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Correlations, Deviations and Expectations
The Extended Principle of the Common Cause

Claudio Mazzola
School of Historical and Philosophical Inquiry
The University of Sydney

Abstract

The Principle of the Common Cause is usually understood to provide causal explanations for probabilistic correlations obtaining between causally unrelated events. In this work, an extended interpretation of the principle is proposed, according to which common causes should be invoked to explain positive correlations whose values depart from the ones that one would expect to obtain in accordance to her probabilistic expectations. In addition, a probabilistic model for common causes is tailored which satisfies the generalized version of the principle, at the same time including the standard conjunctive-fork model as a special case.

1 Introduction

Imagine that you saw two distant geysers spouting at irregular intervals, but always simultaneously to each other: presumably you would not regard this as the unlikely result of mere chance, and you would imagine that both geysers were fed by a common underground reservoir. Similarly, if two or more actors in a stage play simultaneously showed symptoms of food poisoning, you would presumably suppose that they consumed the same poisoned meal; or if two light bulbs went off simultaneously in the same room, you would probably ascribe this to a breakdown interrupting their common power supply [8, pp. 157-158]. Reichenbach's Principle of the Common Cause governs inferences of this type, leading from observed but unlike coincidences to unobserved common causes.

Informally, the principle states that 'if an improbable coincidence has occurred, there must exist a common cause' [8, p. 157]; more precisely, it claims that for any pair of probabilistically correlated random events in a given classical probability space, neither of which is a cause of the other, there must exist a third random event which is a direct cause of both, and conditional on which the given probabilistic correlation disappears. The importance of this principle for contemporary philosophy of science can hardly be underestimated, its applications stemming from econometrics [7, 11] to the foundations of quantum mechanics [13, 5], but its scope is nonetheless generally over-restricted. The aim of this work is to propose an extended version of the principle, based on a natural generalization of its usual interpretation.

In the first place, we shall make a brief examination of the explanatory function which is attributed to common causes according to the usual interpretation of Reichenbach's principle; then, we shall see how that function is in fact a special case of a more general explanatory task. In particular, we shall argue that the function of common causes is to motivate local statistical deviations from the probabilistic behavior that one would expect to be displayed by pairs of random events. In consequence, we shall outline a more general probabilistic model for common causes, which will be proved to fulfill this explanatory role. Finally, we shall prove that a suitable extension of the given classical probability space can always be found, which includes a common cause of that type for any pair of random events whose statistical behavior locally disconfirms our probabilistic expectations.

2 Correlations

Let (Ω, P) be a classical probability space, Ω being a σ-algebra of random events and $P : \Omega \to [0, 1]$ being a probability distribution on Ω. Repeated coincidences between random events are modeled by positive *correlations*:

Definition 1. *Let (Ω, P) be a classical probability space. For any $A, B \in \Omega$, the* correlation *of A and B is the quantity*

$$Corr(A, B) \stackrel{def}{=} P(A \wedge B) - P(A)P(B). \tag{1}$$

Roughly, positive correlations between pairs of random events obtain just in case either event is more likely to occur in the presence of the other than otherwise. For this reason, positive correlations play a key role for the probabilistic account of causation: since it is reasonable to expect that effects are more likely to obtain in the presence of their causes, relations of positive causal dependence between random events should be expected to give rise to positive correlations between them.

The converse implication, however, might fail: positive correlations might well obtain between pairs of random events – such as the spouting of two distant geysers, or two actors simultaneously showing symptoms of food poisoning – neither of which is a positive cause of the other. Following the usual interpretation, the Principle of the Common Cause deals precisely with cases of this kind, offering causal explanations for positive correlations which can be given no straightforward causal meaning. Under this light, the principle is typically conceived as a theoretical device to establish a one-to-one correspondence between positive correlations and relations of positive direct causal dependence.

In accordance with this interpretation, common causes are expected to fulfill a twofold explanatory function: on the one hand, they must restore the probabilistic independence of their effects; on the other hand, they must show how the positive correlation between the two effects could have obtained in spite of their causal independence. These two demands are implicitly raised by the formal statement of the principle, according to which:

> **PCC** *If coincidences of two events A and B occur more frequently than would correspond to their independent occurrence, that is, if A and B are positively correlated, then there exists a common cause C for these events such that the fork (A, C, B) is conjunctive.*

Informally speaking, a conjunctive fork (A, C, B) is 'a fork which makes the conjunction of the two events A and B more frequent than it would be for independent events' [8, p. 159]. More precisely:

Definition 2. *Let (Ω, P) be a classical probability space. For any three distinct $A, B, C \in \Omega$, the ordered triple (A, C, B) is a* conjunctive fork *if and only if A, B and C satisfy the following* *relations:*

$$P(C) \neq 0 \textit{ and } P(\neg C) \neq 0 \tag{2}$$
$$P(A \wedge B|C) - P(A|C)\,P(B|C) = 0 \tag{3}$$
$$P(A \wedge B|\neg C) - P(A|\neg C)\,P(B|\neg C) = 0 \tag{4}$$
$$P(A|C) - P(A|\neg C) > 0 \tag{5}$$
$$P(B|C) - P(B|\neg C) > 0. \tag{6}$$

Conditions (5)-(6) demand that C is a positive cause of both A and B. Conditions (3)-(4), instead, demand that A and B are independent conditional on C and on $\neg C$; in other words, they require that the positive correlation of A and B disappears at the finer-grained level where the occurrence of C and $\neg C$ is respectively taken into account. This demand is usually called the *screening-off criterion*, conditions (3)-(4) accordingly being labeled the *screening-off conditions*. The screening-off criterion has been the focus of extended critiques, on which we shall return later in this paper. Intuitively, its role is to guarantee that no further *explanans* is needed for the positive correlation of A and B over and above the common cause C: in fact, conditional on C and $\neg C$, no positive correlation is left between events A and B for which any request of explanation might possibly be raised.

Reichenbach proved that whenever conditions (3)-(6) are satisfied, C increases the joint probability of A and B [8, pp. 160-161]:

Proposition 1. *Let* (Ω, P) *be a classical probability space. For any* $A, B, C \in \Omega$*, if* (A, C, B) *is a conjunctive fork, then the following inequality holds:*

$$P(A \wedge B|C) - P(A \wedge B|\neg C) > 0. \tag{7}$$

In addition, he demonstrated that conjunctive forks of type (A, C, B) invariably give rise to positive correlations between A and B [8, p. 160]:

Proposition 2. *Let* (Ω, P) *be a classical probability space. For any* $A, B, C \in \Omega$*, if* (A, C, B) *is a conjunctive fork, then*

$$Corr(A, B) = P(A \wedge B) - P(A)P(B) > 0. \tag{8}$$

These results confirm the intuition that non-causal positive correlations may be regarded as by-products of common causes which produce their effects in tandem. Conjunctive forks are thus the probabilistic model that common causes should satisfy to fulfill the desired explanatory task: on the one hand, the screening-off conditions (3)-(4) show the contingent character of the positive correlation between A and B, making it vanishing under the choice of suitable reference classes; on the other hand, Proposition 1 and Proposition 2 show how that correlation could have obtained in spite of the causal independence of the two events.

> When we say that a common cause C *explains* the frequent coincidence, we refer not only to [the] derivability of relation (8), but also to the fact that relative to the cause C the events A and B are mutually independent: a *statistical dependence* [i.e. a positive correlation] is here derived from an independence. The common cause is the connecting link which transforms an independence into a dependence [8, pp. 159-160].

In what follows, we shall see how the conjunctive fork model could be easily generalized so as to derive statistical dependences from correlations of whatever lesser entity.

3 Deviations and Expectations

Not all positive correlations, obviously, need a common-cause explanation; but how to distinguish between those which do, and those which do not? In other words: how do we one know if we can put a given positive correlation down to a genuine causal connection, or if we have rather to go in search of its still unobserved common cause?[1]

Let us go back to the two geysers example: in that case, the implicit assumption was made that no two distant geysers could exert any direct causal influence on each other. By the same token, in the poisoned actors example, the tacit hypothesis was made that symptoms of food poisoning could be attribute to no transmissible infective agent. In these and all similar cases, the demand for a common-cause explanation is raised because the correlated events are in fact *assumed* to be causally unrelated. This suggests that the Principle of the Common Cause, as it stands, cannot be applied in the absence of any prior presuppositions dictating the causal independence of the correlated pair.

Causal assumptions of this kind, in their turn, are typically motivated on statistical grounds: *in general*, geyser spouts or cases of food poisoning display no statistical correlation - so, *in general*, events of this type should be expected to be causally unrelated. Examples of this type accordingly suggest that the Principle of the Common Cause must be applied precisely to those events whose positive correlation locally disconfirms the statistical independence that is generally expected to obtain between events of that kind. The demand for a common-cause explanation is thus a response to the discrepancy between the observed statistical behavior of two random events in a restricted sample, and the probabilistic independence which one may expect to obtain between them in adherence to the available statistical knowledge base. Being unexpected is, accordingly, the demarcating line between positive correlations which call for a common cause explanation, and those which do not.[2] In fact, only those correlations which are implausible in the light of our

[1] At first sight, one may attempt to answer to this question by relying on Bayesian network methodologies, according to which the screening-off criterion should be systematically applied, in the more general form of the Causal Markov Condition, to *any* observed positive correlation: in accordance to this strategy, if no screening-off factor is found, then the observed correlation should be attributed to a genuine case of causal dependence [7, 11]. Unfortunately, this approach relies on a strong assumption of causal completeness: in other words, it assumes that no causal factor can ever be found outside the given range of empirically accessible variables or random events. The Principle of the Common Cause, on the contrary, demands that non-causal positive correlations should be explained by introducing previously unaccounted causal factors: in other words, it relies on the very opposite assumption that observed pairs of positive correlated but causally unrelated events are *not* causally complete. In consequence, the above strategy cannot be consistently employed to determine in what cases the Principle of the Common Cause should be applied. To illustrate this point more clearly, let us suppose that the Principle of the Common Cause *did* apply precisely to those cases which, according to the approach proposed above, are not cases of causal dependence. Since, according to that approach, the cases of direct causal dependence are exactly the ones for which no screening-off common cause is currently given, this would entail that the Principle of the Common Cause should only be invoked for those correlations whose screening-off common causes are already known. In consequence, the Principle of the Common Cause would be openly trivialized.

[2] One may denounce here a shift from ontology to epistemology: what correlations are non-causal is an objective matter of fact, independent of the subjective expectations that one may possibly have relative to their value; so, what correlations need to be explained in terms of common causes, and what do not, should not depend on epistemological criteria such as the conformity of such correlations to our probabilistic expectations. That shift, however, is dictated by unavoidable methodological needs. The Principle of the Common Cause, as we have seen, cannot be effectively employed without making reference to one's prior causal presuppositions. Standard treatments of the principle leave this epistemological prerequisite on the background, but the latter is nonetheless needed not to make the former methodologically empty. In locating the demarcating line between those correlations to which the principle applies, and those to which it does not, in their divergence from the probabilistic expectations underlying one's causal presuppositions, we are then simply making it explicit what any treatment of the principle must – at

background statistical knowledge we regard as 'improbable', and therefore at odds with the causal presuppositions that we develop on the basis of that knowledge: in Reichenbach's own words, correlations of this type give formal shape to the fact that 'the simultaneous happening of A and B is more frequent than can be *expected* for chancy coincidences' [8, p. 158, my emphasis].[3]

To provide this intuition with a more precise statement, let us first introduce the notion of *expected correlation*. For any two random events A and B, the expected correlation $Corr_{Exp}(A, B)$ is the correlation that a well-informed rational individual would expect to obtain between A and B according to her background statistical knowledge. More precisely, $Corr_{Exp}(A, B)$ is the correlation that A and B would exhibit in the ideal classical probability space that one could in principle construct from the entire available statistical information concerning the two events.[4] The difference between $Corr(A, B)$ and $Corr_{Exp}(A, B)$ is then labeled the *deviation* of $Corr(A, B)$:

Definition 3. *Let* (Ω, P) *be a classical probability space. For any* $A, B \in \Omega$*, the* deviation *of* $Corr(A, B)$ *is the quantity*

$$\delta(A, B) \stackrel{def}{=} Corr(A, B) - Corr_{Exp}(A, B). \tag{9}$$

The above examples showed that a common cause explanation should be invoked for $Corr(A, B)$ just case that correlation locally disconfirms our prior expectations concerning the causal independence of A and B. In other words, this means that the Principle of Common Cause applies precisely to observed positive correlations which generate positive deviations from expected correlations whose value is zero.

Under this light, the explanatory role of common causes can be reconceived as the one of showing, on the one hand, how the observed correlation of events A and B could have departed from their expected correlation and, on the other hand, how the latter can be conditionally reestablished. Following this interpretation, the screening-off conditions (3)-(4) demand exactly that the expected independence of A and B is restored conditional on C and $\neg C$, while Proposition 1 and Proposition 2 jointly show how $Corr(A, B)$ could have obtained from $Corr_{Exp}(A, B) = Corr\,(A, B\,|C) = Corr\,(A, B\,|\neg C) = 0$, generating a positive deviation.

least implicitly – do, in order to avoid trivialization.

[3]In fact, two kinds of factors seem to contribute in shaping this type of causal expectations: statistical and metaphysical. Metaphysical constraints include, for example, the assumption that causes must precede their effects, or the hypothesis that effects must be proportioned to their causes. These assumptions can bee seen at work in the well-known barometer-storm example: falls in barometer readings are, in general, positively correlated with later occurrences of storms in nearby areas; nonetheless, their correlation is typically explained by tracing both events back to the occurrence of a previous local fall in atmospheric pressure. In this case, the existence of a direct causal connection between the correlated pair is ruled out on the basis of the tacit hypotheses that (i) small processes, such as meteorological observations, cannot affect huge atmospheric phenomena such as storms, and that (ii) later phenomena cannot retroactively affect previous measurements. On the other hand, since metaphysical assumptions of this kind cannot be modeled probabilistically – they merely 'block' the causal interpretation of positive correlation, without imposing any constraint on their values – we shall not take them into account. For the purposes of our discussion, it will be sufficient to keep in mind that whatever positive correlation, albeit conforming to one's statistical expectations, may nonetheless be precluded any straightforward causal interpretation depending on non-probabilistic metaphysical assumptions.

[4]This picture, obviously, is highly idealized. In fact, one may expect that a positive correlation existed, say, between smoking and developing lung cancer though she had no idea of what the exact value of that correlation should be. Furthermore, no one could ever have access to the entire available statistical information concerning events A and B. However, this kind of idealization seems not to differ substantially from the one which is ordinarily required to model degrees of belief in the Bayesian approach to probabilities, or to obtain exact probability values from finite statistical samples.

This revised interpretation, by itself, adds nothing substantial to the usual understanding of the Principle of the Common Cause – expected independences commonly being concealed under the guise of tacit probabilistic assumptions, and positive deviations consequently be disguised as positive deviations *simpliciter*. However, once the explanatory action of common causes is so reconceived, there is no longer reason to restrict the domain of the Principle of the Common Cause to the sole positive correlations deviating from expected *independences*: in fact, positive deviations from non-zero expected correlations might obtain as well, e.g. in case the effects of a common cause are not causally independent of each other.

4 The Extended Principle of the Common Cause

Let us consider the following example. Encephalopathy is a direct effect of alcohol abuse due to a reduction in thiamine absorption; but on the other hand, encephalopathy may also obtain as consequence of cirrhosis – which is also among the effects of alcoholism – due to the fact that ammonia is not normally processed by the liver. Under normal circumstances, cirrhosis and encephalopathy are therefore positively correlated; but let us suppose that, in a given population, their correlation were greater than expected: in that case, one may suspect the existence of a common cause, such as excessive alcohol consumption, restoring their expected correlation. Cases of this kind are modeled by what we shall call *generalized conjunctive forks*:

Definition 4. *Let* (Ω, P) *be a classical probability space. For any three distinct* $A, B, C \in \Omega$, *the ordered triple* (A, C, B) *is a* generalized conjunctive fork *if and only if* A, B *and* C *satisfy the following relations:*

$$P(C) \neq 0 \text{ and } P(\neg C) \neq 0 \tag{10}$$
$$Corr\,(A, B\,|C\,) = Corr_{Exp}(A, B) \tag{11}$$
$$Corr\,(A, B\,|\neg C\,) = Corr_{Exp}(A, B) \tag{12}$$
$$P\,(A\,|C\,) - P\,(A\,|\neg C\,) > 0 \tag{13}$$
$$P\,(B\,|C\,) - P\,(B\,|\neg C\,) > 0. \tag{14}$$

Reichenbach's common causes were intended to make the conjunction of their effects more frequent than it would be for independent events; common causes satisfying the definition of a generalized conjunctive fork, instead, are meant to make the conjunction of their effects *more frequent than expected*, independently of the value of their expected correlation.

Conditions (10) and (13)-(14) are openly identical to conditions (2) and (5)-(6) in the definition of a conjunctive fork. Conditions (3)-(4), instead, obtain from (11)-(12) as special cases, notably whenever $Corr_{Exp}(A, B) = 0$. Definition 4 is thus a straightforward generalization of Definition 2. Let us now examine its adequacy.

In the first place, common causes satisfying the definition of a generalized conjunctive fork increase the joint probability of their effects, in exactly the same way as Reichenbach's common causes:

Proposition 3. *Let* (Ω, P) *be a classical probability space. For any distinct* $A, B, C \in \Omega$, *if* (A, C, B) *is a generalized conjunctive fork, then*

$$P\,(A \wedge B\,|C\,) - P\,(A \wedge B\,|\neg C\,) > 0. \tag{15}$$

Proof. Let (Ω, P) be a classical probability space, let $A, B, C \in \Omega$ and let (A, C, B) be a generalized conjunctive fork. Then, by (11) and (12), we get

$$P\,(A \wedge B\,|C\,) - P\,(A\,|C\,)\,P\,(B\,|C\,) = P\,(A \wedge B\,|\neg C\,) - P\,(A\,|\neg C\,)\,P\,(B\,|\neg C\,)\,, \tag{16}$$

and therefore

$$P\,(A \wedge B\,|C\,) - P\,(A \wedge B\,|\neg C\,) = P\,(A\,|C\,)\,P\,(B\,|C\,) - P\,(A\,|\neg C\,)\,P\,(B\,|\neg C\,)\,. \tag{17}$$

By (13) and (14), the term on the right-hand side must be strictly greater than zero; and as a consequence, so must be the one on the left.

□

It is worth noticing that inequality (15) was obtained from (16) irrespectively of the exact value of $Corr\,(A, B\,|C\,)$ and $Corr\,(A, B\,|\neg C\,)$. Proposition 1 is therefore but a special case of Proposition 3, obtaining whenever both terms in equality (16) are equal to zero.

In addition to this, generalized conjunctive forks invariably give rise to positive deviations, just like conjunctive forks give rise to positive correlations:

Proposition 4. *Let (Ω, P) be a classical probability space. For any three distinct $A, B, C \in \Omega$, if (A, C, B) is a generalized conjunctive fork, then*

$$\delta(A, B) = Corr(A, B) - Corr_{Exp}(A, B) > 0. \tag{18}$$

Proof. Let (Ω, P) be a classical probability space, let $A, B, C \in \Omega$ and let (A, C, B) be a generalized conjunctive fork. Then, thanks to (11) and (12), we may pose

$$Corr\,(A, B\,|C\,) = Corr\,(A, B\,|\neg C\,) = Corr_{Exp}(A, B) = \varepsilon, \tag{19}$$

that is:

$$P\,(A \wedge B\,|C\,) = P\,(A\,|C\,)\,P\,(B\,|C\,) + \varepsilon \tag{20}$$

and

$$P\,(A \wedge B\,|\neg C\,) = P\,(A\,|\neg C\,)\,P\,(B\,|\neg C\,) + \varepsilon. \tag{21}$$

Since by the theorem of total probability

$$\begin{aligned} Corr(A, B) \;=\; & P\,(A \wedge B\,|C\,)\,P(C) + P\,(A \wedge B\,|\neg C\,)\,P(\neg C) - \\ & -[P\,(A\,|C\,)\,P(C) + P\,(A\,|\neg C\,)\,P(\neg C)] \times \\ & \times[P\,(B\,|C\,)\,P(C) + P\,(B\,|\neg C\,)\,P(\neg C)], \end{aligned} \tag{22}$$

then, according to the above convention:

$$\begin{aligned} Corr(A, B) \;=\; & [P\,(A\,|C\,)\,P\,(B\,|C\,) + \varepsilon]P(C) + \\ & +[P\,(A\,|\neg C\,)\,P\,(B\,|\neg C\,) + \varepsilon]P(\neg C) - \\ & -[P\,(A\,|C\,)\,P(C) + P\,(A\,|\neg C\,)\,P(\neg C)] \times \\ & [P\,(B\,|C\,)\,P(C) + P\,(B\,|\neg C\,)\,P(\neg C)]. \end{aligned} \tag{23}$$

Few calculations then lead to

$$Corr(A,B) - \varepsilon = P(C)P(\neg C)[P(A|C) - P(A|\neg C)][P(B|C) - P(B|\neg C)]. \tag{24}$$

Since (13)-(14) force the term on the right-hand side to be strictly positive, we get:

$$Corr(A,B) - \varepsilon > 0 \tag{25}$$

which, given hypothesis (19) and Definition 3, is in fact nothing but (18).

□

Finally, conditions (11)-(12) were expressly shaped so as to conditionally re-establish the expected correlation between the effects of a common cause, just as the screening-off conditions (3)-(4) were introduced so as to conditionally re-establish their expected independence. Proposition 3, Proposition 4 and (11)-(12) accordingly show that generalized conjunctive forks perfectly emulate the explanatory behavior of conjunctive forks in the more general case in which no constraints are imposed on the value of the violated expected correlations, consequently providing generalized conjunctive forks with as much explanatory power as their special counterparts.

These results give further strength to the intuition underlying the definition of generalized conjunctive forks, accordingly motivating the introduction of a more general version of Reichenbach's principle, which we shall call the Extended Principle of the Common Cause:

> **EPPC** *If coincidences of two events A and B occur more frequently than expected, that is, if the deviation of $Corr(A,B)$ is positive, then there exists a common cause C for these events such that (A,C,B) is a generalized conjunctive fork.*

The advantages in substituting statement PCC with EPCC are many. In the first place, EPCC evidently deals with a broader range of cases than PCC, while PCC can be easily recovered from EPCC whenever the expected correlation between events A and B is null. Moreover, EPCC makes the role of background probabilistic assumptions more evident, incorporating them in the very probabilistic model of common causes under the guise of expected correlations. Finally, EPCC overcomes some of the standard objections which have been moved against PCC due to its commitment to the screening-off requirement.[5]

Probably the best-known objection of this type contends that the screening-off conditions (5)-(6) can only be satisfied by common causes whose effects are produced through independent mechanisms, while effects emerging from a unique indeterministic causal interaction would be positively correlated conditional on the latter [9, **?**]. This objection is easily met by EPCC, which replaces (5)-(6) with the much weaker conditions (13)-(14), according to which pairs of correlated effects do not necessarily become independent conditional on their common cause – as in the encephalopathy-cirrhosis example.

[5] Further objections to the Principle of the Common Cause involve cases of positive correlations for which no non-gerrymandered common causes can possibly be found, irrespectively of the probabilistic constraints that one might be willing to impose on them; in consequence, counterexamples of this type affect EPCC as much as PPC. They include, among others, examples of causally unrelated variables displaying similar laws of evolution [10], indeterministic Markovian processes [1], and equilibrium correlations [4].

One related critique is due to Nancy Cartwright, who accuses the screening-off conditions of imposing too strong probabilistic constraints to causation. In fact, they require that probabilities factorize in the following, unlikely way:

$$P\,(A \wedge B\,|C\,)\,P\,(\neg A \wedge \neg B\,|C\,) = P\,(A \wedge \neg B\,|C\,)\,P\,(\neg A \wedge B\,|C\,)\,, \tag{26}$$

$$P\,(A \wedge B\,|\neg C\,)\,P\,(\neg A \wedge \neg B\,|\neg C\,) = P\,(A \wedge \neg B\,|\neg C\,)\,P\,(\neg A \wedge B\,|\neg C\,)\,. \tag{27}$$

Cartwright condsiders these two conditions over-restrictive, since 'nothing in the concept of causality, or of probabilistic causality, constrains how nature must proceed' in setting what probabilities A and B should jointly take conditional on C and $\neg C$ [**?**, p. 109]. In this case, we should notice that conditions (5)-(6) do not solely imply (26)-(27), but are in fact equivalent to the latter. This is enough to guarantee that (13)-(14), which do not entail (5)-(6), neither entail (26)-(27). In consequence, the above objection does not apply to the more general case in which common causes are modeled by generalized conjunctive forks, as demanded by EPCC.

Finally, the screening-off requirement has been charged of being tacitly committed with a deterministic conception of causation [12]. This critique has presumably been inspired by the fact that deterministic common causes invariably screen-off their effects from each other. That very result, on the other hand, guarantees that non-screening-off common causes are necessarily not deterministic. In consequence, EPCC applies to non-deterministic causes in all those cases in which relations (13)-(14) do not reduce to (5)-(6).

5 Existence

Classical probability spaces may be too small to include a Reichenbachian common cause for each pair of positively correlated events. Hofer-Szabó, Rédei and Szabó proved that any classical probability space (Ω, P) can be embedded in a classical probability space (Ω', P') in which, for each of a finite number of pairs of positively correlated events in (Ω, P), a common cause satisfying conditions (3)-(6) exists [6]. In other words, classical probability spaces can always be extended so that a common cause explanation might be available, in principle,[6] for arbitrarily many positive correlations. The same is proved here for generalized conjunctive forks.

In the first place, we need formal tools for widening classical probability spaces in a rigorous way. So let us recall that, Ω and Ω' being two Boolean algebras, a function $\psi : \Omega \to \Omega'$ is called a *Boolean algebra monomorphism* of Ω in Ω' if and only if it is injective and it preserves all lattice operations, including complements. Then, we can define the *extensions* of classical probability spaces as follows:

Definition 5. *Let (Ω, P) and (Ω', P') be classical probability spaces. The space (Ω', P') is called an* extension *of (Ω, P) if and only if there exists a Boolean algebra monomorphism $\psi : \Omega \to \Omega'$ such that, for all $X \in \Omega$,*

$$P(X) = P'(\psi(X)). \tag{28}$$

Condition (28) demands that all positive correlations are carried over intact while moving from (Ω, P) to (Ω', P'); assuming that expected correlations are left unchanged as well, it follows that all positive deviations are preserved by ψ.

[6]The sole existence of a classical probability space including a common cause for a given positively correlated pair does not, by itself, guarantee that such cause is in fact a genuine, non-gerrymandered event. The same limitation obviously applies to the following demonstration.

Once the notion of extension is given, Hofer-Szabó, Rédei and Szabó's pf can be decomposed into three main steps. In the first place, they define a cluster of mathematical restrictions which the values of probabilities $P(C)$, $P(\neg C)$, $P(A|C)$, $P(A|\neg C)$, $P(B|C)$ and $P(B|\neg C)$ should satisfy so that (A, C, B) is a conjunctive fork. In the second place, they show that for any non-zero value of $Corr(A, B)$ a two-parameter family of real numbers satisfying the given set of constraints exists. Finally, for any finite number n of positively correlated pairs $\{A^i, B^i\}_{i=1}^n$ in a given classical probability space (Ω, P), they build an extension (Ω', P') of (Ω, P) such that, for any $i = 1, ..., n$, events $C^i, \neg C^i$ are included in Ω' and the values of probabilities $P'(C^i)$, $P'(\neg C^i)$, $P'(A^i|C^i)$, $P'(A^i|\neg C^i)$, $P'(B^i|C^i)$ and $P'(B^i|\neg C^i)$ are identical to the real numbers satisfying the required constraints for $Corr(A^i, B^i)$.

Our pf will proceed along very similar tracks, the major differences consisting in changing the set of mathematical constraints which are generally imposed on the values of probabilities $P(C)$, $P(\neg C)$, $P(A|C)$, $P(A|\neg C)$, $P(B|C)$ and $P(B|\neg C)$, and in substituting sets of positive correlations with sets of positive deviations.

Hofer-Szabó, Rédei and Szabó call each set of real numbers satisfying the required conditions for $Corr(A, B)$ *admissible* for that correlation. Similarly,

Definition 6. *Let (Ω, P) be a classical probability space. For any $A, B \in \Omega$, real numbers $a_c, a_{\neg c}, b_c, b_{\neg c}, c, \neg c$ are called* admissible *for $\delta(A, B)$ if and only if $\delta(A, B) > 0$ and they satisfy the following conditions:*

$$0 < c, \neg c < 1 \tag{29}$$
$$0 \leq a_c, a_{\neg c}, b_c, b_{\neg c} \leq 1 \tag{30}$$
$$P(A) = a_c c + a_{\neg c} \neg c \tag{31}$$
$$P(B) = b_c c + b_{\neg c} \neg c \tag{32}$$
$$P(A \wedge B) = a_c b_c c + a_{\neg c} b_{\neg c} \neg c + Corr_{Exp}(A, B) \tag{33}$$
$$a_c > a_{\neg c} \tag{34}$$
$$b_c > b_{\neg c}. \tag{35}$$

Conditions (29)-(35) are all identical to Hofer-Szabó, Rédei and Szabó's conditions for admissible numbers, with the sole exception of (33), namely the sole condition which depends on the screening-off conditions (3)-(4); on the other hand, Hofer-Szabó, Rédei and Szabó's corresponding condition is easily recovered from (33) by posing $Corr_{Exp}(A, B) = 0$. The adequacy of (29)-(35) is testified by the following statement:

Proposition 5. *Let (Ω, P) be a classical probability space. For any $A, B, C \in \Omega$, the ordered triple (A, C, B) is a generalized conjunctive fork if and only if there exist real numbers a_c, $a_{\neg c}$, b_c, $b_{\neg c}$, c, $\neg c$ such that*

$$P(C) = c \tag{36}$$
$$P(\neg C) = \neg c \tag{37}$$
$$P(A|C) = a_c \tag{38}$$
$$P(A|\neg C) = a_{\neg c} \tag{39}$$
$$P(B|C) = b_c \tag{40}$$
$$P(B|\neg C) = b_{\neg c} \tag{41}$$

and a_c, $a_{\neg c}$, b_c, $b_{\neg c}$, c, $\neg c$ are admissible for $\delta(A, B)$.

Proof. Let (Ω, P) be a classical probability space and let $A, B, C \in \Omega$; furthermore, let $a_c, a_{\neg c}, b_c, b_{\neg c}, c, \neg c$ be real numbers satisfying identifications (36)-(41).

If (A, C, B) is a generalized conjunctive fork, then condition (29) is guaranteed by (10), conditions (30)-(32) are guaranteed by the axioms of probability theory, condition (33) obtains due to (11)-(12) and the theorem of total probability, and conditions (34)-(35) are guaranteed by (13)-(14). Hence, by Definition 6, the numbers $a_c, a_{\neg c}, b_c, b_{\neg c}, c, \neg c$ are admissible for $\delta(A, B)$.

Conversely, if the numbers $a_c, a_{\neg c}, b_c, b_{\neg c}, c, \neg c$ are admissible for $\delta(A, B)$, then (10) is guaranteed by (29), (11)-(12) are guaranteed by (29), (33) and the theorem of total probability, while (13)-(14) are guaranteed by (29) and (34)-(35). Therefore, by Definition 4, (A, C, B) is a generalized conjunctive fork.

$\square$

Proposition 5 and Definition 6 jointly determine the set of mathematical constraints that the values of probabilities $P(C)$, $P(\neg C)$, $P(A\,|C)$, $P(A\,|\neg C)$, $P(B\,|C)$ and $P(B\,|\neg C)$ should in principle satisfy so that a generalized conjunctive fork might exist for each positive deviation $\delta(A, B)$. The actual existence of such values, instead, is established by the following:

Lemma 1. *Let (Ω, P) be a classical probability space. For any $A, B \in \Omega$, if $\delta(A, B) > 0$ then a two-parameter family $(a_c(x, y)$, $a_{\neg c}(x, y)$, $b_c(x, y)$, $b_{\neg c}(x, y)$, $c(x, y)$, $\neg c(x, y))$ of admissible numbers exists for $\delta(A, B)$.*

Proof. Let (Ω, P) be a classical probability space, let $A, B \in \Omega$ and let $\delta(A, B) > 0$; furthermore, let us choose $a_c = x$ and $b_c = y$ as parameters. Given the system of three equations (31)-(33), the following equalities obtain:

$$P(C) = \frac{\delta(A, B)}{[x - P(A)][y - P(B)] + \delta(A, B)} \tag{42}$$

$$P(A\,|\neg C) = P(A) - \frac{\delta(A, B)}{y - P(B)} \tag{43}$$

$$P(B\,|\neg C) = P(B) - \frac{\delta(A, B)}{x - P(A)}. \tag{44}$$

Using equations (42)-(44), it would be easy to verify that, choosing x and y within the bounds

$$1 \geq x \geq P(A\,|B) - \frac{Corr_{Exp}(A, B)}{P(B)} \tag{45}$$

$$1 \geq y \geq P(B\,|A) - \frac{Corr_{Exp}(A, B)}{P(A)}, \tag{46}$$

conditions (29)-(35) are satisfied. By Definition 6, numbers $a_c(x, y)$, $a_{\neg c}(x, y)$, $b_c(x, y)$, $b_{\neg c}(x, y)$, $c(x, y)$, $\neg c(x, y)$ are therefore admissible for $\delta(A, B)$.

$\square$

It is interesting to notice that, for $Corr_{Exp}(A, B) = 0$, equations (42)-(46) reduce to Hofer-Szabó, Rédei and Szabó's corresponding equations.

For any positive deviation $\delta(A, B)$, let us say that event C is a common cause of *type* $(a_c, a_{\neg c}, b_c, b_{\neg c}, c, \neg c)$ for $\delta(A, B)$ just in case A, B and C satisfy identities (36)-(41). Then:

Definition 7. *Let* (Ω, P) *be a classical probability space and let* $A, B \in \Omega$ *such that* $\delta(A, B) > 0$. *For any classical probability space* (Ω', P'), *we say that* (Ω', P') *is a* common cause completion of type $(a_c,\ a_{\neg c},\ b_c,\ b_{\neg c},\ c,\ \neg c)$ for $\delta(A, B)$ *if and only if* (Ω', P') *is an extension of* (Ω, P) *and there exists* $C \in \Omega'$ *such that* C *is a common cause of type* $(a_c,\ a_{\neg c},\ b_c,\ b_{\neg c},\ c,\ \neg c)$ *for* $\delta(A, B)$.

Definition 8. *Let* (Ω, P) *be a classical probability space and let* $\{A^i, B^i\}_{i=1}^n$ *be such that, for any* $i = 1, ..., n$, $A^i, B^i \in \Omega$ *and* $\delta(A^i, B^i) > 0$; *we say that* (Ω, P) *is* common cause compleatable for $\{A^i, B^i\}_{i=1}^n$ *if and only if, given any set of admissible numbers* $(a^i_c, a^i_{\neg c}, b^i_c, b^i_{\neg c}, c^i, \neg c^i)$ *for* $\delta(A^i, B^i)$ $(i = 1, ..., n)$, *there exists a classical probability space* (Ω', P') *which is a common cause completion of type* $(a^i_c, a^i_{\neg c}, b^i_c, b^i_{\neg c}, c^i, \neg c^i)$ *for* $\delta(A^i, B^i)$.

Proposition 6. *Every classical probability space is common cause compleatable for any finite set of events the deviation of whose correlation is positive.*

Proof. Proof of Proposition 6 is essentially identical to the pf of Hofer-Szabó, Rédei and Szabó's Proposition 2, the sole difference consisting in replacing families of admissible numbers for positive correlations with sets of admissible numbers for positive deviations, in accordance with Definition 6, Proposition 5 and Lemma 1.

□

6 Conclusion

Moving from a re-examination of the explanatory role of conjunctive common causes, I proposed a revised and extended formulation of the Principle of the Common Cause, along with a suitably generalized probabilistic model for explanatory common causes. This model not only proved to be as much explanatorily satisfactory as conjunctive forks, but also capable to overcome some of the difficulties conjunctive forks are usually faced with due to their commitment with the screening-off condition. Finally I proved that all classical probability spaces can be extended so as to include any finite number of common causes, so revised, just as they were for conjunctive common causes. These results jointly confirm the adequacy of the extended interpretation of the Principle of the Common Cause, according to which common causes should be invoked to explain positive correlations generating non-zero deviations from expected correlations, rather than positive correlations as such.

References

[1] Arntzenius F. (1992) The common cause principle. In: Hull D. L., Forbes M., Okruhlik K. (eds) *PSA 1992: Proceedings of the 1992 Biennial Meeting of the Philosophy of Science Association, Vol. 2: Symposia and Invited Papers*, pp 227-237.. The University of Chicago Press, Chicago

[2] Cartwright, N. (1988). How to tell a common cause: Generalizations of the conjunctive fork criterion. In Fetzer, J. H. (ed) *Probability and causality essays in honor of Wesley C. Salmon*, pp. 181-188. Dordrecht: Reidel.

[3] Cartwright N. (1999) *The dappled world. A study of The boundaries of science*. Cambridge University Press, Cambridge.

[4] Forster M. (1986) Unification and scientific realism revisited. In Fine A., Machamer P. (eds) *PSA 1986: Proceedings of the 1986 Biennial Meeting of the Philosophy of Science Association, Vol. 1: Constributed Papers* pp 394-405. The University of Chicago Press, Chicago.

[5] Grasshoff G., Portmann S., Wüthrich A. (2005) Minimal assumption derivation of a bell-type inequality. *British Journal for the Philosophy of Science* 56(4): 663-680

[6] Hofer-Szabó, G., M. Rédei and L. Szabó (1999). On Reichenbach's common cause principle and Reichenbach's notion of common cause. *British Journal for the Philosophy of Science* 50: 377-399.

[7] Pearl J. (2000) *Causality: Models, reasoning and inference.* Cambridge University Press, Cambridge

[8] Reichenbach H. (1956) *The direction of time.* The University of California Press, Berkeley

[9] Salmon W. C. (1984) *Scientific explanation and the causal structure of the world.* Princeton University Press, Princeton.

[10] Sober, E. (1988). The principle of the common cause. In Fetzer, J. H. (ed) *Probability and causality essays in honor of Wesley C. Salmon*, pp. 211-228. Dordrecht: Reidel.

[11] Spirtes P., Glymour C., Scheines R. (2001) *Causation, prediction, and search* (2nd ed.). Springer, New York.

[12] van Fraassen B. (1980) *The scientific image*. Clarendon Press, Oxford

[13] van Fraassen B. C. (1982) The charybdis of realism: Epistemological implications of BellâĂŹs inequality. *Synthese* 52(1): 25-38