Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2012, Article ID 780740, 17 pages
doi:10.1155/2012/780740

Research Article

Delay-Dependent Fuzzy Hyperbolic Model Based on Data-Driven Guaranteed Cost Control for a Class of Nonlinear Continuous-Time Systems with Uncertainties

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Received 30 August 2012; Accepted 23 November 2012

Academic Editor: Bin Jiang

This paper develops the fuzzy hyperbolic model with time-varying delays guaranteed cost controller design via state-feedback for a class of nonlinear continuous-time systems with parameter uncertainties. A nonlinear quadratic cost function is developed as a performance measurement of the closed-loop fuzzy system based on fuzzy hyperbolic model with time-varying delays. Some sufficient conditions for the existence of such a fuzzy hyperbolic model based on data-driven guaranteed cost controller via state feedback are presented by a set of linear matrix inequalities (LMIs). A simulation example is provided to illustrate the effectiveness of the proposed approach.

1. Introduction

Since time delays are frequently encountered in various areas such as engineering systems, biology, and economics, and the existence of time delays is often the main cause of instability and poor performance of a control system, considerable attention has been paid to the problem of stability analysis and controller synthesis for time-delay systems [1, 2].

Recently, the problem of designing guaranteed cost controllers for uncertain time-delay systems has attracted a number of researchers' attention [3–6]. Guaranteed cost control (GCC) for time-delay systems can also be categorized into delay-independent methods and delay-dependent methods. The recent research trend has been focus on delay-dependent methods. In [3], delay-dependent GCC was first proposed by utilizing model transformation. It was first illustrated that delay-dependent GCC can provide even less guaranteed cost than the delay-independent GCC methods. Reference [4] considered both state delays and input delays, and formulated the optimal guaranteed cost control problem which minimizes the upper bound of the closed-loop cost function. Reference [5] extend the delay-dependent method into the stabilization for time-delay T-S fuzzy systems, delay-dependent GCC problem for nonlinear systems with time-delays represented by the Takagi-Sugeno fuzzy mode was studied.

A novel continuous-time fuzzy model, called fuzzy hyperbolic model with time-varying delays (DFHM), has been proposed in [7]. Fuzzy Hyperbolic Model is essentially a data-driven model. The DFHM based on data-driven has its own distinguishing characteristics. Firstly, neither structure identification nor completeness design of premise variables space is required when the DFHM is used to approximate the nonlinear systems, therefore the computational effort of modeling the DFHM is lower than modeling the T-S models. Secondly, less computational effort is required when DFHM is used since only one LMI needs to be solved. Thirdly, the DFHM based on data-driven we designed is naturally fuzzy nonlinear saturated controller, which is suitable for applying to practical systems. Last but not least, DFHM is a new kind of fuzzy neural networks, whose nodes have clear physical meanings. Therefore, the advantages of the DFHM based on data-driven are more obvious.

In this paper, delay-dependent fuzzy guaranteed cost controller via state feedback design based on DFHM, called delay-dependent fuzzy hyperbolic model based on data-driven guaranteed cost controller (DD-DFHMGCC), is addressed. To the best of our knowledge, this is the first time to study the guaranteed cost controller problem of DFHM. By using the LMI technique, the DD-DFHMGCC design problem is converted into a feasible problem of LMI, which makes the prescribed attenuation level as small as possible, subject to some LMI constraints. A simulation example is finally presented to illustrate the effectiveness of the proposed design procedures.

2. System Description and Preliminaries

The DFHM modeling method for nonlinear systems was given in [7]. The following definition is addressed.

Definition 2.1. Given a plant with n state variables $x(t) = [x_1(t), \dots, x_n(t)]^T$, we call the fuzzy rule base a *hyperbolic type fuzzy rule base* (HFRB) if it satisfies the following conditions.

(1) For each output variable $\dot{x}_r(t)$, $r = 1, 2, \dots, n$, the kth fuzzy rule has the following form: R^k.

If $x_1(t)$ is $A_0^{1,s}$ and $x_1(t - \tau_{11}(t))$ is $A_1^{1,s}$ and $\dots x_1(t - \tau_{1d_1}(t))$ is $A_{d_1}^{1,s}$

$\dots$

and $x_n(t)$ is $A_0^{n,s}$ and $x_n(t - \tau_{n1}(t))$ is $A_1^{n,s}$ and $\dots x_n(t - \tau_{nd_n}(t))$ is $A_{d_n}^{n,s}$,

then

$$\dot{x}_r(t) = c^r_{A_0^{1,s}} + \cdots + c^r_{A_{d_1}^{1,s}} + c^r_{A_0^{2,s}} + \cdots + c^r_{A_{d_2}^{2,s}} + \cdots + c^r_{A_0^{n,s}} + \cdots + c^r_{A_{d_n}^{n,s}} + u^r, \quad s \in \{+, -\}, \tag{2.1}$$

where $A_{i_j}^{j,s}$ is the fuzzy set of $x_j(t - \tau_{ji_j}(t))$, which include $P_{i_j}^j$ (positive) and $N_{i_j}^j$ (negative), respectively, $c^r_{A_{i_j}^{j,s}}$ is a constant term, d_j represents the number of transmission delays associated with x_j, $\tau_{ji_j}(t) > 0$ is the time-varying transmission delay with $\tau_{j0}(t) = 0$, $i_j = 0, 1, \ldots, d_j$, $j = 1, 2, \ldots, n$.

(2) The state variables in the If-part are optional, the same as the constant terms in the Then-part. That is the constant term $c^r_{A^{j,s}_{i_j}}$ in the Then-part is corresponding to $A^{j,s}_{i_j}$ in the If-part.

(3) There are $2^{n+\sum_{j=1}^{n} d_j}$ fuzzy rules in every $\dot{x}_r(t)$, that is, all the possible $P^j_{i_j}$ and $N^j_{i_j}$ combinations of input variables in the "If" part and all the linear combinations of constants in the "Then" part (not including u^r). So there are total $n2^{n+\sum_{j=1}^{n} d_j}$ fuzzy rules in the rule base.

(4) For every $\dot{x}_r$ $(r = 1, 2, \ldots, n)$, we construct the same premise fuzzy subsets, but the conclusion parameters are different.

Lemma 2.2. *Given n HFRBs, if the membership functions of $P^j_{i_j}$ and $N^j_{i_j}$ are defined as*

$$\mu_{P^j_{i_j}}\left(x_j\left(t - \tau_{ji_j}(t)\right)\right) = e^{-(1/2)(x_j(t-\tau_{ji_j}(t))-k_{ji_j})^2}, \qquad \mu_{N^j_{i_j}}\left(x_j\left(t - \tau_{ji_j}(t)\right)\right) = e^{-(1/2)(x_j(t-\tau_{ji_j}(t))+k_{ji_j})^2}, \tag{2.2}$$

where $i_j = 0, 1, \ldots, d_j$, $j = 1, 2, \ldots, n$. $k_{ji_j} > 0$ is a positive constant. Then the system can be derived as

$$\dot{x}(t) = A\tanh(Kx(t)) + \sum_{i=1}^{J} A_i \tanh(K_i x(t - \tau_i(t))) + I, \tag{2.3}$$

where I is a constant vector, A and A_i are constant matrix, (3) is called a fuzzy hyperbolic model with time-varying delays (DFHM).

From Definition 2.1, if we set $c^r_{A^{j,+}_{i_j}}$ and $c^r_{A^{j,-}_{i_j}}$ negative to each other, we can obtain a homogeneous DFHM:

$$\dot{x}(t) = A\tanh(Kx(t)) + \sum_{i=1}^{J} A_i \tanh(K_i x(t - \tau_i(t))). \tag{2.4}$$

Since the difference between (2.3) and (2.4) is only the constant vector term in (2.3), there is essentially no difference between the control of (2.3) and (2.4). In this paper, we will design a fuzzy $H\infty$ guaranteed cost controller based on DFHM described in (2.4).

3. Fuzzy Hyperbolic with Time-Varying Delays Guaranteed Cost Control Design via State-Feedback

The DFHM for the nonlinear time-delay systems with parameter uncertainty is proposed as the following form:

$$\begin{gathered}\dot{x}(t) = (A + \Delta A)\tanh(Kx(t)) + \sum_{i=1}^{J}(A_i + \Delta A_i)\tanh(K_i x(t - \tau_i(t))) + (B + \Delta B)u(t), \\ x(t) = \phi(t), \quad \forall t \in [-\overline{\tau}, 0],\end{gathered} \tag{3.1}$$

where $x(t) \in R^n$ and $u(t) \in R^m$ denote the state vector and input vector, respectively; $A \in R^{n\times n}$, $A_i \in R^{n\times n}$ and $B \in R^{n\times m}$, are known real constant matrices; $\tau_i(t) = [\tau_{1i}(t), \tau_{2i}(t), \ldots, \tau_{ni}(t)]^T$ is the bounded time-varying delay and is assumed to satisfy $0 < \tau_{ji}(t) \leq \overline{\tau} < \infty$ and $\dot{\tau}_{ji}(t) \leq h$, where $\overline{\tau}$ and h are known constant scalars, $j = 1, 2, \ldots, n$, $i = 1, 2, \ldots, J$. The initial condition $\phi(t)$ is given by initial vector function, which is continuous for $-\overline{\tau} \leq t \leq 0$; $\Delta A(t) \in R^{n\times n}$, $\Delta A_i(t) \in R^{n\times n}$ and $\Delta B(t) \in R^{n\times m}$ are time-varying parameter uncertainty matrices and satisfy the condition

$$\begin{bmatrix}\Delta A(t) & \Delta A_i(t) & \Delta B(t)\end{bmatrix} = MF(t)\begin{bmatrix}N & N_i & N_b\end{bmatrix}, \tag{3.2}$$

where M, N, N_i $(i = 1, 2, \ldots, J)$ and N_b are known real constant matrices of appropriate dimension, and $F(t)$ is an unknown matrix function satisfying $F^T(t)F(t) \leq I$ (I is an identity matrix with appropriate dimension). Such parametric uncertainties are said to be admissible.

Definition 3.1. Consider system (3.1) with the following cost function:

$$J = \int_0^\infty \left[\tanh^T(Kx(t))S\tanh(Kx(t)) - u^T(t)Ru(t)\right]dt, \tag{3.3}$$

$$u(t) = G\tanh(Kx(t)), \tag{3.4}$$

where S and R are symmetric, positive-definite matrices; G is the feedback gain. The controller is called fuzzy hyperbolic model with time-varying delays guaranteed cost controller (DFHMGCC) if there exist a fuzzy hyperbolic control $u(t)$ as in (3.4) and a scalar J_0 such that the closed-loop system is asymptotically stable and the closed-loop value of the cost function (3.3) satisfies $J \leq J_0$. J_0 is said to be a guaranteed cost and control law $u(t)$ is said to be a fuzzy hyperbolic with time-varying delays guaranteed cost control law for system (3.1).

With the control law (3.4) the overall closed-loop system can be written as:

$$\begin{gathered}\dot{x}(t) = (A + \Delta A + (B + \Delta B)G)\tanh(Kx(t)) + \sum_{i=1}^{J}(A_i + \Delta A_i)\tanh(K_i x(t - \tau_i(t))), \\ x(t) = \phi(t), \qquad \forall t \in [-\overline{\tau}, 0].\end{gathered} \tag{3.5}$$

For convenience, let $\Delta A := \Delta A(t)$, $\Delta A_i := \Delta A_i(t)(i = 1, \ldots, J)$, $\Delta B := \Delta B(t)$.

Lemma 3.2 (see [8]). *Given appropriate dimension matrices M, E, and F satisfying $F^T F \le I$, for any real scalar $\varepsilon > 0$, the following result holds:*

$$MFE + E^T F^T M^T \le \varepsilon M M^T + \varepsilon^{-1} E^T E. \tag{3.6}$$

Lemma 3.3 (see [9]). *For any constant positive definite matrix $W \in R^{m\times m}$, a scalar $\beta > 0$, a function $\eta : [0,\beta] \to R^+$, and the vector function $\nu : [\beta-\eta(\beta),\beta] \to R^m$ such that the integrations in the following are well defined, then*

$$\eta(\beta)\int_{\beta-\eta(\beta)}^{\beta} \nu^T(s) W \nu(s) ds \ge \left(\int_{\beta-\eta(\beta)}^{\beta} \nu(s) ds\right)^T W \left(\int_{\beta-\eta(\beta)}^{\beta} \nu(s) ds\right). \tag{3.7}$$

Lemma 3.4 (see [10]). *For any vector $\varsigma \in R^n$, $\varsigma^T = [\varsigma_1, \varsigma_2, \dots, \varsigma_n]$, and diagonal positive definite matrix X, the following result holds:*

$$\dot{\tanh}^T(\varsigma) X \dot{\tanh}(\varsigma) \le \dot{\varsigma} X \dot{\varsigma}. \tag{3.8}$$

Then, we have the following results.

Theorem 3.5. *Given scalars $\overline{\tau}$ and h, for the nonlinear system* (3.1) *and associated cost function* (3.3)*, if there exist positive scalars $\varepsilon_{01}, \varepsilon_{02}, \varepsilon_{03}, \varepsilon_{04}, \varepsilon_i$ $(i = 1, 2, \dots, J)$, a positive definite diagonal matrix $\overline{P} = \mathrm{diag}[\overline{p}_1, \dots, \overline{p}_n]$, matrices $\overline{Q}_\lambda > 0$ $(\lambda = 1, 2)$ and L such that the matrix inequality*

$$\begin{bmatrix}
\Xi_{11} & 0 & \Xi_{13} & \Xi_{14} & \overline{P}N^T + L^T N_b^T & 0 & L^T N_b^T & \overline{P}N^T & 0 \\
* & \Xi_{22} & \Xi_{23} & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & \Xi_{33} & \Xi_{34} & 0 & \Xi_{36} & 0 & 0 & \Xi_{39} \\
* & * & * & \Xi_{44} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & -\varepsilon_{04} I & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -\varepsilon_{03} I & 0 & 0 & 0 \\
* & * & * & * & * & * & -\varepsilon_{02} I & 0 & 0 \\
* & * & * & * & * & * & * & -\varepsilon_{01} I & 0 \\
* & * & * & * & * & * & * & * & -\Xi_{99}
\end{bmatrix} < 0 \tag{3.9}$$

holds, then, the control law, $u(t) = G\tanh(Kx(t))$ is a fuzzy hyperbolic with time-varying delays guaranteed cost control law and

$$\begin{aligned}
J_0 &= 2\sum_{i=1}^{n} \overline{p}_i^{-1} \ln\left(\cosh\left(k_{i0}\phi_i(0)\right)\right) \\
&\quad + \sum_{i=1}^{J} \int_{-\tau_i(0)}^{0} \tanh^T\left(K_i\phi(s)\right) \overline{P}^{-1} \overline{Q}_1 \overline{P} \tanh\left(K_i\phi(s)\right) ds \\
&\quad + \sum_{i=1}^{J} \int_{-\overline{\tau}}^{0} \int_{\beta}^{0} \dot{\tanh}^T \left(K_i x(\alpha)\right) \overline{P}\,\overline{Q}_2 \overline{P} \dot{\tanh}\left(K_i x(\alpha)\right) d\alpha\, d\beta,
\end{aligned} \tag{3.10}$$

where

$$
\begin{aligned}
\Xi_{11} &= KA\overline{P} + \overline{P}A^T K + KBL + L^T B^T K + \varepsilon_{01} KMM^T K + \varepsilon_{02} KMM^T K \\
&\quad + \overline{S} + L^T RL + \sum_{i=1}^{J} \varepsilon_i KMM^T K, \\
\Xi_{13} &= \left[KA_1\overline{P} \ \ KA_2\overline{P} \ \ \cdots \ \ KA_J\overline{P} \right], \\
\Xi_{14} &= \left[\left(K_1 A\overline{P} + K_1 BL \right)^T \ \ \cdots \ \ \left(K_J A\overline{P} + K_J BL \right)^T \right], \\
\Xi_{22} &= \operatorname{diag}\left[\underbrace{\overline{Q}_1 - \overline{\tau}^{-1}\overline{Q}_2 \ \ \overline{Q}_1 - \overline{\tau}^{-1}\overline{Q}_2 \ \ \cdots \ \ \overline{Q}_1 - \overline{\tau}^{-1}\overline{Q}_2}_{J} \right], \\
\Xi_{23} &= \operatorname{diag}\left[\underbrace{\overline{\tau}^{-1}\overline{Q}_2 \ \ \overline{\tau}^{-1}\overline{Q}_2 \ \ \cdots \ \ \overline{\tau}^{-1}\overline{Q}_2}_{J} \right], \\
\Xi_{33} &= \operatorname{diag}\left[\underbrace{-(1-h)\ \overline{Q}_1 - \overline{\tau}^{-1}\overline{Q}_2 \ \ \cdots \ \ -(1-h)\overline{Q}_1 - \overline{\tau}^{-1}\overline{Q}_2}_{J} \right], \\
\Xi_{34} &= \operatorname{diag}\left[\overline{P}A_1^T K_1 \ \ \overline{P}A_2^T K_2 \ \ \cdots \ \ \overline{P}A_J^T K_J \right], \\
\Xi_{36} &= \operatorname{diag}\left[\overline{P}N_1^T \ \ \overline{P}N_2^T \ \ \cdots \ \ \overline{P}N_J^T \right], \\
\Xi_{39} &= \operatorname{diag}\left[\overline{P}N_1^T \ \ \overline{P}N_2^T \ \ \cdots \ \ \overline{P}N_J^T \right], \\
\Xi_{44} &= \operatorname{diag}\left[-\overline{\tau}^{-1}\overline{Q}_2 + \varepsilon_{03} K_1 MM^T K_1 \ \ \cdots \ \ -\overline{\tau}^{-1}\overline{Q}_2 + \varepsilon_{03} K_J MM^T K_J \right] \\
&\quad + \varepsilon_{04} \left[M^T K_1 \ \ \cdots \ \ M^T K_J \right]^T \left[M^T K_1 \ \ \cdots \ \ M^T K_J \right], \\
\Xi_{99} &= \operatorname{diag}\left[\varepsilon_1 I \ \ \varepsilon_2 I \ \ \cdots \ \ \varepsilon_J I \right],
\end{aligned}
\tag{3.11}
$$

for any time-varying delay $\tau_i(t) = [\tau_{1i}(t), \tau_{2i}(t), \ldots, \tau_{ni}(t)]^T$ *satisfying* $0 < \tau_i(t) \leq \overline{\tau} < \infty$ *and* $\dot{\tau}_i(t) \leq h$, $i = 1, 2, \ldots, J$, *and* $*$ *denotes the entries induced by symmetry.*

Proof. A Lyapunov-Krasovskii functional candidate for the time-varying delay system (3.5) is chosen as follows:

$$
V(t) = V_1(t) + V_2(t) + V_3(t), \tag{3.12}
$$

where

$$
\begin{aligned}
V_1(t) &= 2\sum_{i=1}^{n} p_i \ln(\cosh(k_{i0}x_i(t))),\\
V_2(t) &= \sum_{i=1}^{J}\int_{t-\tau_i(t)}^{t} \tanh^T(K_i x(s))Q_1 \tanh(K_i x(s))ds,\\
V_3(t) &= \sum_{i=1}^{J}\int_{-\overline{\tau}}^{0}\int_{t+\beta}^{t} \dot{\tanh}^T(K_i x(\alpha))Q_2 \dot{\tanh}(K_i x(\alpha))d\alpha\, d\beta,
\end{aligned}
\tag{3.13}
$$

and scalars $p_i > 0$ $(i = 1, 2, \ldots, n)$, matrices $Q_\lambda > 0$ $(\lambda = 1, 2)$, $P = \mathrm{diag}[p_1, p_2, \ldots, p_n]$ is diagonal positive definite matrix, and k_{i0} $(i = 1, 2, \ldots, n)$ are defined in (2.2).

Taking the derivative of $V(t)$ with respect to t along the trajectory of (3.5) yields

$$
\dot{V}(t)\big|_{(3.5)} = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t),
\tag{3.14}
$$

where

$$
\dot{V}_1(t) = 2\tanh^T(Kx(t))KP\dot{x}(t),
\tag{3.15}
$$

$$
\begin{aligned}
\dot{V}_2(t) \le \sum_{i=1}^{J}\Big(&\tanh^T(K_i x(t))Q_1 \tanh(K_i x(t))\\
&-(1-h)\tanh^T(K_i x(t-\tau_i(t)))Q_1 \tanh(K_i x(t-\tau_i(t)))\Big),
\end{aligned}
\tag{3.16}
$$

$$
\begin{aligned}
\dot{V}_3(t) = \sum_{i=1}^{J}\Bigg(&\overline{\tau}\dot{\tanh}^T(K_i x(t))Q_2 \dot{\tanh}(K_i x(t))\\
&-\int_{t-\overline{\tau}}^{t} \dot{\tanh}^T(K_i x(s))Q_2 \dot{\tanh}(K_i x(s))ds\Bigg).
\end{aligned}
\tag{3.17}
$$

According to $0 < \tau_i(t) \le \overline{\tau} < \infty$ and Lemma 3.3, we have

$$
\begin{aligned}
&-\int_{t-\overline{\tau}}^{t} \dot{\tanh}^T(K_i x(s))Q_2 \dot{\tanh}(K_i x(s))ds\\
&\quad\le -\overline{\tau}^{-1}(\tanh(K_i x(t)) - \tanh(K_i x(t-\tau_i(t))))^T Q_2(\tanh(K_i x(t)) - \tanh(K_i x(t-\tau_i(t)))).
\end{aligned}
\tag{3.18}
$$

From (3.17), (3.18), and Lemma 3.4, we have

$$
\begin{aligned}
\dot{V}_3(t) \le \sum_{i=1}^{J}\Big(&\overline{\tau}\dot{x}^T(t)K_i Q_2 K_i \dot{x}(t) - \overline{\tau}^{-1}(\tanh(K_i x(t)) - \tanh(K_i x(t-\tau_i(t))))^T\\
&\times Q_2(\tanh(K_i x(t)) - \tanh(K_i x(t-\tau_i(t))))\Big).
\end{aligned}
\tag{3.19}
$$

From Lemma 3.2 and (3.2), we have

$$
\begin{aligned}
&2\tanh^T(Kx(t))KP(\Delta A+\Delta BG)\tanh(Kx(t)) \\
&\quad\le \tanh^T(Kx(t))\left(\varepsilon_{01}KPMM^TPK+\varepsilon_{01}^{-1}N^TN\right)\tanh(Kx(t)) \\
&\qquad+\tanh^T(Kx(t))\left(\varepsilon_{02}KPMM^TPK+\varepsilon_{02}^{-1}\left((N_bG)^TN_bG\right)\tanh(Kx(t))\right), \\
&2\tanh^T(Kx(t))KP\sum_{i=1}^{J}\Delta A_i\tanh(K_ix(t-\tau_i(t))) \\
&\quad\le \sum_{i=1}^{J}\Big(\varepsilon_i\tanh^T(Kx(t))KPMM^TPK\tanh(Kx(t)) \\
&\qquad\qquad+\varepsilon_i^{-1}\tanh^T(K_ix(t-\tau_i(t)))N_i^TN_i\tanh(K_ix(t-\tau_i(t)))\Big).
\end{aligned}
\tag{3.20}
$$

So, we have

$$
\dot{V}(t)\big|_{(3.5)}\le\zeta^T(t)\prod\zeta(t)-\tanh^T(Kx(t))S\tanh(Kx(t))-u^T(t)Ru(t), \tag{3.21}
$$

where

$$
\begin{aligned}
&\zeta^T(t)=\left[\tanh^TK(x(t))\quad\varphi^T(t)\quad\eta^T(t)\right], \\
&\varphi^T(t)=\left[\tanh^T(K_1x(t))\quad\tanh^T(K_2x(t))\quad\cdots\quad\tanh^T\left(K_Jx(t)\right)\right], \\
&\eta^T(t)=\left[\tanh^T(K_1x(t-\tau_1(t)))\quad\tanh^T(K_2x(t-\tau_2(t)))\quad\cdots\quad\tanh^T\left(K_Jx\left(t-\tau_J(t)\right)\right)\right], \\
&\prod=\begin{bmatrix}\prod_{11}+\varepsilon_{01}^{-1}N^TN+\varepsilon_{02}^{-1}(N_bG)^TN_bG & 0 & \prod_{13} \\ * & \prod_{22} & \prod_{23} \\ * & * & \prod_{33}+\Psi_3\end{bmatrix}+\begin{bmatrix}\overline{\tau}\Psi_1^TQ_2\Psi_1 & 0 & \overline{\tau}\Psi_1^TQ_2\Psi_2 \\ * & 0 & \overline{\tau}\Psi_2^TQ_2\Psi_2 \\ * & * & 0\end{bmatrix}, \\
&\prod_{11}=KP(A+BG)+(A+BG)^TPK+(\varepsilon_{01}+\varepsilon_{02})KPMM^TPK+\sum_{i=1}^{J}\varepsilon_iKPMM^TPK \\
&\qquad+S+G^TRG, \\
&\prod_{13}=\left[KPA_1\quad KPA_2\quad\cdots\quad KPA_J\right], \\
&\prod_{22}=\operatorname{diag}\Big[\underbrace{Q_1-\overline{\tau}^{-1}Q_2\quad Q_1-\overline{\tau}^{-1}Q_2\quad\cdots\quad Q_1-\overline{\tau}^{-1}Q_2}_{J}\Big],
\end{aligned}
$$

$$\prod_{23} = \operatorname{diag}\left[\underbrace{\overline{\tau}^{-1}Q_2 \ \ \overline{\tau}^{-1}Q_2 \ \ \cdots \ \ \overline{\tau}^{-1}Q_2}_{J}\right],$$

$$\prod_{33} = \operatorname{diag}\left[\underbrace{-(1-h)Q_1 - \overline{\tau}^{-1}Q_2 \ \ -(1-h)Q_1 - \overline{\tau}^{-1}Q_2 \ \ \cdots \ \ -(1-h)Q_1 - \overline{\tau}^{-1}Q_2}_{J}\right],$$

$$\Psi_1 = \left[A + \Delta A + (B + \Delta BG)^T K_1 \ \ \cdots \ \ A + \Delta A + (B + \Delta BG)^T K_J\right],$$

$$\Psi_2 = \left[(A_1 + \Delta A_1)^T K_1 \ \ \cdots \ \ (A_J + \Delta A_J)^T K_J\right],$$

$$\Psi_3 = \operatorname{diag}\left[\varepsilon_1^{-1} N_1^T N_1 \ \ \varepsilon_2^{-1} N_2^T N_2 \ \ \cdots \ \ \varepsilon_J^{-1} N_J^T N_J\right]. \tag{3.22}$$

Let $\theta < 0$. Then $\dot{V}(t)|_{(3.5)} \leq -\tanh^T(Kx(t))S\tanh(Kx(t)) - u^T(t)Ru(t) \leq -\lambda_m(S)\|\tanh(Kx(t))\|^2 < 0$.

Thus, the closed-loop system is asymptotically stable. Furthermore, integrating $\dot{V}(t)|_{(3.5)} \leq -\tanh^T(Kx(t))\,S\tanh(Kx(t)) - u^T(t)Ru(t)$ from 0 to t_f yields

$$\int_0^{t_f} \left[\tanh^T(Kx(t))S\tanh(Kx(t)) - u^T(t)Ru(t)\right]dt \leq V(0) - V(t_f). \tag{3.23}$$

Since $V(t) \geq 0$ and $\dot{V}(t) < 0$, $\lim_{t_f \to \infty} V(t_f) = \mu$ which is a nonnegative constant. When $t_f \to \infty$, (3.23) becomes

$$\begin{aligned}\int_0^{t_f} &\left[\tanh^T(Kx(t))S\tanh(Kx(t)) - u^T(t)Ru(t)\right]dt \leq V(0) \\ &= 2\sum_{i=1}^{n} p_i \ln\left(\cosh\left(k_{i0}\phi_i(0)\right)\right) + \sum_{i=1}^{J}\int_{-\tau_i(0)}^{0} \tanh^T\left(K_i\phi(s)\right)Q_1\tanh\left(K_i\phi(s)\right)ds \\ &\quad + \sum_{i=1}^{J}\int_{-\overline{\tau}}^{0}\int_{\beta}^{0} \dot{\tanh}^T(K_ix(\alpha))Q_2\dot{\tanh}(K_ix(\alpha))\,d\alpha\, d\beta.\end{aligned} \tag{3.24}$$

By Schur complement, $\prod$ can be rewritten as the following form:

$$\begin{bmatrix} \prod_{11} + \varepsilon_{01}^{-1}N^TN + \varepsilon_{02}^{-1}(N_bG)^TN_bG & 0 & \prod_{13} & \prod_{14} \\ * & \prod_{22} & \prod_{23} & 0 \\ * & * & \prod_{33} + \Psi_3 & \prod_{34} \\ * & * & * & \prod_{44} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & \overline{\prod}_{14} \\ * & 0 & 0 & 0 \\ * & * & 0 & \overline{\prod}_{34} \\ * & * & * & 0 \end{bmatrix} < 0, \tag{3.25}$$

where

$$\prod_{14} = \left[(A+BG)^T K_1 \quad (A+BG)^T K_2 \quad \cdots \quad (A+BG)^T K_J\right],$$
$$\overline{\prod_{14}} = \left[(\Delta A+\Delta BG)^T K_1 \quad (\Delta A+\Delta BG)^T K_2 \quad \cdots \quad (\Delta A+\Delta BG)^T K_J\right],$$
$$\prod_{34} = \operatorname{diag}\left[A_1^T K_1 \quad A_2^T K_2 \quad \cdots \quad A_J^T K_J\right],$$
$$\overline{\prod_{34}} = \operatorname{diag}\left[\Delta A_1^T K_1 \quad \Delta A_2^T K_2 \quad \cdots \quad \Delta A_J^T K_J\right],$$
$$\prod_{44} = \operatorname{diag}\left[\underbrace{-\overline{\tau}^{-1} Q_2 \quad -\overline{\tau}^{-1} Q_2 \quad \cdots \quad -\overline{\tau}^{-1} Q_2}_{J}\right]. \tag{3.26}$$

Comparing inequality (3.25) with Lemma 3.2, we can obtain

$$\begin{aligned}
&\begin{bmatrix} 0 & 0 & 0 & \overline{\prod_{14}} \\ * & 0 & 0 & 0 \\ * & * & 0 & \overline{\prod_{34}} \\ * & * & * & 0 \end{bmatrix} \\
&\quad \le \varepsilon_{04}\left[0 \;\; 0 \;\; 0 \;\; \widetilde{\prod_{14}}\right]^T \left[0 \;\; 0 \;\; 0 \;\; \widetilde{\prod_{14}}\right] + \varepsilon_{04}^{-1}\left[N+N_b G \;\; 0 \;\; 0 \;\; 0\right]^T \left[N+N_b G \;\; 0 \;\; 0 \;\; 0\right] \\
&\qquad + \varepsilon_{03}\left[0 \;\; 0 \;\; 0 \;\; \widetilde{\prod_{34}}\right]^T \left[0 \;\; 0 \;\; 0 \;\; \widetilde{\prod_{34}}\right] + \varepsilon_{03}^{-1}\left[0 \;\; 0 \;\; \widetilde{\prod_{34}}' \;\; 0\right]^T \left[0 \;\; 0 \;\; \widetilde{\prod_{34}}' \;\; 0\right],
\end{aligned} \tag{3.27}$$

where

$$\widetilde{\prod_{14}} = \left[M^T K_1 \quad M^T K_2 \quad \cdots \quad M^T K_J\right],$$
$$\widetilde{\prod_{14}} = \operatorname{diag}\left[M^T K_1 \quad M^T K_2 \quad \cdots \quad M^T K_J\right],$$
$$\widetilde{\prod_{34}}' = \operatorname{diag}\left[N_1 \quad N_2 \quad \cdots \quad N_J\right]. \tag{3.28}$$

Thus, the necessary and sufficient condition for inequality (3.25) to hold is that there exists a positive constant $\varepsilon_{03} > 0$ and $\varepsilon_{04} > 0$ such that

$$\begin{aligned}&\begin{bmatrix}\prod_{11} + \varepsilon_{01}^{-1} N^T N + \varepsilon_{02}^{-1} (N_b G)^T N_b G & 0 & \prod_{13} & \prod_{14} \\ * & \prod_{22} & \prod_{23} & 0 \\ * & * & \prod_{33} + \Psi_3 & \prod_{34} \\ * & * & * & \overline{\prod_{44}}\end{bmatrix} \\ &+ \varepsilon_{04}^{-1} \left[N + N_b G \;\; 0 \;\; 0 \;\; 0\right]^T \left[N + N_b G \;\; 0 \;\; 0 \;\; 0\right] \\ &+ \varepsilon_{03}^{-1} \left[0 \;\; 0 \;\; \widetilde{\prod_4}' \;\; 0\right]^T \left[0 \;\; 0 \;\; \widetilde{\prod_4}' \;\; 0\right] < 0,\end{aligned} \tag{3.29}$$

where

$$\overline{\overline{\prod_{44}}} = \overline{\prod_{44}} + \varepsilon_{04} \widetilde{\prod_{14}}^T \widetilde{\prod_{14}} + \varepsilon_{03} \widetilde{\prod_{34}}^T \widetilde{\prod_{34}}. \tag{3.30}$$

By Schur complement, (3.29) is equivalent to

$$\begin{bmatrix}\prod_{11} & 0 & \prod_{13} & \prod_{14} & (N + N_b G)^T & 0 & (N_b G)^T & N^T & 0 \\ * & \prod_{22} & \prod_{23} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \prod_{33} & \prod_{34} & 0 & \widetilde{\prod_{34}}'^T & 0 & 0 & \widetilde{\prod_{34}}'^T \\ * & * & * & \overline{\prod_{44}} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_{04} I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_{03} I & 0 & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_{02} I & 0 & 0 \\ * & * & * & * & * & * & * & -\varepsilon_{01} I & 0 \\ * & * & * & * & * & * & * & * & -\Theta\end{bmatrix} < 0, \tag{3.31}$$

where

$$\Theta = \operatorname{diag}\left[\varepsilon_1 I \;\; \varepsilon_2 I \;\; \cdots \;\; \varepsilon_J I\right]. \tag{3.32}$$

Pre- and post-multiplying $\operatorname{diag}[\underbrace{P^{-1} \; P^{-1} \; \cdots \; P^{-1}}_{2J+1}, \underbrace{I \; \cdots \; I}_{3J+3}]$ to (3.31), and letting $\overline{P} = P^{-1}, \overline{Q}_\lambda = P^{-1} Q_\lambda P^{-1}$ $(\lambda = 1, 2)$, $\overline{S} = P^{-1} S P^{-1}$ and $L = G P^{-1}$, (3.9) can be obtained by Schur complement. When LMI (3.9) is feasible, the guaranteed cost controller designed ensures the closed-loop system to be asymptotically stable and an upper bound of the closed-loop cost function given by

$$\begin{aligned}
J_0 &= 2\sum_{i=1}^{n} p_i \ln\left(\cosh\left(k_{i0}\phi_i(0)\right)\right) + \sum_{i=1}^{J}\int_{-\tau_i(0)}^{0} \tanh^T\left(K_i\phi(s)\right)Q_1 \tanh\left(K_i\phi(s)\right)ds \\
&\quad + \sum_{i=1}^{J}\int_{-\overline{\tau}}^{0}\int_{\beta}^{0} \dot{\tanh}^T(K_i x(\alpha))Q_2\dot{\tanh}(K_i x(\alpha))d\alpha\, d\beta \\
&= 2\sum_{i=1}^{n}\overline{p}_i^{-1} \ln\left(\cosh\left(k_{i0}\phi_i(0)\right)\right) + \sum_{i=1}^{J}\int_{-\tau_i(0)}^{0} \tanh^T\left(K_i\phi(s)\right)\overline{P}^{-1}\overline{Q}_1\overline{P}^{-1} \tanh\left(K_i\phi(s)\right)ds \\
&\quad + \sum_{i=1}^{J}\int_{-\overline{\tau}}^{0}\int_{\beta}^{0} \dot{\tanh}^T(K_i x(\alpha))\overline{P}^{-1}\overline{Q}_2\overline{P}^{-1}\dot{\tanh}(K_i x(\alpha))d\alpha\, d\beta.
\end{aligned} \tag{3.33}$$

The proof is completed. □

In fact, any feasible solution to (3.9) yields a suitable robust guaranteed cost controller. A better robust guaranteed cost control law minimizes the upper bound J_0. Then, we can obtain Theorem 3.6.

Theorem 3.6. *Consider the nonlinear system* (3.1) *and its associated cost function* (3.3). *If the optimization problem*

$$\begin{aligned}
&\min_{\varepsilon_i,\overline{P},\overline{Q}_1,\overline{Q}_2,V,G} \quad \delta + \alpha\cdot \mathrm{Tr}(V) \\
&\textit{Subject to} \quad (1)\ \textit{LMIS}\ (3.5), \\
&\qquad (2) \begin{bmatrix} -V & \Phi^T & \Omega^T \\ * & -\overline{P}^{-1}\overline{Q}_1\overline{P} & 0 \\ * & * & -\overline{P}^{-1}\overline{Q}_2\overline{P} \end{bmatrix} < 0
\end{aligned} \tag{3.34}$$

has a solution $\widehat{\varepsilon}_i, \widehat{\overline{P}}, \widehat{\overline{Q}}_1, \widehat{\overline{Q}}_2, V, G$, *where*

$$\Phi\Phi^T = \int_{-\tau_i(0)}^{0} \tanh\left(K_i\phi(s)\right)\tanh^T\left(K_i\phi(s)\right)ds,$$

$$\Omega\Omega^T = \sum_{i=1}^{J}\int_{-\overline{\tau}}^{0}\int_{\beta}^{0} \dot{\tanh}^T(K_i x(\alpha))\dot{\tanh}(K_i x(\alpha))d\alpha\, d\beta.$$

$\mathrm{Tr}(\cdot)$ *denotes the trace of the matrix* $(\cdot)$, $\delta = 2\sum_{i=1}^{n}\overline{p}_i^{-1}\ln(\cosh(k_{i0}\phi_i(0)))$, *then, the corresponding guaranteed cost control law,* $u(t) = G\tanh(Kx(t))$ *is an optimal guaranteed cost control. Under this control law the closed-loop cost function* (3.2) *is minimized.*

Proof. By Theorem 3.6, the control law constructed in terms of any feasible solution of (3.9) is a guaranteed cost control law. According to Schur complement, the condition (2)) is equivalent to $\Phi^T\overline{P}^{-1}\overline{Q}_1\overline{P}\Phi + \Omega^T\overline{P}^{-1}\overline{Q}_2\overline{P}\Omega < V$.

Since $\mathrm{Tr}(AB) = \mathrm{Tr}(BA)$, we have

$$\begin{aligned}
&\sum_{i=1}^{J}\int_{-\tau_i(0)}^{0} \tanh^T(K_i\phi(s))\overline{P}^{-1}\overline{Q}_1\overline{P}\tanh(K_i\phi(s))ds \\
&\quad+\sum_{i=1}^{J}\int_{-\overline{\tau}}^{0}\int_{\beta}^{0} \dot{\tanh}^T(K_ix(\alpha))\overline{P}\,\overline{Q}_2\overline{P}\dot{\tanh}(K_ix(\alpha))d\alpha\, d\beta \\
&\quad=\sum_{i=1}^{J}\int_{-\tau_i(0)}^{0}\mathrm{Tr}\Big[\tanh^T(K_i\phi(s))\overline{P}^{-1}\overline{Q}_1\overline{P}\tanh(K_i\phi(s))\Big]ds \\
&\qquad+\sum_{i=1}^{J}\int_{-\overline{\tau}}^{0}\int_{\beta}^{0}\mathrm{Tr}\Big[\dot{\tanh}^T(K_ix(\alpha))\overline{P}\,\overline{Q}_2\overline{P}\dot{\tanh}(K_ix(\alpha))\Big]d\alpha\, d\beta \\
&\quad=\mathrm{Tr}\Big[\Phi\Phi^T\overline{P}^{-1}\overline{Q}_1\overline{P}\Big]+\mathrm{Tr}\Big[\Omega\Omega^T\overline{P}^{-1}\overline{Q}_2\overline{P}\Big] \\
&\quad=\mathrm{Tr}\Big[\Phi^T\overline{P}^{-1}\overline{Q}_1\overline{P}\Phi\Big]+\mathrm{Tr}\Big[\Omega^T\overline{P}^{-1}\overline{Q}_2\overline{P}\Omega\Big] \\
&\quad<\mathrm{Tr}(V).
\end{aligned}\tag{3.35}$$

So it follows that

$$\begin{aligned}
J_0 &= 2\sum_{i=1}^{n}\overline{p}_i^{-1}\ln(\cosh(k_{i0}\phi_i(0))) + \sum_{i=1}^{J}\int_{-\tau_i(0)}^{0}\tanh^T(K_i\phi(s))\overline{P}^{-1}\overline{Q}_1\overline{P}^{-1}\tanh(K_i\phi(s))ds \\
&\quad+\sum_{i=1}^{J}\int_{-\overline{\tau}}^{0}\int_{\beta}^{0}\dot{\tanh}^T(K_ix(\alpha))\overline{P}^{-1}\overline{Q}_2\overline{P}^{-1}\dot{\tanh}(K_ix(\alpha))d\alpha\, d\beta \\
&\le \delta + \mathrm{Tr}(V).
\end{aligned}\tag{3.36}$$

Therefore, the guaranteed cost controller subject to (3.34) is an optimal guaranteed cost control. Under this controller the closed-loop cost function (3.3) is minimized.

This completes the proof. □

4. Simulation

In the following, we will give an example to demonstrate the effectiveness of the obtained results.

Example 4.1. We apply the above analysis technique to a continuous stirred tank reactor (CSTR) in which the first-order irreversible exothermic reaction $A \rightarrow B$ occurs [11, 12].

The material and energy balance equations are

$$\begin{aligned}
\dot{x}_1(t) &= \left(\frac{-1}{\lambda} + \Delta A_0\right)x_1(t) + \left(\frac{1}{\lambda} - 1\right)x_1(t - \tau_{11}(t)) + D_\alpha(1 - x_1(t))\exp\left[\frac{x_2(t)}{1 + x_2(t)/\gamma_0}\right], \\
\dot{x}_2(t) &= -\left(\frac{1}{\lambda} + \beta\right)x_2(t) + \left(\frac{1}{\lambda} - 1\right)x_2(t - \tau_{21}(t)) + HD_\alpha(1 - x_1(t))\exp\left[\frac{x_2(t)}{1 + x_2(t)/\gamma_0}\right] + \beta u(t),
\end{aligned} \tag{4.1}$$

where ΔA_0 denotes the parameter uncertainty of original nonlinear systems. Constants H, β, D_α, and γ_0 are all positive. Here, the model parameters are given as

$$\gamma_0 = 20, \qquad H = 8, \qquad \beta = 0.3, \qquad D_\alpha = 0.072, \qquad \lambda = 0.8. \tag{4.2}$$

Suppose that we have the following hyperbolic type fuzzy rule bases:

If $x_1(t)$ is $A_0^{1,+}$, $x_1(t-\tau_{11}(t))$ is $A_1^{1,+}$, $x_1(t-\tau_{12}(t))$ is $A_2^{1,+}$, $x_2(t)$ is $A_0^{2,+}$, then $\dot{x}_1 = C_{x_1}^1 + C_{dx_{11}} + C_{dx_{12}} + C_{x_2}^1$;

...

If $x_1(t)$ is $A_0^{1,-}$, $x_1(t-\tau_{11}(t))$ is $A_1^{1,-}$, $x_1(t-\tau_{12}(t))$ is $A_2^{1,-}$,$x_2(t)$ is $A_0^{2,-}$, then $\dot{x}_1 = -C_{x_1}^1 - C_{dx_{11}} - C_{dx_{12}} - C_{x_2}^1$;

If $x_1(t)$ is $A_0^{1,+}$, $x_2(t)$ is $A_0^{2,+}$, $x_2(t-\tau_{21}(t))$ is $A_1^{2,+}$, $x_2(t-\tau_{22}(t))$ is $A_2^{2,+}$, then $\dot{x}_2 = C_{x_1}^2 + C_{x_2}^2 + C_{dx_{21}} + C_{dx_{22}}$;

...

If $x_1(t)$ is $A_0^{1,-}$, $x_2(t)$ is $A_0^{2,-}$, $x_2(t-\tau_{21}(t))$ is $A_1^{2,-}$, $x_2(t-\tau_{22}(t))$ is $A_2^{2,-}$, then $\dot{x}_2 = -C_{x_1}^2 - C_{x_2}^2 - C_{dx_{21}} - C_{dx_{22}}$.

Here we choose membership functions of $P_{i_j}^j$ and $N_{i_j}^j$ $(i_j = 0, 1, 2, j = 1, 2)$ as (2.2).

Then, we have the following model:

$$\dot{x}(t) = A\tanh(Kx(t)) + \sum_{i=1}^{2} A_i \tanh(K_i x(t - \tau_i(t))) + Bu(t), \tag{4.3}$$

where $x(t) = [x_1(t), x_2(t)]^T$, $x(t - \tau_i(t)) = [x_1(t - \tau_{1i}(t)), x_2(t - \tau_{2i}(t))]^T$,

$$\begin{aligned}
&A = \begin{bmatrix} C_{x_1}^1 & C_{x_2}^1 \\ C_{x_1}^2 & C_{x_1}^2 \end{bmatrix}, \qquad A_1 = \begin{bmatrix} C_{dx_{11}} & 0 \\ 0 & C_{dx_{21}} \end{bmatrix}, \qquad A_2 = \begin{bmatrix} C_{dx_{12}} & 0 \\ 0 & C_{dx_{22}} \end{bmatrix}, \\
&K = \operatorname{diag}\begin{bmatrix} k_{10} & k_{20} \end{bmatrix}, \qquad K_i = \operatorname{diag}\begin{bmatrix} k_{1i} & k_{2i} \end{bmatrix}, \qquad C = \begin{bmatrix} 0 & D_{x_2} \end{bmatrix}, \quad i = 1, 2.
\end{aligned} \tag{4.4}$$

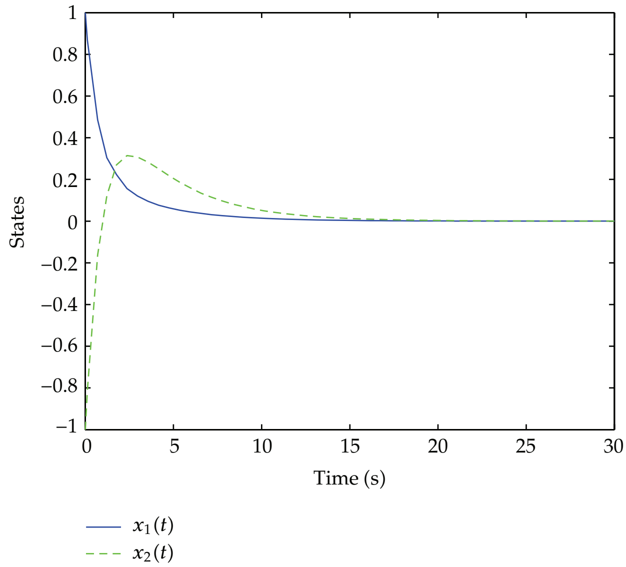

Figure 1: States response of the closed-loop system.

We choose that $\tau_{i1}(t) = 0.8|1.1\sin^2(t) - 0.6|$, $\tau_{i2}(t) = 0.5|1.1\sin^2(t) - 0.6|, i = 1, 2$, and the initial condition $\phi(t) = [1 \;\; -1]^T$, for all $t \in [-0.8 \;\; 0]$. The parameters of DFHM (4.3) can be obtained using neural network BP algorithm [13, 14] as follows:

$$A = \begin{bmatrix} -3.6117 & 0.4922 \\ 5.0821 & 0.1119 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0.5193 & 0 \\ 0 & -2.3141 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.0589 & 0 \\ 0 & -0.1868 \end{bmatrix},$$
$$K = \operatorname{diag}[0.3165 \;\; 0.2801], \quad K_1 = \operatorname{diag}[0.6211 \;\; 0.1207], \quad K_2 = \operatorname{diag}[0.0562 \;\; 0.0003]. \tag{4.5}$$

Other parameters are given as follows:

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}. \tag{4.6}$$

Consider the modeling errors, we assume $M = [0.2 \;\; 0.2]^T$, $N = [0.1 \;\; 0.2]$, $N_1 = [0.2 \;\; 0.1]$, $F(t) = \sin t$. Solve the LMI problem in (3.34). We obtain

$$u = \begin{bmatrix} -0.0536 & -0.1082 \\ -6.0117 & -33.9723 \end{bmatrix} \tanh(Kx(t)) \tag{4.7}$$

and corresponding $J = 80.9812$. Figure 1 depicts the behavior of the closed-loop system based on the DFHM for the initial conditions $x(0) = [1 \;\; -1]^T$. Figure 2 shows that the control input u. Simulation result demonstrates the effectiveness of the fuzzy hyperbolic with time-varying delays guaranteed control approach.

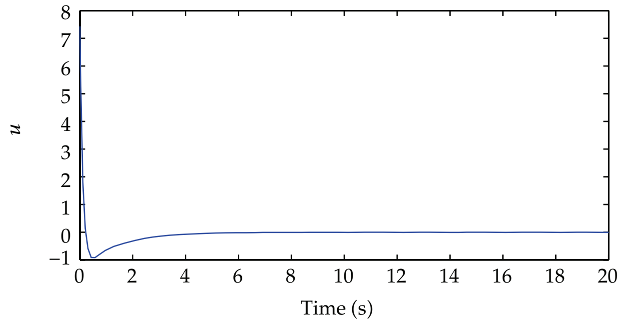

Figure 2: The trajectories of control input.

5. Conclusion

In this paper, the delay-dependent fuzzy hyperbolic guaranteed cost control for nonlinear uncertain systems with time delay using DFHM has been considered. The design problem of DD-DFHMGCC is converted into linear matrix inequalities. The controller designed achieves closed-loop asymptotic stability and provides an upper bound on the closed-loop value of cost function. Simulation example is provided to illustrate the design procedure of the proposed method.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (61104021), the National High Technology Research and Development Program of China (2012AA040104), and the Fundamental Research Funds for the Central Universities of China (N100304007).

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