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Can Discrete Time Make Continuous Space Look Discrete?

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Abstract

Van Bendegem has recently offered an argument to the effect that that, if time is discrete, then there should exist a correspondence between the motions of massive bodies and a discrete geometry. On this basis, he concludes that, even if space is continuous, it should nonetheless appear discrete. This paper examines the two possible ways of making sense of that correspondence, and shows that in neither case van Bendegem's conclusion logically follows.

Keywords: Chronon, Hodon, Jerky Motion, Teleportation.

1. Introduction

Physics textbooks tell us that time is *continuous*. This is to say that all finite temporal intervals possess more than denumerably many proper subintervals – or, which is the same, that time is composed of durationless instants, which can be put into a one-one correspondence with the totality of the real numbers. Since the advent of quantum mechanics, on the other hand, physicists have started entertaining the idea that time could be *discrete*: that is, that every finite interval of time possess only finitely many proper subintervals (Kragh and Carazza 1994). This hypothesis has most frequently taken the form of a temporal kind of atomism, according to which time is composed by extended yet indivisible *temporal atoms*, also known as *chronons*.[1] Contrary to instants, chronons can be put into a one-one correspondence with a (possibly improper) subset of the natural numbers, and (unless in the

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[1] A different view is offered by Aronov (1971), according to whom space and time are discrete in the sense that different physical laws are predominant within spatio-temporal regions of different scales. Newton-Smith (1980: 113) discusses the idea that time could be discrete in the sense of being a lattice of finitely many durationless instants. More recently, Dummett (2000) has outlined, though not endorsed, a fuzzy account of discrete time aimed at avoiding the possibility of abrupt instantaneous changes. However, we shall thereafter restrict our attention to the sole atomistic conception.

degenerate case in which there is only one chronon), for every chronon there is one chronon immediately preceding it or one chronon immediately following it.

The idea of discrete time has eventually become the subject of systematic investigations in the groundbreaking field of quantum gravity, whose attempt to reconcile the theory of relativity with quantum mechanics has called into questions the very fundamental character of time and space (Meschini et al. 2005).[2] These recent developments call for an urgent philosophical examination into the metaphysical possibility of discrete time, which contemporary philosophers have rarely taken into serious consideration (notable exceptions include Whithrow 1961; Forrest 1995; Faris 1996).

Certainly, part of the widespread scepticism towards discrete time is motivated by the fact that the very idea of the finite divisibility of durations generates conceptual difficulties on its own (see for example Swinburne 1968: 208; Zwart 1976: 140-141; Capek 1991: 45-47). But on the other hand, part of that scepticism stems from the fact that discrete time is generally considered inseparable from discrete space, and in particular from the hypothesis that, just like time, space may be composed of minimal yet extended intervals, called *hodons* (see for example Margenau 1950: 155-159; Benardete 1964: 242-245; Newton-Smith 1980: 115-117). The problems with atomistic or finitist geometries, on the other hand, are notorious (van Bendegem 2010). Thus, it seems that discrete time should necessarily inherit the same difficulties that burden discrete space. But is this really the case? Does discrete time really require that space should be discrete as well?

Van Bendegem (2011) has recently offered an argument to the effect that, if time is discrete, then space must at the very least *look* discrete. Though van Bendegem himself is inclined to believe that discrete time is possible, he acknowledges that this result imposes a serious constraint to that possibility, for it ties it down to the logical consistency of discrete space. So, are the supporters of discrete time really obliged to accept the possibility of discrete space? This paper is intended to show that van Bendegem's argument fails to demonstrate that they are.

To begin with, few terminological remarks are in order, not to let words carry any hidden presuppositions concerning the microscopic structure of time. By *a time*, let us denote any minimal unit of time, whether it is extended or not. So understood, the notion of a time equally attaches to instants and chronons, and it is accordingly neutral as to whether or not time is infinitely divisible. Similarly, by a *place*, we shall understand any minimal unit of space, whether it is extended or not. This notion attaches to unextended spatial points as well as to hodons, and is therefore neutral as to the infinite divisibility of space. Every extended portion of time or space, whether it is divisible or not, we shall call

[2] I shall be using 'time' and 'space' is a somewhat wide sense, so as to include the temporal and spatial dimensions of space-time, respectively.

an *interval*. Chronons accordingly qualify both as intervals and as times, while hodons qualify both as intervals and places.[3] With this in mind, let us have a look at van Bendegem's argumentation.

Let time be completely made of chronons, let space be continuous, and let us consider any arbitrary moving object x. Moreover, let us suppose that no object can be at more than one place at one time. Clearly, at each chronon, x will have to be at no more than one place. Hence, to allow for the motion of x, there should exist at least two subsequent chronons t_1 and t_2 such that x is at a place s_1 at one chronon and at a different place s_2 at the other. Since space is supposed to be continuous, there will be infinitely many points between s_1 and s_2, but by hypothesis there is no time between t_1 and t_2 during which x could be at any of those points. This means that x will have to move abruptly between non-adjacent places, displaying what van Bedegem calls a *jerky motion*.

Unfortunately, at this point van Bendegem's argumentation becomes relatively obscure. He contends that jerky motion 'implies [that] even if the background is supposed to be continuous, nevertheless movements of masses correspond to a discrete geometry in the sense that 'during' one chronon, nothing can change'. Then, he straightforwardly concludes that '[i]n summary, discrete time does not exclude continuous space, but it reduces continuous space to something that is similar to a discrete space and in that sense, one may conclude that discrete time forces continuous space not so much to become discrete, but at least to appear so' (van Bendegem 2011: 153).[4]

Two questions, then, need to be answered. Firstly, what does he mean when he claims that there is a correspondence between the movement of masses and a discrete geometry? Secondly, how should we understand the claim that space can be continuous yet appear discrete? Finding an answer to these two questions will be necessary not only to understand the real significance of van Bendegem's conclusion, but also to discover what kind of inference could have led him from the former claim to the latter one. Let us consider the latter question first.

2. *The Way Space Looks Is Not the Way It Is.*

Similarity presupposes communion of properties. Hence, one may be inclined to think that, in order to be 'reduced to something similar to discrete space', continuous space should display at least some of the properties of the latter. However, if understood in this sense, van Bendegem's conclusion would be rather uninteresting, if not trivial. For, there necessarily exist some features that continuous space

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[3] For the purposes of the discussion to follow, it is immaterial if times and places are set-theoretical *members* of intervals or rather mereological *parts* of them; accordingly, let us prefer the more neutral *constituents*.

[4] Things get even more complicated if we consider that, few lines after having formulated his argument, van Bendegem explicitly says that discrete time *implies* discrete space (van Bendegem 2011: 153). This claim is evidently at odds with his purported conclusion that discrete time forces continuous space not to become discrete, but only to appear so. However, this might just be taken as a much unfortunate way of phrasing the same conclusion, so we shall not consider it any further.

shares with discrete space, independently of the microscopic structure of time: for example, being divisible into intervals, being capable of hosting physical objects, being the dependent variable of motion, and in general all the properties that qualify space as such.

One may then try and specify a restricted set of properties that continuous spaces do not generally possess, but which discrete spaces generally have. However, that would not make van Bendegem's conclusion any stronger. For example, discrete spaces are most likely, if not forced, to be anisotropic (van Bendegem 2010). This is to say that most if not all discrete spaces are not isomorphic under arbitrary rotations or, in plain words, that they are not the same in all directions. Continuous spaces, on the other hand, can be anisotropic, but they are not necessarily so. Let us suppose, then, that discrete time forced continuous space to be anisotropic. Would this make it be or appear any more discrete? Perhaps. Nonetheless, this would not bother the supporter of discrete time anyway, since evidently continuous space could be anisotropic even though it turned out that discrete space is in fact not conceptually possible. So, if looking discrete only entailed displaying properties such as anisotropy, then one could make sense of the conjunction of discrete time and continuous space without being forced to demonstrate that space could be discrete in its turn.

Finally, one may suggest that, in order to look discrete, continuous space would have to display some or all of the *distinguishing* features of discrete space. This would clearly be a desperate move, since how could any property characterize discrete space as such, if it could be had by non-discrete spaces as well?

How, then, can we make sense of the claim that space could be not discrete yet appear so? Two intuitions seem to underlie that claim. Firstly, that there may be a discrepancy between the way space *is* and the way it *looks*. Secondly, and consequently, that there should be someone *to whom* space may look other than the way it is in reality. Those observers might well be actual or ideal, but it is evident that, in either case, they would not be able to directly perceive the microscopic structure of space. For otherwise, how could they be so systematically deceived about it? This means that, in either case, they would have to infer the topological properties of space from the observation of something else. The question, then, is what that something could be.

Now, there are only three ingredients in van Bendegem's argumentation: time, space and motion. If observing time could tell anything about the microscopic properties of space, then there would presumably be no reason to introduce motion into the picture. Hence, we can reasonably conclude that motion is what our hypothetical observers should look at, to gather any information about the micro-structure of space. This suggests that we should understand the claim that discrete time makes continuous space look discrete as the claim that, if time is discrete, then any observer who will look at the motions of the physical bodies will thereby infer that space is discrete.

This is presumably also why van Bendegem speaks of a *correspondence* between the movements of masses and a discrete geometry. This result leads us naturally to the first of our questions: that is, what kind of correspondence should that be? There are two possibilities at hand, but none of these possibilities, as we shall see, will cause any trouble for the supporters of discrete time.

3. The Geometry of Physical Positions

First, one may understand the correspondence in question as the fact that, if our hypothetical observers employed the subsequent positions of the physical objects as the building blocks of some geometry, then that geometry would have to be discrete. This means that no dense or discrete geometry is possible, whose set of basic constituents has the same geometrical properties as the set of occupied places in a world whose time is discrete. The problem that this interpretation immediately faces, however, is that it is unclear from van Bendegem's argument why that should be the case. Let us examine some possibilities.

Firstly, van Bendegem suggests that we should understand the correspondence between the movements of masses and a discrete geometry 'in the sense that [...] nothing can change' within the lapse of a chronon (van Bendegem 2011: 153), where change is clearly to be thought of in the restricted sense of a change in spatial position. However, if *that* was the reason why the geometry constructed on the positions of the physical objects would have to be discrete, then the surprising consequence would follow that continuous time should make space appear discrete, too.

The fact that nothing can change within a chronon, in fact, is a straightforward consequence of the hypothesis that nothing can be at more than one place at a time. Given that hypothesis, no spatial change can clearly be possible within a time, but that is true independently of whether that time is extended or not. That is, if no change is to be allowed within a chronon, then by the same reason no change should be allowed at an instant. So, either the absence of change at a chronon is *not* the reason why the geometry constructed by our observers would have to be discrete, or they would get a discrete geometry even if time was continuous. The latter option, on the other hand, is arguably false.

Secondly, van Bendegem argues that the position of every moving object at a chronon 'corresponds to a position that cannot be further analysed' into proper sub-intervals, so he proposes that, 'in analogy with chronons', that position be called a hodon (van Bendegem 2011: 153). This may suggest that the reason why the positions of the physical bodies should generate a discrete geometry is the fact that such positions, like chronons, cannot be further subdivided.

The problem with this solution is much similar to the problem we encountered in the previous case. Surely, unextended spatial points cannot be further subdivided either, but certainly not all geometries whose basic elements are points need to be discrete. Hence, the sole fact of being indivisible does not explain why the positions of the physical bodies at a chronon should make them the building blocks of a discrete geometry. Nor, evidently, simply baptising them 'hodons' will make them any more extended, if they are not.[5] Hence, there must exist some other reason, not explicitly mentioned by van Bendegem, why the movement of masses should correspond to a discrete geometry. But what?

Here is one possible explanation. Granted that nothing can be at more than one place at a time, the physical objects will occupy at most as many spatial positions as there are times. On the other hand, if time is discrete, then chronons will surely not be more than denumerably many. Hence, there will be not more than denumerably many occupied positions, and therefore any geometry constructed on those positions will have to have not more than denumerably many constituents. Hence, purportedly, it will be discrete.

Neither this solution, however, would do the job. For, let us suppose that time is not only discrete, but also infinite, in the sense that there exist no first nor last chronons. Then, there will be infinitely many chronons, even though only denumerably so, and hence there will be no strictly finite upper bound to the number of occupied spatial positions. Denumerably many spatial places do not suffice to construct a continuous geometry, but they can nonetheless be employed to construct a *dense* one: that is to say, a geometry whose intervals have exactly denumerably many proper sub-intervals. Dense spaces are not discrete, so it is false that the movements of masses in a universe with discrete time should correspond to a discrete geometry in the sense that discrete geometries are the sole geometries whose constituents can be put in a one-one correspondence with the occupied places in such a universe.[6]

One may object, at this point, that nothing precludes our hypothetical observer from having access to the ordinal or metric properties of the set of occupied spatial positions. These properties would presumably be analogous to the ordinal or metric properties of discrete spaces, but not to those of dense spaces; hence, taking them into account would presumably force our observers to construct a discrete geometry even if the number of occupied places was denumerably infinite.[7] This objection presents us with another possible answer to the question why, in a universe with discrete time, the geometry constructed on the occupied places would necessarily be discrete: the reason is that only a discrete geometry can at the same time possess as many basic constituents as such places *and* preserve the ordinal or metric relations that hold between them. But is this really the case?

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[5] Notice that, having assumed that space is continuous, and since we are dealing with pure conceptual possibilities, there is nothing which could prevent us from imagining x as a massive point. Given the hypothesis that nothing could be at more than one place at a time, the position of x at each time will then be unanalysable *because* dimensionless. Notice, too, that this would be the case independently of the microscopic structure of time.

[6] Admittedly, discrete spaces and dense spaces are both endowed with a discrete topology. Thus, one could preserve van Bendegem's thesis from this objection by contending that all geometrical properties are reducible to topological properties, and that in consequence geometry itself is in fact nothing more than topology. This claim, however, would be highly debatable, as recently illustrated by Maudlin (2010).

[7] This objection was suggested to me by an anonymous referee.

Let us consider the case of metric properties first. Suppose that time is discrete but infinite, and consider any massive point that progressively decelerates so that, at each time, its speed is half the speed it had at the previous chronon. Notice that, given that space is continuous, there is no reason why this should not be possible, unless one makes the unjustified assumption that there exists some minimal non-zero speed. The spatial distances traversed by the above mentioned object will form an infinite series, each term of which will be half the length of its immediate predecessor. Now, if we assume that our hypothetical observers can measure the relative distances between occupied places, they will be able in principle to measure the decreasing distances covered by the above decelerating object. This way, they will manage to construct an infinite series of ever smaller spatial lengths. However, no discrete geometry can be compatible with the existence of any such series. This means that any geometry that preserved the metric relations between the positions occupied by the above mentioned object would have to be either dense or continuous. Therefore, the movement of masses in the world just depicted cannot correspond to a discrete geometry in the sense that only a discrete geometry can reflect the metric properties of the set of occupied places.

The case of ordinal properties is analogous. Consider the same situation as above, but this time let us suppose that the above massive point is moving in the direction of some other object, which is at rest according to the reference frame of our hypothetical observers. Suppose, further, that the initial positions of the two objects are so far apart from each other that the object in motion will never reach or pass the one at rest. Then, at each time, the two objects will be closer than they were at the previous chronon, and there will be no place *s* in the trajectory of the moving body such that no further occupied place lies between *s* and the position of the resting object. By observing the order of the subsequent positions of the two objects, therefore, our observers will be able to construct an infinite series, one of whose terms (the location of the body at rest) has infinitely many predecessors but no immediate predecessor. However, no discrete geometry exists whose constituents are arranged in such a way. So, as before, we must conclude that the movement of masses in a world with discrete time and continuous space cannot correspond to a discrete geometry in the sense that only a discrete geometry could reflect the ordinal properties of the set of places in that world that are occupied by some object.

4. *From Discrete Geometries to Discrete Spaces*

The arguments in the preceding section show that there is apparently no reason supporting the claim that, if time was discrete, then the *geometry* constructed on the positions of physical bodies would necessarily be discrete. But let us concede, for the sake of the argument, that van Bendegem could actually offer a satisfactory explanation of why that should be the case. How could that suffice to make *space* look discrete, too?

The most plausible answer is that, at any possible world, the positions of massive bodies at that world offer the only base of information that any hypothetical observer could gather in order to infer the micro-structure of space. Since we have conceded that that information is strictly compatible with the construction of a discrete geometry, any such observer will be accordingly be inclined to conclude that space itself should be discrete. The problem with this answer, however, is that it is based on two unwarranted assumptions.

The first such assumption is that the inhabitants of a world with discrete time and continuous space could not acquire information about the micro-structure of space by any other means than observing the positions of massive bodies. However, there is no reason to exclude that they could gain such information by, say, observing the *shapes* of physical bodies. For instance, they may easily infer that space is not discrete by, say, observing the existence of perfectly spherical objects, which could not exist in a discrete space. Notice that the existence of similar objects is guaranteed by the fact that space is supposed to be continuous and by the fact that we are dealing with purely metaphysical possibilities. Similarly, there is no metaphysical constraint prohibiting that those observers could directly perceive the shape of physical objects with absolute precision.

The second unwarranted assumption is that the inhabitants of the above world could not go *beyond* the empirical evidence at their disposal and argue that space is continuous even if the positions of physical objects could only suffice to construct a discrete geometry. This means, in other words, that they could not interpolate their empirical data in the way we ordinarily do, thereby hypothesising that infinitely many points exist between the places that are actually observed to be occupied by the moving bodies.

Clearly, we do not *know* that space is continuous. The reason why we do not *know* that is that the empirical evidence that is available to us is unavoidably finite and imprecise: we can only measure a finite number of the infinitely many positions supposedly occupied by moving bodies in the course of their motions. Nonetheless, most of us do not consequently think that space is necessarily discrete. How is it possible? The reason is that we suppose that ever more sophisticated experimental devices could *in principle* allow us to further analyse the motions of physical objects into ever more numerous positions (notice that this assumption is not undermined by quantum indeterminacy). This way, we can attribute the fact that we possess only finite information about the locations of physical bodies not to the fact that space is discrete, but to a limitation in our observational powers.

On the other hand, there seems to be no reason why the inhabitants of a world whose time is discrete could not do the same, and construct continuous models of space in the same way as we do, *in spite of* the finite character of the empirical evidence at their disposal. Thus, even if it was true that only a discrete geometry could be constructed out of the occupied positions in a world with discrete time, there would be no reason why the physicists of that world should take into consideration *exclusively* those positions while modelling physical space.

But, it may be objected, there is a substantive difference between our epistemic condition and the epistemic condition of those physicists. The reason why we can coherently conjecture that ever more sophisticated experimental devices would allow us to indefinitely subdivide space is that, in our world, time is not *ex hypothesi* discrete. This allows us, in principle, to observe closer and closer spatial positions by reducing the temporal interval occurring between any two subsequent observations of the location of a moving body; and this arguably supports our supposition that space is at least infinitely divisible.

This is not possible, on the other hand, for the physicists inhabiting the discrete time world. There is a finite upper limit to the precision of their observations, for they can at best observe the positions of moving bodies at two subsequent chronons. Their efforts to observe spatial positions at shorter intervals of time will necessarily be frustrated, thus making it reasonable for them to recognise that time is discrete, and thus prohibiting them from hypothesising that ever more precise measurement tools could allow them to observe closer and closer spatial locations.

There are diverse possible replies to this objection. Let us consider just three. Firstly, it is possible to imagine a discrete time world in which physicists cannot avail themselves of sufficiently sophisticated measurement devices, and who cannot as a consequence realise that the time of the world they live in is discrete. For all they know, the inhabitants of that world will be in the same epistemic position as ours, so they can justifiably suppose that, in principle, successive refinements in their measurement techniques could allow them to observe closer and closer spatial positions. Therefore, they will be as entitled as we are to suppose that space is continuous.

Secondly, there is no reason to exclude that *we* are the inhabitants of such a world: after all, we do not *know* that time is continuous, just as we do not know that space is. This is to say that we cannot exclude that there is some finite upper limit to the precision of our measurements of spatial distances and temporal durations, nor we can exclude that such a limit would be due to the discrete character of space and time. Nonetheless, this does not prohibit us to assume otherwise, and to ordinarily conceive of time and space as continuous even if the empirical evidence at our disposal is irremediably finite.

Finally, even if they knew that they inhabit a discrete time world, the physicists of that world could still be entitled to think that ever more *numerous* (even though not more precise) measurements will support the hypothesis that the space of their world is continuous. To substantiate this claim, let us consider the following thought experiment.

Suppose that those physicists, whose observational abilities we have supposed to be finite, could nonetheless empirically determine the position reached in one chronon by some inertial mover m having initial position x and moving on a plane p. Since space is by hypothesis continuous, they will be free to repeat the same kind of measurement with respect to any arbitrary finite number of movers having the same speed and initial position as *m*, but travelling in some different and arbitrarily chosen directions on the same plane. Supposing for the sake of simplicity that space is isotropic, the collection of the final positions so determined will progressively approximate a circumference with centre in *x*. Then, what could prevent the physicists performing this experiment from supposing that, if they repeated the same kind of measurement in *all* possible directions on the plane *p*, the collection of the final positions so detected would be a perfect circumference? But, in that case, what could prevent them from supposing that space is continuous? The fact that time is discrete in their world, and that they could not measure any shorter temporal interval than a chronon, would make no difference at all in this case.

To sum up: even granting that the positions of the massive bodies in a universe with discrete time could be only compatible with a discrete geometry, the inhabitants of such a universe could still recognise that space is continuous, either by directly observing the shapes of physical objects, or by interpolating the empirical data at their disposal. Therefore, even if the movement of masses in a discrete time world really corresponded to a discrete geometry in the above specified sense, van Bendegem's conclusion would not follow.

5. Constraining Motion

The second way one may interpret the claim that the motions of the massive bodies should correspond to a discrete geometry is in the sense that such motions should satisfy the constraints that they would have to obey if space was actually discrete. This interpretation, as we shall see, is apparently supported by the role that jerky motion plays in van Bendegem's argumentation. But first, we should ask what those constraints are.

To that purpose, let us assume that both time and space are discrete. Then, moving at a speed of one hodon per chronon will be relatively unproblematic. But what about moving at other speeds? Objects moving at a lower speed than one hodon per chronon will apparently have to cover only part of a chronon per unit of time, which is not possible, since hodons cannot have proper parts. Symmetrically, objects moving at a speed higher than that will apparently have to cover a unit of space per a fraction of a chronon, which is not possible either. So, it seems that if both time and space are discrete, then everything should move at one hodon per chronon.

Van Bendegem (1995: 142-145) suggests two different ways of evading this unpalatable constraint. The former solution is that objects should display alternate patterns of motions and rest. This could allow them to move at an average rate of less than one hodon per chronon by, saying, moving at one hodon per chronon within a certain interval, resting for a while, and then continuing their motion at one hodon per chronon until reaching their final destination. This kind of motion has occasionally been called *staccato run* (Grünbaum 1973: 632). Van Bendegem calls it jerky motion – though, as we shall see, this is not the kind of jerky motion involved in his argument.

The latter solution envisaged by van Bendegem is meant to make room for motions faster than one hodon per chronon: it consists in allowing objects to move between distant places without crossing all the places in between. He calls this a case of annihilation and recreation, since it requires that objects moving that way should disappear from the place they are at a chronon and reappear at a distant spatial location at the immediately subsequent time. We may call it *teleportation*. These are the two kinds of motion that our observer should presumably observe, in order to be induced to think that space is discrete. So, will she?

One may be tempted to answer affirmatively, since what van Bendegem argument shows, if anything, is that if time is discrete and space is continuous, then motion must be jerky. But notice that the kind of jerky motion that objects will display in that case is rather different from the one that is required to move at a speed lower than one hodon per chronon, i.e. the staccato run.

Probably the easiest way of showing this is by appealing to the so-called *at-at* conception of motion, according to which motion is best analysed in terms of 'a correlation [...] between places and times', such that 'when different times, throughout any period however short, are correlated with different places, there is motion', and conversely, 'when different times, throughout some period however short, are all correlated with the same place, there is rest' (Russell 1938: 473).[8]

Following this account, being at rest requires being at the same place at more than one time, just as being in motion requires being at different places at more than one time. Therefore, doing the staccato run if time is discrete requires at least *four* chronons: one chronon to be in the starting position, one chronon to be at a different place, one chronon to persist in that position, and one chronon to get elsewhere. The first and second chronons jointly allow for the initial motion, the second and the third chronons jointly allow for the intermediate rest, and the third and fourth chronons make room for the final motion.

Moving in the jerky way implied by the conjunction of continuous space and discrete time, instead, only requires being at a place at one chronon and at a different place at the immediately subsequent chronon. Moving this way is surely not incompatible with the staccato run, but it does not entail it either, so our observer may not see any jerky motion of the latter kind.

What she will certainly see, on the other hand, is teleportation. Moving in continuous space if time is discrete, as we have seen, requires being at two distant places at subsequent chronons without ever being at any of the denumerably many intervening points, which is precisely the kind of abrupt

[8] Russell explicitly demands that a request for continuity should be in-built into this analysis of motion. However, he also recognizes that 'this is an entirely new assumption, having no kind of necessity, but serving merely the purpose of giving a subject akin to rational dynamics' (Russell 1938: 473). Extending the at-at account of motion to the present case only requires dropping this admittedly non-necessary assumption.

displacement in which teleportation consists. This may suggest that motion should appear to our observer as if time and space were both discrete, thereby supporting van Bendegem's conclusion. There is a good reason, however, why that conclusion should be resisted.

Teleportation is strictly implied by the conjunction of discrete time and continuous space, but not by the conjunction of discrete space and discrete time: in the latter case, in fact, objects travelling at an average rate not higher than one hodon per chronon must move between non-adjacent hodons without skipping the intermediate ones at least for part of their motion. Conceding that time is discrete, this means that teleportation is a characteristic feature of all continuous spaces in which motion is possible, but not of the discrete ones. Hence, there seems to be no reason why observing teleportation on continuous spaces should make them appear discrete. On the very contrary, it is reasonable to argue that, *at best*, if one could observe only teleportation, then she could have good grounds to think that space is continuous even though it is in fact discrete.

Perhaps a more precise way of making the same point is the following. Given that time is discrete, then in at least some discrete spaces there will exist a minimum *non-zero* speed such that all the objects moving at a speed equal to or lower than that one will have to move, at least in part, not by teleportation. No such lower limit, however, could ever be found in any continuous space. Faced with the problem of determining the microscopic structure of space, our observers will thus have to go in search for a similar threshold: if they could find it, then they will be certain that space is discrete; otherwise, they will have at least reliable inductive grounds to infer that it is not. Though they could never be sure if space is really continuous, nonetheless this certainly excludes that space should necessarily appear discrete to them. But this is all one needs to resist van Bendegem's conclusion.

6. Conclusion

Van Bendegem argues that, if time is discrete and space is continuous, then the motions of the massive bodies should correspond to a discrete geometry. From this claim, he concludes that, given discrete time, space does not properly become discrete, but it must nonetheless appear so. I have analysed two different ways of interpreting the above correspondence, and neither of them proved to support this conclusion.

On the one hand, one may interpret the declared correspondence between motions and a discrete geometry in the sense that the geometry constructed on the subsequent positions of the moving bodies is necessarily discrete. However, that interpretation would suffer from two major shortcomings. Firstly, van Bendegem's premises do not suffice to establish the existence of that correspondence. Secondly, drawing the desired conclusion from that kind of correspondence would require making unwarranted assumptions concerning the epistemic powers of any hypothetical observer.

On the other hand, the above correspondence may be interpreted in the sense that motions should obey the same constraints that they would have to adhere to if space was discrete. Contrary to the previous one, this correspondence is supported by van Bendegem's premises, which entail that, in the mixed case in which time is discrete and space is continuous, all displacements must take place through teleportation. However, this result does not entail that continuous space would then appear discrete. On the very contrary, it may be employed to support the converse claim that some discrete spaces could appear continuous.

The supporters of discrete time can thus freely contemplate the wonders of the mixed worlds where time is discrete but space continuous, without bothering about whether or not discrete spaces could exist as well.

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