# When Do Consumers Purchase Online?: Based on Inter-Purchase Time 

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# WHEN DO ONLINE CONSUMERS PURCHASE?: BASED ON INTER-PURCHASE TIME 

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#### Abstract

This study is motivated by the premise that online consumers can make a purchase at any time of day if they have even a tiny time slot along with Internet access. To identify the increased shopping time flexibility, we first characterize the patterns of online purchase timing in comparison to those in the offline market. The results show (1) the breakdown of purchase timing regularity and (2) the change of weekly spike purchase occurrence. Second, we build online inter-purchase time model and estimate it with the data collected from one of the premier online vendors in Korea. We verify new theories in online consumers' purchase timing, particularly, about (1) how online consumers' hazard rates change, (2) how online consumers respond to price promotions, and (3) what factors affect online consumers' purchasing timing decision. Both marketing researchers and practitioners can gain significant new insight to understanding the nature of online commerce on the time space.


Purchase timing is an important element in a consumer's offline purchase decision (Gupta 1991). By comparison, online consumers are less concerned about the decision of when to buy because they can buy at any time of day if they have even a tiny time slot along with Internet access. The literature shows that the online channel is more attractive than the off-line channel because consumers can reduce time costs such as travel time and in-store shopping time (Chintagunta et al. 2012; Forman et al. 2009; Keeney 1999). As a result, consumers can take advantage of shopping time flexibility via online commerce. Here, shopping time flexibility means that consumers can select their purchase time more flexibly relative to offline shopping.

The increased shopping time flexibility can change consumers' purchasing behavior. Let us consider hypothetical situations. If a consumer is time pressed on weekdays, presumably with the high opportunity cost of a shopping trip, the consumer can shop only on weekends, making a regularity of shopping day and inter-purchase times. In contrast, the consumer can shop online during a coffee break on weekdays. As another example, when an offline retailer offers special promotions on specific days, some consumers will not be able to take advantage of the promotions if the transaction costs including travel, transportation costs and in-store shopping time on the specific days are larger than the amount of money to be saved (Saini et al. 2010). However, in the online market, consumers may immediately respond to the promotions at negligible transaction costs.

Over the last 10 to 15 years, many studies have analyzed the nature of online shopping and its impact on consumers' purchase decisions. Their target observations cover diverse decisions (or situations): (1) lowered search cost, (2) price dispersion across vendors, (3) online marketing and differentiation strategies, (4) impact of product characteristics and (5) online activities involved in purchase decisions (Brynjolfsson and Smith 2000; Clay et al. 2002; Clemons et al. 2002; Khan et al. 2009; Kuruzovich et al. 2010; Lal and Sarvary 1999; Lynch and Ariely 2002). However, online consumer's shopping timing has not been exploited. We first aim to identify the increased shopping time flexibility in the online market and accordingly characterize the changed purchase timing decisions. Particularly, many previous studies documented shopping time regularities that result from the optimized shopping trip schedule in the offline market. We examine how these regularities have changed in the online market.

Shopping time regularity has been highlighted in the offline setting since around three decades ago: (1) the regularity of inter-purchase times (as measured by the amount of time elapsed since the previous purchase) and (2) shopping day regularity. Dunn et al. (1983) reported a particular phenomenon for toilet tissue data that the histograms of purchase times exhibit spikes at multiples of 7 days. Since this first observation of inter-purchase time regularity, Kahn and Schmittlein (1989) found a similar pattern in cracker sales, Jain and Vilcassim (1991) for coffee sales, and Chiang et al. (2001) for several grocery products. More recently, Bijwaar (2005) reported the same striking pattern in the data on yogurt. The regularity in the inter-purchase times such as 7 -day peaks are created by strong preferences for shopping on a specific day of the week rather than when products are used up (Kahn and Schmittlein 1989).

We hypothesize that shopping time regularity collapses in the online market. Previous studies classify consumers' shopping trips into major and fill-in trips (Frisbie, 1980; Kahn and Schmittlein, 1989). Major shopping trips are regularly planned to purchase many diverse products at the cost of one shopping trip, whereas fill-in shopping is made from instantaneous decisions. But, a shopping trip itself does not exist in the online market and online consumers cannot save transaction costs per item by simultaneously purchasing multiple products (major shopping). Consequently, online consumers are expected to acquire most products through fill-in shopping anytime when it is most convenient. The increase of fill-in shopping in more random fashion reduces shopping time regularity in turn (Kim and Park 1997). Similarly, we also conjecture the collapse of weekly-based shopping day regularity because the weekly patterns may simply be the by-product of consumers' regular shopping trip schedules (Kahn and Schmittlein 1989).

As the second research objective, we aim to verify the existing theories related to purchase timings (e.g., inventory effects) under the new paradigm of online commerce. We build on previous studies a comprehensive stochastic model at an individual level, focusing on inter-purchase times and price promotion effects. Our inter-purchase time model incorporates the state dependence for diverse factors and also allows for their change over time and the unobserved heterogeneity (e.g., the rate of consumption and beginning inventory). Our analysis has a theoretical contribution on the online purchase decision process. Also, our empirical results enable us to predict online consumers' purchase timing and thus take more effective marketing actions (e.g., when to advertise and promote products).
The outline of the paper is arranged as follows. In the next section, we describe distinctive online purchase timings based on the collected data. In $\S 3$, we build an inter-purchase time model. In the subsequent section, empirical results are presented along with its theoretical and managerial implications. Finally, we conclude with limitations and future research directions.

## 2 DATA AND ONLINE PURCHASE TIMING PATTERNS

Clay et al. (2002) brought up the question of "how does consumer behavior change in electronically mediated markets from what is observed in physical markets?" that is the issue of particular interest to both practitioners and academics. This section is devoted to investigating whether there are systematic differences in consumers' purchase timing between the online and offline market and to articulating the theoretical basis for the differences. Specifically, we compare the patterns documented in offline market-based literature with online purchase timing patterns identified through the data we collected.
We collected the transaction data of 10,103 randomly selected consumers from one of the premier online shopping malls in Korea. It sells diverse products (e.g., upscale apparel, fashion goods, home furnishings, food, and electronics) as does Amazon.com, unlike online vendors selling certain product categories (e.g., neweggs.com for electronics and bluenile.com for jewelry). Therefore, our target observation is the online consumers' purchase behavior from various product categories in an online retailer (Jen et al. 2003). The collected data enable us to identify consumer, product, product price, discount, and purchase date information on the 361,263 transactions occurring from January 2002 to June 2006 and so we can trace individual consumers' purchase history over the four and half years. Additionally we acquired consumers' demographic profile (age and gender).

### 2.1 Regularity in Inter-Purchase Times

### 2.1.1 Frequency Distribution of Inter-Purchase Times

A stream of shopping trip research has examined various types of shopping time regularity. The best way to present a single snapshot of purchase timings is a frequency distribution of inter-purchase times. The right figure in Figure 1 shows the histogram of inter-purchase times acquired from our data. To compare online with offline patterns, we quoted the frequency distribution of inter-purchase times of the panelists in the IRI that were reported by Kahn and Schmittlein (1989) in the offline market (see the left figure). In both histograms, an inter-purchase time is measured in days between $m^{\text {th }}$ and $m+l^{\text {th }}$ purchase days.



Figure 1. Frequency Distribution of Inter-Purchase Times
The left histogram clearly shows the evidence of pronounced 7 day cycles (or multiples of 7 days) except for the large frequency in 2-4 days that results from fill-in purchases. Our target observation is the consumer's purchasing behavior from various product categories at an online retailer. Fill-in (spur-of-the-moment) purchases may occur for any product class and independently for different
categories. Therefore, a large peak at 2-4 days represents a superposition of fill-in purchases from all the individual products, thus accumulating to a relatively large number of fill-in purchase occurrences (Kahn and Schmittlein 1989). By contrast, the right histogram from the online market does not show any clue pertaining to regular interval 7 day cycles. Rather, the histogram looks like an exponential distribution. If the purchase interval is exponentially distributed, the consumer's propensity to go shopping at time $t$ is independent of the elapsed time since the last shopping trip. This implies that online shopping may randomly occur without any regularity.

### 2.1.2 Consecutive Inter-Purchase Times

Besides the frequency distributions of inter-purchase times at an aggregate level, we examine the relationship between consecutive inter-purchase times (i.e., elapsed days between $m-1^{\text {th }}$ and $m^{\text {th }}$ purchase dates and elapsed days between $m^{\text {th }}$ and $m+l^{\text {th }}$ purchase dates) on an individual consumer level. Table 1 shows their distribution within a 2 week time window. The figures in each cell indicate the proportion of the sequence of inter-purchase times that fall under each category (the sum over all cells is 1 ). For example, the cell of the third row and column shows the probability that both interpurchase times (between $m-l^{\text {th }}$ and $m^{\text {th }}$ and between $m^{\text {th }}$ and $m+l^{\text {th }}$ ) are three days.

| Day |  | Elapsed day between $m^{\text {th }}$ and $m+1^{\text {th }}$ purchase dates |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | Sum |
| Elapsed day between $m-I^{\text {th }}$ and $m^{\text {th }}$ purchase dates | 1 | 0.051 | 0.032 | 0.023 | 0.018 | 0.015 | 0.014 | 0.011 | 0.009 | 0.007 | 0.007 | 0.006 | 0.006 | 0.006 | 0.204 |
|  | 2 | 0.030 | 0.020 | 0.016 | 0.013 | 0.012 | 0.009 | 0.008 | 0.006 | 0.006 | 0.005 | 0.004 | 0.005 | 0.004 | 0.139 |
|  | 3 | 0.021 | 0.015 | 0.012 | 0.011 | 0.008 | 0.007 | 0.007 | 0.006 | 0.005 | 0.004 | 0.004 | 0.004 | 0.003 | 0.109 |
|  | 4 | 0.017 | 0.013 | 0.011 | 0.008 | 0.007 | 0.006 | 0.006 | 0.005 | 0.004 | 0.004 | 0.004 | 0.003 | 0.003 | 0.092 |
|  | 5 | 0.014 | 0.011 | 0.008 | 0.007 | 0.006 | 0.005 | 0.005 | 0.005 | 0.004 | 0.004 | 0.003 | 0.003 | 0.003 | 0.078 |
|  | 6 | 0.013 | 0.009 | 0.007 | 0.006 | 0.006 | 0.005 | 0.005 | 0.005 | 0.003 | 0.003 | 0.003 | 0.003 | 0.002 | 0.069 |
|  | 7 | 0.011 | 0.008 | 0.006 | 0.006 | 0.005 | 0.005 | 0.005 | 0.004 | 0.003 | 0.003 | 0.003 | 0.003 | 0.003 | 0.065 |
|  | 8 | 0.009 | 0.006 | 0.005 | 0.005 | 0.005 | 0.005 | 0.004 | 0.003 | 0.003 | 0.003 | 0.002 | 0.002 | 0.003 | 0.053 |
|  | 9 | 0.007 | 0.005 | 0.004 | 0.004 | 0.004 | 0.004 | 0.003 | 0.003 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.045 |
|  | 10 | 0.006 | 0.005 | 0.004 | 0.004 | 0.004 | 0.003 | 0.003 | 0.003 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.042 |
|  | 11 | 0.005 | 0.004 | 0.004 | 0.003 | 0.003 | 0.003 | 0.003 | 0.002 | 0.002 | 0.002 | 0.002 | 0.001 | 0.002 | 0.037 |
|  | 12 | 0.006 | 0.004 | 0.004 | 0.003 | 0.003 | 0.003 | 0.003 | 0.002 | 0.002 | 0.002 | 0.002 | 0.001 | 0.001 | 0.035 |
|  | 13 | 0.005 | 0.004 | 0.003 | 0.003 | 0.003 | 0.002 | 0.003 | 0.002 | 0.002 | 0.002 | 0.001 | 0.001 | 0.001 | 0.033 |
|  | Sum | 0.194 | 0.136 | 0.108 | 0.093 | 0.080 | 0.070 | 0.067 | 0.054 | 0.046 | 0.042 | 0.039 | 0.037 | 0.034 | 1 |

## Table 1. Relationship of Consecutive Inter-Purchase Times

The table shows that the probability that a consumer has shopped for three days in a row is the highest, accounting for $5.1 \%$ of all the three consecutive purchase incidents. In comparison, the cell of the $7^{\text {th }}$ row and column (implying 7 day shopping cycles) occupies only $0.5 \%$, which is the most common shopping trip schedule in the offline markets. The cell of the $14^{\text {th }}$ row and column accounts for less than $0.1 \%$. These findings indicate that the marked peaks at 7 days and 14 days in inter-purchase times disappear in the online market.

The table also shows that online consumers purchase highly irregularly. Note that if all the sequences of purchase incidents are made at regular intervals, the off-diagonal cells of the table should be empty. Moreover, if all the consumers have the same shopping rate as well, then only one cell should be nonzero. In Table 1, there is no concentration in the diagonal cells and also all the figures are fairly equally distributed except for the first and second row and column, supporting the irregularity of inter-purchase times. Another distinctive pattern is that the highest proportion is at the top and left corner and the figures become smaller as it moves to either the right or the bottom. The likelihood that a consumer will shop for three consecutive days (four consecutive days / five consecutive days) is $5.1 \%$ $(13.3 \% / 22.0 \%)$. Also purchase for two consecutive days occupies $35 \%$ of the sequence of purchase incidents (the summation of first row and first column), indicating that inter-purchase times have substantially shortened in the online market. We attribute this shorter interval of inter-purchase times to decreased transaction costs and accordingly the increase of fill-in purchases (along with the disappearance of major shopping trips).

## 3 MODEL FORMULATION

To estimate an inter-purchase time model, we consider the hazard rate, $h(t)$, which is the conditional probability that an consumer makes a purchase at inter-purchase time $t$, given that the consumer did not make a purchase during the interval $(0, t)$. Here, we measure inter-purchase time $t$ as the days
elapsed from the previous transaction. Then, the hazard function is formulated: $h(t)=\lim _{\Delta t \rightarrow 0} \frac{\operatorname{Prob}(t \leq T \leq t+\Delta t \mid t \leq T)}{\Delta t}=\frac{f(t)}{1-F(t)}=\frac{f(t)}{S(t)}$, where $f(t), F(t)$, and $S(t)$ are the probability density function, the cumulative distribution function, and the survival function of $t$, respectively (Kalbfleisch and Prentice 1980).
Let $h(t \mid \mathrm{X}, \theta)$ represent the hazard function of $t$ conditional on a vector of covariates (X) and an unobserved heterogeneity component ( $\theta$ ). Modifying the representation of Jain and Vilcassim (1991), the hazard function at time $t$ can be factorized as $h(t \mid X, \theta)=h_{0}(t) \cdot \psi(\mathrm{X}) \cdot \Phi(\theta)$.

The first component, $h_{0}(t)$ is a baseline hazard rate at time $t$ that captures the common dependence of the hazard on the elapsed time since the previous purchase. We use three alternative parametric specifications of the baseline hazard function (Exponential, Weibull, and Expo-power distributions). Most of the frequently used probability distributions for inter-purchase times are nested within the Expo-power hazard function so that we can test among competing probability distributions for interpurchase times. The second component is a regression factor and reflects the effect of explanatory variables $\mathrm{X}(t)$ facing individual at time $t$. The third term of $\Phi(\theta)$ denotes the specification for the unobserved heterogeneity. The most popular method of accounting for heterogeneity in applied stochastic models is the multiplicative frailty model (Massy et al. 1970). The hazard function is modified as the product of an individual-specific random effect $\theta$ representing the individual's frailty and a baseline hazard: $h(t \mid \mathrm{X}, \theta)=h_{0}(t) \cdot \psi(\mathrm{X}) \cdot \theta$.
Estimation of the parameters at an individual level can proceed as follows. Each of these interpurchase times is associated with a covariate vector, yielding a stacked set of covariates given by $\left(\mathrm{X}_{i, 1}, \mathrm{X}_{i, 2}, \ldots \mathrm{X}_{i, M_{i}}\right) .[S(t \mid \theta)]^{\left(1-\delta_{m_{i}}\right)}$ accounts for the right censoring in the likelihood function. The conditional likelihood function for the $i^{\text {th }}$ consumer conditional on $\theta$ is given by:

$$
L_{i}(\boldsymbol{\Theta} \mid \theta)=\prod_{m_{i}}[f(t \mid \mathrm{X}, \theta)]^{\delta_{m_{i}}}[S(t \mid \mathrm{X}, \theta)]^{\left(1-\delta_{m_{i}}\right)}
$$

, where $\delta_{m_{i}}=1$ if the $m_{i}^{\text {th }}$ spell ends in a purchase, 0 otherwise.
We use a parametric mixing distribution and the unconditional likelihood function can be obtained by integrating the conditional likelihood function over all values of $\theta$, weighted by their appropriate probabilities. The parameter estimates are obtained by maximizing the likelihood function across all N individuals in the sample.

$$
L(\boldsymbol{\Theta})=\prod_{i=1}^{N} L_{i}(\boldsymbol{\Theta})=\int L_{i}(\boldsymbol{\Theta} \mid \theta) g(\theta) d \theta
$$

The symbols used throughout the paper and the variables they represent are listed on Table 2.

| Variable | Operational Definition and Managerial Implication |
| :---: | :--- |
| $i$ | Consumer index: $N$ is the total number of consumers |
| $m_{i}$ or $j$ | Transaction sequence index of individual $i$ : the multiple transactions <br> occurring in a day are classified into a transaction, $j$ is used in the subscript <br> instead of $m_{i}$ |
| $t$ | Inter-purchase time: elapsed days since previous transaction |
| $\mathbf{X}, \boldsymbol{\beta}$ | Vector of covariates and corresponding parameters |
| Products $_{i j-1}$ | Number of products purchased by individual $i$ at the $j-1$ <br> inventory effect) |
| Products $_{i j}$ | Number of products purchased by individual $i$ at the $j^{\text {th }}$ transaction day (for a <br> consumer's online purchase tendency) |
| Money $_{i j-1}$ | Amount of money used by individual $i$ at the $j-1$ <br> inventory effects in terms of money) |
| SavedMonetion day (for $_{i j-l}$ | Amount of money saved through price promotion by individual $i$ at the $j-1$ <br> transaction day (for the indirect effect of price promotion) |
| PricePromoExp $_{i j-1}$ | 1 if there is a product(s) purchased with price promotion at the $j-I^{\text {th }}$ |


|  | transaction day, 0 otherwise (for indirect effect of price promotion) |
| :---: | :--- |
| PricePromoRatio $_{i j}$ | The ratio of products with price promotion over all sold products (for direct <br> effect of price promotion) |
| OnlineShoppingExp ${ }_{i j}$ | Number of cumulative online shopping transactions by consumer $i$ at the <br> research site through $j^{\text {th }}$ transaction (for the change of consumers' attitude to <br> online commerce or retailer) |
| Diversity ${ }_{i j}$ | Number of product categories purchased by individual $i$ at the $j$ th <br> day, 22 product categories clarified (for consumers' online purchase <br> tendency) |
| PurchaseFrequency $_{i}$ | Average number of transactions by individual $i$ per month (for consumer <br> type) |
| Age $_{j}$ | Consumer $i$ 's age |
| Sex $_{i}$ | 1 if consumer $i$ is male, 0 otherwise |
| Day $_{j}$ | Day over the week of $j^{\text {th }}$ transaction |

Table 2. Variables and Operational Definition

### 3.1 Covariates

The purchase timing decision is influenced by a broad spectrum of covariates that draw on diverse theoretical backgrounds. The identification of the covariates is important in many managerial situations such as predicting purchase occasion and sales forecasting (Bawa and Ghosh 1991). In this section, we classify covariates into three categories (state dependency, price promotion, and consumer profile) and discuss their theoretical backgrounds and managerial implications.
We develop some covariates from individual consumer's market baskets to characterize a consumer's purchase behavior in the online market. A market basket is generally defined as the set of products purchased in a shopping trip. However, the definition is not applicable to online shopping because the boundary of one shopping trip is ambiguous. Consumers can visit an online shopping mall many times at intervals even in a day. In this study, we define a market basket as the set of item(s) purchased in a day. This setup corresponds with the daily-based time horizon to measure interpurchase times. All the covariates remain constant during spells.

### 3.1.1 State Dependency

The development and verification of an inter-purchase time model requires the history of consumers' purchases in order to account for inventory because purchase timing and quantity decisions are correlated over time. The more a consumer purchases for future consumption on the previous transaction, the longer the inter-purchase time between the previous and current transactions would be (Chiang et al. 2001). Similarly, a customer can compensate for late orders by ordering larger quantities (Jen et al. 2009). The inventory effect has been mostly reported on a specific product level, but it is also observed in purchasing behavior from diverse product categories (Jen et al. 2003).

In a proportional hazards model, the inventory level is captured by a market basket size at the previous transaction along with a duration term and unobserved heterogeneity. We developed two variables to calibrate the inventory accumulated from the previous transaction. First, Products $s_{i j-1}$ is the number of products purchased by individual $i$ at the $j-1^{\text {th }}$ transaction day, which is the traditional way of measuring a new supply for inventory. Second, we measure the amount of money spent by individual $i$ at the $j-1^{\text {th }}$ transaction day ( Mone $_{i j-1}$ ). Focusing on specific products such as coffee, diaper and tomato ketchup the money spent for the items is relatively small. But, when we expand our observation into consumers' purchases from diverse categories including high-priced products such as LCD TV and expensive cloths, we need to consider budget constraint. We add Money ij-l in the model to approximate the consumption of money allocated to shopping.

### 3.1.2 Price Promotion

We hypothesize that a price promotion directly and indirectly affects consumers' purchase timing in the online market. First, when a product is available at a discounted price, consumers can make an unplanned purchase (even if they don't buy the product at the original price) or purchase a product earlier than scheduled. This stockpiling behavior indicates that a present price promotion accelerates the current transaction and thus reduces the inter-purchase time between previous and current transactions (the direct effect of a price promotion). To test the direct effect, we calculate the ratio of products with a price promotion over all products available with the retailer on the $j^{\text {th }}$ transaction day of consumer $i$ (PricePromoRatio ${ }_{i j}$ ). We cannot identify which consumer is persuaded to purchase what products due to price promotion. But PricePromoRatio ${ }_{i j}$ is based on the reasonable assumption that products for price promotions are randomly selected over time.

Second, consumers who take advantage of a price promotion at the current transaction are likely to accelerate the next purchase time. We call this shortened inter-purchase time between the current and the next transactions the indirect effect of a price promotion. This indirect effect is justified by the increased loyalty to a retailer. If a consumer has a good impression of a retailer because they benefit from price promotions, they are likely to re-visit the retailer (while expecting other price promotions) when they need to buy. Particularly this explanation should be true when the re-visit does not incur any extra cost as shown in the online market. The consumers' experience of price promotions might systematically change their shopping/browsing behavior by selecting the retailer as the home page in their web browser or registering the retailer as one of their favorite websites. To assess the indirect effect of a price promotion, we develop SavedMoney $y_{i j-1}$, which is defined as the amount of money saved through price promotions at the $j-1^{\text {th }}$ transaction day of consumer $i$. This measure is implicitly based on the assumption that the more consumers save, the better impression they have of the retailer.
The impact of SavedMoney ${ }_{i j-1}$ on inter-purchase time (between the $j$-1 th and the $j^{\text {th }}$ transaction days) may have compounding effects. In addition to the positive influence on hazard rate that is derived from the increased loyalty to a retailer, the stockpiling effects of promoted products may delay the next purchase later than expected, causing a negative impact. Given these compelling forces, we examine which explanation is valid.

### 3.1.3 Consumer Profile

Consumer's profile includes both time-invariant and time-variant factors. The first three variables are developed to capture the transition of consumers' propensity in the online market and the latter ones are demographic profile and weekday seasonality. First, our model includes the current market basket size ( Products $_{i j}$ ). Products ${ }_{i j}$ is coded as the number of products purchased by consumer $i$ at the $j^{\text {th }}$ transaction day across all the product categories. In contrary to Products ${ }_{i j-1}$ to test the inventory effect, Products $_{i j}$ measures how many items a consumer purchases simultaneously in a day. Because consumers who tend to purchase multiple products in a day would be online heavy shoppers, they may purchase more frequently (or equivalently, show shorter inter-purchase times). As an opposite force, the increase of a market basket size may result from delaying the instant purchase of some products. This behavior may partly be rationalized based on the saving of delivery cost (Hossain and Morgan 2006; Lewis et al. 2006; Rosen and Howard 2000) or reducing the procedures requiring special attention such as inputting credit card numbers. Second, we add cumulative online shopping experience in the model. Online shopping experience (OnlineShoppingExp ${ }_{i j}$ ) is measured as the number of cumulative online transactions made by consumer $i$ at our research site through $j^{\text {th }}$ transaction date. Third, we consider how many kinds of products a consumer purchase online. Kim and Krishnan (2010) show the evolution of product categories purchased online (from tangible products to intangible products) as consumers adapt themselves to online commerce. Consumers who purchase products over a diverse product spectrum including intangible products are likely to more frequently visit and make a transaction because their set of products of interest is correspondingly large. To identify the impact of the diversity of product categories, we clarify products into 22 categories with the assistance of category managers at our research site (e.g., home improvement, clothing, and electronics). The diversity of product category (Diversity ${ }_{i j}$ ) is coded as the number of
product categories purchased by individual $i$ at the $j^{\text {th }}$ transaction day. Finally, we include dummy variables for the day of over the week in the regression model to control weekly-based seasonality.

## 4 EMPIRICAL RESULTS AND DISCUSSION

### 4.1 Baseline Hazard Function

We can understand how the hazard rate varies with time by identifying the baseline hazard function, which represents the hazard rate with all covariate variables and unobserved heterogeneity equal to zero. If a consumer's inter-purchase time is exponentially distributed, hazard rate is constant over time (there is no duration dependency). Also, the Weibull distribution enables us to model monotonically increasing/decreasing hazard rates. In addition to Exponential and Weibull distributions, we consider the Expo-power distribution to take into account any possible shape of time dependence. Table 3 contains the estimation results for Exponential, Weibull, and Expo-power baseline hazard specifications.

|  | Baseline hazard specification |  |  |
| :---: | :---: | :---: | :---: |
| Variable | Exponential | Weibull | Expo-power |
| $\gamma$ (scale parameter) | 0.009***(37.16) | $0.0102 * * *(36.74)$ | $0.0230 * * *(564.78)$ |
| $\alpha$ (shape parameter) | - | 0.7996***(822.46) | $0.9033 * * *(1609.00)$ |
| $\omega$ (shape parameter) | - | - | $-0.0080 * * *(-152.23)$ |
| Products $_{\text {jij-1 }}$ | $0.039 * * *(34.41)$ | $0.0299 * * *(26.66)$ | $0.0289 * * *(34.00)$ |
| Money $_{\text {ij-I }}$ | 0.000*** (3.43) | 0.0000*** (3.71) | $0.0000 * * *(3.14)$ |
| SavedMoney $_{i j-1}$ | $0.002 * * *(21.90)$ | $0.0018 * * *(15.51)$ | $0.0016^{* * *}(13.43)$ |
| PricePromoRatio ${ }_{i j}$ | 5.647***(189.16) | 4.4552***(150.38) | $4.3922 * * *(157.40)$ |
| Products $_{\text {ij }}$ | $0.027 * * *(21.28)$ | $0.0193 * * *(15.19)$ | $0.0193 * * *(21.63)$ |
| OnlineShoppingExp ${ }_{i j}$ | 0.003***(51.97) | $0.0026 * * *(44.78)$ | $0.0026^{* * *}(76.10)$ |
| Diversity $_{i j}$ | $0.072 * * *(49.54)$ | $0.0579 * * *(49.29)$ | $0.0523 * * *(330.48)$ |
| $A g e_{j}$ | $0.006 * * *(9.19)$ | 0.0047***(9.63) | $0.0044 * * *(87.45)$ |
| Sex ${ }_{i}$ | $-0.112 * * *(-11.33)$ | -0.0913***(-11.67) | -0.0818*** (-20.33) |
| Sunday | $-0.057 * * *(-7.88)$ | $-0.0478 * * *(-6.72)$ | $-0.0471^{* * *(-8.26) ~}$ |
| Monday | $-0.029 * * *(-4.37)$ | $-0.0303 * * *(-4.68)$ | $-0.0316 * * *(-6.72)$ |
| Tuesday | $-0.041^{* * *}(-6.2)$ | $-0.0370 * * *(-5.73)$ | $-0.0380 * * *(-8.15)$ |
| Wednesday | $-0.014 * *(-2.19)$ | -0.0136**(-2.11) | $-0.0160 * * *(-3.41)$ |
| Thursday | 0.007(1.00) | $0.0038(0.59)$ | $0.0017(0.35)$ |
| Friday | 0.001(0.20) | -0.0002(-0.03) | -0.0017(-0.35) |
| $\lambda$ (scale parameter) | $6.083 * * *(59.03)$ | $11.0587 * * *(50.68)$ | $12.7600 * * *(513.94)$ |
| $1 / \lambda$ (shape parameter) | $0.164 * * *(59.03)$ | $0.0904 * * *(50.68)$ | $0.0784 * * *(513.94)$ |
| N | 361263 | 361263 | 361263 |
| Log Likelihood | -1625500 | -1580100 | -1543200 |
| Number of parameters | 17 | 18 | 19 |
| BIC | -3251310 | -3160430 | -3086643 |
| AIC | 3251127 | 3160236 | 3086438 |
| $t$-statics are shown in parentheses |  |  |  |

*Significant at $\mathrm{p}<0.05{ }^{* *}$ significant at $\mathrm{p}<0.01 \quad * * *$ significant at $\mathrm{p}<0.001$
Table 3. Hazard Rate Estimation Results
Because prior information on the correct model is limited, we evaluate the overall fit of parametric forms based on Bayesian information criterion (BIC) and Akaike information criterion (AIC): BIC $=2 \cdot \operatorname{Ln}(L L)-k \cdot \ln (N)$ and AIC $=2 \cdot k-2 \cdot \operatorname{Ln}(L L)$, where $L L$ is the log likelihood, $N$ is the sample size, and $k$ is the number of parameters in the model. BIC is always negative and so the maximum BIC is the least negative value while a lower value of AIC suggests a better fit of the model. Given that Expopower distribution fits the data the best based on both criteria, our main discussion is based on Expopower baseline hazard specification.
This monotonic decreasing hazard rate is a distinguishable pattern compared to that of the offline market. In general, the offline hazard rate temporally exhibits positive duration dependence since the previous transaction and after a certain point, negative duration dependence is observed. The estimated inverted U-shaped hazard rate is attributed to the optimized shopping trip schedule encompassing the utility and costs from a shopping trip. When a consumer wants to buy a product, the consumer must delay the purchase if the transaction cost for the product purchase (including time and travelling costs) exceeds the utility to be acquired. As time passes, the consumer may need to buy other products, presumably due to depleted inventory, and thus can acquire more utility at the same transaction cost. The negative duration dependence after the peak of the inverted U-shape indicates that if the consumer does not utilize the retailer for a long time, the consumer might leave for another retailer.

### 4.2 No Inventory Effect

In PHMs, the sign of the coefficients indicates how covariates affect a hazard rate. A positive (negative) coefficient increases (decreases) a hazard rate and, therefore, reduces (increases) the expected duration. The positive and significant coefficient of Products ${ }_{j i-1}$ shows that the more a consumer purchases at the previous transaction, the sooner the consumer makes another transaction. This accelerating effect of Products $_{j_{j-1}}$ indicates that the market basket size at the previous transaction does not reflect the supply of inventory. Because consumers can buy from other online and offline retailers, the number of products previously purchased from an online retailer is not well representative of the inventory level. Also, no stockpiling effect might result from our target observation on consumers' purchase from diverse product categories. Even though a consumer purchases more items in a product category (e.g., diaper and socks) than needed, the consumer might need to purchase products in other categories (e.g., electronics and pens).

Similarly, the positive coefficient of Money $_{i j-1}$ also shows that there is no inventory effect in terms of money. The money spent at an online retailer does not represent the decrement of the money allocated to shopping. Our data show that consumers on average have purchased the products corresponding to around $\$ 150$ a month. This amount of money is far less than the average income of city employees in Korea (around \$4000), who are expected to occupy the significant portion of online shoppers. Given the significant and positive coefficients of Products $_{i j-l}$ and Money $_{i j-l}$, we conclude that both variables rather represent a consumer type such as the big online spender versus the small online spender.

### 4.3 Price Promotion Effect

Many studies have reported that price promotions can effectively accelerate purchase timing. But Chiang et al. (2001) showed a conflicting result that (price) promotions have little effect because consumers are unlikely to change their shopping trip schedule to take advantage of advertised promotions in the offline market. But this explanation is not pertinent in the online market. Online consumers can select their purchase time without any restriction and so they are expected to be more responsive to price promotions. The coefficient of PricePromoRatio ${ }_{i j}(4.3922, p<0.000)$ supports this direct effect of a price promotion: price promotions induce immediate purchases.
The indirect effect is also supported. The positive coefficient of SavedMoney $_{i j-1}(0.0016, p<0.000)$ indicates that as the money saved through price promotions at the previous transaction increases,
consumers tend to make the next transaction sooner than expected. The more frequent visits induced from the good impression (increased loyalty to the retailer or consumers' lock in the retailer) make consumers better aware of the other products worth buying and new price promotions, leading to accelerated purchasing timing. Actually, this indirect effect of a price promotion supports the rational basis of loss leaders increasing store traffic and ultimately sales revenue (Walters and MacKenzie 1988).

### 4.4 Market Basket Feature and Individual Profile

The coefficient of Products $_{i j}(0.0193, p<0.001)$ shows that consumers who simultaneously purchase multiple items are likely to be frequent consumers. We confirm through the positive and significant coefficient of Diversity $_{i j}(0.0579, p<0.000)$ that as consumers buy online from diverse product categories, they show more frequent purchases. Also, online shopping experience accelerates interpurchase timing, indicating that online purchase frequency itself increases along with online shopping experience.

The results show that younger (female) consumers are more frequent shoppers compared to older (male) consumers. Consumers are more likely to purchase on Thursday through Saturday than Sunday through Wednesday - we selected Saturday as the basis to examine the weekly seasonality of purchase days. The estimation results also show that the hazard rates on Thursday and Friday do not significantly differ from that on Saturday which is the most favorite offline shopping day. This is partly consistent to the frequency distribution of purchase days over the week based on an aggregated level, supporting the disappearance of preferred shopping days over the week.

## 5 CONCLUSIONS, LIMITATIONS AND FUTURE RESEARCH DIRECTIONS

We identify increased shopping time flexibility in the online market by examining the patterns of online purchase timing in comparison with those documented in the offline market. Furthermore, based on our econometric models on inter-purchase time, we verify new theories in online consumers' purchase timing, particularly, about (1) how online consumers' hazard rates change, (2) how online consumers respond to price promotions, and (3) what factors affect online consumers' purchasing timing decision.

One important limitation is that we cannot observe marketing activities (besides price promotions) that were deployed during our research period, which might explain some variation of inter-purchase times. In addition to newspaper and TV advertisements exposed to the public, our research site has used email campaigns for individual consumers. Our field study shows that the email campaigns were not customized and so the same contents were sent to all consumers. Also they were mainly focus on the enhancement of their image, not to separately advertise specific products. As a result, the unobservable marketing activities will not be viable factors to cause bias in comparing the promotion effects across consumer types. However, the inclusion of the relevant variables (e.g., when advertisement is exposed and their frequency) would increase our knowledge of the marketing activity effects on online consumers' purchasing timing.

Consumers' purchase decisions in a product category can be affected by their purchase decisions in other product categories (Chung and Rao 2003; Russell and Kamakura 1997). However, our model does not allow for the economic relationships between products purchased either in a market basket or across market baskets constructed over time. In the online market, multi-category purchase behavior should be another interesting subject.

We examine the effects of present price promotions in accelerating both current purchase timing (direct effect of price promotions) and next purchase timing (when a consumer experiences the price promotion). However, we cannot identify the type of price promotions. They can be (1) retailergenerated or manufacturer-generated coupons, (2) the discount of a specific sum of money or a certain percent, (3) the applicability or non-applicability of multiple coupons and (4) discount of product prices or extra benefits such as free delivery. Further research could attempt to assess potentially
different price promotion effects depending on how they are implemented. Such research would suggest a more effective strategy formulation of price promotions.

Finally, we find online consumers' evolution/learning progress in the hazard rates and the utilization of price promotions with online purchasing experience. But our model cannot explicitly address the dynamics of consumers' purchase timings. Similarly, we assume that consumer's type is static and constant over time. Future studies can overcome this limitation.

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