Research Article

Experimental Study on Forecasting Mathematical Model of Drying Shrinkage of Recycled Aggregate Concrete

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On the basis of basic law in AASHTO2007 model, the forecasting mathematical model of drying shrinkage of recycled aggregate concrete (RAC) is established by regression analysis and experimental study. The research results show that (1) with the replacement rate of RCA increases, the drying shrinkage value of RAC increases; this trend is even more obvious in the early drying time. (2) The addition of fly ash can inhibit the drying shrinkage of RAC, but the effect is not very obvious. Specifically, the addition of fly ash will increase the shrinkage to some extent when the mixing amount is 20%. (3) The addition of expansive agent can obviously inhibit the shrinkage of RAC; the inhibition affection is better than that of fly ash. (4) The forecasting mathematical models of drying shrinkage of RAC established in this paper have high accuracy and rationality according to experiment validation and error analysis.

1. Introduction

Forecasting mathematical models of concrete drying shrinkage is a mathematical equation which represents the relationship of concrete shrinkage and its curing age of [1]. The models are generally fitted by regression analysis of experimental data. The common functional expressions are hyperbolic function, exponential function, and logarithmic function, and so on.

In recent years, many domestic and foreign researchers are committed to the study of forecasting mathematical models and had found a lot of computational model. However, it is very difficult to estimate the shrinkage value and its development in different age because of experimental value of concrete shrinkage having great dispersion [2]. Researchers have introduced a variety of concrete prediction computational model, but these models have some limitations and application scope. Currently, more typical forecasting mathematical models of drying shrinkage of natural aggregate concrete are shown below.

The main factors considered in forecasting mathematical model proposed by CEB/FIP [3] are relative humidity, water-cement ratio, cement content, dry age, and size of components, the proposed expression is shown in the following:

$$\varepsilon_{\rm sh}(t) = \varepsilon_c \cdot K_b \cdot K_t \cdot K_e, \tag{1.1}$$

where $\varepsilon_{sh}(t)$ is concrete shrinkage strain at any age t, (10^{-6}) ; ε_c is shrinkage strain relative to humidity (10^{-6}) ; K_b is correction coefficient of concrete mix proportion affection; K_t is of concrete drying time; K_e is correction coefficient of affection of concrete members.

Main factors what ACI proposed [4] also considered the influence of curing time before drying, curing way, aggregates content, and concrete's air-containing except ones considered by CEB/FIP forecasting mathematical model. ACI's recommended formula is shown in (1.1) and (1.2).

Under wet curing conditions,

$$\varepsilon_{\rm sh}(t) = \left(\frac{t}{35+t}\right)\varepsilon_u.$$
 (1.2)

Under steam curing conditions,

$$\varepsilon_{\rm sh}(t) = \left(\frac{t}{55+t}\right)\varepsilon_u,$$

$$\varepsilon_u = 780 \times 10^{-6} \gamma_{\rm sh},$$
(1.3)

where $\varepsilon_{sh}(t)$ is shrinkage strain drying time age t; ε_u is limit shrinkage strain with the relative humidity 40% which is recommended by ACI; γ_{sh} is product of correction factors of a series of consideration factors, such as wet curing cycle, exposed time, and relative humidity.

This model accords with engineering practice well, for example, last days of before drying, *t* is considered, but the parameter *t* is very difficult to estimate.

The forecasting mathematical model proposed by Gardner and Lockman [5] is shown in the following:

$$\varepsilon_{\rm sh}(t,t_c) = \varepsilon_{\rm sh}(t,t_c)_0 f_{\rm sh}(\rm RH) f_{\rm sh}(f_{cu,k}) f_{\rm sh}(M_{\rm cure}) f_{\rm sh}(T_{\rm cement}),$$

$$f_{\rm sh}(\rm RH) = 1 - 1.18 \,\rm RH^4,$$
(1.4)

where $\varepsilon_{sh}(t, t_c)$ is concrete shrinkage from beginning observational age t_c to age t; $\varepsilon_{sh}(t, t_c)_0$ is shrinkage of concrete under standard conditions; V/S is volume to surface area of concrete component; RH is relative humidity; $f_{sh}(RH)$ is the impact of relative humidity; $f_{sh}(f_{cu,k})$ is the impact of strength grade of concrete; $f_{sh}(M_{cure})$ is the impact coefficient of curing method, taking 1.0 under the standard curing of 1.0; $f_{sh}(T_{cement})$ is influence coefficient of cement type.

In addition to the factors of humidity, component size of concrete, the model also considers the influence of concrete strength grade and cement type on concrete shrinkage. Mathematical Problems in Engineering

Based on more recent experimental data, AASHTO [6] puts forward a forecasting mathematical model of concrete shrinkage by means of modifying ACI209 model and CEB/ FIP model, the formula as can be seen from he following:

$$\varepsilon_{\rm sh} = k_s \, k_{hs} \, k_f \, k_{td} \, 0.48 \times 10^{-3}, \tag{1.5}$$

where ε_{sh} is shrinkage strain; k_s is effect coefficient of component size between them:

$$k_s = \left[\frac{t/(26\exp(0.0142(V/S)) + t)}{t/(t+45)}\right] \left[\frac{1064 - 3.70(V/S)}{923}\right].$$
 (1.6)

V/S is volume to surface area of concrete component; $k_{hs} = (2.00 - 0.014 H)$, is humidity correlation coefficient; H is humidity; $k_f = 35/(7 + f'_{ci})$ is correlation coefficient of concrete strength; $k_{td} = (t/(61 - 0.58f'_{ci} + t))$ is correlation coefficient with the development of time t; f'_{ci} is compressive strength of concrete.

This model accords well with engineering practice, and the parameter selection is simple and convenient. Therefore, the model is used widely since being proposed.

In summary, the considered factors of the existing mathematical models of concrete shrinkage are emphasized particularly, but all of these models have not considered the influence of superplasticizer, fly ash, expansive agent, and other commonly used admixtures in projects on shrinkage and also does not reflect the their positive and negative effects on drying shrinkage of concrete. So they cannot accurately predict the shrinkage of concrete mixed with additives. In addition, there is no consideration of influence of aggregate properties on concrete shrinkage.

2. Parameters Considered in the Model and Its Basic Formula

2.1. Parameters Considered in the Model

Besides the environmental humidity, specimen geometry, strength, and other factors, the model mainly considered the influence of replacement rate of recycled concrete aggregate (Hereinafter referred to as RCA) and the mixing amount of fly ash (FA), expansive agent on drying shrinkage of recycled aggregate concrete (hereinafter referred to as RAC) in this paper.

2.2. Determination of the Basic Formula

On the basis of some basic law in AASHTO2007 model, the influence of replacement rate of RCA and the mixing amount of fly ash, expansive agent on drying shrinkage were considered simultaneously in the model of this paper. The forecasting mathematical model of drying shrinkage of RAC is obtained by fitting analysis of test results. The basic formula of the model is shown in he following:

$$\varepsilon'_{\rm sh} = k_s \, k_{hs} \, k'_f \, k_{td} \, k_r \, k_k \, 0.48 \times 10^{-3}, \tag{2.1}$$

Project	Group number	Cement	NCA	RCA	NFA	Water	FDN	Sand ratio	W/C	FA	UEA
RCA test	RC0	426	1000	0	560	200	0	0.36	0.47	0	0
	RC70	426	300	700	560	200	0	0.36	0.47	0	0
	RC100	426	0	1000	560	200	0	0.36	0.47	0	0
FA test	FA0	426	0	1000	560	200	0	0.36	0.47	0	0
	FA10	426	0	1000	560	200	0	0.36	0.47	43	0
	FA20	426	0	1000	560	200	0	0.36	0.47	85	0
	FA30	426	0	1000	560	200	0	0.36	0.47	128	0
UEA test	UEA0	426	0	1000	560	200	0	0.36	0.47	0	0
	UEA8	426	0	1000	560	200	0	0.36	0.47	0	34.1
	UEA12	426	0	1000	560	200	0	0.36	0.47	0	51.1
	UEA15	426	0	1000	560	200	0	0.36	0.47	0	63.9

Table 1: Mix design of RAC (kg/m^3) .

Table 2: Influence coefficient of strength grade.

Variety of concrete	Strength grade (MPa)	k'_f
	20	0.78
RAC	30	1
	50	1.44

where k_r is impact coefficient of RCA; k_k is impact coefficient of additive; $k_k = k_{fa} \cdot k_e$, k_{fa} is impact coefficient of fly ash; k_e is impact coefficient of expansive agent; the rest of parameters meaning is same to (1.5).

3. Experimental Designs

3.1. Experimental Materials

Cement: P·S 32.5. Construction waste (hereinafter referred to as CW): generating from building demolition in Kunming city, china. UEA-Expansive agent (hereinafter referred to as UEA): the recommended dosage is 8–12%. I grade product fly ash (hereinafter referred to as FA): the fineness is 7.3%.

3.2. Experimental Methods and Mix Proportion

Drying shrinkage test of RAC is made according to GBJ 82–85 "ordinary concrete test method of long-term performance and durability." In the test, mix design of C30 NAC is used as reference. Specimen size is $100 \text{ mm} \times 100 \text{ mm} \times 515 \text{ mm}$ mix proportion is shown in Table 1.

4. Experimental Results and Analysis

4.1. RAC Strength Grade and Its Impact on Drying Shrinkage of RAC

The influence coefficient k'_f [7] (the influence of strength grade on drying shrinkage) is shown in Table 2.

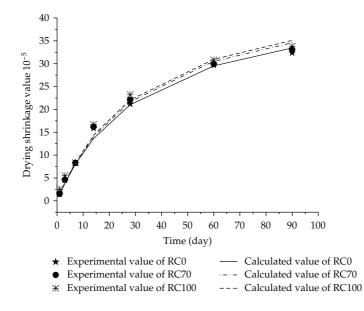


Figure 1: The influence of different RCA replacement rate on RAC drying shrinkage.

4.2. Replacement Rate of RCA and Its Impact on the Drying Shrinkage of RAC

The result of the effects of replacement rate of RCA on drying shrinkage of RAC is shown in Figure 1.

The formula of forecasting mathematical model of concrete drying shrinkage considering the influence of RCA replacement rate is shown in the following:

$$\varepsilon_{\rm shr}(t) = k_r \cdot \varepsilon'_{\rm sh}.\tag{4.1}$$

As can be seen from Figure 1, the concrete shrinkage curve of RAC with different RCA replacement rate is in good agreement with the power function. The influence coefficient of RCA according to power function is shown in (4.2), and the corresponding shrinkage model is shown in (4.3):

$$k_r = \exp(a \cdot v_r + b), \tag{4.2}$$

$$\varepsilon_{\rm shr} = \exp(a \cdot v_r + b) \cdot \varepsilon'_{\rm sh'} \tag{4.3}$$

where v_r is replacement rate of RCA; *a*, *b* are constant which are derived from experimental data fitting.

Mathematical model	Regression coefficients		Fitting equation	Correlation	Mean square deviation	F statistics	
	а	b	Thing equation	coefficients	deviation	1 5101151105	
Equation (4.3)	0.046	-0.003	Equation (4.5)	0.99459	1.56085	2565	
Equation (4.8)	-3.88	1.18	Equation (4.10)	0.99324	3.15600	1390	
Equation (4.13)	-1.85	-0.026	Equation (4.15)	0.99253	1.45512	1720	

Table 3: Table of fitting parameters of mathematical model.

The function expression derived from regression fitting using Levenberg-Marquardt method + general global optimization method is shown in (4.4), (4.5), and the fitting parameters are shown in Table 3:

$$k_r = \exp(0.046v_r - 0.003), \tag{4.4}$$

$$\varepsilon_{\rm shr} = \exp(0.046v_r - 0.003) \cdot \varepsilon_{\rm sh}^{\prime}. \tag{4.5}$$

When replacement rates of RCA are 0%, 70%, 100%, the concrete shrinkage values measured in the experimental at 3d age are 4.51×10^{-5} , 4.6×10^{-5} , 5.51×10^{-5} , which is that the shrinkage strain of RAC with 70% RCA is 1.02 times of the natural coarse aggregate concretes (NACs) at 3 d age; the shrinkage strain of RAC with 100% recycled coarse aggregate is 1.2217 times of NACs at 3 d age. The concrete shrinkage values measured in the experimental at 28 d age are 21.2×10^{-5} 22.1×10^{-5} , 23.2×10^{-5} , which is that the shrinkage strain of RAC with 70% recycled coarse aggregate is 1.02 times of the natural coarse aggregate concretes (NACs) at 3 d age; the shrinkage strain of RAC with 70% recycled coarse aggregate is 1.02 times of NACs at 28 d age, the shrinkage strain of RAC with 100% recycled coarse aggregate is 1.094 times of NACs at 28 d age; the concrete shrinkage values measured in the experimental at 60 d age are 29.7×10^{-5} , 29.9×10^{-5} , 30.85×10^{-5} , which is that the shrinkage strain of RAC with 70% recycled coarse aggregate is 1.007 times of the natural coarse aggregate concretes (NACs) at 60 d age; the shrinkage strain of RAC with 100% recycled coarse aggregate is 1.009 times of NACs at 60 d age; the concrete shrinkage values measured in the experimental at 90 d age are 32.4 \times 10⁻⁵, 33.1 \times 10⁻⁵, 33.8 \times 10⁻⁵, which is that the shrinkage strain of RAC with 70% recycled coarse aggregate is 1.02 times of the natural coarse aggregate concretes (NACs) at 90 d age, the shrinkage strain of RAC with 100% recycled coarse aggregate is 1.043 times of NACs at 90 d age. It can clearly be seen that with the dosage of RCA increases, the shrinkage values of RAC were gradually increasing, which is similar to the research results of Gómez-Soberón [8], Poon and Kou [9], Khatib [10]. This paper also found that the shrinkage values' increasing trend of RAC with 100% RCA is more obvious; the increasing trend of RAC shrinkage values in early age larger than late stages, but with the drying time increase, this trend gradually becomes flat.

We can see that the increase effect of drying shrinkage caused by RCA is larger in the early time than late stage's; the shrinkage strain of RAC with 100% RCA is larger significantly than NAC's, This may be because RCA itself has larger shrinkage deformation and lower elastic modulus [11].

In regression (4.4), when v_r is from 0 to 0.7, 1, k_r is from 0.9974 to 1.031, 1.045, it is 1.033 and 1.048 times of the original, which tallies with the experimental results. As can be seen from Table 3, the model correlation coefficients are over 0.99, which explains that the forecasting mathematical model (4.5) in this paper is with high accuracy.

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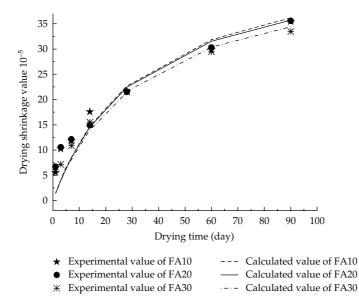


Figure 2: Drying shrinkage value of different content of fly ash RAC at various drying time.

4.3. Mixing Amount of Fly Ash and Its Impact on the Drying Shrinkage of RAC

The result of the effects of the mixing amount of fly ash on drying shrinkage of RAC is shown in Figure 2.

The formula of forecasting mathematical model of concrete drying shrinkage considering the influence of the mixing amount of fly ash is shown in the following:

$$\varepsilon_{\mathrm{sh}\,fa}(t) = 1.044 \cdot k_{fa} \cdot \varepsilon_{\mathrm{sh}'}^{\prime} \tag{4.6}$$

where "1.044" is the impact coefficient of RCA when the replacement rate of RCA is 100%.

As can be seen from Figure 2, the concrete shrinkage curve of RAC with different mixing amount of fly ash is in good agreement with the composite exponential function. The influence coefficient of fly ash according to composite exponential function is shown in (4.7), and the corresponding shrinkage model is shown in the following:

$$k_{fa} = 1 - \exp\left(a \cdot (1 - v_{fa})^b\right),$$
 (4.7)

$$\varepsilon_{\mathrm{sh}\,fa}(t) = 1.044 \cdot k_{\mathrm{fa}} \cdot \varepsilon'_{\mathrm{sh}'} \tag{4.8}$$

where v_{fa} is the mixing amount of fly ash; *a*, *b* are constant which derived from experimental data fitting.

The function expression derived from regression fitting using Levenberg-Marquardt method + general global optimization method is shown in (4.9), (4.10), and the fitting parameters are shown in Table 3:

$$k_{fa} = 1 - \exp\left(-3.38(1 - v_{fa})^{1.18}\right),\tag{4.9}$$

$$\varepsilon_{\mathrm{sh}\,fa}(t) = 1.044 \cdot \left(1 - \exp\left(-3.38 \cdot \left(1 - \upsilon_{fa}\right)^{1.18}\right)\right) \cdot \varepsilon_{\mathrm{sh}}'.\tag{4.10}$$

As can be seen from Figure 2, when the mixing amount of fly ash is 10%, 20%, 30%, the concrete shrinkage values measured in the experimental at 3 d age are 10.26×10^{-5} , 10.55×10^{-5} 10^{-5} , 7.17 × 10^{-5} , which is that the shrinkage strain of RAC with 20% fly ash is 1.208 times of the RACs with 10% fly ash at 3d age; the shrinkage strain of RAC with 30% fly ash is 0.6988 times of RACs with 10% fly ash at 3d age. The concrete shrinkage values measured in the experimental at 28 d age are 21.6×10^{-5} , 21.55×10^{-5} , 21.46×10^{-5} , which is that the shrinkage strain of RAC with 20% fly ash is 0.9977 times of the RACs with 10% fly ash at 28 d age; the shrinkage strain of RAC with 30% fly ash is 0.9935 times of the RACs with 10% fly ash at 28 d age. the concrete shrinkage values measured in the experimental at 60 d age are 29.91×10^{-5} , 30.31×10^{-5} , 29.44×10^{-5} , which is that the shrinkage strain of RAC with 20% fly ash is 1.013 times of the RACs with 10% fly ash at 60 d age; the shrinkage strain of RAC with 30% fly ash is 0.9843 times of RACs with 10% fly ash at 60 d age. The concrete shrinkage values measured in the experimental at 90 d age are 35.46×10^{-5} , 35.45×10^{-5} , 33.47×10^{-5} , which is that the shrinkage strain of RAC with 20% fly ash is 0.9997 times of the RACs with 10% fly ash at 90 d age; the shrinkage strain of RAC with 30% fly ash is 0.9439 times of the RACs with 10% fly ash at 90 d age.

The results show that the addition of fly ash will slightly increase the shrinkage of RAC in early drying time, but, generally, the addition of fly ash can inhibit the drying shrinkage of RAC; however, this effect is not very obvious. The study found that the addition of fly ash will increase the shrinkage to some extent when the mixing amount is 20%, the phenomenon is more apparent in the early time, whose conclusion is slightly different from the research result of Mandal and Gupta [12]. This maybe because the total porosity of RAC with 20% fly ash slightly higher than that with 10%, 30% fly ashes.

In regression (4.9), when v_{fa} is from 0.1 to 0.2, 0.3, k_{fa} is from 0.9675 to 0.9493, 0.9217, it is 0.9812 and 0.9527 times of the original, which tallies with the experimental results. As can be seen from Table 3, the model correlation coefficients are over 0.99, which explains that the forecasting mathematical model (4.10) in this paper is with high accuracy.

4.4. Mixing Amount of Expansive Agent and Its Impact on the Drying Shrinkage of RAC

The result of the effects of the mixing amount of expansive agent on drying shrinkage of RAC is shown in Figure 3.

The formula of forecasting mathematical model of concrete drying shrinkage considering the influence of the mixing amount of expansive agent is shown in the following:

$$\varepsilon_{\operatorname{sh}e}(t) = 1.044 \cdot k_e \cdot \varepsilon'_{\operatorname{sh}}.\tag{4.11}$$

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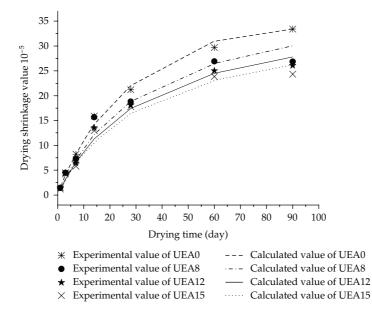


Figure 3: Drying shrinkage value of different content of expansion agent RC at various drying time.

As can be seen from Figure 3, the concrete shrinkage curve of RAC with different mixing amount of expansive agent is in good agreement with the power function. The influence coefficient of fly ash according to power function is shown in (4.12), and the corresponding shrinkage model is shown in (4.13):

$$k_e = \exp(a \cdot v_e + b), \tag{4.12}$$

$$\varepsilon_{\mathrm{sh}\,e}(t) = 1.044 \cdot k_e \cdot \varepsilon'_{\mathrm{sh}}.\tag{4.13}$$

The function expression derived from regression fitting using Levenberg-Marquardt method + general global optimization method is shown in (4.14), (4.15), and the fitting parameters are shown in Table 3:

$$k_e = \exp(-1.85v_e - 0.026), \tag{4.14}$$

$$\varepsilon_{\rm sh\,e}(t) = 1.044 \cdot \exp(-1.85v_e - 0.026) \cdot \varepsilon'_{\rm sh}.$$
 (4.15)

As can be seen from Figure 3, when the mixing amount of expansive agent are 0%, 8%, 12%, and 15%, the concrete shrinkage values measured in the experimental at 3 d age are 4.51 \times 10⁻⁵, 4.47 \times 10⁻⁵, 4.42 \times 10⁻⁵, 4.24 \times 10⁻⁵, which is that the shrinkage strain of RAC with 8% expansive agent is 0.9911 times of the RACs with 0% expansive agent at 3 d age; the shrinkage strain of RAC with 12% expansive agent is 0.9800 times of the RACs with 0% expansive agent at 3 d age; the shrinkage strain of RAC with 15% expansive agent is 0.9401 times of the RACs with 0% expansive agent at 3 d age. The concrete shrinkage values measured in the experimental at 28 d age are 21.2 \times 10⁻⁵, 18.81 \times 10⁻⁵, 18.12 \times 10⁻⁵, 17.81 \times 10⁻⁵, which is that the shrinkage strain of RAC with 8% expansive agent is 0.9873 times of the RACs with 0%

expansive agent at 28 d age; the shrinkage strain of RAC with 12% expansive agent is 0.8547 times of the RACs with 0% expansive agent at 28 d age; the shrinkage strain of RAC with 15% expansive agent is 0.8401 times of the RACs with 0% expansive agent at 28 d age. The concrete shrinkage values measured in the experimental at 60 d age are 29.7×10^{-5} , 26.92×10^{-5} , 25.03×10^{-5} , 23.84×10^{-5} , which is that the shrinkage strain of RAC with 8% expansive agent is 0.9064 times of the RACs with 0% expansive agent at 60 d age; the shrinkage strain of RAC with 12% expansive agent is 0.8519 times of the RACs with 0% expansive agent at 60 d age; the shrinkage strain of RAC with 15% expansive agent is 0.8027 times of the RACs with 0% expansive agent at 90 d age are 32.4×10^{-5} , 26.85×10^{-5} , 26.08×10^{-5} , 24.32×10^{-5} , which is that the shrinkage strain of RAC with 8% expansive agent at 90 d age; The shrinkage strain of RAC with 12% expansive agent is 0.8287 times of the RACs with 0% expansive agent at 90 d age; The shrinkage strain of RAC with 12% expansive agent at 90 d age; the shrinkage strain of RAC with 12% expansive agent at 90 d age; The shrinkage strain of RAC with 12% expansive agent at 90 d age; The shrinkage strain of RAC with 12% expansive agent at 90 d age; The shrinkage strain of RAC with 12% expansive agent at 90 d age; The shrinkage strain of RAC with 12% expansive agent at 90 d age; The shrinkage strain of RAC with 12% expansive agent is 0.8049 times of the RACs with 0% expansive agent at 90 d age; the shrinkage strain of RAC with 15% expansive agent is 0.7506 times of the RACs with 0% expansive agent at 90 d age.

It can clearly be seen that the addition of expansive agent can obviously inhibit the shrinkage of RAC and the inhibition affection is better than that of fly ash, which is similar to the research result of Bissonnette et al. [13].

In regression (4.14), when v_e is from 0 to 0.08, 0.12, 0.15, k_e is from 0.9743 to 0.7804, 0.7382, it is 0.8624, 0.8009, and 0.7577 times of the original, which tallies with the experimental results. As can be seen from Table 3, the model correlation coefficients are over 0.99, which explains that the forecasting mathematical model (4.15) in this paper is with high accuracy.

4.5. Ambient Humidity, Specimen Size, and Its Impact on the Drying Shrinkage of RAC

According to AASHTO2007 model (1.5), $k_{hs} = (2.00 - 0.014 H)$, when relative humidity is from 60% to 100%, k_{hs} value changes from 1.16 to 0.6, which decreases by 48.3%. The result show that the change of humidity having have a great impact on drying shrinkage of concrete materials.

In general, the effect of specimen size on concrete shrinkage is mainly embodied in the spread speed of internal moisture to the outside and the impact mechanism on shrinkage has the similar law to that of relative humidity [13]. The size of shrinkage test mode in this paper is $100 \text{ mm} \times 515 \text{ mm}$, then V/S = 22.79. The effect coefficient of specimen size is directly calculated from AASHTO2007 model (1.6).

4.6. Forecasting Mathematical Models of Concrete Drying Shrinkage of RAC

With (2.1) as basic formula, in the comprehensive study in Sections 4.1, 4.2, 4.3, 4.4, and 4.5 in this paper, the forecasting mathematical models of drying shrinkage of RAC are shown in the following:

$$\varepsilon_{\rm sh}' = 0.00048 \, k_s \cdot k_{hs} \cdot k_{td} \cdot k_f' \, \exp(0.046v_r - 0.003) \cdot \left(1 - \exp\left(-3.38(1 - v_{\rm fa})^{1.18}\right)\right)$$

$$\cdot \exp(-1.85v_e - 0.026), \qquad (4.16)$$

where ε'_{sh} is shrinkage strain, 10^{-5} ; k_s is effect coefficient of component size (as can be seen from (1.6)); $k_{hs} = (2.00 - 0.014 H)$ is humidity correlation coefficient; H is humidity; k'_f is

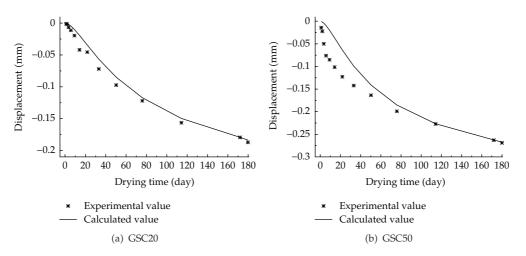


Figure 4: Calculated value of displacement of RAC contrasted with experimental value.

Table 4: Table of mix design in verification test.

Project	Strength grade	Cement (kg/m^3)	RCA (kg/m^3)	Sand (kg/m^3)	Water (kg/m^3)	Sand ratio (kg/m ³)	W/C	FDN (kg/m^3)	UEA (kg/m^3)	FA (kg/m^3)
						0.42				
GSC50	C50	476	950	510	200	0.35	0.36	2.38	27.8	80

correlation coefficient of concrete strength grade (as can be seen from Table 2); $k_{td} = (t/(61 - 0.58f'_{ci}+t))$ is correlation coefficient with the development of time $t; f'_{ci}$ is compressive strength of concrete at 28 d age; v_r is replacement rate of RCA; v_{fa} is the mixing amount of fly ash; v_e is the mixing amount of expansive agent. The rest of parameters' meaning is same to those in (1.5).

5. Error Analysis and Model Validation

In order to verify the accuracy of the model, GSC20 and GSC50 (the strength grade is, resp., C20, and C50), two different groups shrinkage specimen are devised and prepared in this paper. The mix design of verification experiment is shown in Table 4.

One point located on the top surface of the specimen selected as object, transforming the shrinkage values of the point into displacement, the comparison chart of calculated and experimental values of GSC20, GSC50, is shown in Figures 4(a) and 4(b)

Absolute error and relative error are selected as indexes to do error analysis of displacement calculated values of RAC; the formula is shown in the following:

$$e_a = x_c - x_e, \tag{5.1}$$

$$e_r = \frac{e_a}{x_e} \times 100\%,$$
 (5.2)

where e_a is absolute error value; x_e is experimental value of displacement, mm; x_c is calculated value of displacement, mm; e_r is relative error value, 100%.

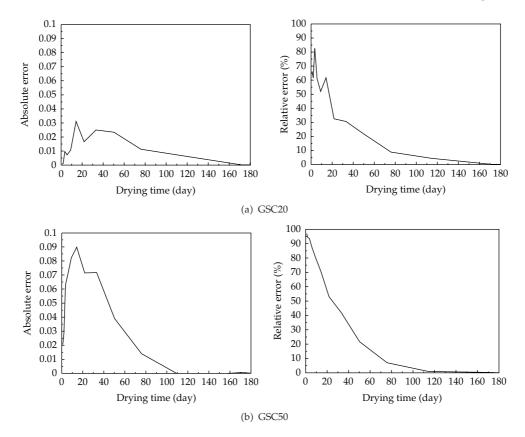


Figure 5: Error analysis chart of displacement calculated value and experimental value of RAC.

The absolute error values and relative error values are calculated according to (5.1) and (5.2). The error distribution curve drew by Matlab is shown in Figure 5.

Figures 5(a) and 5(b) is error chart (including absolute error and relative error) of concrete of GSC20 and GSC50. As can be seen from Figures 5(a) and 5(b), the absolute error values between calculated shrinkage value (calculated by the model built in this paper) and experimental value have small fluctuation in 180-day drying time. Especially after 60 d drying time, the absolute error values gradually approach zero, which indicates that the model built in this paper has high accuracy after 60 d drying time. The absolute error values have large fluctuation in the drying time of $0 \sim 28$ d, the maximum absolute error value arises in the drying time of 14 d, absolute error value is about 0.035. The relative error values between calculated shrinkage value and experimental value have the similar law to the absolute error ones in 180-day drying time. The relative error values have large fluctuation in the early drying time, the relative maximum error value is even up to 95%, but, with the drying time increase, the relative error values are becoming smaller and smaller. Especially after 60 d drying time, the error values gradually approach zero.

So, as can be seen from Figures 4 and 5, the drying shrinkage values calculated in the use of the model of this paper are slightly smaller than those of experimental values, but, with the drying time increase, the calculated values have higher coincide degree with the experimental results. Calculated values and experimental values of RAC shrinkage are almost

the same in 60 d age especially, which also shows that the forecasting mathematical models of drying shrinkage of RAC builded in this paper have high accuracy and rationality.

6. Conclusion

(1) Under the same experimental condition, the drying shrinkage values of RAC are higher than those of NAC because of the reduce of RAC inhibition effect and old cement mortar's further shrinkage after water absorption, and this trend is even more obvious in the early drying time. With the replacement rate of RCA increases, the drying shrinkage value of RAC increases too.

(2) The addition of fly ash can inhibit the drying shrinkage of RAC, but the effect is not very obvious. Specifically, the addition of fly ash will increase the shrinkage to some extent when the mixing amount is 20%, the phenomenon is more apparent in the early time. This is because the total porosity of RAC with 20% fly ash is slightly higher than that with 10%, 30% fly ashes.

(3) The addition of expansive agent can obviously inhibit the shrinkage of RAC, the inhibition affection is better than that of fly ash.

(4) Through regression analysis of large number of experimental data, the forecasting mathematical models of drying shrinkage of RAC shown as follow:

$$\varepsilon_{\rm sh}' = 0.048 \times 10^{-3} \times k_s \cdot k_{hs} \cdot k_{td} \cdot k_f' \exp(0.046v_r - 0.003) \cdot \left(1 - \exp(-3.38(1 - v_{\rm fa})^{1.18})\right)$$

$$\cdot \exp(-1.85v_e - 0.026).$$
(6.1)

(5) The forecasting mathematical models of drying shrinkage of RAC built in this paper have high accuracy and rationality according to experiment validation and error analysis.

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