl): the Consumer Price Index;
9. *Long-term return* (ltr): return on long-term government bonds;
10. *Equity risk premium volatility* (rvol): moving standard deviation of the monthly returns on the S&P 500 index (Mele, 2007):

$$\hat{\sigma}_t = \sqrt{\frac{\pi}{2}} \sum_{i=1}^{12} \frac{|r_{t+1-i}|}{\sqrt{12}}.$$

I use this volatility measure rather than the realized volatility in Goyal and Welch (2008) to avoid two severe outliers in Octobers of 1987 and 2008.

[7]The predictive R^2s with compounded returns are generally larger than simple arithmetic returns. The results are available upon request.

To calculate the upper bounds of the predictive R^2, I need state variables. The most popular state variable in consumption-based asset pricing models is the consumption growth rate: the percentage change in the seasonally adjusted, aggregate, real per capita consumption expenditures on nondurable goods and services. The data are reported by the Bureau of Economic Analysis (BEA). In addition, I consider the linear factor models where the market index or the Fama-French three-factors are used as the state variables. I also use data on ten size portfolios, ten book-to-market portfolios, ten momentum portfolios, and ten industry portfolios, for cross-sectional predictability.

What is the reasonable maximum risk aversion has been and will continue to be a debate for a long time, although researchers admit that it should not be large. Mehra and Prescott (1985) argue that a reasonable upper bound of risk aversion is around 10. Ross (2005) uses the insurance premium to explain that a value of 5 is large enough. Barro and Ursúa (2012) think that "a γ [risk aversion] of 6 seems implausibly high." Empirically, Guiso, Sapienza and Zingales (2011) find that the average risk aversion increases from 2.85 before the 2008 crisis to 3.27 after the collapse of the financial market. Paravisini, Rappoport and Ravina (2012) estimate the risk aversion from investors' financial decisions and find that the average risk aversion is 2.85 with a median of 1.62. I follow Ross (2005) by setting the maximum risk aversion to be 5. Also, in the application, I assume that the optimal wealth for the marginal investor who has the maximum risk aversion is the market portfolio. During the sample period, the market portfolio has an annual risk premium 5.31% and a volatility 15.44%.

When the market Sharpe ratio is used to bound the predictive R^2, I follow Ross (1976) and Cochrane and Saá-Requejo (2000) by setting h equal to 2. I find that the upper bound with this value is close to that with the maximum risk aversion bound with a value of 5. This result indirectly supports Ross (2005) that the upper bound of risk aversion should not exceed 5.

4.2 Estimation and test

The parameters to calculate the predictive R^2 and its upper bounds involve only the mean and covariance of $y_{t+1} = (r_{t+1}, z_t, r_{t+1}z_t, x'_{t+1})'$, where x_{t+1} could be multi-dimensional. The moment conditions are

$$h(y_{t+1},\theta)=\left(\begin{array}{c} y_{t+1}-\mu_y \\ y_{t+1}y'_{t+1}-(\Sigma_y+\mu_y\mu'_y) \end{array}\right), \tag{23}$$

where $\mu_y = E(y_{t+1})$ and $\Sigma_y = Cov(y_{t+1})$. The econometric specification in (23) is exactly identified, the GMM estimator of $\theta = (\mu'_y, \Sigma_y)$ is the value that sets $1/T\sum_{t=1}^{T} h(y_{t+1},\theta)$ equal to zero.

The distribution of $\hat{\theta}$ takes the form

$$\sqrt{T}(\hat{\theta}-\theta) \xrightarrow{d} N(0,S), \tag{24}$$

where $S = \sum_{j=-\infty}^{\infty} E[h(y_{t+1},\theta)h(y_{t+1-j},\theta)']$.

We use a Wald test to evaluate whether $R^2 \leq \bar{R}^2_{RA}$ or $\bar{R}^2_{SR}$. This is equivalent to a one-sided test for $g(\theta_{RA}) = 0$ or $g(\theta_{Sh}) = 0$, where θ_{RA} and θ_{Sh} are the parameters used in $g(\theta_{RA}) = R^2 - \bar{R}^2_{RA}$ for the bound with risk aversion and $g(\theta_{Sh}) = R^2 - \bar{R}^2_{SR}$ for the bound with Sharpe ratio. Let Σ_{RA} and Σ_{Sh} be the corresponding covariances of θ_{RA} and θ_{Sh}. The Wald statistic is

$$W_{RA} = Tg(\hat{\theta}_{RA})\left[\frac{dg}{d\theta_{RA}}\hat{\Sigma}_{RA}\frac{dg}{d\theta_{RA}}\right]^{-1} g(\hat{\theta}_{RA}) \xrightarrow{d} \chi^2(1) \tag{25}$$

for the bound with risk aversion, and

$$W_{Sh} = Tg(\hat{\theta}_{Sh})\left[\frac{dg}{d\theta_{Sh}}\hat{\Sigma}_{Sh}\frac{dg}{d\theta_{Sh}}\right]^{-1} g(\hat{\theta}_{Sh}) \xrightarrow{d} \chi^2(1) \tag{26}$$

for the bound with Sharpe ratio.

The approach here is slightly different from the typical GMM estimation and testing by imposing the constraint $R^2 = \bar{R}^2_{RA}$ or $\bar{R}^2_{SR}$ in the econometric specification directly. With the property of GMM, the two approaches are asymptotically equivalent. The choice of this paper makes it easy to compare the difference between the predictive R^2 and the theoretical upper bounds apparently.

4.3 R^2 bounds on market portfolio predictability

Table 1 reports the predictive R^2 and its bounds for the regression model, $r_{t+1} = \alpha+\beta z_t+\varepsilon_{t+1}$, where r_{t+1} is the excess return of the market portfolio and z_t is the predictor given in the table's first column. The value of R^2 and its bounds are all in percentage points. The state variable is the consumption growth rate and the default SDF m_0 is constructed by the market portfolio.[8] Panel A shows the results when the maximum risk aversion is 5. The predictive R^2s are given in the second column, which range from 0.04% for the net equity issues (ntis) to 1.23% for the equity risk premium volatility (rvol). Positive R^2s suggest that the excess return of the market portfolio is predictable and the degree of predictability varies across predictors. The upper bound of Ross (2005), $\bar{R}^2_{Ross}$, is 4.78% reported in Column 3 regardless of what the predictor is. Because this bound exceeds any R^2 in Column 2, time-varying expected return appears to be a perfect explanation of return predictability. To the best of my knowledge, however, there is no single predictor in the literature generating an R^2 as large as 4.78% with monthly data. The reason is that Ross' bound implicitly assumes a correlation of 1 between the excess return and the consumption growth rate. Hence, it is too loose to be meaningful.

Column 4 reports the correlation between the state variable and the default SDF and Column 5 reports the bound developed by Zhou (2010). Since the correlation is 0.17, the bound in Zhou (2010) is 0.13%, and thereby improves approximately 37 times relative to Ross (2005). Out of ten predictors, eight exhibit significantly higher predictive R^2s than this bound. The two exceptions are the earnings-price ratio (e/p) and the net equity issues (ntis).

Column 6 shows that the correlations between the state variable and the excess returns with trading strategy z_t. Surprisingly, all the correlations are pretty small and range from 0.02 to 0.06. Recall that the key parameter in the upper bounds in (20) and (22) is $\phi^2_{x,rz} = \rho^2_{x,rz}\mathrm{Var}(r_{t+1}z_t)/\mathrm{Var}(r_{t+1})$. $\mathrm{Var}(r_{t+1}z_t)/\mathrm{Var}(r_{t+1})$ is larger than one but less than 4 for any z_t of the ten predictors. This implies that small value of $\rho_{x,rz}$ makes the upper bounds of the

[8] Other portfolios, such as the Fama-French three factors or Fama-French 25 size book-to-market portfolios, can be easily used to construct m_0. This will change the multiple correlation ρ^2_{x,m_0}, but the change is very small. The results are available upon request.

predictive R^2 small. Actually, both bounds with the maximum risk aversion and the market Sharpe ratio are approximately zero. As a result, the proposed bounds are significantly less than the predictive R^2s. The low bound of R^2 is consistent with Hansen and Singleton (1983) who explore the joint dynamics of stock returns and consumption growth and find that the predictability of stock returns is proportional to that of consumption growth. The weak predictability of the consumption growth rate in turn implies that stock returns are almost unpredictable. This result is confirmed by Kirby (1998) with a formal GMM test.

Panel B considers the case when the maximum risk aversion is 10. In this case, the upper bounds R^2 can be obtained by multiplying the bounds in Panel A by 4. Due to the small value of $\rho_{x,rz}$, the increase in the risk aversion does not change the upper bounds significantly. This insensitivity of the R^2 bounds implies that changing the maximum risk aversion is not promising to reconcile the violations of the bounds. On the other hand, since the bound with the maximum risk aversion of 5 is close to the bound with the market Sharpe ratio (as shown in Panel A), I believe that 5 is a reasonable upper bound of risk aversion. In this sense, I will report results with the maximum risk aversion of 5 in the sequel.

One may be curious that the results in Table 1 are only valid to consumption-based asset pricing models since I only consider the consumption growth rate as the state variable of the SDF. In the literature, there are many factor-based asset pricing models. Table 2 reports the bounds with alternative state variables. In particular, Panel A assumes that the state variable is the market portfolio (the state variable of CAPM) and Panel B considers the Fama-French three factors. With these two cases, since the correlation ρ_{x,m_0} is approaching one, the bound of Zhou (2010) reduces to Ross (2005) and exceeds the predictive R^2s. However, the bounds proposed in this paper still work well. When the state variable is the market portfolio, eight predictors violate the bounds, either with the maximum risk aversion or the market Sharpe ratio. When the state variables are the Fama-French three factors, six predictors violate the bounds with the maximum risk aversion and seven violate the bounds with the market Sharpe ratio. When the momentum factor is added to the Fama-French three factors, the upper bounds of R^2 do not change significantly, and therefore, to conserve space, the results are not reported.

In summary, the predictive R^2s from the predictive regression are larger than the maximum predictability permitted by asset pricing models. Since $\mathrm{Var}(r_{t+1}z_t)/\mathrm{Var}(r_{t+1})$ has nothing with the asset pricing model, the failure to explain predictability is clearly due to the correlation between the excess return and the state variables of the SDF, the maximum risk aversion, or the volatility of the marginal investor' wealth. In the bound with the market Sharpe ratio, the parameter of the maximum risk aversion is replaced by the parameter h which is the threshold of excluding arbitrage opportunities. Among them, I have already considered the case of a risk aversion of 10. The marginal investor's wealth is assumed to be the market portfolio, which may be more volatile than the real wealth with other non-financial assets. Therefore, the only reason is that the correlation between the excess return and the state variables in the SDF is too low (as shown in Column 6 in Tables 1 and 2). This explanation is obvious when the state variables are the Fama-French three factors, which have a much higher correlation with the excess returns and so generate higher bounds on the predictive R^2s. The findings of this section suggest that the state variables are more important than investor's preferences in explaining return predictability. This explanation is consistent with Cochrane and Hansen (1992), Campbell and Cochrane (1999), and Albuquerque, Eichenbaum, and Rebelo (2012) who attribute the failure of consumption-based asset pricing models to the low correlation between asset returns and the state variables of the SDF.

4.4 R^2 bounds with recently developed models

This subsection discusses whether the habit formation model, the long-run risk model, or the rare disaster model can explain the predictability of the market portfolio when the state variables are from one of these three asset pricing models.

4.4.1 Habit formation

The state variables in the habit formation model are the consumption growth rate and the surplus consumption ratio s_t that is unobservable since the habit level is latent. I follow Campbell and Cochrane (1999) by extracting s_t from the model and calculate the multiple correlation between the state variables $x_t = (\triangle c_t, \triangle s_t)$ and the excess return with

z_t units of investment in the market portfolio. The results are reported in Panel A of Table 3. With an additional state variable $\triangle s_t$, the correlation between the excess return and the state variables approximately doubles relative to the traditional Consumption-based models. However, it is still very small. Nine out of ten correlations (since different predictor implies different correlation) are less than 0.1. As a result, both bounds with the maximum risk aversion and the market Sharpe ratio are still close to zero, significantly less than the predictive R^2s.

4.4.2 Long-run risk

The long-run risk model focuses on the low-frequency properties of the time series of dividends and aggregate consumption, and can explain simultaneously the equity risk premium puzzle, the risk-free rate puzzle, and the high level of market volatility. The key assumptions in the long-run risk model are that the consumption growth rate and the dividend growth rate follow the following joint dynamics:

$$\begin{aligned}
\triangle c_{t+1} &= \mu_c + \mu_{c,t} + \sigma_t \epsilon_{c,t+1}, \\
\mu_{c,t+1} &= \rho_\mu \mu_{c,t} + \psi_c \sigma_t \epsilon_{\mu,t+1}, \\
\sigma_{t+1}^2 &= (1-\nu)\bar{\sigma}^2 + \nu \sigma_t^2 + \sigma_w \epsilon_{\sigma,t+1}, \\
\triangle d_{t+1} &= \mu_d + \phi \mu_{c,t} + \phi \sigma_t \epsilon_{d,t+1},
\end{aligned}$$

where c_{t+1} is the log aggregate consumption and d_{t+1} is the log dividends. The shocks $\epsilon_{c,t+1}$, $\epsilon_{\mu,t+1}$, $\epsilon_{\sigma,t+1}$, and $\epsilon_{d,t+1}$ are assumed to be i.i.d. normally distributed.[9]

With log-affine approximation, the SDF is

$$\log m_{t+1} = A_0 + A_1 \mu_{c,t} + A_2 \sigma_t^2 + A_3 \triangle c_{t+1} + A_4 \mu_{c,t+1} + A_5 \sigma_{t+1}^2, \tag{27}$$

where $A_0, \cdots, A_5$ are parameters to be estimated. There are two latent state variables in the SDF, the conditional mean of the consumption growth rate y_t and the conditional variance of its innovation σ_t^2, which are difficult to be measured in the data. Motivated by Dai and

[9] I use $\mu_{c,t}$ rather than x_t to denote the persistent component of consumption since x_t has been used as the state variables of the SDF.

Singleton (2000), Constantinides and Ghosh (2011) bridge this gap and find that these two latent variables can be projected on the log risk-free rate $r_{f,t}$ and the log dividend-price ratio dp_t:

$$\begin{aligned}\mu_{c,t} &= \alpha_0 + \alpha_1 r_{f,t} + \alpha_2 dp_t,\\ \sigma_t^2 &= \beta_0 + \beta_1 r_{f,t} + \beta_2 dp_t.\end{aligned}$$

In this way, the log SDF is an affine function of the log risk-free rate, the log dividend-price ratio, and the consumption growth rate:

$$\log m_{t+1} = B_0 + B_1 r_{f,t} + B_2 dp_t + B_3 r_{f,t+1} + B_4 dp_{t+1} + B_5 \triangle c_{t+1}.$$

Panel B of Table 3 shows the R^2 bounds when the state variables in the SDF are

$$x_{t+1} = (\triangle c_{t+1}, r_{f,t+1}, dp_{t+1})'. \tag{28}$$

The correlations between x_{t+1} and the excess returns conditional information z_t are around 0.1, implying that the R^2 bounds will be significantly larger than that in the habit formation model. However, both bounds are still less than the predictive R^2s. These results are consistent with Constantinides and Ghosh (2011) and Bansal, Kiku, and Yaron (2012) who find that the permitted degree of predictability is extremely low in the long-run risk framework.

4.4.3 Rare disaster

The rare disaster model revived by Barro (2006) is intended to solve the equity risk premium puzzle and does not accommodate time-varying expected returns. Gabaix (2012) allows for time-varying probability and size of disasters, thereby generating volatility of price-dividend ratios and implying return predictability to some extent. Gourio (2008) exclusively studies whether the predictability generated by the rare disaster model can match the magnitude of predictability observed in market data. In so doing, he introduces an exogenous, persistent, time-varying disaster probability in the rare disaster framework. With numerical simulation, to best match the predictive power of the dividend-price ratio, the model needs to have an average equity premium as high as 13.71%, which is obviously not reasonable. As a result, Gourio concludes "with Epstein-Zin utility, the model can fit the facts qualitatively, and to some extent quantitatively, if we allow for a highly variable probability of disaster, leverage and an IES above unity" to explain return predictability.

The basic assumption for the rare disaster model is that the consumption growth rate follows the stochastic process:

$$\triangle c_{t+1} = \begin{cases} \mu_c + \sigma \epsilon_{t+1}, & \text{with probability } 1 - p_t; \\ \mu_c + \sigma \epsilon_{t+1} + \log(1-b), & \text{with probability } p_t. \end{cases} \tag{29}$$

where ϵ_{t+1} is i.i.d. $N(0,1)$ and $0 < b < 1$ is the size of the disaster. The crucial question is to find a variable to proxy the unobservable probability of disasters. Wachter (2012) considers the rare disaster model in a continuous-time setting and find that the dividend-price ratio is a strictly increasing function of the disaster probability, which implies that one can invert this function to find the disaster probability given the observations of the dividend-price ratio. Hence, in addition to the consumption growth rate, the dividend-price ratio can be used as an observable state variable for the rare disaster model. That is,

$$x_{t+1} = (\triangle c_{t+1}, dp_{t+1})'.$$

The predictive R^2 bounds are exhibited in Panel C of Table 3. Again, ten predictive R^2s exceed the bounds significantly. These results are approximately the same as that in the habit formation model. Therefore, consistent with Gourio's (2008) numerical simulation, it is difficult for the rare disaster model to match the observed return predictability.

4.5 R^2 bounds with market frictions

One interesting question is what happens when the market is not frictionless. The proposed bounds in this paper assume that investors can trade freely without transaction costs and constraints. It may be the case that the profits documented in the literature are not attainable for investors because of transaction costs and constraints. The limit of arbitrage forces investors to deviate from the trading strategy that seeks to exploit predictability in the market.

Market frictions refer to trading costs that can be the transaction cost in He and Modest (1995), the marginal value of liquidity services of tradeable assets in Holmström and Tirole (2001), the transaction cost in Acharya and Pedersen (2005), the funding liquidity in Brunnermeier and Pedersen (2009), or the execution cost in Hasbrouck (2009).[10] Nagel (2012) reviews these models and finds that the SDF in frictionless market can be augmented with a factor Λ_t that captures the state of transaction costs

$$\tilde{m}^F_{t+1} = \tilde{m}_{t+1} \frac{\Lambda_t}{\Lambda_{t+1}}. \tag{30}$$

Let $\triangle\lambda_{t+1} = \log(\Lambda_{t+1}/\Lambda_t)$. Then I can rewrite $\tilde{m}^F_{t+1}$ as

$$\tilde{m}^F_{t+1} = \tilde{m}^F(x_{t+1}, \triangle\lambda_{t+1}). \tag{31}$$

In this way, a higher $\triangle\lambda_{t+1}$ means a higher transaction cost, and an asset paying well in the state of higher $\triangle\lambda_{t+1}$ earns a lower expected return. The bounds in this paper can be adjusted easily by including $\triangle\lambda_{t+1}$ into the state variables.

I use the liquidity factor constructed by Pástor and Stambaugh (2003) as the proxy of transaction cost. Table 4 reports the R^2 bounds on the market portfolio forecasts with the ten macroeconomic variables. Panel A considers the case when the state variable is the consumption growth rate. In this case, the bound with either the maximum risk aversion or the market Sharpe ratio is marginally improved relative to that without considering transaction (Panel A of Table 1). All the ten R^2s are significantly larger than the two bounds. Where the Fama-French three factors are used as the state variables in Panel B, the results are almost the same as Panel B of Table 2. Six R^2s exceed the two proposed bounds significantly. The results in Table 4 are in contrast to de Roon and Szymanowska (2012) who point out that the finding in Kirby (1998) can be reconciled with transaction costs. The reason is that they consider fixed transaction cost while I focus on time-varying cost.

[10] Amihud, Mendelson, and Pedersen (2005) give an excellent literature review on the relationship between transaction costs of different dimensions and asset prices.

4.6 R^2 bounds on cross-sectional portfolio predictability

One interesting question is whether the proposed bounds work well for cross-sectional portfolio forecasts. Theoretically, Propositions 2 and 3 show that individual portfolios should have different predictability since they have different correlations with the state variables. For this reason, I report results on ten size portfolios, ten value portfolios (formed based on the book-to-market ratio), ten momentum portfolios, and ten industry portfolios. I consider two cases for the state variables: the consumption growth rate and the Fama-French three factors. The maximum risk aversion is assumed to be 5. To save space, I report the results when the predictor is the dividend-price ratio or the term spread. The results for the other predictors exhibit similar characteristics and are available upon request.

4.6.1 Portfolio forecasts with dividend-price ratio

Size portfolios The predictability of size portfolios (i.e., portfolios formed based on market capitalization) has been extensively investigated (Ferson and Harvey, 1991; Ferson and Korajczyk, 1995; Kirby, 1998). The basic pattern is that portfolios with small size are more predictable than portfolios with large size. Table 5 reports the predictive R^2s when the dividend-price ratio is used as the predictor, and the upper bounds proposed in this paper. Panel A considers the case when the state variable is the consumption growth rate. Surprisingly, the predictability of size portfolios in Column 2 does not show the monotonic pattern reported by Kirby (1998).[11] The minimum predictability is the smallest size portfolio with an R^2 of 0.09%. The maximum predictability is the 4th smallest size portfolio with an R^2 of 0.48%. The predictive R^2 for the largest size portfolio is 0.25%. The bound developed by Ross (2005) is 4.78%, larger than any predictive R^2, suggesting that the predictability of size portfolios can be explained. However, the bound in Zhou (2010) is 0.13%, smaller than all R^2s except for the smallest size portfolio. With respect to the proposed bounds in this paper, both bounds with the maximum risk aversion and the market Sharpe ratio are close

[11] Kirby (1998) forecasts the size portfolios by using five predictors simultaneously (the excess return on the equally weighted NYSE index, a dummy variable for the month of January, the 1-month 90-day Treasury bill rate less than the 30-day Treasury bill rate, the yield on Moody's Baa rated bonds less the yield on Moody's Aaa rated bonds, and the dividend yield on the S&P 500 stock index less the 30-day Treasury bill rate).

to zero and significantly smaller than the corresponding R^2s.

Panel B of Table 5 considers the case when the Fama-French three factors are used as the state variables. Since the Fama-French three factor model includes the market portfolio as a factor, which has high correlations with component portfolios, the bound in Zhou (2010) loses the power to diagnose the predictability. However, my bounds remain valid. Except for the first two smallest size portfolios, the other eight portfolios display R^2s larger than the bounds, either the bound with the maximum risk aversion or the bound with the Sharpe ratio. Cross-sectionally, the proposed bounds are monotonically decreasing in firm size. The inability of the dividend-price ratio to generate monotonic predictive R^2s may be due to the fact that the dividend-price ratio uses the sum of dividends paid on the S&P 500 index. Big firms usually pay more dividends than small firms. As a result, the dividend-price ratio is more informative for large size portfolios, exhibiting higher predictive power.

Value portfolios The predictability of the value premium reported in the literature is mixed. Lettau and Ludvigson (2001) show some positive evidence, but Lewellen and Nagel (2006) find that the time-variation in the expected value premium is marginal and hence unpredictable. Table 6 reports the results when the dividend-price ratio is used to forecast the ten value portfolios formed based on the book-to-market ratio. The predictive R^2s are 0.11% for the 1st decile portfolio (growth portfolio) and 0.32% for the 10th decile portfolio (value portfolio). This suggests that when the difference between the value and the growth portfolio is used as a proxy of the value premium, the value premium should be significantly predicted by the dividend-price ratio. Panel A shows that the proposed bounds, as well as those in Zhou (2010), are less than the predictive R^2s when the state variable is the consumption growth rate. When the Fama-French three factors are used, the two proposed bounds are still less than the predictive R^2s. While the predictive R^2s are more than 0.25% except for the growth portfolio. The difference between the R^2s for the value portfolios versus the size portfolios is that the proposed bounds do not show a monotonic pattern with respect to the book-to-market ratio.

Momentum portfolios When the dividend-price ratio is used to forecast the ten decile momentum portfolios, the predictive R^2s vary significantly, ranging from 0.10% for the 5th portfolio to 0.83% for the 9th portfolio, as shown in Table 7. The R^2s for the lowest- and the highest-momentum portfolios are 0.19% and 0.24%, respectively. Again, all R^2s exceed the proposed bounds when the state variable is the consumption growth rate. When the Fama-French three factors are used, the predictive R^2s, except for the 5th portfolio, exceed the two bounds.

Industry portfolios Ferson and Harvey (1991) and Ferson and Korajczky (1995) show significant predictability for industry portfolios. In Table 8, when the dividend-price ratio is used as the predictor, eight out of ten industries (with two exceptions, manufacturing and energy) show strong performance. The most predictable industry is nondurable goods with an R^2 of 0.64%. Panel A shows that all the predictive R^2s except Enrgy exceed the proposed bounds when the state variable is the consumption growth rate. Panel B identifies that five industries that have larger R^2s than the bounds when the state variables are the Fama-French three factors. This result indicates that asset pricing models can generate more predictability for some industry portfolios than others.

4.6.2 Portfolio forecasts with term spread

This section discusses the results when the term spread is used to forecast cross-sectional portfolios. While the term spread exhibits stronger predictive ability, the overall pattern is similar to the case when the predictor is the dividend-price ratio. When the consumption growth rate is used as the state variable, asset pricing models do not show any hope of explaining return predictability. Instead, when the Fama-French three factors are used, they generate larger bounds. Here I only report the results for the case of the Fama-French three factors (see Tables 9 and 10), which are summarized as follows. First, the predictive ability of the term spread is too strong to be explained by asset pricing models. That is, all the predictive R^2s, with three exceptions, exceed the proposed bounds. Second, the predictability varies significantly across different portfolios. Third, the failure of current asset pricing models lies in the poor ability of the state variable in capturing the cross-sectional characteristics of individual portfolios.

The results are robust to the habit formation model, the long-run risk model, and the rare disaster model, and also robust to the case with transaction costs. Overall, the cross-sectional results echo the market portfolio forecast that time-varying expected return can only explain a small fraction of predictability.

5 Conclusion

This paper asks whether the overall pattern of return predictability is consistent with asset pricing models. To answer this question, I develop two upper bounds on the predictive R^2. When one of ten established macroeconomic variables in Goyal and Welch (2008) is used to forecast the excess returns of the market portfolio and cross-sectional portfolios, the predictive R^2s almost always exceed the upper bounds, implying that return predictability cannot be fully explained by extant asset pricing models. The reason is the low correlation between the forecasted excess return and the state variables used in the SDF.

There are also many other reasons to explain why the predictive R^2s violate the upper bounds. There may be structural breaks in the specific models over the long-term period investigated in this study. For example, Goyal and Welch (2008), Rapach, Strauss, and Zhou (2010), Henkel, and Martin and Nardari (2011) find strong evidence of fairly frequent breaks in the predictive regression. Most macro fundamental variables exhibit significant power of return predictability during economic recessions, but perform badly during economic expansions. It may be necessary to incorporate regime changes into the upper bounds. Also, an alternative explanation is behavioral bias that leads investors to under- or over-react to private or public news, generating return predictability.

This paper focuses on the stock market. It will be of interest to investigate whether any asset pricing model can explain return predictability on the bond market, housing market, commodity market, currency market, and international markets.

Appendix

Proof of Proposition 1. Since $\mathrm{E}_t(\varepsilon_{t+1}|x_{t+1}) = 0$ in the regression $r_{t+1}z_t = a + bx_{t+1} + \varepsilon_{t+1}$, I have

$$\mathrm{Cov}(\varepsilon_{t+1}, \tilde{m}(x_{t+1}) = \mathrm{E}[\mathrm{E}(\varepsilon_{t+1}|x_{t+1})\tilde{m}(x_{t+1})] = 0.$$

Then

$$\mathrm{Cov}(r_{t+1}z_t, \tilde{m}(x_{t+1})) = \mathrm{Cov}[b'x_{t+1}, \tilde{m}(x_{t+1})] = b'\Sigma_{x\tilde{m}}.$$

The Cauchy-Schwarz inequality generates

$$\mathrm{Cov}[r_{t+1}z_t, \tilde{m}(x_{t+1})]^2 = (b'\Sigma_{xx}^{1/2}\Sigma_{xx}^{-1/2}\Sigma_{x\tilde{m}})^2 \leq (b'\Sigma_{xx}b)(\Sigma'_{x\tilde{m}}\Sigma_{xx}^{-1}\Sigma_{x\tilde{m}}).$$

From the regression $r_{t+1} = \alpha + \beta z_t + \varepsilon_{t+1}$, the R^2 is

$$\begin{aligned}
R^2 &= \frac{\beta \mathrm{Var}(z_t)\beta}{\mathrm{Var}(r_{t+1})} \leq \frac{\mathrm{Cov}^2(r_{t+1}, z_t)}{\mathrm{Var}(r_{t+1})} \\
&= \frac{\mathrm{Cov}^2(\tilde{m}_{t+1}, r_{t+1}z_t)}{\mathrm{Var}(r_{t+1})} \\
&\leq \frac{b'\Sigma_{xx}b}{\mathrm{Var}(r_{t+1}z_t)} \frac{\mathrm{Var}(r_{t+1}z_t)(\Sigma'_{x\tilde{m}}\Sigma_{xx}^{-1}\Sigma_{x\tilde{m}})}{\mathrm{Var}(r_{t+1})} \\
&\leq \rho^2_{x,rz}\frac{\mathrm{Var}(r_{t+1}z_t)}{\mathrm{Var}(r_{t+1})}\mathrm{Var}(\tilde{m}_{t+1}) \\
&= \phi^2_{x,rz}\mathrm{Var}(\tilde{m}_{t+1}).
\end{aligned}$$

This completes the proof.

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Table 1: R^2 bounds on market portfolio forecast

The table reports the bounds of the predictive R^2 from the regression $r_{t+1} = \alpha + \beta z_t + \varepsilon_{t+1}$, where r_{t+1} is the excess return of the market portfolio and z_t is one of the ten predictors given in the first column. The state variable x in the SDF is the consumption growth rate. The marginal investor's risk aversion is 5 in Panel A and 10 in Panel B. $\bar{R}^2_{Ross}$ and $\bar{R}^2_{Zhou}$ denote the bounds proposed by Ross (2005) and Zhou (2010). $\bar{R}^2_{RA}$ and $\bar{R}^2_{SR}$ denote the bounds developed in this paper with the maximum risk aversion and the market Sharpe ratio, respectively. ρ_{x,m_0} and $\rho_{x,rz}$ denote the multiple correlations, where the default SDF m_0 is constructed by the market portfolio. Statistical significance is assessed by the Wald statistic for testing that the predictive R^2 is less than the theoretical upper bound. ***, **, and * indicate significance at the 1%, 5%, and 10% level, respectively.

z	$R^2(\%)$	$\bar{R}^2_{Ross}(\%)$	ρ_{x,m_0}	$\bar{R}^2_{Zhou}(\%)$	$\rho_{x,rz}$	$\bar{R}^2_{RA}(\%)$	$\bar{R}^2_{SR}(\%)$
Panel A: maximum risk aversion $\gamma = 5$							
d/p	0.27	4.78	0.17	0.13***	0.02	0.00***	0.00***
e/p	0.11	4.78	0.17	0.13	0.03	0.01***	0.01***
dy	0.33	4.78	0.17	0.13***	0.02	0.00***	0.00***
tbl	0.21	4.78	0.17	0.13***	0.06	0.02***	0.02***
dfy	0.21	4.78	0.17	0.13***	0.02	0.00***	0.00***
tms	0.56	4.78	0.17	0.13***	0.04	0.01***	0.01***
ntis	0.04	4.78	0.17	0.13	0.02	0.00***	0.00***
infl	0.42	4.78	0.17	0.13***	0.02	0.00***	0.00***
ltr	1.04	4.78	0.17	0.13***	0.02	0.00***	0.00***
rvol	1.23	4.78	0.17	0.13***	0.03	0.00***	0.00***
Panel B: maximum risk aversion $\gamma = 10$							
d/p	0.27	19.11	0.17	0.54	0.02	0.01***	0.00***
e/p	0.11	19.11	0.17	0.54	0.03	0.03***	0.01***
dy	0.33	19.11	0.17	0.54	0.02	0.01***	0.00***
tbl	0.21	19.11	0.17	0.54	0.06	0.07***	0.02***
dfy	0.21	19.11	0.17	0.54	0.02	0.01***	0.00***
tms	0.56	19.11	0.17	0.54**	0.04	0.03***	0.01***
ntis	0.04	19.11	0.17	0.54	0.02	0.01**	0.00***
infl	0.42	19.11	0.17	0.54	0.02	0.01***	0.00***
ltr	1.04	19.11	0.17	0.54***	0.02	0.01***	0.00***
rvol	1.23	19.11	0.17	0.54***	0.03	0.01***	0.00***

Table 2: R^2 bounds on market portfolio forecast

The table reports the bounds of the predictive R^2 from the regression $r_{t+1} = \alpha + \beta z_t + \varepsilon_{t+1}$, where r_{t+1} is the excess return of the market portfolio and z_t is one of the ten predictors given in the first column. The state variables x_t in the SDF are the market portfolio in Panel A or the Fama-French three factors in Panel B. $\bar{R}^2_{Ross}$ and $\bar{R}^2_{Zhou}$ denote the bounds proposed by Ross (2005) and Zhou (2010). $\bar{R}^2_{RA}$ and $\bar{R}^2_{SR}$ denote the bounds developed in this paper with the maximum risk aversion and the market Sharpe ratio, respectively. ρ_{x,m_0} and $\rho_{x,rz}$ denote the multiple correlations, where the default SDF m_0 is constructed by the market portfolio. Statistical significance is assessed by the Wald statistic for testing that the predictive R^2 is less than the theoretical upper bound. ***, **, and * indicate significance at the 1%, 5%, and 10% level, respectively.

z	$R^2(\%)$	$\bar{R}^2_{Ross}(\%)$	ρ_{x,m_0}	$\bar{R}^2_{Zhou}(\%)$	$\rho_{x,rz}$	$\bar{R}^2_{RA}(\%)$	$\bar{R}^2_{SR}(\%)$
Panel A: x_t is the return of the market portfolio							
d/p	0.27	4.78	1.00	4.76	0.10	0.06***	0.05***
e/p	0.11	4.78	1.00	4.76	0.05	0.02***	0.02***
dy	0.33	4.78	1.00	4.76	0.06	0.02***	0.02***
tbl	0.21	4.78	1.00	4.76	0.07	0.03***	0.02***
dfy	0.21	4.78	1.00	4.76	0.25	0.47	0.40
tms	0.56	4.78	1.00	4.76	0.03	0.00***	0.00***
ntis	0.04	4.78	1.00	4.76	0.19	0.24	0.21
infl	0.42	4.78	1.00	4.76	0.10	0.07***	0.06***
ltr	1.04	4.78	1.00	4.76	0.00	0.00***	0.00***
rvol	1.23	4.78	1.00	4.76	0.25	0.32***	0.27***
Panel B: x_t are the Fama-French three-factors							
d/p	0.27	4.78	1.00	4.76	0.11	0.07***	0.06***
e/p	0.11	4.78	1.00	4.76	0.12	0.10	0.09**
dy	0.33	4.78	1.00	4.76	0.08	0.04***	0.03***
tbl	0.21	4.78	1.00	4.76	0.25	0.35	0.30
dfy	0.21	4.78	1.00	4.76	0.32	0.75	0.64
tms	0.56	4.78	1.00	4.76	0.18	0.16***	0.13***
ntis	0.04	4.78	1.00	4.76	0.23	0.33	0.28
infl	0.42	4.78	1.00	4.76	0.16	0.16***	0.14***
ltr	1.04	4.78	1.00	4.76	0.09	0.06***	0.04***
rvol	1.23	4.78	1.00	4.76	0.28	0.40***	0.34***

Table 3: R^2 bounds on market portfolio forecast with recently developed models

The table reports the bounds on the R^2 from the regression $r_{t+1} = \alpha + \beta z_t + \varepsilon_{t+1}$, where r_{t+1} is the excess return of the market portfolio and z_t is one of the ten predictors in Column 1. $\bar{R}^2_{Ross}$ and $\bar{R}^2_{Zhou}$ denote the bounds proposed by Ross (2005) and Zhou (2010). $\bar{R}^2_{RA}$ and $\bar{R}^2_{SR}$ denote the bounds developed in this paper with the maximum risk aversion and the market Sharpe ratio, respectively. ρ_{x,m_0} and $\rho_{x,rz}$ denote the multiple correlations, where the default SDF m_0 is constructed by the market portfolio. Statistical significance is assessed by the Wald statistic for testing that the predictive R^2 is less than the theoretical upper bound. ***, **, and * indicate significance at the 1%, 5%, and 10% level, respectively.

z	R^2(%)	$\bar{R}^2_{Ross}$(%)	ρ_{x,m_0}	$\bar{R}^2_{Zhou}$(%)	$\rho_{x,rz}$	$\bar{R}^2_{RA}$(%)	$\bar{R}^2_{SR}$(%)
Panel A: Habit formation model							
d/p	0.27	4.78	0.18	0.15***	0.07	0.03***	0.02***
e/p	0.11	4.78	0.18	0.15	0.04	0.01***	0.01***
dy	0.33	4.78	0.18	0.15***	0.06	0.02***	0.02***
tbl	0.21	4.78	0.18	0.15***	0.06	0.02***	0.02***
dfy	0.21	4.78	0.18	0.15***	0.11	0.10***	0.08***
tms	0.56	4.78	0.18	0.15***	0.04	0.01***	0.01***
ntis	0.04	4.78	0.18	0.15	0.02	0.00***	0.00***
infl	0.42	4.78	0.18	0.15***	0.03	0.00***	0.00***
ltr	1.04	4.78	0.18	0.15***	0.04	0.01***	0.01***
rvol	1.23	4.78	0.18	0.15***	0.03	0.00***	0.00***
Panel B: Long-run risk model							
d/p	0.27	4.78	0.17	0.14***	0.11	0.07***	0.06***
e/p	0.11	4.78	0.17	0.14	0.07	0.04***	0.03***
dy	0.33	4.78	0.17	0.14***	0.12	0.09***	0.07***
tbl	0.21	4.78	0.17	0.14***	0.08	0.03***	0.03***
dfy	0.21	4.78	0.17	0.14***	0.09	0.06***	0.05***
tms	0.56	4.78	0.17	0.14***	0.09	0.04***	0.03***
ntis	0.04	4.78	0.17	0.14	0.11	0.08	0.07
infl	0.42	4.78	0.17	0.14***	0.13	0.10***	0.08***
ltr	1.04	4.78	0.17	0.14***	0.10	0.06***	0.05***
rvol	1.23	4.78	0.17	0.14***	0.12	0.07***	0.06***
Panel C: Rare disaster model							
d/p	0.27	4.78	0.17	0.14***	0.08	0.04***	0.03***
e/p	0.11	4.78	0.17	0.14	0.06	0.02***	0.02***
dy	0.33	4.78	0.17	0.14***	0.08	0.04***	0.03***
tbl	0.21	4.78	0.17	0.14***	0.05	0.02***	0.01***
dfy	0.21	4.78	0.17	0.14***	0.08	0.05***	0.04***
tms	0.56	4.78	0.17	0.14***	0.07	0.03***	0.02***
ntis	0.04	4.78	0.17	0.14	0.06	0.02***	0.02***
infl	0.42	4.78	0.17	0.14***	0.03	0.01***	0.00***
ltr	1.04	4.78	0.17	0.14***	0.09	0.05***	0.04***
rvol	1.23	4.78	0.17	0.14***	0.04	0.01***	0.01***

Table 4: R^2 bounds on market portfolio forecast with transaction cost

The table reports the bounds on the R^2 from the regression $r_{t+1} = \alpha + \beta z_t + \varepsilon_{t+1}$, where r_{t+1} is the excess return of the market portfolio and z_t is one of the 10 predictors in Column 1. The state variables x_t in the SDF are the consumption growth rate in Panel A and the Fama-French three factors in Panel B. The transaction cost is measured by the liquidity factor of Pástor and Stambaugh (2003). $\bar{R}^2_{Ross}$ and $\bar{R}^2_{Zhou}$ denote the bounds proposed by Ross (2005) and Zhou (2010). $\bar{R}^2_{RA}$ and $\bar{R}^2_{SR}$ denote the bounds developed in this paper with the maximum risk aversion and the market Sharpe ratio, respectively. ρ_{x,m_0} and $\rho_{x,rz}$ denote the multiple correlations, where the default SDF m_0 is constructed by the market portfolio. Statistical significance is assessed by the Wald statistic for testing that the predictive R^2 is less than the theoretical upper bound. ***, **, and * indicate significance at the 1%, 5%, and 10% level, respectively.

z	$R^2(\%)$	$\bar{R}^2_{Ross}(\%)$	ρ_{x,m_0}	$\bar{R}^2_{Zhou}(\%)$	$\rho_{x,rz}$	$\bar{R}^2_{RA}(\%)$	$\bar{R}^2_{SR}(\%)$
Panel A: x_t is consumption growth rate							
d/p	0.29	4.88	0.34	0.55	0.05	0.01***	0.01***
e/p	0.10	4.88	0.34	0.55	0.03	0.01***	0.01***
dy	0.31	4.88	0.34	0.55	0.06	0.02***	0.02***
tbl	0.24	4.88	0.34	0.55	0.04	0.01***	0.01***
dfy	0.20	4.88	0.34	0.55	0.01	0.00***	0.00***
tms	0.53	4.88	0.34	0.55	0.04	0.01***	0.01***
ntis	0.04	4.88	0.34	0.55	0.06	0.02**	0.02**
infl	0.55	4.88	0.34	0.54	0.09	0.05***	0.04***
ltr	1.10	4.88	0.34	0.55***	0.09	0.05***	0.04***
rvol	1.19	4.88	0.34	0.55***	0.03	0.00***	0.00***
Panel B: x_t are the Fama-French three-factors							
d/p	0.29	4.88	1.00	4.86	0.14	0.12***	0.11***
e/p	0.10	4.88	1.00	4.86	0.13	0.12	0.10
dy	0.31	4.88	1.00	4.86	0.12	0.08***	0.07***
tbl	0.24	4.88	1.00	4.86	0.26	0.39	0.33
dfy	0.20	4.88	1.00	4.86	0.34	0.84	0.72
tms	0.53	4.88	1.00	4.86	0.18	0.16***	0.14***
ntis	0.04	4.88	1.00	4.86	0.22	0.32	0.28
infl	0.55	4.88	1.00	4.86	0.17	0.19***	0.16***
ltr	1.10	4.88	1.00	4.86	0.12	0.09***	0.08***
rvol	1.19	4.88	1.00	4.86	0.29	0.42***	0.36***

Table 5: R^2 bounds on size portfolio forecasts

The table reports the bounds of the predictive R^2 from the regression $r_{j,t+1} = \alpha_j + \beta_j z_t + \varepsilon_{j,t+1}$, where $r_{j,t+1}$ is the excess return on one of the ten size portfolios and z_t is the dividend-price ratio. The state variables x_t in the SDF are the consumption growth rate (Panel A) or the Fama-French three factors (Panel B). $\bar{R}^2_{Ross}$ and $\bar{R}^2_{Zhou}$ denote the bounds proposed by Ross (2005) and Zhou (2010). $\bar{R}^2_{RA}$ and $\bar{R}^2_{SR}$ denote the bounds developed in this paper with the maximum risk aversion and the market Sharpe ratio, respectively. ρ_{x,m_0} and $\rho_{x,rz}$ denote the multiple correlations, where the default SDF m_0 is constructed by the market portfolio. Statistical significance is assessed by the Wald statistic for testing that the predictive R^2 is less than the theoretical upper bound. ***, **, and * indicate significance at the 1%, 5%, and 10% level, respectively.

r	R^2(%)	$\bar{R}^2_{Ross}$(%)	ρ_{x,m_0}	$\bar{R}^2_{Zhou}$(%)	$\rho_{x,rz}$	$\bar{R}^2_{RA}$(%)	$\bar{R}^2_{SR}$(%)
Panel A: x_t is the consumption growth rate							
Small	0.09	4.78	0.17	0.13	0.08	0.04***	0.03***
2	0.20	4.78	0.17	0.13***	0.05	0.02***	0.02***
3	0.32	4.78	0.17	0.13***	0.06	0.02***	0.02***
4	0.48	4.78	0.17	0.13***	0.04	0.01***	0.01***
5	0.43	4.78	0.17	0.13***	0.03	0.01***	0.01***
6	0.43	4.78	0.17	0.13***	0.03	0.01***	0.00***
7	0.29	4.78	0.17	0.13***	0.03	0.01***	0.00***
8	0.28	4.78	0.17	0.13***	0.00	0.00***	0.00***
9	0.24	4.78	0.17	0.13***	0.02	0.00***	0.00***
Large	0.25	4.78	0.17	0.13***	0.01	0.00***	0.00***
Panel B: x_t are the Fama-French three-factors							
Small	0.09	4.78	1.00	4.76	0.23	0.31	0.26
2	0.20	4.78	1.00	4.76	0.23	0.34	0.29
3	0.32	4.78	1.00	4.76	0.20	0.23***	0.19***
4	0.48	4.78	1.00	4.76	0.19	0.18***	0.16***
5	0.43	4.78	1.00	4.76	0.17	0.18***	0.15***
6	0.43	4.78	1.00	4.76	0.14	0.12***	0.10***
7	0.29	4.78	1.00	4.76	0.15	0.14***	0.11***
8	0.28	4.78	1.00	4.76	0.14	0.12***	0.10***
9	0.24	4.78	1.00	4.76	0.12	0.09***	0.08***
Large	0.25	4.78	1.00	4.76	0.09	0.06***	0.05***

Table 6: R^2 bounds on value portfolio forecasts

The table reports the bounds of the predictive R^2 from the regression $r_{j,t+1} = \alpha_j + \beta_j z_t + \varepsilon_{j,t+1}$, where $r_{j,t+1}$ is the excess return on one of the ten value portfolios and z_t is the dividend-price ratio. The state variables x_t in the SDF are the consumption growth rate (Panel A) or the Fama-French three factors (Panel B). $\bar{R}^2_{Ross}$ and $\bar{R}^2_{Zhou}$ denote the bounds proposed by Ross (2005) and Zhou (2010). $\bar{R}^2_{RA}$ and $\bar{R}^2_{SR}$ denote the bounds developed in this paper with the maximum risk aversion and the market Sharpe ratio, respectively. ρ_{x,m_0} and $\rho_{x,rz}$ denote the multiple correlations, where the default SDF m_0 is constructed by the market portfolio. Statistical significance is assessed by the Wald statistic for testing that the predictive R^2 is less than the theoretical upper bound. ***, **, and * indicate significance at the 1%, 5%, and 10% level, respectively.

r	R^2(%)	$\bar{R}^2_{Ross}$(%)	ρ_{x,m_0}	$\bar{R}^2_{Zhou}$(%)	$\rho_{x,rz}$	$\bar{R}^2_{RA}$(%)	$\bar{R}^2_{SR}$(%)
Panel A: x_t is the consumption growth rate							
Low	0.11	4.78	0.17	0.13	0.04	0.01***	0.01***
2	0.33	4.78	0.17	0.13***	0.03	0.01***	0.00***
3	0.28	4.78	0.17	0.13***	0.03	0.01***	0.00***
4	0.25	4.78	0.17	0.13***	0.03	0.01***	0.01***
5	0.19	4.78	0.17	0.13***	0.03	0.01***	0.01***
6	0.39	4.78	0.17	0.13***	0.02	0.00***	0.00***
7	0.23	4.78	0.17	0.13***	0.04	0.01***	0.01***
8	0.28	4.78	0.17	0.13***	0.02	0.00***	0.00***
9	0.41	4.78	0.17	0.13***	0.04	0.01***	0.01***
High	0.32	4.78	0.17	0.13***	0.06	0.02***	0.02***
Panel B: x_t are the Fama-French three-factors							
Low	0.11	4.78	1.00	4.76	0.11	0.09**	0.08***
2	0.33	4.78	1.00	4.76	0.13	0.09***	0.08***
3	0.28	4.78	1.00	4.76	0.15	0.14***	0.12***
4	0.25	4.78	1.00	4.76	0.16	0.15***	0.13***
5	0.19	4.78	1.00	4.76	0.15	0.14***	0.12***
6	0.39	4.78	1.00	4.76	0.12	0.08***	0.07***
7	0.23	4.78	1.00	4.76	0.17	0.16***	0.14***
8	0.28	4.78	1.00	4.76	0.18	0.18***	0.15***
9	0.41	4.78	1.00	4.76	0.15	0.12***	0.10***
High	0.32	4.78	1.00	4.76	0.18	0.16***	0.14***

Table 7: R^2 bounds on momentum portfolio forecasts

The table reports the bounds of the predictive R^2 from the regression $r_{j,t+1} = \alpha_j + \beta_j z_t + \varepsilon_{j,t+1}$, where $r_{j,t+1}$ is the excess return on one of the ten momentum portfolios and z_t is the dividend-price ratio. The state variables x_t in the SDF are the consumption growth rate (Panel A) or the Fama-French three factors (Panel B). $\bar{R}^2_{Ross}$ and $\bar{R}^2_{Zhou}$ denote the bounds proposed by Ross (2005) and Zhou (2010). $\bar{R}^2_{RA}$ and $\bar{R}^2_{SR}$ denote the bounds developed in this paper with the maximum risk aversion and the market Sharpe ratio, respectively. ρ_{x,m_0} and $\rho_{x,rz}$ denote the multiple correlations, where the default SDF m_0 is constructed by the market portfolio. Statistical significance is assessed by the Wald statistic for testing that the predictive R^2 is less than the theoretical upper bound. ***, **, and * indicate significance at the 1%, 5%, and 10% level, respectively.

r	R^2(%)	$\bar{R}^2_{Ross}$(%)	ρ_{x,m_0}	$\bar{R}^2_{Zhou}$(%)	$\rho_{x,rz}$	$\bar{R}^2_{RA}$(%)	$\bar{R}^2_{SR}$(%)
Panel A: x_t is the consumption growth rate							
Low	0.19	4.78	0.17	0.13***	0.01	0.00***	0.00***
2	0.23	4.78	0.17	0.13***	0.04	0.01***	0.01***
3	0.48	4.78	0.17	0.13***	0.04	0.01***	0.01***
4	0.24	4.78	0.17	0.13***	0.06	0.02***	0.02***
5	0.10	4.78	0.17	0.13	0.08	0.03***	0.01***
6	0.56	4.78	0.17	0.13***	0.07	0.03***	0.02***
7	0.31	4.78	0.17	0.13***	0.07	0.03***	0.03***
8	0.31	4.78	0.17	0.13***	0.03	0.01***	0.01***
9	0.83	4.78	0.17	0.13***	0.02	0.00***	0.00***
High	0.24	4.78	0.17	0.13***	0.01	0.00***	0.00***
Panel B: x_t are the Fama-French three-factors							
Low	0.19	4.78	1.00	4.76	0.12	0.08***	0.07***
2	0.23	4.78	1.00	4.76	0.10	0.06***	0.05***
3	0.48	4.78	1.00	4.76	0.12	0.13***	0.11***
4	0.24	4.78	1.00	4.76	0.18	0.18***	0.15***
5	0.10	4.78	1.00	4.76	0.17	0.17	0.15
6	0.56	4.78	1.00	4.76	0.19	0.22***	0.19***
7	0.31	4.78	1.00	4.76	0.21	0.27**	0.23***
8	0.31	4.78	1.00	4.76	0.19	0.24***	0.20***
9	0.83	4.78	1.00	4.76	0.16	0.17***	0.14***
High	0.24	4.78	1.00	4.76	0.15	0.16***	0.13***

Table 8: R^2 bounds on industry portfolio forecasts

The table reports the bounds of the predictive R^2 from the regression $r_{j,t+1} = \alpha_j + \beta_j z_t + \varepsilon_{j,t+1}$, where $r_{j,t+1}$ is the excess return on one of the ten industry portfolios and z_t is the dividend-price ratio. The state variables x_t in the SDF are the consumption growth rate (Panel A) or the Fama-French three factors (Panel B). $\bar{R}^2_{Ross}$ and $\bar{R}^2_{Zhou}$ denote the bounds proposed by Ross (2005) and Zhou (2010). $\bar{R}^2_{RA}$ and $\bar{R}^2_{SR}$ denote the bounds developed in this paper with the maximum risk aversion and the market Sharpe ratio, respectively. ρ_{x,m_0} and $\rho_{x,rz}$ denote the multiple correlations, where the default SDF m_0 is constructed by the market portfolio. Statistical significance is assessed by the Wald statistic for testing that the predictive R^2 is less than the theoretical upper bound. ***, **, and * indicate significance at the 1%, 5%, and 10% level, respectively.

r	$R^2(\%)$	$\bar{R}^2_{Ross}(\%)$	ρ_{x,m_0}	$\bar{R}^2_{Zhou}(\%)$	$\rho_{x,rz}$	$\bar{R}^2_{RA}(\%)$	$\bar{R}^2_{SR}(\%)$
Panel A: x_t is the consumption growth rate							
NoDur	0.64	4.78	0.17	0.13^{***}	0.09	0.05^{***}	0.04^{***}
Durbl	0.19	4.78	0.17	0.13^{***}	0.06	0.02^{***}	0.02^{***}
Manuf	0.03	4.78	0.17	0.13	0.03	0.01^{***}	0.01^{***}
Enrgy	0.01	4.78	0.17	0.13	0.00	0.00	0.00
HiTec	0.10	4.78	0.17	0.13	0.01	0.00^{***}	0.00^{***}
Telcm	0.52	4.78	0.17	0.13^{***}	0.02	0.00^{***}	0.00^{***}
Shops	0.47	4.78	0.17	0.13^{***}	0.04	0.01^{***}	0.01^{***}
Hlth	0.25	4.78	0.17	0.13^{***}	0.08	0.04^{***}	0.03^{***}
Utils	0.16	4.78	0.17	0.13^{***}	0.00	0.00^{***}	0.00^{***}
Other	0.44	4.78	0.17	0.13^{***}	0.05	0.01^{***}	0.01^{***}
Panel B: x_t are the Fama-French three-factors							
NoDur	0.64	4.78	1.00	4.76	0.27	0.42^{***}	0.35^{***}
Durbl	0.19	4.78	1.00	4.76	0.09	0.04^{***}	0.03^{***}
Manuf	0.03	4.78	1.00	4.76	0.16	0.16	0.14
Enrgy	0.01	4.78	1.00	4.76	0.20	0.29	0.24
HiTec	0.10	4.78	1.00	4.76	0.21	0.32	0.27
Telcm	0.52	4.78	1.00	4.76	0.16	0.16^{***}	0.13^{***}
Shops	0.47	4.78	1.00	4.76	0.18	0.18^{***}	0.15^{***}
Hlth	0.25	4.78	1.00	4.76	0.23	0.32	0.27
Utils	0.16	4.78	1.00	4.76	0.25	0.40	0.34
Other	0.44	4.78	1.00	4.76	0.22	0.28^{***}	0.24^{***}

Table 9: R^2 bounds on size and value portfolio forecasts

The table reports the bounds on R^2 from the regression $r_{j,t+1} = \alpha_j + \beta_j z_t + \varepsilon_{j,t+1}$, where z_t is the term spread and $r_{j,t+1}$ is the excess return on one of the 10 size portfolios (Panel A) or the 10 value portfolios (Panel B). The state variables x_t in the SDF are the Fama-French three factors. $\bar{R}^2_{Ross}$ and $\bar{R}^2_{Zhou}$ denote the bounds proposed by Ross (2005) and Zhou (2010). $\bar{R}^2_{RA}$ and $\bar{R}^2_{SR}$ denote the bounds developed in this paper with the maximum risk aversion and the market Sharpe ratio, respectively. ρ_{x,m_0} and $\rho_{x,rz}$ denote the multiple correlations, where the default SDF m_0 is constructed by the market portfolio. Statistical significance is assessed by the Wald statistic for testing that the predictive R^2 is less than the theoretical upper bound. ***, **, and * indicate significance at the 1%, 5%, and 10% level, respectively.

r	$R^2(\%)$	$\bar{R}^2_{Ross}(\%)$	ρ_{x,m_0}	$\bar{R}^2_{Zhou}(\%)$	$\rho_{x,rz}$	$\bar{R}^2_{RA}(\%)$	$\bar{R}^2_{SR}(\%)$
Panel A: Size portfolios							
Small	0.83	4.78	1.00	4.76	0.23	0.26***	0.22***
2	0.61	4.78	1.00	4.76	0.20	0.19***	0.16***
3	0.54	4.78	1.00	4.76	0.17	0.15***	0.13***
4	0.63	4.78	1.00	4.76	0.17	0.16***	0.13***
5	0.58	4.78	1.00	4.76	0.17	0.14***	0.12***
6	0.81	4.78	1.00	4.76	0.16	0.13***	0.11***
7	0.69	4.78	1.00	4.76	0.16	0.14***	0.12***
8	0.52	4.78	1.00	4.76	0.16	0.14***	0.12***
9	0.58	4.78	1.00	4.76	0.18	0.18***	0.13***
Large	0.54	4.78	1.00	4.76	0.18	0.17***	0.14***
Panel B: Value portfolios							
Low	0.52	4.78	1.00	4.76	0.18	0.16***	0.14***
2	0.51	4.78	1.00	4.76	0.13	0.08***	0.07***
3	0.73	4.78	1.00	4.76	0.12	0.08***	0.06***
4	0.68	4.78	1.00	4.76	0.14	0.10***	0.09***
5	0.65	4.78	1.00	4.76	0.12	0.08***	0.07***
6	0.43	4.78	1.00	4.76	0.14	0.10***	0.09***
7	0.28	4.78	1.00	4.76	0.11	0.06***	0.05***
8	0.14	4.78	1.00	4.76	0.16	0.14***	0.11***
9	0.29	4.78	1.00	4.76	0.16	0.14***	0.12***
High	0.49	4.78	1.00	4.76	0.20	0.24***	0.21***

Table 10: R^2 bounds on momentum and industry portfolio forecasts

The table reports the bounds on R^2 from the regression $r_{j,t+1} = \alpha_j + \beta_j z_t + \varepsilon_{j,t+1}$, where z_t is the term spread and $r_{j,t+1}$ is the excess return on one of the 10 momentum portfolios (Panel A) or the 10 industry portfolios (Panel B). The state variables x_t in the SDF are the Fama-French three factors. $\bar{R}^2_{Ross}$ and $\bar{R}^2_{Zhou}$ denote the bounds proposed by Ross (2005) and Zhou (2010). $\bar{R}^2_{RA}$ and $\bar{R}^2_{SR}$ denote the bounds developed in this paper with the maximum risk aversion and the market Sharpe ratio, respectively. ρ_{x,m_0} and $\rho_{x,rz}$ denote the multiple correlations, where the default SDF m_0 is constructed by the market portfolio. Statistical significance is assessed by the Wald statistic for testing that the predictive R^2 is less than the theoretical upper bound. ***, **, and * indicate significance at the 1%, 5%, and 10% level, respectively.

r	R^2(%)	$\bar{R}^2_{Ross}$(%)	ρ_{x,m_0}	$\bar{R}^2_{Zhou}$(%)	$\rho_{x,rz}$	$\bar{R}^2_{RA}$(%)	$\bar{R}^2_{SR}$(%)
Panel A: Momentum portfolios							
Low	0.54	4.78	1.00	4.76	0.22	0.25***	0.21***
2	0.43	4.78	1.00	4.76	0.18	0.16***	0.14***
3	0.51	4.78	1.00	4.76	0.15	0.12***	0.10***
4	0.60	2.78	1.00	4.76	0.15	0.11***	0.10***
5	0.50	4.78	1.00	4.76	0.17	0.14***	0.12***
6	0.80	4.78	1.00	4.76	0.15	0.13***	0.11***
7	1.11	4.78	1.00	4.76	0.14	0.11***	0.09***
8	0.56	4.78	1.00	4.76	0.16	0.14***	0.12***
9	0.37	4.78	1.00	4.76	0.15	0.13***	0.11***
High	0.33	4.78	1.00	4.76	0.19	0.21***	0.18***
Panel B: Industry portfolios							
NoDur	0.40	4.78	1.00	4.76	0.16	0.14***	0.12***
Durbl	1.41	4.78	1.00	4.76	0.21	0.24***	0.20***
Manuf	0.80	4.78	1.00	4.76	0.15	0.12***	0.10***
Enrgy	0.16	4.78	1.00	4.76	0.12	0.08***	0.07***
HiTec	0.61	4.78	1.00	4.76	0.21	0.21***	0.18***
Telcm	0.21	4.78	1.00	4.76	0.21	0.21	0.18*
Shops	0.43	4.78	1.00	4.76	0.10	0.05***	0.04***
Hlth	0.00	4.78	1.00	4.76	0.12	0.06	0.05
Utils	0.19	4.78	1.00	4.76	0.03	0.00***	0.00***
Other	0.40	4.78	1.00	4.76	0.16	0.14***	0.12***