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Review Article

Robust Stabilization for Continuous Takagi-Sugeno Fuzzy System Based on Observer Design

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This paper investigates the influence of a new parallel distributed controller (PDC) on the stabilization region of continuous Takagi-Sugeno (T-S) fuzzy models. Using a nonquadratic Lyapunov function, a new sufficient stabilization criterion is established in terms of linear matrix inequality. The criterion examines the derivative membership function; an approach to determine state variables is given based on observer design. In addition, a stabilization condition for uncertain system is given. Finally, numeric simulation is given to validate the developed approach.

1. Introduction

Fuzzy control systems have experienced a big growth of industrial applications in the recent decades, because of their reliability and effectiveness. Many researches have investigated the Takagi-Sugeno models [1–3] during the last decades. Two classes of Lyapunov functions are used to analyze these systems: quadratic Lyapunov functions and nonquadratic Lyapunov ones which are less conservative than the first class. Many researches have investigated with nonquadratic Lyapunov functions [4–10].

As the information about the time derivatives of membership function is considered by the PDC fuzzy controller, it allows the introduction of slack matrices to facilitate the stability analysis. The relationship between the membership function of the fuzzy model and the fuzzy controllers is used to introduce some slack matrix variables. The boundary information of the membership functions is brought to the stability condition and thus offers some relaxed stability conditions [6]. In order to determine the state variables many approaches of observer design are given [9–11].

In this paper, new stabilization conditions for Takagi-Sugeno uncertain fuzzy models based on the use of fuzzy Lyapunov function are presented. This criterion is expressed in terms of linear matrix inequalities (LMIs) which can be efficiently solved by using various convex optimization algorithms [12, 13]. The presented method is less conservative than existing results.

The organization of the paper is as follows. In Section 2, we present the system description and problem formulation and we give some preliminaries which are needed to derive results. Section 3 will be concerned with stabilization analysis for continuous *T-S* fuzzy systems by the use of new PDC controller based on derivative membership functions. An observer approach design is derived to estimate state variables. In Section 4, a new stabilization condition for uncertain system is given. Next, a new robust PDC controller design approach is presented. Illustrative examples are given in Section 5 for a comparison of previous results to demonstrate the advantage of the proposed method. Finally Section 6 makes the conclusion.

Notation. Throughout this paper, a real symmetric matrix $S > 0$ denotes S being a positive definite matrix. The superscript "T" is used for the transpose of a matrix.

2. System Description and Preliminaries

Consider an uncertain *T-S* fuzzy continuous model for a nonlinear system as follows:

$$\begin{aligned} &\text{IF} \quad z_1(t) \text{ is } M_{i1}, \dots, z_p(t) \text{ is } M_{ip}, \\ &\text{THEN} \quad \dot{x}(t) = (A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t) \quad i = 1, \dots, r, \end{aligned} \tag{2.1}$$

where M_{ij} $(i = 1, 2, \dots, r, j = 1, 2, \dots, p)$ is the fuzzy set and r is the number of model rules, $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re^m$ is the input vector, $A_i \in \Re^{n\times n}$, $B_i \in \Re^{n\times m}$ are constant real matrices, and $z_1(t), \dots, z_p(t)$ are known premise variables. ΔA_i, and ΔB_i are time-varying matrices representing parametric uncertainties in the plant model. These uncertainties are admissibly norm-bound and structured.

The final outputs of the fuzzy systems are:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))\{(A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t)\}, \tag{2.2}$$

where

$$\begin{aligned} z(t) &= [z_1(t) z_2(t) \cdots z_p(t)], \\ h_i(z(t)) &= \frac{w_i(z(t))}{\sum_{i=1}^{r} w_i(z(t))}, \\ w_i(z(t)) &= \prod_{j=1}^{p} M_{ij}(z_j(t)) \quad \forall t. \end{aligned} \tag{2.3}$$

The term $M_{i1}(z_j(t))$ is the grade of membership of $z_j(t)$ in M_{i1}

$$\text{Since } \begin{cases} \sum_{i=1}^{r} w_i(z(t)) \succ 0 \\ w_i(z(t)) \geq 0, \quad i = 1, 2, \ldots, r \end{cases} \quad \text{we have } \begin{cases} \sum_{i=1}^{r} h_i(z(t)) = 1, \\ h_i(z(t)) \geq 0, \quad i = 1, 2, \ldots, r \end{cases} \tag{2.4}$$

for all t.

We have the following property:

$$\sum_{k=1}^{r} \dot{h}_k(z(t)) = 0. \tag{2.5}$$

This study investigates the PDC controller influence on the closed-loop stability region and gives robustness analysis of uncertain Takagi-Sugeno fuzzy system. Thus, we consider a PDC fuzzy controller which examines the derivative membership function and it is given by

$$u(t) = -\sum_{i=1}^{r} h_i(z(t)) F_i x(t) - \sum_{\rho=1}^{r} \dot{h}_\rho(z(t)) \left(K_\rho + R\right) x(t). \tag{2.6}$$

The fuzzy controller design consists to determine the local feedback gains F_i, K_ρ, and R in the consequent parts. The state variables are determined by an observer and are detailed in the next section.

The open-loop system is given by

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t)) (A_i + \Delta A_i) x(t). \tag{2.7}$$

By substituting (2.6) into (2.2), the closed-loop fuzzy system can be represented as:

$$\dot{x}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) \left\{ A_{\Delta i} - B_{\Delta i} F_j - \sum_{\rho=1}^{r} \dot{h}_\rho(z(t)) B_{\Delta i} \left(K_\rho + R\right) \right\} x(t), \tag{2.8}$$

where $A_{\Delta i} = A_i + \Delta A_i$ and $B_{\Delta i} = B_i + \Delta B_i$.

Assumption 2.1. The time derivative of the premises membership function is upper bound such that $|\dot{h}_k| \leq \phi_k$, for $k = 1, \ldots, r$, where, ϕ_k, $k = 1, \ldots, r$ are given positive constants.

Assumption 2.2. The matrices denote the uncertainties in the system and take the form of

$$\begin{aligned} \Delta A_i &= D_{a_i} F_{a_i}(t) E_{a_i}, \\ \Delta B_i &= D_{b_i} F_{b_i}(t) E_{b_i}, \end{aligned} \tag{2.9}$$

where $D_{a_i}, D_{b_i}, E_{a_i}$, and E_{b_i} are known constant matrices and $F_{a_i}(t)$ and $F_{b_i}(t)$ are unknown matrix functions satisfying:

$$F_{a_i}^T(t)F_{a_i}(t) \leq I, \quad \forall t,$$
$$F_{b_i}^T(t)F_{b_i}(t) \leq I, \quad \forall t, \tag{2.10}$$

where I is an appropriately dimensioned identity matrix.

Lemma 2.3 (Boyd et al. Schur complement [12]). *Given constant matrices Ω_1, Ω_2, and Ω_3 with appropriate dimensions, where $\Omega_1 = \Omega_1^T$ and $\Omega_2 = \Omega_2^T$, then*

$$\Omega_1 + \Omega_3^T \Omega_2^{-1} \Omega_3 \prec 0 \tag{2.11}$$

if and only if

$$\begin{bmatrix} \Omega_1 & \Omega_3^T \\ & -\Omega_2 \end{bmatrix} \succ 0 \quad or \quad \begin{bmatrix} -\Omega_2 & \Omega_3 \\ & \Omega_1 \end{bmatrix} \succ 0. \tag{2.12}$$

Lemma 2.4 (Peterson and Hollot [2]). *Let $Q = Q^T, H, E$, and $F(t)$ satisfying $F^T(t)F(t) \leq I$ are appropriately dimensional matrices then the following inequality*

$$Q + HF(t)E + E^T F^T(t) H^T \prec 0 \tag{2.13}$$

is true, if and only if the following inequality holds for any $\lambda \succ 0$

$$Q + \lambda^{-1} H H^T + \lambda E^T E \prec 0. \tag{2.14}$$

The aim of the next section is to find conditions for the stabilization of the closed-loop T-S fuzzy system by using the Lyapunov theory.

3. Main Results

Consider the closed-loop system without uncertainties

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t)) \{ A_i x(t) + B_i u(t) \}. \tag{3.1}$$

In order to give stability conditions, the slack matrix variables and the membership function boundary Mozelli et al. [14] are used. Consider the following null product that will serve stability analysis purposes:

$$2\left[x^T(t)M + \dot{x}^T(t)\mu M\right] \times \left[\dot{x}(t) - \sum_{i=1}^{r} h_i(z(t)) \{ A_i x(t) + B_i u(t) \}\right] = 0. \tag{3.2}$$

3.1. A PDC Controller with Derivative Membership Function

By substituting (2.6) into (3.1), the closed-loop fuzzy system can be represented as:

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))\left\{A_i - B_iF_j - \sum_{m=1}^{r}\dot{h}_m(z(t))B_i(K_m + R)\right\}x(t). \tag{3.3}$$

The next Theorem gives sufficient conditions to guarantee stability of system (3.3).

Theorem 3.1. *Under Assumptions 2.1 and 2.2, and for $\mu > 0$, $\varepsilon \geq 0$, the Takagi-Sugeno fuzzy system* (3.1) *is stabilizable with the PDC controller* (2.6), *with gains given by $F_i = S_i^T H^{-T}$, $K_\rho = V_\rho^T H^{-T}$, and $R = V^T H^{-T}$, if there exist positive definite symmetric matrices T_k, $k = 1, 2, \ldots, r$, Y, and any matrices H, S_i, V_ρ, and V with appropriate dimensions such that the following LMIs hold.*

$$\begin{gathered} T_i > 0, \\ T_i + Y > 0 \quad (i = 1, 2, \ldots, r), \\ \Lambda_{ii} < 0, \\ \overline{\Lambda}_{ij} < 0, \end{gathered} \tag{3.4}$$

$$\overline{\Lambda}_{ij} = \Lambda_{ij} + \Lambda_{ji}, \tag{3.5}$$

where

$$\begin{gathered} \Lambda_{ij} = \begin{bmatrix} T_\phi - A_iH^T - HA_i^T + B_iS_j^T + S_jB_i^T + B_i\overline{V}_\phi^T + \overline{V}_\phi B_i^T & * \\ T_i - \mu\left(A_iH^T - B_iS_j^T - B_i\overline{V}_\phi^T\right) + H & \mu\left(H^T + H\right) \end{bmatrix}, \\ \overline{V}_\phi = \sum_{\rho=1}^{r}\phi_\rho\left(V_\rho^T + V^T\right), \\ \overline{T}_\phi = \sum_{k=1}^{r}\phi_k(T_k + Y). \end{gathered} \tag{3.6}$$

Proof of Theorem 3.1. Let's consider the fuzzy weighting-dependent Lyapunov-Krasovskii functional as:

$$V(x(t)) = \sum_{k=1}^{r} h_k(z(t)) \cdot V_k(x(t)), \tag{3.7}$$

with $V_k(x(t)) = x^T(t)(P_k + \varepsilon X)x(t)$, $k = 1, 2, \ldots, r$, where $P_k = P_k^T$, $X = X^T$, $\varepsilon \geq 0$, and $(P_k + \varepsilon X) \geq 0$.

This candidate Lyapunov function satisfies

(i) $V(x(t))$ is C^1,

(ii) $V(0) = 0$ and $V(x(t)) \geq 0$ for $x(t) \neq 0$,

(iii) $\|x(t)\| \to \infty \Rightarrow V(x(t)) \to \infty$.

The time derivative of $V(x(t))$ is given by:

$$\dot{V}(x(t)) = \sum_{k=1}^{r} \dot{h}_k(z(t)) V_k(x(t)) + \sum_{k=1}^{r} h_k(z(t)) \dot{V}_k(x(t)). \tag{3.8}$$

Adding the null product, then

$$\begin{aligned}\dot{V}(x(t)) = {} & \sum_{k=1}^{r} \dot{h}_k(z(t)) V_k(x(t)) + \sum_{k=1}^{r} h_k(z(t)) \dot{V}_k(x(t)) + 2\left[x^T(t)M + \dot{x}^T(t)\mu M\right] \\ & \times \left[\dot{x}(t) - \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) \left\{ A_i - B_i F_j - \sum_{\rho=1}^{r} \dot{h}_\rho(z(t)) B_i \left(K_\rho + R\right) \right\} x(t) \right].\end{aligned} \tag{3.9}$$

Equation (3.9) can be rewritten as,

$$\dot{V}(x(t)) = \Upsilon_1(x, z) + \Upsilon_2(x, z), \tag{3.10}$$

where

$$\begin{aligned}\Upsilon_1(x, z) = {} & x^T(t) \left(\sum_{k=1}^{r} \dot{h}_k(z(t)) \cdot (P_k + \varepsilon X) \right) x(t) + 2\left[x^T(t)M + \dot{x}^T(t)\mu M\right] \\ & \times \left[\sum_{i=1}^{r}\sum_{\rho=1}^{r} h_i(z(t)) \dot{h}_\rho(z(t)) B_i \left(K_\rho + R\right) x(t) \right], \\ \Upsilon_2(x, z) = {} & \sum_{i=1}^{r} h_i(z(t)) \left\{ 2x^T(t)(P_i + \varepsilon X)\dot{x}(t) \right\} + 2\left[x^T(t)M + \dot{x}^T(t)\mu M\right] \\ & \times \left[\dot{x}(t) - \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) \left\{ A_i - B_i F_j \right\} x(t) \right].\end{aligned} \tag{3.11}$$

Then, based on Assumption 2.1, an upper bound of $\Upsilon_1(x, z)$ is obtained as:

$$\begin{aligned}\Upsilon_1(x, z) \leq {} & \sum_{i=1}^{r} h_i(z(t)) \left\{ \sum_{k=1}^{r} \phi_k \cdot x(t)^T (P_k + \varepsilon X) x(t) + 2\left[x^T(t)M + \dot{x}^T(t)\mu M\right] \right. \\ & \left. \times \left[\sum_{\rho=1}^{r} \phi_\rho B_i \left(K_\rho + R\right) x(t) \right] \right\}.\end{aligned} \tag{3.12}$$

Based on (2.5), it follows that $\sum_{k=1}^{r} \dot{h}_k(z(t))(1-\varepsilon)X = 0$ where X is any symmetric matrix of proper dimension.

Suppose that $\overline{X} = \sum_{k=1}^{r} \dot{h}_k(z(t))(1-\varepsilon)X$ and adding $\overline{X}$ to (3.12), then

$$\begin{aligned}\Upsilon_1(x,z) &\leq \sum_{i=1}^{r} h_i(z(t)) \left\{ \sum_{k=1}^{r} \phi_k \cdot x(t)^T (P_k + X) x(t) + 2\left[x^T(t)M + \dot{x}^T(t)\mu M\right] \right.\\ &\qquad \left. \times \left[\sum_{\rho=1}^{r} \phi_\rho B_i (K_\rho + R) x(t)\right]\right\} \\ &= \sum_{i=1}^{r} h_i(z(t)) \left\{ x(t)^T \overline{P}_\phi x(t) + 2x^T(t)M \cdot B_i \overline{K}_\phi x(t) + 2\dot{x}^T(t)\mu M \cdot B_i \overline{K}_\phi x(t) \right\},\end{aligned} \tag{3.13}$$

where

$$\overline{P}_\phi = \sum_{k=1}^{r} \phi_k \cdot (P_k + X), \qquad \overline{K}_\phi = \sum_{\rho=1}^{r} \phi_\rho (K_\rho + R). \tag{3.14}$$

Then,

$$\begin{aligned}\dot{V}(x(t)) \leq &\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) \\ &\times \left\{ x(t)^T \overline{P}_\phi x(t) + 2x^T(t)M \cdot B_i \overline{K}_\phi x(t) + 2\dot{x}^T(t)\mu M \cdot B_i \overline{K}_\phi x(t) \right.\\ &\quad + 2x^T(t)(P_i + \varepsilon R)\dot{x}(t) + 2x^T(t)M\dot{x}(t) + 2\dot{x}^T(t)\mu M \dot{x}(t) \\ &\quad \left. -2x^T(t)M(A_i - B_i F_j)x(t) - 2\dot{x}^T(t)\mu M(A_i - B_i F_j)x(t) \right\}.\end{aligned} \tag{3.15}$$

Using vector $\eta^T = [x^T(t) \;\; \dot{x}^T(t)]^T$, (3.15) can be rewritten as

$$\begin{aligned}\dot{V}(x(t)) &\leq \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) \eta^T \Xi_{ij} \eta \\ &= \sum_{i=1}^{r} h_i^2(z(t)) \eta^T \Xi_{ii} \eta + \sum_{i=1}^{r}\sum_{i<j} h_i(z(t)) h_j(z(t)) \eta^T (\Xi_{ij} + \Xi_{ji}) \eta,\end{aligned} \tag{3.16}$$

where

$$\Xi_{ij} = \begin{bmatrix} \left\{ \overline{P}_\phi - M\left(A_i - B_i F_j - B_i \overline{K}_\phi\right) - \left(A_i - B_i F_j - B_i \overline{K}_\phi\right)^T M^T \right\} & * \\ (P_i + \varepsilon X) - \mu M\left(A_i - B_i F_j - B_i \overline{K}_\phi\right) + M^T & \mu(M + M^T) \end{bmatrix}. \tag{3.17}$$

If $\Xi_{ii} \prec 0$ and $(\Xi_{ij} + \Xi_{ji}) \prec 0$, then $\dot{V}(x(t)) \prec 0$ and (3.3) is stable. Pre- and post-multiplying $\Xi_{ii} \prec 0$ and $(\Xi_{ij} + \Xi_{ji}) \prec 0$ by nonsingular matrices $\mathrm{diag}(M^{-1}, M^{-1})$ and $\mathrm{diag}(M^{-T}, M^{-T})$, respectively, and pre- and postmultiplying by M^{-1} and M^{-T}, respectively, then we obtain

$$\begin{gathered} M^{-1}(P_i + \varepsilon X)M^{-T} \succ 0 \quad (i = 1, 2, \dots, r), \\ M^{-1}(P_i + X)M^{-T} \succ 0 \quad (i = 1, 2, \dots, r), \\ \Lambda_{ii} \prec 0 \quad (i = 1, 2, \dots, r), \\ \overline{\Lambda}_{ij} \prec 0 \quad (i <= 1, 2, \dots, r), \end{gathered} \tag{3.18}$$

where $\overline{\Lambda}_{ij} = \Lambda_{ij} + \Lambda_{ji}$, and

$$\Lambda_{ij} = \begin{bmatrix} \begin{gathered} \Big\{ M^{-1}\overline{P}_\phi M^{-T} - \Big(A_i - B_i F_j - B_i \overline{K}_\phi\Big) M^{-T} \\ -M^{-1}\Big(A_i - B_i F_j - B_i \overline{K}_\phi\Big)^T \Big\} \end{gathered} & * \\ \Big\{ M^{-1}(P_i + \varepsilon X) M^{-T} - \mu\Big(A_i - B_i F_j - B_i \overline{K}_\phi\Big) M^{-T} + M^{-1} \Big\} & \mu\big(M^{-T} + M^{-1}\big) \end{bmatrix} \tag{3.19}$$

for the following variables definition:

$$\begin{gathered} H = M^{-1}, \qquad T_i = H(P_i + \varepsilon X)H^T, \qquad T_\phi = H\overline{P}_\phi H^T, \qquad S_j = HF_j^T, \\ V_j = HK_\rho^T, \qquad V = HR^T, \qquad Y = HXH^T. \end{gathered} \tag{3.20}$$

If LMI in (3.4) holds then the closed-loop continuous fuzzy system (3.3) is asymptotically stable. The control gains are given by $F_i = S_i^T H^{-T}$, $K_\rho = V_\rho^T H^{-T}$, and $R = V^T H^{-T}$. This completes the proof. □

Remark 3.2. The selection of ϕ_k given in Assumption 2.1 is performed by using a simple procedure given in [15] .

Remark 3.3. The major contribution of the Theorem 3.1 is represented by the proposed PDC controller given by (2.6). The contribution appears in the gains $(K_\rho + R)$ introduced in the controller term based on derivative membership functions. The stabilization condition proposed is less conservative than some of those in the literature, as is shown in the example below.

3.2. Observer Design

In order to determine state variables of the system, this section gives a solution by the means of fuzzy observer design. The following condition is to be satisfied by the observer:

$$x(t) - \hat{x}(t) \longrightarrow 0 \quad \text{as } t \longrightarrow \infty, \tag{3.21}$$

where $\widehat{x}(t)$ denotes the state vector estimated by a fuzzy observer. This condition guarantees that the steady-state error between $x(t)$ and $\widehat{x}(t)$ converge to 0 and we denote this error by $e(t) = x(t) - \widehat{x}(t)$.

A stabilizing observer-based controller can be formulated as follow:

$$\begin{gathered}\dot{\widehat{x}}(t) = \sum_{j=1}^{r} h_i(z(t))\{A_i\widehat{x}(t) + B_iu(t) + L_j(C_i\widehat{x}(t) - y(t))\},\\ y(t) = -\sum_{i=1}^{r} h_i(z(t))C_i\widehat{x}(t),\end{gathered} \tag{3.22}$$

where $y(t)$ denotes the output vector.

We consider the proposed PDC controller given by (2.6):

$$u(t) = -\sum_{i=1}^{r} h_i(z(t))F_i\widehat{x}(t) - \sum_{\rho=1}^{r} \dot{h}_\rho(z(t))(K_\rho + R)\widehat{x}(t). \tag{3.23}$$

Replacing the fuzzy controller (2.6) in fuzzy observer (3.22) we obtain the closed-loop fuzzy system as:

$$\begin{aligned}\dot{x}(t) = {}& \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))\left\{(A_i - B_iF_j) - \sum_{\rho=1}^{r}\dot{h}_\rho(z(t))(K_\rho + R)\right\}x(t)\\ &+ \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))\left\{B_iF_j + \sum_{\rho=1}^{r}\dot{h}_\rho(z(t))(K_\rho + R)\right\}e(t)\\ \dot{e}(t) = {}& \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))\{A_i - L_iC_j\}e(t).\end{aligned} \tag{3.24}$$

The augmented system is represented as follows:

$$\begin{aligned}\dot{x}_a(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))\widetilde{G}_{ij}x_a(t)\\ &= \sum_{j=1}^{r} h_i(z(t))h_j(z(t))\widetilde{G}_{ii}x_a(t) + 2\sum_{i=1}^{r}\sum_{i<j}^{r} h_i(z(t))h_j(z(t))\left\{\frac{\widetilde{G}_{ij} + \widetilde{G}_{ji}}{2}\right\}x_a(t),\end{aligned} \tag{3.25}$$

where

$$x_a(t) = \begin{bmatrix} x(t) \\ e(t) \end{bmatrix},$$

$$\widetilde{G}_{ij} = \begin{bmatrix} A_i - B_iF_j - \sum_{\rho=1}^{r} \dot{h}_\rho B_i(K_\rho + R) & B_iF_j + \sum_{\rho=1}^{r} \dot{h}_\rho B_i(K_\rho + R) \\ 0 & A_i - L_iC_j \end{bmatrix}. \tag{3.26}$$

By applying Theorem 1 in [16] in the augmented system (3.25) we derive the following Theorem.

Theorem 3.4. *Under Assumptions 2.1 and 2.2, and for* $0 \leq \mu \leq 1$, *the Takagi-Sugeno fuzzy system* (3.1) *is stable if there exist positive definite symmetric matrices* P_k, $k = 1, 2, \ldots, r$, *and* R, *matrices* $F_1, \ldots, F_r$ *such that the following LMIs holds.*

$$\begin{gathered} P_k + R \succ 0, \quad k \in \{1, \ldots, r\}, \\ P_j + \mu R \geq 0, \quad j = 1, 2, \ldots, r, \\ P_\phi + \left\{ \widetilde{G}_{ii}^T (P_k + \mu R) + (P_k + \mu R)\widetilde{G}_{ii} \right\} \prec 0, \quad i, k \in \{1, \ldots, r\}, \\ \left\{ \frac{\widetilde{G}_{ij} + \widetilde{G}_{ji}}{2} \right\}^T (P_k + \mu R) + (P_k + \mu R) \left\{ \frac{\widetilde{G}_{ij} + \widetilde{G}_{ji}}{2} \right\} \prec 0, \quad \text{for } i, j, k = 1, 2, \ldots, r \text{ such that } i \prec j, \end{gathered} \tag{3.27}$$

where

$$\begin{gathered} \widetilde{G}_{ij} = \begin{bmatrix} A_i - B_iF_j - \sum_{\rho=1}^{r} \dot{h}_\rho B_i(K_\rho + R) & B_iF_j + \sum_{\rho=1}^{r} \dot{h}_\rho B_i(K_\rho + R) \\ 0 & A_i - L_iC_j \end{bmatrix}, \\ P_\phi = \sum_{k=1}^{r} \phi_k (P_k + R). \end{gathered} \tag{3.28}$$

Proof of Theorem 3.4. The result follows immediately from the proof of Theorem 1 in [16]. □

4. Robust Stability Condition with PDC Controller

Consider the uncertain closed-loop system (2.8). A sufficient robust stability condition is given as follows.

Theorem 4.1. *Under Assumptions 2.1 and 2.2, and for* $\mu \succ 0$, $\varepsilon \geq 0$, *the Takagi-Sugeno fuzzy system* (2.2) *is stabilizable with the PDC controller* (2.6), *with gains given by* $F_i = S_i^T H^{-T}$, $K_\rho = V_\rho^T H^{-T}$, *and $R = V^T H^{-T}$, if there exist positive definite symmetric matrices $T_k,\ k = 1,2,\ldots,r,\ Y$, and any matrices H, S_i, V_ρ, and V with appropriate dimensions such that the following LMIs hold.*

$$\begin{gathered}T_i \succ 0,\\ T_i + Y \succ 0 \quad (i = 1,2,\ldots,r),\\ \sum_{ii} \prec 0,\\ \overline{\sum_{ij}} \prec 0,\end{gathered} \tag{4.1}$$

$$\begin{gathered}\overline{\sum_{ij}} = \sum_{ij} + \sum_{ji},\\ T_\phi = \sum_{k=1}^{r} \phi_k (T_k + Y),\end{gathered} \tag{4.2}$$

where

$$\Sigma_{ij} = \begin{bmatrix} \Phi_{11} & \Phi_{12} & MD_{ai} & MD_{bi} & E_{a_i}^T & -\left(E_{bi}\left(F_i + \overline{K}_\phi\right)\right)^T \\ & \Phi_{22} & \mu MD_{ai} & \mu MD_{bi} & 0 & 0 \\ & * & -\lambda I & 0 & 0 & 0 \\ & * & * & -\lambda I & 0 & 0 \\ & * & * & * & -\lambda^{-1} I & 0 \\ & * & * & * & * & -\lambda^{-1} I \end{bmatrix}, \tag{4.3}$$

with

$$\begin{gathered}\Phi_{11} = \overline{P}_\phi + \left[M \cdot B_i \overline{K}_\phi + \overline{K}_\phi^T B_i^T M\right] - \left[M G_{ii} + {G_{ii}}^T M^T\right],\\ \Phi_{12} = (P_i + \varepsilon X) - \mu\left(G_{ii} - B_i \overline{K}_\phi\right)^T M^T + M,\\ \Phi_{22} = \mu\left(M + M^T\right),\end{gathered} \tag{4.4}$$

where $G_{ij} = A_i - B_i F_j$, $G_{ii} = A_i - B_i F_i$, and $P_\phi = \sum_{k=1}^{r} \phi_k (P_k + R)$.

Proof. [Proof of Theorem 4.1] The result follows immediately from the proof of Theorem 3.1 by replacing in the matrix inequality A_i with $A_i + D_{ai}F(t)E_{ai}$ and B_i with $B_i + D_{bi}F(t)E_{bi}$, we obtain the following inequality:

$$\begin{aligned}
\dot{V}(x(t)) \le \sum_{i=1}^{r}\sum_{j=1}^{r} & h_i(z(t))h_j(z(t)) \\
& \times \Big\{ x(t)^T\overline{P}_\phi x(t) + 2x^T(t)M\cdot B_i\overline{K}_\phi x(t) + 2\dot{x}^T(t)\mu M\cdot B_i\overline{K}_\phi x(t) \\
& \quad + 2x^T(t)(P_i+\varepsilon R)\dot{x}(t) + 2x^T(t)M\dot{x}(t) + 2\dot{x}^T(t)\mu M\dot{x}(t) \\
& \quad - 2x^T(t)M(A_i - B_iF_j)x(t) - 2x^T(t)M \\
& \quad \times \left(\begin{bmatrix} D_{ai} & D_{bi} \end{bmatrix} \begin{bmatrix} F_{a_i}(t) & 0 \\ 0 & F_{b_i}(t) \end{bmatrix} \begin{bmatrix} E_{ai} \\ -E_{bi}\left(F_j + \overline{K}_\phi\right) \end{bmatrix} \right) x(t) \\
& \quad - 2\dot{x}^T(t)\mu M(A_i - B_iF_j)x(t) - 2\dot{x}^T(t)\mu M \\
& \quad \times \left(\begin{bmatrix} D_{ai} & D_{bi} \end{bmatrix} \begin{bmatrix} F_{a_i}(t) & 0 \\ 0 & F_{b_i}(t) \end{bmatrix} \begin{bmatrix} E_{ai} \\ -E_{bi}\left(F_j + \overline{K}_\phi\right) \end{bmatrix} \right) x(t) \Big\}.
\end{aligned} \tag{4.5}$$

Using vector $\eta^T = [x^T(t) \ \ \dot{x}^T(t)]^T$, (4.5) can be rewritten as

$$\begin{aligned}
\dot{V}(x(t)) &\le \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))\eta^T\tilde{\Xi}_{ij}\eta \\
&= \sum_{i=1}^{r} h_i^2(z(t))\eta^T\tilde{\Xi}_{ii}\eta + \sum_{i=1}^{r}\sum_{i<j} h_i(z(t))h_j(z(t))\eta^T\left(\tilde{\Xi}_{ij} + \tilde{\Xi}_{ji}\right)\eta,
\end{aligned} \tag{4.6}$$

where

$$\begin{gathered}
\tilde{\Xi}_{ij} = \Xi_{ij} + \begin{bmatrix} \left\{ \begin{array}{l} M\left(\begin{bmatrix} D_{ai} & D_{bi} \end{bmatrix} \begin{bmatrix} F_{a_i}(t) & 0 \\ 0 & F_{b_i}(t) \end{bmatrix} \begin{bmatrix} E_{ai} \\ -E_{bi}\left(F_j + \overline{K}_\phi\right) \end{bmatrix} \right) \\ + \left(\begin{bmatrix} D_{ai} & D_{bi} \end{bmatrix} \begin{bmatrix} F_{a_i}(t) & 0 \\ 0 & F_{b_i}(t) \end{bmatrix} \begin{bmatrix} E_{ai} \\ -E_{bi}\left(F_j + \overline{K}_\phi\right) \end{bmatrix} \right)^T M^T \end{array} \right\} & * \\ \mu M\left(\begin{bmatrix} D_{ai} & D_{bi} \end{bmatrix} \begin{bmatrix} F_{a_i}(t) & 0 \\ 0 & F_{b_i}(t) \end{bmatrix} \begin{bmatrix} E_{ai} \\ -E_{bi}\left(F_j + \overline{K}_\phi\right) \end{bmatrix} \right) & 0 \end{bmatrix}, \\
\tilde{\Xi}_{ij} = \Xi_{ij} + \begin{bmatrix} M\begin{bmatrix} D_{ai} & D_{bi} \end{bmatrix} \\ \mu M\begin{bmatrix} D_{ai} & D_{bi} \end{bmatrix} \end{bmatrix} \begin{bmatrix} F_{a_i}(t) & 0 \\ 0 & F_{b_i}(t) \end{bmatrix} \begin{bmatrix} E_{ai} & 0 \\ -E_{bi}\left(F_j + \overline{K}_\phi\right) & 0 \end{bmatrix} \\
+ \begin{bmatrix} E_{a_i}^T & -\left(E_{bi}\left(F_j + \overline{K}_\phi\right)\right)^T \\ 0 & 0 \end{bmatrix} \begin{bmatrix} F_{a_i}(t) & 0 \\ 0 & F_{b_i}(t) \end{bmatrix}^T \begin{bmatrix} D_{a_i}^T M^T & \mu D_{a_i}^T M^T \\ D_{b_i}^T M^T & \mu D_{b_i}^T M^T \end{bmatrix}.
\end{gathered} \tag{4.7}$$

Then, based on Lemma 2.4, an upper bound of $\tilde{\tilde{\Xi}}_{ij}$ obtained as:

$$\begin{aligned}\tilde{\tilde{\Xi}}_{ij} = \Xi_{ij} + \lambda^{-1} M \begin{bmatrix} D_{ai} & D_{bi} \\ \mu D_{ai} & \mu D_{bi} \end{bmatrix} \begin{bmatrix} D_{a_i}^T & \mu D_{a_i}^T \\ D_{b_i}^T & \mu D_{b_i}^T \end{bmatrix} M^T \\ + \lambda \begin{bmatrix} E_{a_i}^T & -\left(E_{bi}\left(F_j + \overline{K}_\phi\right)\right)^T \\ 0 & 0 \end{bmatrix} \begin{bmatrix} E_{ai} & 0 \\ -E_{bi}\left(F_j + \overline{K}_\phi\right) & 0 \end{bmatrix} \prec 0\end{aligned} \tag{4.8}$$

by Schur complement, we obtain,

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & MD_{ai} & MD_{bi} & E_{a_i}^T & -\left(E_{bi}\left(F_j + \overline{K}_\phi\right)\right)^T \\ & \Phi_{22} & \mu MD_{ai} & \mu MD_{bi} & 0 & 0 \\ & * & -\lambda I & 0 & 0 & 0 \\ & * & * & -\lambda I & 0 & 0 \\ & * & * & * & -\lambda^{-1} I & 0 \\ & * & * & * & * & -\lambda^{-1} I \end{bmatrix} \prec 0, \tag{4.9}$$

with

$$\begin{aligned}\Phi_{11} &= \overline{P}_\phi + \left[M \cdot B_i \overline{K}_\phi + \overline{K}^T{}_\phi B_i^T M\right] - \left[M\left(A_i - B_i F_j\right) + \left(A_i - B_i F_j\right)^T M^T\right], \\ \Phi_{12} &= \left(P_i + \varepsilon X\right) - \mu\left(A_i - B_i F_j - B_i \overline{K}_\phi\right)^T M^T + M, \\ \Phi_{22} &= \mu\left(M + M^T\right).\end{aligned} \tag{4.10}$$

If the LMI (4.1) holds then the system (2.8) is stable. This completes the proof. □

The following theorem gives sufficient conditions for robust PDC controller design.

Theorem 4.2. *Under Assumptions 2.1 and 2.2, and for $\mu > 0$, $\varepsilon \geq 0$, the Takagi-Sugeno robust fuzzy system* (2.2) *is stabilizable with the PDC controller* (2.6), *with gains given by $F_i = \tilde{S}_i^T \widetilde{H}^{-T}$,*

$K_\rho = \widetilde{V}_\rho^T \widetilde{H}^{-T}$, and $R = \widetilde{V}^T \widetilde{H}^{-T}$, if there exist positive definite symmetric matrices $\widetilde{T}_k$, $k = 1,2,\ldots,r$, $\widetilde{Y}$, and any matrices $\widetilde{H}, \widetilde{S}_i, \widetilde{V}_\rho$, and $\widetilde{V}$ with appropriate dimensions such that the following LMIs hold.

$$\begin{gathered}\widetilde{T}_i \succ 0,\\ \widetilde{T}_i + \widetilde{Y} \succ 0 \quad (i = 1,2,\ldots,r),\\ \widetilde{\Lambda}_{ii} \prec 0,\\ \overline{\widetilde{\Lambda}}_{ij} \prec 0,\end{gathered} \tag{4.11}$$

$$\begin{gathered}\overline{\widetilde{\Lambda}}_{ij} = \widetilde{\Lambda}_{ij} + \widetilde{\Lambda}_{ji},\\ \widetilde{T}_\phi = \sum_{k=1}^{r} \phi_k \left(\widetilde{T}_k + \widetilde{Y}\right),\\ \widetilde{\widetilde{V}}_\phi = \sum_{\rho=1}^{r} \phi_\rho \left(\widetilde{V}_\rho^T + \widetilde{V}^T\right),\end{gathered} \tag{4.12}$$

where

$$\widetilde{\Lambda}_{ij} = \begin{bmatrix} \widetilde{\Phi}_{11} & \widetilde{\Phi}_{12} & D_{ai} & D_{bi} & HE_{a_i}^T & -\widetilde{S}_j E_{bi}^T - \widetilde{\widetilde{V}}_\phi E_{bi}^T \\ & \widetilde{\Phi}_{22} & \mu D_{ai} & \mu D_{bi} & 0 & 0 \\ & * & -\lambda I & 0 & 0 & 0 \\ & * & * & -\lambda I & 0 & 0 \\ & * & * & * & -\lambda^{-1} I & 0 \\ & * & * & * & * & -\lambda^{-1} I \end{bmatrix}, \tag{4.13}$$

with

$$\begin{gathered}\widetilde{\Phi}_{11} = \widetilde{T}_\phi - A_i \widetilde{H}^T - \widetilde{H} A_i^T + B_i \widetilde{S}_j^T + \widetilde{S}_j B_i^T + B_i \widetilde{\widetilde{V}}_\phi^T + \widetilde{\widetilde{V}}_\phi B_i^T,\\ \widetilde{\Phi}_{12} = \widetilde{T}_i - \mu\left(A_i \widetilde{H}^T - B_i \widetilde{S}_j^T - B_i \widetilde{\widetilde{V}}_\phi^T\right) + \widetilde{H},\\ \widetilde{\Phi}_{22} = \mu\left(H^T + H\right).\end{gathered} \tag{4.14}$$

Proof of Theorem 4.2. Consider (4.8) and pre- and postmultiplying by M^{-1} and M^{-T}, respectively, then we obtain

$$\begin{aligned}\widetilde{\Lambda}_{ij} &= \Lambda_{ij} + \lambda^{-1} \begin{bmatrix} D_{ai} & D_{bi} \\ \mu D_{ai} & \mu D_{bi} \end{bmatrix} \begin{bmatrix} D_{a_i}^T & \mu D_{a_i}^T \\ D_{b_i}^T & \mu D_{b_i}^T \end{bmatrix}\\ &\quad + \lambda M^{-1} \begin{bmatrix} E_{a_i}^T & -\left(E_{bi}\left(F_j + \overline{K}_\phi\right)\right)^T \\ 0 & 0 \end{bmatrix} \begin{bmatrix} E_{ai} & 0 \\ -E_{bi}\left(F_j + \overline{K}_\phi\right) & 0 \end{bmatrix} M^{-T} \prec 0,\end{aligned} \tag{4.15}$$

with Λ_{ij} defined by (3.5).

By Schur complement, we obtain,

$$\begin{bmatrix} \widetilde{\Phi}_{11} & \widetilde{\Phi}_{12} & D_{ai} & D_{bi} & M^{-1}E_{a_i}^T & -M^{-1}\left(E_{bi}\left(F_j+\overline{K}_\phi\right)\right)^T \\ & \widetilde{\Phi}_{22} & \mu D_{ai} & \mu D_{bi} & 0 & 0 \\ & * & -\lambda I & 0 & 0 & 0 \\ & * & * & -\lambda I & 0 & 0 \\ & * & * & * & -\lambda^{-1}I & 0 \\ & * & * & * & * & -\lambda^{-1}I \end{bmatrix} \prec 0, \tag{4.16}$$

with

$$\begin{gathered} \widetilde{\Phi}_{11} = M^{-1}\overline{P}_\phi M^{-T} - \left(A_i - B_iF_j - B_i\overline{K}_\phi\right)M^{-T} - M^{-1}\left(A_i - B_iF_j - B_i\overline{K}_\phi\right)^T, \\ \widetilde{\Phi}_{12} = M^{-1}(P_i+\varepsilon X)M^{-T} - \mu\left(A_i - B_iF_j - B_i\overline{K}_\phi\right)M^{-T} + M^{-1}, \\ \widetilde{\Phi}_{22} = \mu\left(M^{-T}+M^{-1}\right) \end{gathered} \tag{4.17}$$

for the following variables definition:

$$\begin{gathered} \widetilde{H} = M^{-1}, \qquad \widetilde{T}_i = \widetilde{H}(P_i+\varepsilon X)\widetilde{H}^T, \qquad \widetilde{T}_\phi = \widetilde{H}\widetilde{P}_\phi\widetilde{H}^T, \\ \widetilde{S}_j = \widetilde{H}F_j^T, \qquad \widetilde{V}_j = \widetilde{H}K_\rho^T, \qquad \widetilde{V} = \widetilde{H}R^T, \qquad \widetilde{Y} = \widetilde{H}X\widetilde{H}^T. \end{gathered} \tag{4.18}$$

If LMI in (4.11) holds then the closed-loop continuous fuzzy system (2.8) is asymptotically stable.

The control gains are given by $F_i = \widetilde{S}_i^T\widetilde{H}^{-T}$, $K_\rho = \widetilde{V}_\rho^T\widetilde{H}^{-T}$, and $R = \widetilde{V}^T\widetilde{H}^{-T}$. This completes the proof. □

5. Numerical Examples

In order to show the improvements of the proposed approaches over some existing results, in this section, we present a numerical example in which we present the feasible area for a *T-S* fuzzy system. Indeed, we compare the proposed fuzzy Lyapunov approaches (Theorem 3.4) with result provided by [17], and in [14, Theorem 6]. A second example is given to improve the given gains of robust PDC controller.

Example 5.1. Consider the following continuous *T-S* fuzzy system:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))A_ix(t), \tag{5.1}$$

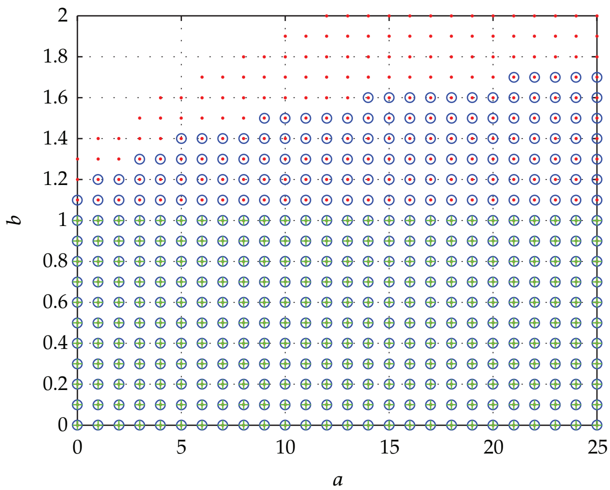

Figure 1: Feasible area provided by [17] (+), [14] (◯) and Theorem 3.1 (•).

with

$$r = 2; \qquad A_1 = \begin{bmatrix} 3.6 & -1.6 \\ 6.2 & -4.3 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -a & -1.6 \\ 6.2 & -4.3 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} -0.45 \\ -3 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} -b \\ -3 \end{bmatrix}, \tag{5.2}$$

where $a \in [0, 25]$, $b \in [0, 2]$, considering $\mu = 0.04$ and $\phi_{1,2} = 1$.

The proposed approach (Theorem 3.4) gives less conservative stabilization conditions (Figure 1) than some recent results provided by [14, 17].

Example 5.2. Consider the uncertain continuous *T-S* fuzzy system given by (2.8) with

$$r = 2; \qquad A_1 = \begin{bmatrix} 3.6 & -1.6 \\ 6.2 & -4.3 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -a & -1.6 \\ 6.2 & -4.3 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} -0.45 \\ -3 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} -b \\ -3 \end{bmatrix},$$
$$D_{a1} = E_{a1} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0 \end{bmatrix}, \qquad D_{b1} = E_{b1} = \begin{bmatrix} 0.3 \\ 0 \end{bmatrix}, \tag{5.3}$$
$$D_{a2} = E_{a2} = \begin{bmatrix} -0.2 & 0 \\ 0 & 0.3 \end{bmatrix}, \qquad D_{b2} = E_{b2} = \begin{bmatrix} -0.2 \\ 0.3 \end{bmatrix}$$

for $a = 1$, $b = 0.5$, considering $\mu = 0.04$, $\lambda = 0.1$ and $\phi_{1,2} = 1$, we find the following gains values:

$$\begin{gathered} F_1 = [-5.03401.5230], \qquad F_2 = [3.7868 - 0.8082], \\ K_1 = [-180.233465.7330], \qquad K_2 = [-180.949465.9073], \qquad R = [201.9216 - 71.7345]. \end{gathered} \tag{5.4}$$

6. Conclusion

This paper provided new conditions for the stabilization with a class of PDC controller of Takagi-Sugeno fuzzy systems in terms of a combination of the LMI approach and the use of nonquadratic Lyapunov function as fuzzy Lyapunov function. In addition, the time derivative of membership function is considered by the PDC fuzzy controller and the slack matrix variables are introduced in order to facilitate the stability analysis. An approach to design an observer is derived in order to estimate variable states. In addition, a new condition of the stabilization of uncertain system is given in this paper.

The stabilization condition proposed in this paper is less conservative than some of those in the literature, which has been illustrated via examples.

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