Visual patterns and the development of creativity and functional reasoning[6]

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Abstract

Creativity has been recently recognized as an important factor of progress. However, its development has not been a priority of the school and, in particular, of Mathematics. Also, it has not been a topic greatly studied, especially in what concerns 'normally' (not gifted) students. On the other hand, students often reveal many difficulties in functional reasoning. Some studies have concluded that an adequate exploration of visual patterns may contribute to overcoming these difficulties. However, they suggest that we should continue to study their impact on other publics and in other contexts. Thus, we developed a qualitative (exploratory) case study aiming to find out to what extent the exploitation of tasks focused on visual patterns contributed to the development of creativity and functional reasoning in students of the 8th grade. Participant observation, inquiry and documental analysis were the main sources of data collection, supported by various instruments. The data collected were submitted to content analysis, guided by categories. Some of them emerged from the research questions and others were defined during the analysis. Generally, it was found that students improved their performance in tasks which resolution required the mobilization of functional reasoning. There was also a remarkable improvement of creativity in what concerns fluency, flexibility and originality. Some of the representations about creativity also evolved positively.

Keywords: Creativity, visual patterns, Algebra, Functional reasoning

Introduction

Life in modern society requires that people be creative, thus capable of producing innovative solutions to the problems they deal with. Unexpectedly creativity was found to be a transversal competence, shared by all content areas. School in general and Mathematics in particular are not aware of this reality and thus do not contribute to the development of creativity in the pupils, because they control excessively their reactions (Robinson & Aronica, 2009).

In Portugal, recent guidelines on education tend to value algebra a lot (ME – DEB, 2001; Ponte, Serrazina, Guimarães, Breda, Guimarães, Sousa, Menezes, Martins & Oliveira, 2007)

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and, consequently, functional reasoning. As a matter of fact, developing functional reasoning is one of the most important goals of algebra. Nevertheless, students reveal much difficulty in functional reasoning (Warren, 2000; Barbosa, 2010; Tanisli, 2011).

According to several authors (e.g. Blanton & Kaput, 2004; Warren & Cooper, 2008; Vale, Barbosa, Barbosa, Borralho, Cabrita, Fonseca & Pimentel, 2011), making the students *see* visual arrangements and detect the underlying structure could help them to improve their performance in functional reasoning. Moreover, visual representations could also lead to more creative findings (Barbosa, 2010).

These were the basic ideas of a study we conducted, mainly focused on: i) developing creativity, ii) improving functional reasoning and iii) using visual patterns exploration as a tool to develop crossover and specific mathematical competences. Its main objective was to identify the representations of students attending the 8th grade on creativity and to evaluate the impact of the implementation of a didactic sequence about "Sequences and regularities", using tasks focused on the exploration of visual patterns and the discussion of the several solutions found, on the development of creativity and functional reasoning.

Theoretical Framework

Teachers are aware of the declining of the interest for Mathematics and the abilities it develops in the students. For Vale et al. (2011), this reality results from the fact that more and more students see Mathematics as an ensemble of procedures one must know by heart.

But an effective learning of Mathematics demands that the students are actively engaged in diverse and significant tasks (Doyle, 2007; Stein & Smith, 2009). According to NCTM (2000), students must do routine tasks, but they should also be involved in good tasks – the ones that make them aware of essential mathematical ideas by presenting them as challenges to overcome.

At the same time, mathematicians and researchers on mathematic education defend that exploring patterns is the essence of Mathematics (e.g. Davis & Hersh, 1995; Orton & Orton, 1999; NCTM, 2000; Devlin, 2002). Several studies were conducted showing that tasks involving repeating, growth, linear, nonlinear, numeric and figurative patterns promote algebraic thinking, including symbolism attached to it, and the development of the ability to generalize and thus the exercise of functional reasoning (e.g. Stacey, 1989; Rivera & Becker, 2005; Lee & Freiman, 2006; Amit & Neria, 2008; Radford, 2008; Vale & Cabrita, 2008).

Thus, tasks focused on pattern exploration may be an interesting tool for the development of mathematical abilities. Visualization is recognized by many experts in education in mathematics as an essential ability to develop. According to Vale et al. (2012), it must not be seen as mere illustration; it must be considered as an important piece in reasoning, problem solving and proof.

Seeing that a characteristic of the data is repeated may help to identify a pattern. Lee & Freiman (2006) state that *seeing* a pattern is the first step towards the ability to identify and explore patterns.

Seeing in different ways implies, for example, the ability to identify disjointed sets that, once they are assembled, compose the original figure. This is called *constructive generalization* (Rivera & Becker, 2008). One strategy suggested by Rivera (2007) in this context is supported by a kind of symmetric numbering: the students identify the symmetry in the figures they are presented, count the elements in one of the parts and multiply the number of elements of that part by the number of equal parts. But *seeing* may also imply the observation of superposed subsets, counting some of those elements several times and then subtract. This is a *deconstructive generalization* (Rivera & Becker, 2008). Barbosa (2010), based on Rivera & Becker (2008) and Taplin (1995), concludes that the pupils tend to use more frequently constructive than deconstructive generalizations, because the latter imply a greater level of visualization.

Creativity is the ability to produce something that is original and useful at the same time (Sternberg & Lubart, 1999). It is not exclusive of scientists or artists: we all use it in everyday life (Pehkonen, 1997).

Although there is no consensual definition for creativity, we chose the one presented by Torrance (1974), which includes four elements: fluency, flexibility, originality and elaboration. Fluency is the ability to generate a great number of ideas and refers to the continuity of those ideas, use of basic knowledge and flow of associations. It can be measured by the number of correct responses, solutions, proposed by the student during the same task (Silver, 1997; Conway, 1999). Flexibility is the ability to produce different categories or perceptions, whereby there is a variety of different ideas about the same problem or thing. It reflects when students show the capacity of changing ideas among solutions. It can be measured with the number of different categories of solutions that the student can produce. Originality is the ability to create unique, unusual, totally new or extremely different ideas or products. It can be measured analyzing the number of responses in the categories that were identified as original, by comparison with the number of students in the same group that could produce the same solutions. With regard to Mathematics, originality may be manifested when a student analyzes many solutions to a problem, methods or answers and then creates a different one (Silver, 1997; Leikin, 2009; Vale et al., 2012). Elaboration is related to the presentation of a large amount of details in one idea (Adams & Hamm, 2010).

Method

The method used to accomplish the investigation was a qualitative one (Bogdan & Biklen, 1994), focused on an exploratory case study (Yin, 2010). The data collection was directed to twenty-five 8th grade students, the whole class and, in particular, to three pairs of students: Manuel and Gonçalo, because Manuel's vision of creativity proved to be completely different from his classmates; Joana and António, because this pair used unique and more complex methods in the tasks' resolution; Margarida and Daniela, because their resolutions were quite similar to the remaining pairs of students' resolutions.

The main sources of data collection were: i) participant observation by the teacher/researcher, supported by audio and photographic records of the work done in class, field notes and logbook, ii) inquiry, through questionnaires and interviews with the case students and iii) a documentary analysis of a variety of documents - the students' tasks resolutions, the test implemented at the beginning and the end of the study and some official documents produced by the school.

To begin with, we passed a questionnaire, divided into two parts: i) characterization and ii) representations on creativity in Mathematics.

Then, we passed a pre-test, previously validated with students from another class of the 8^{th} grade, in the same school, similar to the one that took part in our study. This validation procedure showed that it was not necessary to change anything neither in the test, nor in the conditions of its application.

The test included six questions. The first one presented three sequences: two pictorial ones and a numeric one. The students were asked to mention the two following terms for each of them. In the second question, they were invited to explore different ways of counting symbols present in a picture and write down the corresponding numeric expressions. The third question presented a situation opposite to this one. The students were given a picture and a numeric expression and they had to draw a way of "seeing" corresponding to the expression presented. The fourth and fifth questions concerned respectively the recognition of an ABCCD, … repeating pattern and a growth pattern. Both asked the students to write an algebraic expression referring to a distant element. In the last question, the pupils are asked to create a sequence of drawings, using a certain formation law.

Then, we implemented the didactic intervention, consisting of a sequence of tasks previously validated and presented (Vale et al., 2011).

The first two tasks was based on the idea that visual arrangement plays an important role in finding calculation strategies more simple and intuitive (Vale et al., 2011) and it were related to visual count. First of all, we presented visual arrangements and we asked the students: i) to explore different ways of counting the symbols included in those visual arrangements and ii) to write the corresponding numeric expressions. Still within these tasks, the students should find a way of *seeing* a visual arrangement using the corresponding numeric expression as a start and other ways of seeing it.

In the following session, the students were given a document presenting the solutions some of them had proposed and a scale of creativity (see Figure 1) and asked to use letters to indicate the assessment they made of each solution and to justify their answer.

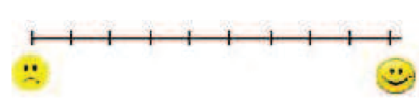


Fig. 1. Scale of creativity

Then, there were five tasks focused on: the identification and description of repeating patterns and growth patterns and their continuation; the identification of the position of certain elements of the module; using functional reasoning to determine distant elements; writing the algebraic expression that allows to determine the position of a certain element of the module; creating visual representations of a given sequence.

After the sixth task, the students were given another document presenting the solutions some of them had proposed and invited to assess them using the scale of creativity.

During all the sessions, the students were organized in pairs (although there was a group of three elements), because they had been working like this since the beginning of the school year. They had to present their solutions to the teacher and the other pupils and the underlying strategies were discussed in the class, in order that everybody reflected on the work done by each pair. The main ideas were registered.

By the end of each class, we collected the students' productions. The field notes were analyzed as soon as possible and they were used to improve the logbook. All these documents and the audio registers were analyzed before the following session, so that the plan could be changed, if necessary.

Two months after the didactic intervention, we passed the post-test and the questionnaire the students had answered in the beginning of the study. This repetition of the questionnaire was intended to collect data allowing us to determine if there had been any changes in their representations on creativity. The pre-test and the post-test had double aims: the initial one gave us an image of the knowledge and competences the pupils had before the didactic intervention and helped to adapt it to them, and the final one allowed us to assess what they had learned concerning sequences and regularities.

All the data collected were the object of content analysis using categories related to: i) the dimensions of creativity – fluency, flexibility and originality –, ii) representations on creativity – novelty, originality, simplicity and others found during the didactic intervention and iii) reasoning – functional and nonfunctional.

We also tried to analyze the main strategies used to see the patterns – constructive and deconstructive. We equally took into account aspects such as the reading direction – horizontal, vertical, oblique and mixed –, the form – rectangular, square, triangular, … – and the existence or absence of symmetry.

Results

In what concerns the representations on creativity, taking into account the analysis of the answers to the first questionnaire, we concluded that the concept of creativity presented by Gonçalo, Joana, António, Margarida and Daniela was related to the idea of generating something new, original and different from the usual: *"For me being creative means making something that does not exist yet, i.e. creating something original or even completely new."* and *"For me being creative means being imaginative, to create something unusual."*

The analysis of the answers given by those students to the second questionnaire revealed that they were associating creativity to complexity, an idea emphasized by Meissner (2011). That idea was already present in the tasks involving the use of the scale of creativity, probably because the students were taking into account strategies related to deconstructive resolution (Rivera & Becker, 2008) presented by their colleagues.

However, Manuel, whose representation stayed the same from the beginning to the end of the study, related creativity to simplicity (*"For me, being creative is being capable of solving* *everyday problems in a simple and fast although effective way."*), thus going against the idea expressed by the literature on this topic (Meissner, 2011).

In the beginning, only Gonçalo and Margarida considered that one could be creative in every subject. In their answers to the final questionnaire, all the other students revealed they were aware of this fact: Joana and Margarida mentioned all the subjects, António excluded the mother tongue and Daniela included Mathematics.

The six kept considering that teacher could be creative in Mathematics, but Joana and António thought this was not possible for the students. Their answers to the final questionnaire have shown that their opinion had changed.

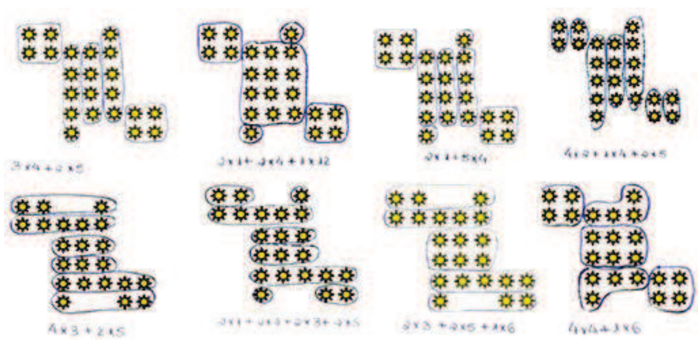
Thanks to this study, these students got aware of the fact that creativity can be a result of team work (Levenson, 2011). We must emphasize that only Joana did not think so in her answers to the first questionnaire.

Most of these students believed since the beginning that creativity may be developed at school, but they thought this institution could be responsible for its underdevelopment (cf. Robinson & Aronica, 2009). Only Gonçalo disagreed with the idea that school did not allow the development of creativity in his answer to the final questionnaire.

There were different opinions in what concerned the possibility of assessing students' creativity: Manuel, Margarida and Daniela thought this was possible, but the others disagreed, according to their answers to both questionnaires.

This happened also for the following statements: "In Mathematics, everything is created, no one can create anything new.", "In Mathematics, you cannot be creative: there is only one answer.", "Mathematics is a creative subject, such as music and arts.", "Creativity must be present in Mathematics classes, so that the pupils can learn better." In their answers to the final questionnaire, all these students disagreed with the two first statements and agreed with the other two. Nevertheless, in the initial questionnaire, Joana and António had agreed with the two first statements, Daniela with the second and Gonçalo e Margarida with the third. All of them agreed with the fourth statement in both questionnaires.

In what concerns the modalities of creativity, the analysis of the answers given to the pre-test and the post-test revealed that all the students improved in terms of fluency: they presented more and more ways of *seeing* (cf. figures 2 and 3).



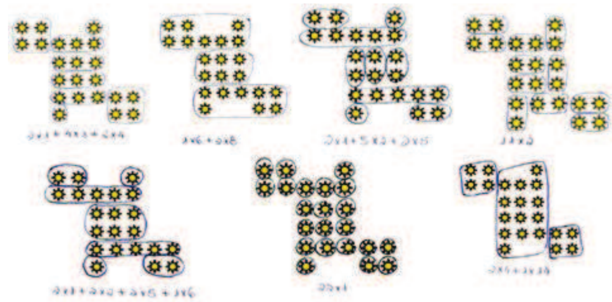


Fig. 2. Ways of counting and corresponding numeric expressions presented by Manuel in his answer to the second question in the pre-test

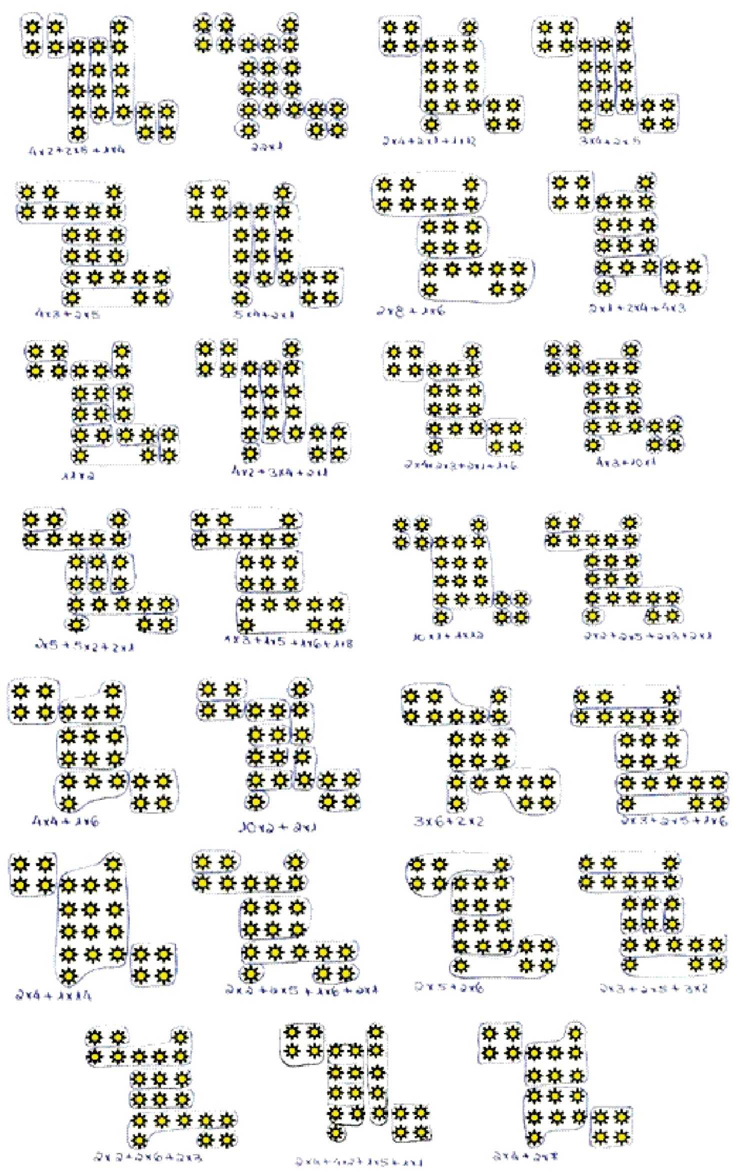


Fig. 3. Ways of counting and corresponding numeric expressions presented by Manuel in his answer to the second question in the post-test

Besides, Gonçalo, António and Margarida revealed a better performance in terms of flexibility, using more deconstructive strategies in the post-test (cf. figure 4) than they had been using during the study.

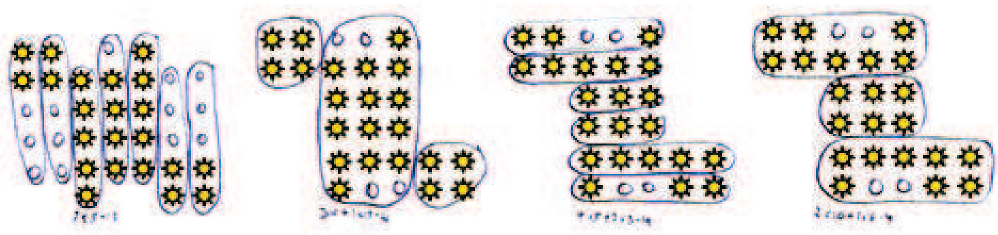


Fig. 4. Examples of ways of counting and corresponding numeric expressions presented by Margarida in her answer to the second question in the post-test

As for originality, Gonçalo, Joana and António presented ways of counting that were referred by very few pupils in the whole class. Joana's ways of counting were exclusive of her (cf. figure 5).

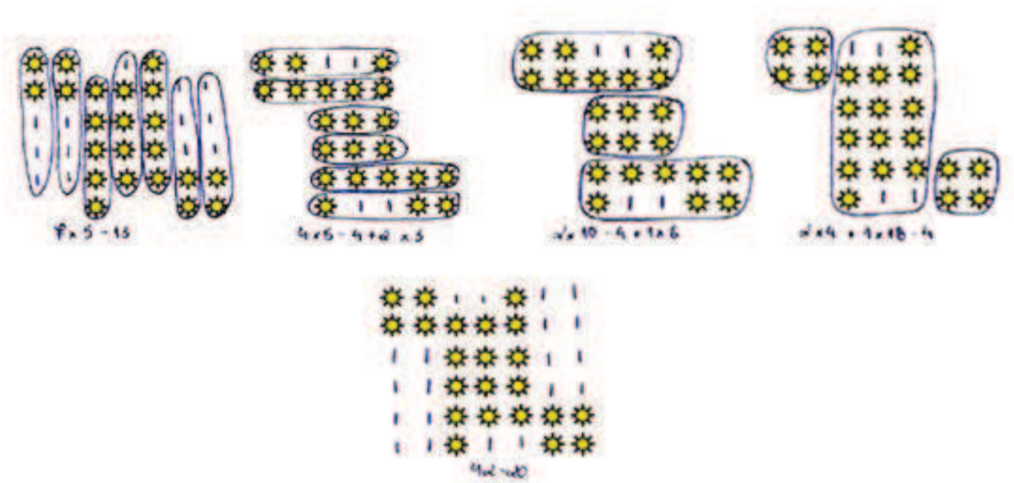


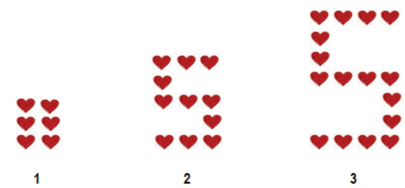
Fig. 5. Examples of ways of counting and corresponding numeric expressions presented by Joana in her answer to the second question in the post-test

Most visual presentations included symmetry, horizontal, vertical and mixed reading, and rectangular, quadrangular and hexagonal geometrical forms.

In terms of reasoning, the analysis of the answers given to the three questions concerning this aspect in the pre-test and the post-test revealed that there was a positive evolution – from recursive reasoning to functional reasoning – in Manuel and Daniela and especially António. Thus, evolution occurred gradually, as the different tasks proposed were solved. For example, in the resolution of the third task, Joana/António and Margarida/Daniela used alternatively recursive and functional reasoning, while in the fourth task all of them used only functional reasoning.

In figure 6 are presented two examples of the answers given to this question:

"Francisco and Madalena met five years ago… and it was love at first sight. In Valentine's Day, both draw figures made of hearts. (…) In figures 2 and 3, you must draw different ways of seeing the forms they have drawn. Please explain how you can obtain the number of hearts featured in the seventh figure without drawing it, just using these ways of 'seeing' "



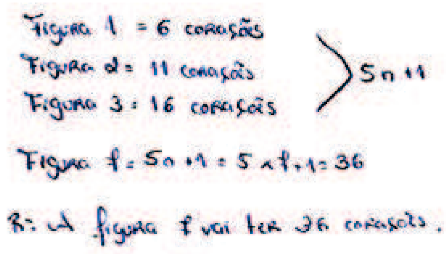


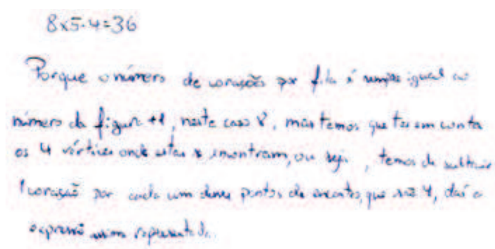
Figure 1 = 6 hearts

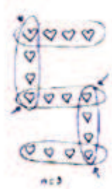
Figure 2 = 11 hearts *5n + 11*

Figure 3 = 16 hearts

Figure 7 = 5n + 1 = 5 x 7 + 1 = 36

A: Figure 7 will have 36 hearts.





8 x 5 = 36

Because the number of hearts per rang is always equal to the number of the figure + 1, thus 8 in the present example, but we must consider the 4 vertices to which they converge, i.e. we must subtract 1 heart in each point they meet and they are 4, thus the numeric expression we presented.

Fig. 6. Examples of answers given to the fourth question in task 4

Final remarks

Our study revealed considerable improvement in what concerned: i) the different dimensions of creativity – fluency, flexibility and originality –; ii) representations on creativity and iii) the use of functional reasoning.

Thus, we conclude that the effect of the implementation of the didactic sequence on "Sequences and regularities" was very positive and we relate this success to the fact that the tasks proposed were focused on visual patterns and also the dynamics developed in the classroom, which included the resolution of the tasks by the students followed by their presentation and discussion and the assessment of the different solutions that had been found.

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