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Magnetic branes in Gauss–Bonnet gravity with nonlinear electrodynamics: correction of magnetic branes in Einstein–Maxwell gravity

Seyed Hossein Hendi[1,2,a] , **Shahram Panahiyan**[1,b], **Behzad Eslam Panah**[1,c]

[1] Physics Department and Biruni Observatory, College of Sciences, Shiraz University, 71454 Shiraz, Iran
[2] Research Institute for Astronomy and Astrophysics of Maragha (RIAAM), Maragha, Iran

Abstract   In this paper, we consider two first order corrections to both the gravity and the gauge sides of the Einstein–Maxwell gravity: Gauss–Bonnet gravity and quadratic Maxwell invariant as corrections. We obtain horizonless magnetic solutions by implying a metric representing a topological defect. We analyze the geometric properties of the solutions and investigate the effects of both corrections, and find that these solutions may be interpreted as magnetic branes. We study the singularity condition and find a non-singular spacetime with a conical geometry. We also investigate the effects of different parameters on the deficit angle of spacetime near the origin.

1 Introduction

Magnetic strings/branes may be interpreted as topological defects that were formed during a phase transition of the early universe [1,2]. These defects contain information regarding the early structure of the universe and also its evolution [3–9]. On the other hand, considering the AdS/CFT correspondence, these horizonless magnetic solutions in the presence of negative cosmological constant may contain information regarding a quantum field theory on the boundary of the AdS spacetime [10–13]. These topological defects have wide variety of applications in quantum gravity and has been studied in different context such as hadron dynamics [14], anti-ferromagnetic crystals [15], Yang–Mills plasma [16] and also, in studying quantum criticality [17]. These magnetic branes/strings have been studied in some papers and their properties for different cases of gravity and nonlinear elec-

tromagnetic fields have been derived [18–25]. Motivated by these facts, we study magnetic branes in the presence of the two corrections mentioned and investigate the effects of these two corrections on the properties of the magnetic branes.

From the other point of view, Einstein (EN) gravity has been a successful theory for describing many phenomena, whereas in some aspects it confronts some fundamental problems [26–28]. In order to overcome these problems, alternative theories of gravity or a generalization of EN gravity have been introduced [29–39]. One of these theories is the generalization of the EN gravity to the well-known Gauss–Bonnet (GB) theory. This generalization solves some of the shortcomings of EN gravity and gives a renewed view and properties in a gravitational context [40–51]. This theory of gravity has been studied for different astrophysical objects [52–56]. The properties of GB gravity, may attract one to the idea of not to consider GB gravity as a generalization, but as a correction to EN gravity. In other words, one can consider the first order of the GB parameter, α, as a correction and study its effects on the properties of the solutions. This fact shows that one can take small values of the GB parameter into account and interpret the effects of the GB correction as a perturbation to EN gravity. This consideration enables us to study the effects of the GB parameter in more detail. In this paper, we consider GB gravity not as a generalization but as a correction or perturbation to EN theory [57].

Naturally, most of the systems that we are studying are nonlinear or they have nonlinear properties. In order to have more realistic results, one should take into account the nonlinear behavior of these systems. The Maxwell theory of electrodynamics is a linear theory which works well in many aspects but fails regarding some important issues. In order to overcome its problems, different theories of nonlinear electrodynamics (NED) were introduced [58–62]. Among them, Born–Infeld (BI) type ones are quite interesting due to their

[a] e-mail: hendi@shirazu.ac.ir

[b] e-mail: ziexify@gmail.com

[c] e-mail: behzad_eslampanah@yahoo.com

properties and the fact that these theories may arise from the low energy limits of the effective string theory [63–68]. For large values of the nonlinearity parameter, these BI type theories lead to the same behavior as Maxwell theory. In order to avoid the complexity that nonlinear electromagnetic fields pose, one can consider the effects of nonlinearity as a correction to the Maxwell field. In other words, one can add the quadratic Maxwell invariant to the Lagrangian of the Maxwell theory [57,69,70] to obtain NED as a correction. On the other hand, it is arguable that in order to obtain physical results that are consistent with experiments, one should consider the weak effects of nonlinearity. Using the series expansion of BI type theories in the weak field limit, one finds that the first term is the Lagrangian of the Maxwell theory and the second term is proportional to the quadratic Maxwell invariant [71]. Therefore, in this paper, we are considering a quadratic Maxwell invariant as a correction of the Maxwell electrodynamics and study its effects on the properties of solution.

The structure of the paper will be as follows. In the next section, we will present the field equations and obtain the metric function for the case of magnetic branes. We will plot some graphs in order to study the effects of the corrections on the metric function. Also we will study the geometrical structure of the obtained solutions and investigate the effects of different parameters on the deficit angle and conical structure of the magnetic branes through graphs. The last section is devoted to closing remarks.

2 Static solutions

In order to study horizonless magnetic branes, we consider the following metric for d dimensions [72]:

$$ds^2 = -\frac{\rho^2}{l^2}dt^2 + \frac{d\rho^2}{f(\rho)} + l^2 f(\rho)d\phi^2 + \frac{\rho^2}{l^2}dX^2, \quad (1)$$

where l is a scale factor related to the cosmological constant, $f(\rho)$ is a function of the coordinate ρ, and $dX^2 = \sum_{i=1}^{d-3} dx_i^2$ is the Euclidean metric on the $(d-3)$-dimensional submanifold. The angular coordinate ϕ is dimensionless and ranges in $[0, 2\pi]$, while x_i ranges in $(-\infty, \infty)$. Due to the fact that we are interested in solutions that contain GB gravity and a correction to Maxwell field, we consider the following field equations [57,69]:

$$\partial_\mu \left(\sqrt{-g} \mathcal{L}_\mathcal{F} F^{\mu\nu} \right) = 0, \quad (2)$$

$$\Lambda g_{\mu\nu} + G^{(1)}_{\mu\nu} + \alpha G^{(2)}_{\mu\nu} + O\left(\alpha^2\right)$$
$$= \frac{1}{2}g_{\mu\nu}\mathcal{L}(\mathcal{F}) - 2\mathcal{L}_\mathcal{F} F_{\mu\lambda} F_\nu^\lambda, \quad (3)$$

where $\mathcal{L}_\mathcal{F} = \frac{d\mathcal{L}(\mathcal{F})}{d\mathcal{F}}$, in which $\mathcal{L}(\mathcal{F})$ is the Lagrangian of NED; $\Lambda = -\frac{(d-1)(d-2)}{2l^2}$ and $G^{(1)}_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ are, respectively the cosmological constant and the EN tensor; α is the GB coefficient and

$$G^{(2)}_{\mu\nu} = 2(R_{\mu\sigma\kappa\tau}R_\nu^{\ \sigma\kappa\tau} - 2R_{\mu\rho\nu\sigma}R^{\rho\sigma} - 2R_{\mu\sigma}R^\sigma_{\ \nu} + RR_{\mu\nu})$$
$$-\frac{\mathcal{L}^{(2)}}{2}g_{\mu\nu}, \quad (4)$$

where $\mathcal{L}^{(2)}$ denotes the Lagrangian of GB gravity, given as

$$\mathcal{L}^{(2)} = R_{\mu\nu\gamma\delta}R^{\mu\nu\gamma\delta} - 4R_{\mu\nu}R^{\mu\nu} + R^2. \quad (5)$$

We consider the following Lagrangian for the electromagnetic field [57,69,70]:

$$\mathcal{L}(\mathcal{F}) = -\mathcal{F} + \beta\mathcal{F}^2 + O\left(\beta^2\right), \quad (6)$$

where β is nonlinearity parameter and the Maxwell invariant $\mathcal{F} = F_{\mu\nu}F^{\mu\nu}$, in which $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field tensor, and A_μ is the gauge potential. It is easy to show that the electric field comes from the time component of the vector potential (A_t), while the magnetic field is associated with the angular component (A_ϕ). The black hole solutions of GB gravity in the presence of this nonlinear electromagnetic field were obtained previously [57]. In this paper we are looking for horizonless solutions with a conical singularity which are not interpreted as black holes but as magnetic brane solutions. Since we are looking for the magnetic solutions, we consider the following form of the gauge potential:

$$A_\mu = h(\rho)\delta_\mu^\phi. \quad (7)$$

Using Eq. (7) with the mentioned NED, one can show that the electromagnetic field equation, (2), reduces to the following differential equation:

$$\left[(d-2)E + E'\rho\right]l^2 + 4E^2\left[(d-2)E + 3E'^2\rho\right]\beta$$
$$+O(\beta^2) = 0, \quad (8)$$

where a prime denotes the first derivative with respect to ρ and $E = h'(\rho)$. Solving Eq. (8), one obtains

$$E(\rho) = \frac{2ql^2}{\rho^{d-2}} - \frac{32q^3l^4\beta}{\rho^{3(d-2)}} + O\left(\beta^2\right), \quad (9)$$

where q is an integration constant related to the electric charge. We should note that for small values of β all relations reduce to the corresponding relations of Maxwell theory.

In order to obtain the metric function, $f(\rho)$, one should solve all components of the gravitational field equation (3) simultaneously. After cumbersome calculations, we find that

there are two different differential equations with the following explicit forms:

$$e_{\rho\rho} = \mathcal{K}_1 + \alpha \mathcal{K}_2 = 0, \tag{10}$$

$$e_{tt} = \mathcal{K}_{11} + \alpha \mathcal{K}_{22} = 0, \tag{11}$$

where

$$\mathcal{K}_1 = \Lambda + \frac{(d-2)(d-3)}{2\rho^2} f + \frac{4q^2 l^2}{\rho^{2d-4}}$$
$$+ \frac{(d-2)f'}{2\rho} - \frac{32l^4 q^4 \beta}{\rho^{4d-8}} + O(\beta^2),$$

$$\mathcal{K}_2 = -\frac{(d-2)(d-3)(d-4)ff'}{\rho^3}$$
$$- \frac{(d-2)(d-3)(d-4)(d-5)f^2}{2\rho^4},$$

$$\mathcal{K}_{11} = -\Lambda - \frac{(d-3)(d-4)f}{2\rho^2} - \frac{(d-3)f'}{\rho}$$
$$- \frac{f''}{2} + \frac{4l^2 q^2}{\rho^{2d-8}} - \frac{96l^4 q^4 \beta}{2\rho^{4d-12}} + O(\beta^2),$$

$$\mathcal{K}_{22} = \frac{(d-3)(d-4)(d-5)}{2\rho^4}$$
$$\left((d-6)f^2 + 4\rho ff'' + \frac{2\rho^2}{d-5} \left(f'^2 + ff'' \right) \right).$$

Equations (10) and (11) correspond to the $\rho\rho$ and tt components of the gravitational field equation (3). It is easy to show that the $\phi\phi$ and $x_i x_i$ components of Eq. (3) are, respectively, similar to $e_{\rho\rho}$ and e_{tt}, and therefore, it is sufficient to solve Eqs. (10) and (11), simultaneously. Since we desire to obtain higher dimensional magnetic brane solutions with GB as a correction for EN gravity and the quadratic Maxwell invariant as a correction to the Maxwell theory, we ignored $\alpha\beta$, α^2, and β^2 terms and higher orders. Interestingly, the results for consideration of these two corrections will be as follows:

$$f(\rho) = f_{\text{EN}} - \frac{64q^4 l^4}{(d-2)(3d-7)\rho^{4d-10}} \beta$$
$$+ \frac{(d-3)(d-4)f_{\text{EN}}^2}{\rho^2} \alpha + O\left(\alpha\beta, \alpha^2, \beta^2\right), \tag{12}$$

with

$$f_{\text{EN}} = \frac{2ml^3}{\rho^{d-3}} - \frac{2\Lambda}{(d-1)(d-2)}\rho^2$$
$$+ \frac{8q^2 l^2}{(d-2)(d-3)\rho^{2d-6}}, \tag{13}$$

where m is an integration constant related to the mass. As one can see for the case of $\alpha = \beta = 0$, the effects of corrections are canceled and the obtained results will be magnetic solutions of EN gravity. In order to study the effects of these two corrections on the obtained metric function, we plot some graphs in the presence (absence) of these two corrections (see Fig. 1).

As one can see, in the absence of Maxwell and GB corrections, the plotted graph for the metric function versus ρ is quite different comparing to when one of the considered corrections (GB or Maxwell) is present. We consider that at least one of these corrections will modify the behavior of the metric function. This modification is more evident and stronger for small values of ρ. The metric function will have root(s) for specific value of ρ (namely ρ_0). The function $f(\rho)$ has two extrema at ρ_{ext_1} and ρ_{ext_2} ($\rho_{\text{ext}_1} < \rho_{\text{ext}_2}$) in which for $\rho_0 \leq \rho \leq \rho_{\text{ext}_1}$, the metric function is an increasing function of ρ. For $\rho_{\text{ext}_1} \leq \rho \leq \rho_{\text{ext}_2}$, $f(\rho)$ is a decreasing function of ρ and finally, in the case of $\rho \geq \rho_{\text{ext}_2}$, it is an increasing function of radial coordinate (Fig. 1; left and middle). The root of $f(\rho)$ is an increasing (or decreasing) function of the β (or the α) parameter (Fig. 1; left and middle). ρ_{ext_1} is a decreasing function of the GB parameter and $f(\rho_{\text{ext}_1})$ is increasing functions of the GB parameter. As for the nonlinearity parameter, ρ_{ext_1} is an increasing function of β, but interestingly $f(\rho_{\text{ext}_1})$ is a decreasing function of it. It is worthwhile to mention that the variations of α and β do not have a reasonable effect on ρ_{ext_2} and $f(\rho_{\text{ext}_2})$. It is notable that for small values of the nonlinearity parameter the metric function has no root. Contrary to this effect, for large values of the GB parameter, the metric function will be without any root. It simply shows the fact that GB and nonlinearity parameters have opposite effects on the behavior of the metric function.

Next, we are going to discuss the geometric properties of the solutions. To do this, we look for possible black hole solutions on obtaining the curvature singularities and their horizons. We usually calculate the Kretschmann scalar, $R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$, to achieve the essential singularity. Considering the mentioned spacetime, it is easy to show that

$$R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} = f''^2 + 2(d-2)\left(\frac{f'}{\rho}\right)^2$$
$$+ 2(d-2)(d-3)\left(\frac{f}{\rho^2}\right)^2. \tag{14}$$

Inserting the metric function, $f(\rho)$, in Eq. (14) and using a numerical analysis, one finds that the Kretschmann scalar diverges at $\rho = 0$ and it is finite for $\rho > 0$ and naturally one may think that there is a curvature singularity located at $\rho = 0$. In what follows, we state an important point, in which one confirms that the spacetime never reaches $\rho = 0$. As one can easily see, the metric function has a positive value for $\rho > r_+$. So two cases may occur. For the first case, $f(\rho)$ is a positive definite function with no root and therefore the singularity is called a naked singularity, which we are not interested in. We consider the second case, in which the metric function has one or more real positive root(s). We denote r_+ as the largest real positive root of $f(\rho)$. The metric function is negative for $\rho < r_+$ and positive for $\rho > r_+$ and hence the metric signature
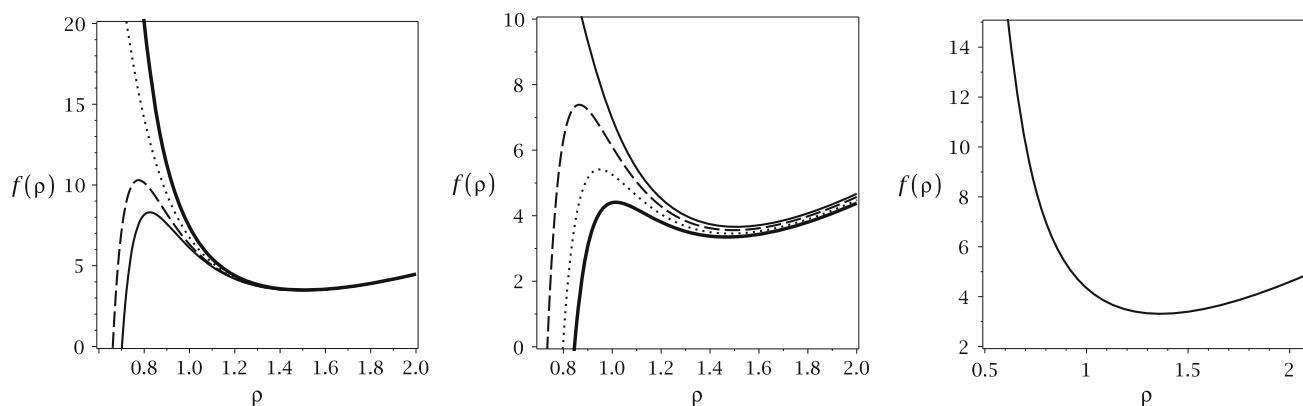
Fig. 1 f(ρ) versus ρ for $m = 0.1$, $q = 2$, $l = 1$, and $d = 5$. *Left diagram*: $\alpha = 0.01$, $\beta = 0$ (*bold line*), $\beta = 0.015$ (*dotted line*), $\beta = 0.025$ (*dashed line*) and $\beta = 0.030$ (*continuous line*). *Middle dia-gram*: $\beta = 0.05$, $\alpha = 0$ (*bold line*), $\alpha = 0.01$ (*dotted line*), $\alpha = 0.02$ (*dashed line*) and $\alpha = 0.03$ (*continuous line*). *Right diagram*: $\beta = 0$ and $\alpha = 0$

may change from $(-+++++\cdots+)$ to $(---+++\cdots+)$ in the range $\rho < r_+$.

We should note that this situation is different from that of black hole solutions. Consider a typical d-dimensional black hole metric $ds^2 = -f(\rho)dt^2 + \frac{d\rho^2}{f(\rho)} + \rho^2 d\Omega^2$ with $(-+++++\cdots+)$ signature. Denoting r_+ as largest real positive root of $f(\rho)$, we know that for $\rho > r_+$ the metric function is positive definite and the signature does not change. For $\rho < r_+$, although the mentioned signature changes to $(+-++++\cdots+)$, the number of positive and negative signs remains unchanged. In other words, for the entire spacetime, the black hole metric has one negative (temporal coordinate) sign and $(d-1)$ positive (spatial coordinates) signs. In this case, the change in sign merely signifies that this is not the right coordinate system to study the $\rho < r_+$ region. But for our magnetic metric, Eq. (1), the number of positive and negative signs will change for $\rho < r_+$. Taking into account this apparent change of signature of the metric, we conclude that one cannot extend the spacetime to $\rho < r_+$. In order to get rid of this incorrect extension, one may use the following suitable transformation on introducing a new radial coordinate r:

$$r^2 = \rho^2 - r_+^2,$$
$$\rho \geq r_+ \Longleftrightarrow r \geq 0. \qquad (15)$$

Using the mentioned transformation with $d\rho = \frac{r}{\sqrt{r^2+r_+^2}}dr$, one finds that the metric (1) should change to

$$ds^2 = -\frac{r^2 + r_+^2}{l^2}dt^2 + \frac{r^2}{(r^2+r_+^2)f(r)}dr^2$$
$$+l^2 f(r)d\phi^2 + \frac{r^2 + r_+^2}{l^2}dX^2. \qquad (16)$$

It is worthwhile to mention that with this new coordinate, the metric function will be of the following form:

$$f(r) = f_{EN} - \frac{64q^4l^4}{(d-2)(3d-7)(r^2+r_+^2)^{2d-5}}\beta$$
$$+\frac{(d-3)(d-4)f_{EN}^2}{r^2+r_+^2}\alpha + O\left(\alpha\beta, \alpha^2, \beta^2\right), \qquad (17)$$

where

$$f_{EN} = \frac{2ml^3}{(r^2+r_+^2)^{\frac{d-3}{2}}} - \frac{2\Lambda}{(d-1)(d-2)}\left(r^2+r_+^2\right)$$
$$+\frac{8q^2l^2}{(d-2)(d-3)(r^2+r_+^2)^{d-3}}. \qquad (18)$$

We should note that the function $f(r)$ given in Eq. (17) is a non-negative function in the whole spacetime. Although the Kretschmann scalar does not diverge in the range $0 \leq r < \infty$, one can show that there is a conical singularity at $r = 0$. One can investigate the conic geometry by using the *circumference/radius ratio*. Using the Taylor expansion, in the vicinity of $r = 0$, we find

$$f(r) = f(r)\,|_{r=0} + \left(\frac{df}{dr}\,|_{r=0}\right)r$$
$$+\frac{1}{2}\left(\frac{d^2f}{dr^2}\,|_{r=0}\right)r^2 + O(r^3) + \cdots, \qquad (19)$$

where

$$f(r)|_{r=0} = \frac{df}{dr}\bigg|_{r=0} = 0, \qquad (20)$$

$$\frac{d^2f}{dr^2}\bigg|_{r=0} \equiv f'' = \frac{-2\Lambda}{d-2} - \frac{8l^2q^2}{(d-2)r_+^{2d-4}}$$
$$+\frac{64l^4q^4}{(d-2)r_+^{4d-8}}\beta + O(\beta^2), \qquad (21)$$

and hence

$$\lim_{r \longrightarrow 0^+} \frac{1}{r}\sqrt{\frac{g_{\phi\phi}}{g_{rr}}} = \lim_{r \longrightarrow 0^+} \frac{\sqrt{r^2 + r_+^2}\, lf(r)}{r^2}$$
$$= \frac{lr_+}{2} f'' \neq 1, \qquad (22)$$

which confirms that as the radius r tends to zero, the limit of the *circumference/radius ratio* is not 2π and, therefore, the spacetime has a conical singularity at $r = 0$. This conical singularity may be removed if one identifies the coordinate ϕ with the period

$$\text{Period}_\phi = 2\pi \left(\lim_{r \longrightarrow 0} \frac{1}{r}\sqrt{\frac{g_{\phi\phi}}{g_{rr}}}\right)^{-1} = 2\pi (1 - 4\mu), \qquad (23)$$

where μ is given by

$$\mu = \frac{1}{4}\left(1 - \frac{2}{lr_+ f''}\right). \qquad (24)$$

In other words, the near origin limit of the metric (16) describes a locally flat spacetime which has a conical singularity at $r = 0$ with a deficit angle $\delta\phi = 8\pi\mu$. Using the Vilenkin procedure, one can interpret μ as the mass per unit volume of the magnetic brane [73]. It is obvious that the nonlinearity of electrodynamics can change the value of the deficit angle $\delta\phi$. Taking into account Eqs. (21) and (24), we can write

$$\frac{d(\delta\phi)}{d\beta} = \frac{-4\pi}{lr_+}\frac{d\left(f''^{-1}\right)}{d\beta} = \frac{256\pi l^3 q^4}{(d-2)r_+^{4d-7} f''^2} > 0. \qquad (25)$$

Equation (25) indicates that $\delta\phi$ is an increasing function of β. In addition, considering Eqs. (21) and (24), one finds that the deficit angle does not depend on the GB coefficient. In order to investigate the effects of nonlinearity, r_+, q, l, and dimensionality, we plot $\delta\phi$ versus β and r_+.

As one can see, the deficit angle can be affected by changing the values of q, β, d, and r_+. In order to provide additional clarification, we plot Figs. 2, 3 and 4. Considering these figures, the left panels indicate the variation of the deficit angle versus β, whereas the right panels correspond to the behavior of the deficit angle with respect to r_+. The left panels of Figs. 2, 3 and 4 confirm that the deficit angle is an increasing function of β. Moreover, Fig. 2 (left) shows that for $\beta \longrightarrow 0$ (Maxwell case), the deficit angle is a decreasing function of q. In addition, there is a q_c, in which for $q > q_c$ the deficit angle is negative for $\beta = 0$ (one can obtain q_c in such a way that $\delta\phi \big|_{\beta=0,q=q_c} = 0$).

In the case of the deficit angle versus r_+ for different values of the charge parameter (Fig. 2, right), the plotted graph is divided into three distinguishable regions. We find that variations of q, β, and q do not affect, significantly,

the behavior of the deficit angle for sufficiently small (large) r_+ (see right panel of Figs. 2, 3 and 4). In addition Fig. 2 (right) shows that there is an extremum point $r_{+\text{ext}}$, in which for $r_+ \leq r_{+\text{ext}}$ ($r_{+\text{ext}} \leq r_+$), the deficit angle is a decreasing (an increasing) function of r_+. The mentioned $r_{+\text{ext}}$ and its corresponding deficit angle are increasing functions of the charge parameter. In other words, for adequately small charge the minimum value of the deficit angle is negative (Fig. 2, right). By increasing the charge the region of negativity and hence the distance between the roots of the deficit angle decreases.

Now, we plot the deficit angle versus r_+ for various β (Fig. 3, right) to show an interesting behavior. This figure indicates two divergencies for the deficit angle when $\beta < \beta_c$. In other words, these singularities, r_{+_1} and r_{+_2} ($r_{+_1} < r_{+_2}$) can be removed by increasing β. For $\beta > \beta_c$, the deficit angle has a minimum and for $\beta < \beta_c$, we cannot obtain a physical ($\delta\phi \leq 2\pi$) deficit angle for $r_{+_1} < r_+ < r_{+_2}$. The deficit angle before the first singular point is a decreasing function of r_+, whereas it is an increasing function after the second singular point. Variation of the nonlinearity parameter changes the distance between these two singular points. In other words, increasing β leads to a decrease of the mentioned interval. Figure 3 (left) indicates that the deficit angle is an increasing function of r_+ in this parameter region.

Finally, we consider the effects of the dimensions on the deficit angle (Fig. 4). As one can see, for certain dimensions, there is a region for β where the deficit angle is negative (Fig. 4, left) and increasing dimensions leads to vanishing of this region. Figure 4 shows that the deficit angle is an increasing function of the dimensions in this parameter region. As for the deficit angle versus r_+ (Fig. 4, right), there is a similar behavior to Fig. 2 (right) and the deficit angle has a minimum.

In order to explain the negative deficit angle, we first describe a positive one. Cut a segment of a certain angular size of a two dimensional plane and then sewing together the edges to obtain a conical surface. This conical space is flat but has a singular point corresponding to the apex of the cone. The segment deleted from the plane is known as a deficit angle with positive values. According to the previous statement, here we imagine a new situation: that a segment is added to the new plane to obtain a flat surface with a saddle-like cone (for more details see Fig. 2 in Ref. [74]). This added segment corresponds to a negative deficit angle (or surplus angle).

It is worthwhile to mention that although the deleted segment is bounded by the value of 2π the added segment is unbounded. Therefore, one can conclude that the range of deficit angles is from $-\infty$ to 2π. Positive/negative deficit angles may be related to the positive/negative torsion of the space or attractive-type/repulsive-type gravitational potentials; more details of a negative deficit angle with its physical interpretations can be found in [74–79].
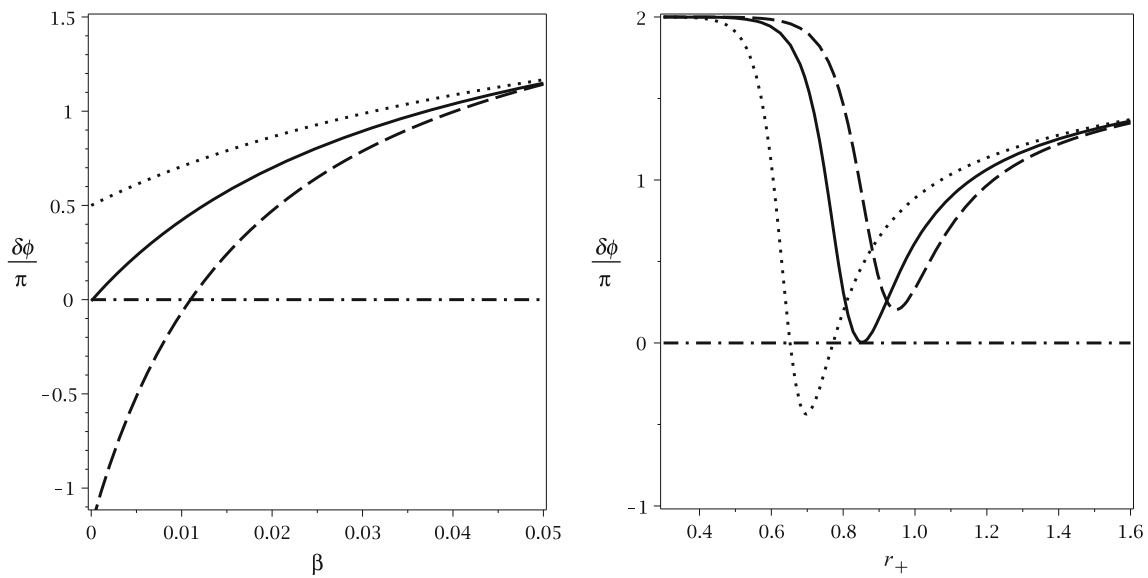
Fig. 2 $\delta\phi/\pi$ versus β (*left*) and $\delta\phi/\pi$ versus r_+ (*right*) for $d = 5$ and $l = 1$. *Left diagram*: $r_+ = 2$, $q = 8$ (*dotted line*), $q = 8.5$ (*continuous line*) and $q = 9$ (*dashed line*). *Right diagram*: $\beta = 0.05$, $q = 0.4$ (*dotted line*), $q = 0.73$ (*continuous line*), and $q = 1$ (*dashed line*)
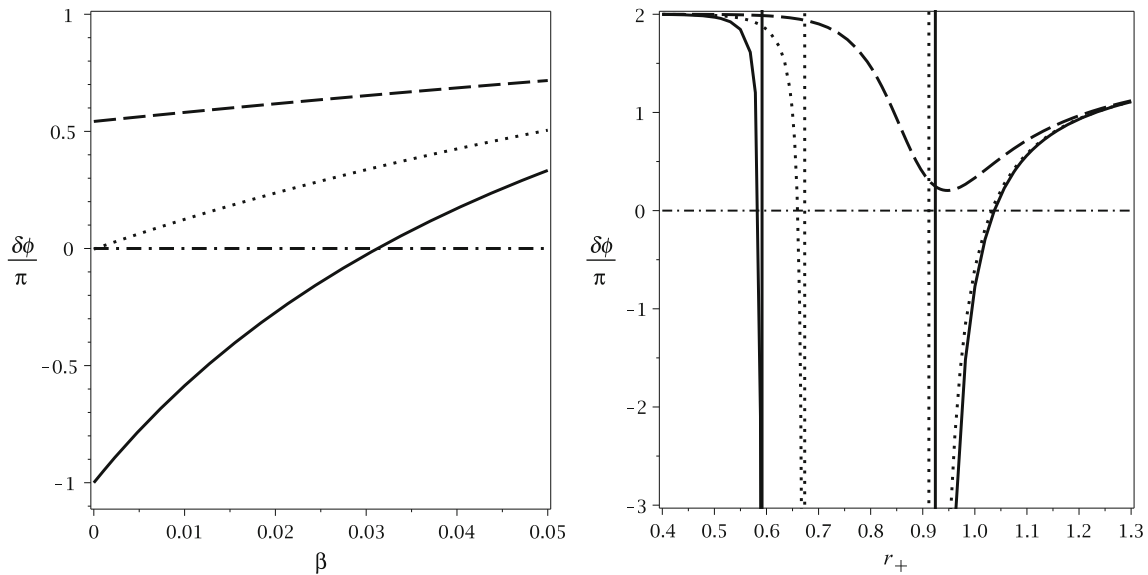

Fig. 3 $\delta\phi/\pi$ versus β (*left*) and $\delta\phi/\pi$ versus r_+ (*right*) for $d = 5$, $l = 1$, and $q = 1$. *Left diagram*: $r_+ = 1$ (*continuous line*), $r_+ = 1.04$ (*dotted line*) and $r_+ = 1.1$ (*dashed line*), respectively. *Right diagram*: $\beta = 0.005$ (*continuous line*), $\beta = 0.01$ (*dotted line*), and $\beta = 0.05$ (*dashed line*), respectively

3 Closing remarks

In this paper, we have considered two different kinds of corrections to both matter and gravitational fields. For the gravitational aspect, we have considered GB gravity as a correction to EN gravity whereas for the electromagnetic aspect, we have regarded a quadratic power Maxwell invariant in addition to the Maxwell Lagrangian as a correction to the electromagnetic field. Remarkably it was seen that, in the absence of these two corrections, the behavior of the metric

function is completely different comparing to a consideration of at least one of them.

Interestingly, the root(s) of the metric function was (were) also modified by considering these corrections. The place of this (these) root(s) was a decreasing (an increasing) function of the GB (nonlinearity) parameter. For small values of the nonlinearity parameter and large values of the GB parameter, the behavior of the metric function was similar to the case of Einstein–Maxwell. In addition, we found that the contribution of the considered matter field was opposite to the
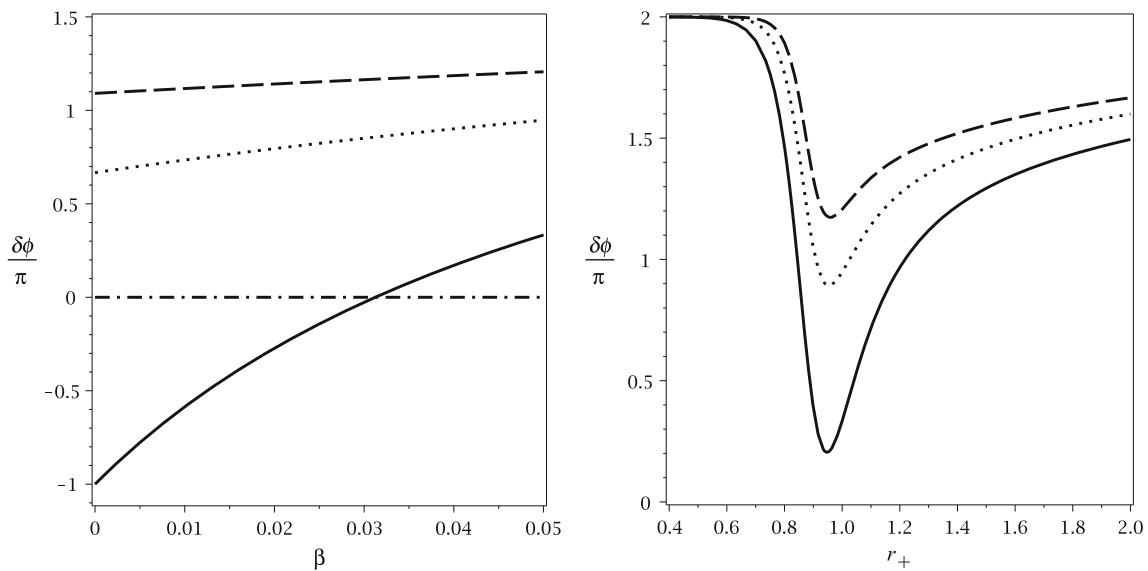
Fig. 4 $\delta\phi/\pi$ versus β (*left*) and $\delta\phi/\pi$ versus r_+ (*right*) for $q = 1$ and $l = 1$. *Left diagram*: $r_+ = 1$, $d = 5$ (*continuous line*), $d = 6$ (*dotted line*), and $d = 7$ (*dashed line*). *Right diagram*: $\beta = 0.05$, $d = 5$ (*continuous line*), $d = 6$ (*dotted line*), and $d = 7$ (*dashed line*)

gravitational field. We found that for large values of the radial coordinate these corrections do not have a significant effect on the metric function. In other words, the dominant region in which these two corrections modify the metric function meaningfully was for small values of ρ.

Next, in order to avoid a change of signature, we used a suitable radial transformation and found that there is no curvature singularity through the whole of spacetime, but there is a conical singularity located at the origin. We have studied the behavior of the deficit angle and the effects of different parameters on it. We have plotted two kinds of diagrams. One is for the deficit angle versus β and the other one is the deficit angle versus r_+. For the case of the deficit angle versus β, due to the fact that we considered the nonlinearity as a correction, we only have taken small values of it into account. In general the behavior of the left graphs were monotonic and the calculated deficit angles were increasing functions of the nonlinearity parameter. As for the deficit angle versus r_+, interestingly, the behavior was completely different from the other case. In this case, for the variation of q and d, we found that for sufficiently small or large r_+ the deficit angle is independent of the value of other parameters. In addition, we showed that there is a minimum value for the deficit angle in a specific r_+. Moreover, we found that the only parameter that modified the general behavior of these graphs significantly was the nonlinearity parameter. For small values of β two singular points were seen. Interestingly, as one increases the nonlinearity parameter the distance between these two singular points decreases. In other words, by increasing the nonlinearity parameter, a compactification occurs which decreases the region between two singular points to a level that the mentioned singularities vanish. This behavior shows the fact

that small values of β have a stronger contribution comparing to other parameters drastically.

The existence of a root, the region of negativity and divergency for the deficit angle are other important issues that must be taken into account. In studying the deficit angle, we consider a second order derivation of the metric function with respect to the radial coordinate. Considering the fact that the metric function could be interpreted as a potential (see for example chapter 9 of Ref. [80]), it is arguable that the singular point may be indicated as a phase transition. On the other hand, the geometrical structure of the solutions in the case of positive and negative values of the deficit angle is different. In the case of a positive deficit angle, the geometrical structure of the object is cone-like with a deficit angle whereas in the case of the negative angle the structure will be saddle-like with a surplus angle. We found that for a positive deficit angle, there is an upper limit, whereas for a negative deficit angle there is no limit. Therefore, one may state that due to these differences in the structure of the solutions, the root of the deficit angle may represent a phase transition. These two arguments could be discussed in more detail if the physical concept of a negative deficit angle has become more clear. Moreover, we should note that spacetime has no deficit angle for vanishing $\delta\phi$. In other words, the geometrical structure of the solutions in this case represents no defect. Therefore, one may argue that these cases represent the magnetic brane without conical structure.

Finally, we will be interested in analyzing the theory of gravity that was proposed in this paper in more detail and calculating the conserved quantities of this theory. Also one can consider higher orders of the Lovelock gravity as corrections to the EN gravity and study their effects on magnetic branes

and their deficit angle [81,82]. Phase transitions, structural properties, and physical behavior of different objects have been studied through these defects. Since the obtained solutions are asymptotically AdS, it is worthwhile to consider these solutions in the context of the AdS/CFT correspondence and study different phenomena through these solutions. In addition, one can make some modifications regarding the geometry of the solutions and remove the mentioned conical singularity, and then use the copy-and-paste method to obtain geodesically complete spacetime with a minimum value for $\rho_{min} = r_+$ as a throat [83–87]. We left these issues for future work.

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