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# A Delegated Agent Asset-pricing model

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"A man's got to know his limitations."

Harry Callahan

# A Delegated Agent Asset-pricing model

#### Abstract

Asset pricing theory has traditionally made predictions about risk and return, but has been silent on the actual process of investment. Today most investors delegate major investment decisions to financial professionals. This suggests that the instructions given by investors to their delegated agents and the compensation of those agents might be important determinants of capital market equilibrium. In the extreme when all investment decisions are delegated, the preferences and beliefs of individuals would be completely superseded by the objective functions of agent/managers. A provocative illustration of the difference between direct and delegated investing is provided based on active asset management relative to a benchmark index, a common objective function in practice. With the growing preponderance of delegated investing, future asset pricing theory will not only have to describe risk and return but, to be complete, must also be able to explain the observed objective functions used by professional managers.

#### 1. Introduction

Since the path breaking working of Sharpe (1964) and Lintner (1965) one of the primary goals of financial research has been to extend the basic capital asset-pricing model (CAPM) to make it more theoretically complete and more empirically accurate. In broad terms, efforts to extend the CAPM have advanced along three fronts.

The first might be called "raw empiricism." This approach begins with documented deviations from predictions of the CAPM and suggests extensions of the model to explain them without necessarily going back to theoretical first principles. For instance, Basu (1977) and Banz (1981) originally discovered that the returns to value firms (low P/E) and small firms (low market capitalization) were greater than predicted by the CAPM. Their discoveries, subsequently confirmed by others, led eventually to the three-factor model of Fama and French (1992, 1993) in which book/market and size are introduced as priced risk factors. Unfortunately, raw empiricism is difficult to distinguish from data snooping, (Cf. Lo and MacKinlay (1990)), so Campbell (1996) and others argue that multi-factor extensions of the CAPM should use only factors that are implied by pre-specified theoretical models.

The second, or theoretical, approach extends the conceptual foundation of the CAPM by extending the framework to develop intertemporal (continuous time or multiperiod) models with more general utility functions and probability distributions. The seminal contributions here are by Merton (1973) and Ross (1976). Their work has been extended by dozens of scholars including Breeden (1979) and Grossman and Shiller (1981), who highlight the role of consumption, Campbell (1996,) who substitutes asset returns and returns on human capital for consumption, and Hansen and Singleton (1983), who develop the analysis in terms of the stochastic discount factor. More recently, relatively exotic utility functions have been proposed to explain such anomalies as the equity premium puzzle, Campbell and Cochrane

(1999), Constantinidies and Duffie (1996). Cochrane (2001) provides a succinct overview of many of these models. A more limited approach with the same objective has been to develop conditional CAPMs that allow betas to change over time as in Harvey (1989) and Jagannathan and Wang (1996), among others.

The third approach is "behavioral." It attempts to explain deviations from the CAPM and other anomalies by hypothesizing that investors depart from rational expected utility maximization in systematic ways that can be understood, at least in part, by research in psychology. Specific models are constructed to investigate the impact of various assumed forms of less than perfectly rational behavior. Fama (1998) provides a critical review of this literature. Barberis and Thaler (2002) offer a more recent, and more favorable, survey. Although behavioral research emphasizes the cognitive and psychological shortcomings of investors, it has not dwelled much on how investors should deal with their shortcomings, or how they actually do deal with them. One way to cope is by hiring professional agents, money managers, to invest on their behalf. The evidence indicates this is precisely what a growing majority of investors do.

When it comes to evaluating the models developed by each of the three existing approaches, an appeal is typically made to Friedman's (1953) argument that the scientific worth of a model depends on its predictions, not on the reality of its assumptions. Accordingly, tests of the models use data on asset returns and supposedly related variables such as consumption, labor income and innovations in the investment opportunity set. What, if anything, the models predict about the institutional structure of investment management, or how and why investors choose to delegate the stock selection decision, is generally ignored.

This paper is devoted to a fourth manner in which the CAPM can be extended. It is based on the notion that the evolution of market institutions provides an insight into how an

asset-pricing model should be constructed. In particular, it builds on the salient institutional fact that investors delegate stock selections to agents, to professional money managers and financial advisers. Although a variety of papers including, some of which we discuss below, recognize that the impact of delegated investing is likely to be empirically important, virtually no research has been done on the objective functions actually used by institutional investors. Nor have attempts been made to derive the implications of such objective functions for asset pricing.

As Allen (2001) stresses, there are obvious reasons why it is sensible for investors to delegate portfolio decision-making. To begin with, there are economies of scale in investment analysis and transacting. Second, investors who are at least quasi-rational will recognize their own limited capacity for information gathering and processing. Finally, investors may be wary of the psychological biases about which behaviorists warn and believe, perhaps wrongly, but believe nonetheless, that professionals are less susceptible to such predispositions.

As the principal-agent literature makes clear, significant trade-offs arise when agents are employed. In specifying an objective function for the agent, there is always a trade-off between a complete reflection of the principal's desires and sufficient clarity and transparency so that objectives can be codified, monitored and enforced at reasonable cost. Fortunately, in the case of investment management, the objective function that emerges from such trade-offs does not have to be theoretically derived because it can be empirically observed. To the extent that delegated agents are the marginal investors, and results reported by Allen (2001) and by Gompers and Metrick (2001) indicate that this is probably already the case, it is these

objective functions, not individual utility functions, which determine relative asset prices.<sup>1</sup> We believe this fact has important implications for asset pricing that have yet to be adequately explored, even in the context of the basic mean-variance model.

Figure 1 gives a comparison between the delegated agent/institutional approach and more traditional asset pricing. The traditional approaches either take "investor" preferences and probabilities as primitives and develop models, or in the case of raw empiricism jump directly to the final model. An appeal is then made to Friedman's (1953) argument to test the theories using asset returns. Unfortunately, because asset returns are very noisy and possibly non-stationary, the tests are often ambiguous. As a result, the number of models has proliferated, each attempting to become more realistic by extending or complicating the underlying assumptions.

Our suggested approach begins with a recognition that "real world complications" such as economies of scale in investing, limited investor information, limited investor cognitive capacities, investor irrationality, highly noisy asset returns, significant non-stationarity of probability distributions, and possible market inefficiencies lead to the delegation of investing. More specifically, the delegation process serves as a filter on two dimensions. First, investors must determine what information is sufficiently important that it must be incorporated into the objective function mandated for professional managers. Second, that objective function must be further refined to reflect the costs of specifying, monitoring and enforcing the contracts that define it.

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<sup>&</sup>lt;sup>1</sup> It should be noted that data on the fraction of securities held and traded by institutions, such as that reported by Gompers and Metrick, undoubtedly understates the importance of delegated agent investing. For instance, very wealthy individuals such as Bill Gates are treated as individual investors in the data set even though they

Only the investment criteria that make it through this two-step filtering process end up affecting the investment allocation decisions of delegated agents. From a research perspective the filter is an invaluable tool. It provides direct evidence on the criteria that investors consider of first order importance. By studying that contracting process and the resulting contracts between investors and money managers, and more specifically, the resulting manager objective functions, financial economists should be able to develop more realistic asset-pricing models.

In this paper, we develop a mean-variance delegated agent asset-pricing model to illustrate the impact of delegation on market equilibrium and to make some initial empirical predictions. In doing so we hope to promote a shift of emphasis in asset pricing research. Cochrane (2001, p. 130) states that one of the major developments in finance theory has been the transition from mean-variance frontiers and beta analysis to discount factor analysis specifying preferences and budget constraints over state contingent consumption. He argues that this is a more natural mapping of microeconomics into finance. The problem with this view is that most consumption decisions, whether to buy a television or take a vacation, are made by consumers. The decision whether to buy IBM or Intel is delegated.

The failure to incorporate delegated investing into asset pricing is rather odd. After all, the corporate finance literature has long recognized that corporate decisions are more appropriately analyzed in terms of the objective functions of managers, not the utility functions of investors.<sup>2</sup> One possible explanation for the oversight is that while professional

privately delegate their investment activities to specific personal advisers. Furthermore, many less wealthy individual investors rely on stockbrokers, or other financial advisers, when making investment decisions.

<sup>&</sup>lt;sup>2</sup> Diamond and Verrecchia (1982) is an excellent early example. Brennan (1996) provides a useful collection of the most notable research.

agents have managed firms for a long time, the rise of delegated investing has been relatively recent. Gompers and Metrick report that aggregate institutional ownership of common stock increased from less than 10 percent in 1950 to over 50 percent in 1994. Xu and Malkiel (2003) report that by 1998 the volume on the New York Stock Exchange accounted for by institutions, which was minimal in 1950, was as great as 90 percent on selected days.

There have, nonetheless, been a handful of papers about delegated investing. Perhaps the best known is Allen's (2001) presidential address to American Finance Association. Allen emphasizes the importance of financial institutions for financial markets, but he does not analyze the objective functions of institutional investors and does not develop a delegated agent asset-pricing model. Bhattacharya and Pfleiderer (1985) consider how a principal can elicit truthful information about a delegated agent's investment abilities (assuming the agent knows them.) Roll (1992) provides a normative framework for an investment manager striving to outperform a benchmark portfolio; he shows that the manager will generally select a sub-optimal portfolio from the client's perspective because of the imposed objective function. Admati and Pfleiderer (1997) reach a similar negative conclusion by showing that a manager who possesses superior information, but receives benchmark-related compensation, will deliver to investors only a small fraction of the value of his information and will sometimes do even worse than a passive (index fund) manager. Ross (2000), continuing the tradition of negative findings, demonstrates that investors will not often be able to distinguish between managers with superior ability and other managers with no ability at all.

Dybvig, Farnsworth and Carpenter (2004) show that given particular information structures, agent compensation based on performance relative to a benchmark may be optimal. They do not, however, analyze the impact of such optimal contractual structures for asset pricing. Goldman and Slezak (2003) also develop a model that is based on delegated

investing. Their analysis is based on various assumptions about the objective function and they do not consider the role of benchmark portfolios in determining the behavior of delegated agents.

The paper most similar to ours is Brennan (1993). He assumes that delegated agent/managers possess risk aversion with respect to the volatility of tracking error; i.e., the difference between the managed portfolio and the specified benchmark. Some investors, however, do not hire managers but simply handle their own investments (and are mean/variance optimizers.) Agents and self-directed investors share common beliefs about expected returns and covariances. Based on these assumptions, Brennan goes on to derive market equilibrium conditions that depend on the benchmark chosen for managers and the proportion of funds managed by the principals directly. He tests these conditions using proxies for the benchmark (the S&P500 index) and for the market portfolio (the value-weighted US equity index) but finds only "limited support."

Following Brennan, below we present an alternative asset pricing model that does not depend on tracking error risk aversion, but simply and pragmatically on the tracking error mandate specified by client/principals. In our framework, principals divide their funds between passive managers who simply match the benchmark and active managers who strive to outperform. For simplicity, we assume that principals know nothing about returns and thus realize that they should delegate investment decisions to either active or passive managers.

It should be noted that the impact of delegation, at least to date, is not the same across all aspects of the investment decision-making process. Currently, individuals play a more active role in deciding how to allocate wealth across broad asset classes than in making individual security decisions. For instance, many employees play a role in deciding how

much to invest in their retirement fund, but the actual investment of the pension portfolio is turned over to a professional investment firm.<sup>3</sup> This suggests that there may be a natural split within the overall investment process. Individuals seem to decide what fraction of their wealth will be invested in risky assets and probably have something to say about allocations to various broad asset classes. Accordingly, there may be a natural division in asset pricing theory between models that investigate pricing across categories of assets and those that focus on the relative pricing of individual equities. Traditional intertemporal pricing models that ignore the delegation process could be appropriate for analyzing questions related to the relative pricing of assets classes, such as the equity premium puzzle. However, individual stock selection, and therefore, questions related to the relative pricing of individual common stocks, might be analyzed more successfully within the framework of a delegated agent asset-pricing model.

The delegation process also has important implications for research in behavioral finance. Instead of simply noting a limitation on rational decision-making, such as overconfidence, and then developing an asset-pricing model that incorporates the limitation, behaviorists could analyze how the limitation influences the delegation process. As noted previously, overcoming such cognitive limitations is one of the reasons that delegation occurs in the first place. As an analogy, consumers face similar cognitive limitations when deciding how to cope with a serious illness. However, the choice of medical treatment does not usually

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<sup>&</sup>lt;sup>3</sup>Even here, however, delegated agents are beginning to play a larger role. Many individuals employ financial advisers to help make broad asset allocation decisions and some firms offer this service to employees. The *Wall Street Journal* (Simon, 2004) reports that one of the most profitable and fastest growing services offered by diversified financial firms like American Express is financial planning.

reflect the direct decision of the patient but is, instead, filtered through the medical professionals that the patient hires as agents. The investment process is similar.

To develop this idea, the remainder of the paper is organized as follows. The next section describes the assumed institutional environment. The key issue is the depiction of the delegation process and the determination of the agent's objective function. Given the observed institutional environment, a simple yet illustrative asset-pricing model is derived in section three. It turns out that even a simplistic delegated agent model has provocative implications for asset pricing. Section four discusses complexities and possible extensions in future research of the basic idea of delegated agent asset pricing. Our conclusions are summarized in the final section.

## 2. The institutional environment: Manager compensation and behavior

To develop a delegated agency asset-pricing model, we assume an institutional environment with the following stylized but realistic features. Rather than manage their own portfolios, investors entrust their funds to money managers. These managers fall into two categories: passive managers who mimic well-known indexes and active managers who employ various strategies in an attempt to provide superior performance. More specifically, active managers choose securities so as to maximize the objective function that arises out of negotiations with investors.

Investment managers are compensated in a variety of ways, but most compensation contracts have certain common features. For example, there is usually an agreement that managers will be paid a fixed percentage of the assets under management, perhaps based on a trailing average of asset values. Less commonly, there are "incentive" compensation schemes which reward managers by the total return performance, net of fees, relative to the total return of a particular benchmark index. Such incentive contracts usually feature both a minimum

(again based on the value of assets under management) and a maximum.

Even the more common contracts without incentive clauses almost always involve a benchmark. This makes sense because the client's alternative to hiring an active manager is to employ a passive manager who simply matches a widely followed benchmark such as the S&P500. In every case, the client investor has a choice between a passive fund and an active manager with the passive fund as the benchmark. There are many possible benchmarks including diversified international indexes, high-tech indexes, bond indexes, and various specialized indexes ranging from industrial sectors through sin-free stocks. Although specialized benchmarks are employed more than occasionally, the most common benchmarks are well-known indexes designed to measure movements in the overall market, such as the S&P 500, the Wilshire 5000, or the MSCI (Morgan Stanley Capital International.)

Once an active manager has been hired, his compensation continues quarterly so long as he remains employed. Termination is, however, always a looming worry for the manager. There are two basic reasons for terminating a manager; (1) the funds are withdrawn because they are required for distribution to beneficiaries or because the client has made a strategic decision to invest in alternative asset classes and (2) the manager's performance is inadequate. Managers can do little about the first reason for termination, so they focus mainly on an effort to perform well.

Clients generally recognize that asset returns are noisy, which makes performance difficult to judge. There are, however, some more or less common evaluation practices. Few if any managers are terminated when their total return performance over the last several years exceeds the benchmark's return. This is true even though other managers have done even better. On the other hand, few managers are retained if return performance is well below the benchmark for several years. This is true even though they might be well ahead of the

benchmark over, say, the last decade. Recent performance is what really matters when a manager is reviewed.

The essence of this behavior is to induce an asymmetry into manager compensation. When performance is just passably better than the benchmark, the present value of the annuity of fees is very large. However, compensation can go suddenly to zero when performance slackens for only a relatively short time.

Because manager compensation depends greatly on <u>retaining</u> clients, thereby receiving the periodic annuity in fees for as long as possible, managers strive to maintain a precautionary strategy that makes it unlikely to under perform their benchmark for several years in a row. In other words, they not only strive to have higher total returns than the benchmark on average, but to minimize the volatility of tracking error (the return difference between the managed portfolio and the benchmark.)

Clients have similar preferences. Naturally they want high returns relative to their passive alternative, the benchmark, but they also want the tracking error to have low volatility. The client's latter preference arises because one of his most important decisions, perhaps the most important, is whether to retain or replace the active manager. A low volatility of tracking error implies much more precise information about whether the manager is adding value or is simply lucky (or unlucky.) In fact, many clients would agree that an ideal manager should outperform the benchmark every period by the same margin; i.e., with zero tracking error volatility. After only a few periods, the client could be confident in the manager's value added.

As a consequence, average performance and tracking error volatility are often both discussed during the negotiations prior to a manager's engagement. The manager typically offers estimates of both; e.g., 200bp per annum extra return over the benchmark on average

and a tracking error standard deviation of 300bp per annum. Gross deviations from either target are cause for discussion in periodic manager reviews.

#### 3. A mean-variance delegated agent asset-pricing model.

This section presents a mean-variance model of delegated agent behavior and uses it to draw implications about capital market equilibrium. For tractability and transparency, we ignore the intertemporal nature of manager retention outlined in the previous subsection and assume a simple compensation function based on a single period. Making our analysis multi-period would increase its complexity and reality but might not provide many additional insights.

#### 3.A. Representative Agent Managers.

We first assume the existence of just two representative agent managers, one active and one passive, who invest all available funds. There is a single benchmark managed by the passive agent.<sup>4</sup> The active agent/manager has the same benchmark, but his compensation depends on the relative performance of the actively managed portfolio and on tracking error volatility. The active manager is assumed to have his own beliefs about the expected returns of all assets and about the variances and covariance of those returns. In our framework, it is not necessary for either the passive representative manager or the client to have any particular set of beliefs.

Schematically, the active manager's anticipated compensation is something like

E(Compensation) = E(Fees|Keeping the Client)\*Prob(Keeping the Client)
so we specify the active manager's revenues, S, to depend positively on the excess return of
the actively managed portfolio, A, over the benchmark, B, and negatively on the volatility of
tracking error (since reduced tracking error increases the probability of client retention.)

Specifically, let R<sub>i</sub> denote the total return (net of fees) of portfolio j (j=A, B). Then the

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beliefs.

<sup>&</sup>lt;sup>4</sup>Later, we extend the analysis to multiple managers and benchmarks but only under an assumption of common

expected excess return of the representative active manager is

$$G = E(R_A-R_B)$$

while the variance of tracking error is

$$T = Var(R_A - R_B)$$
.

The active manager's expected revenue is assumed to be

$$E(S) = vG-qT, (1)$$

where the positive constants v and q are compensation parameters negotiated between the client and the active manager. Note that the expected performance and tracking error are prospective; i.e., determined in advance. This makes sense because the manager has to select an active portfolio based on his beliefs and cannot choose whatever turns out to be the best portfolio *ex post*. Actual compensation could be determined after the fact, but the manager is obliged to make his portfolio decisions *a priori* based on what he expects will generate the highest revenue.

Since the active manager expects to outperform the passive benchmark on average, one might wonder why investors would place any funds at all with the passive manager. There are at least three reasons. First, although investors might think the active manager will perform better, they are not 100% sure and thus sensibly hedge their uncertainty about any potential value added by active management. Second, active managers charge higher fees, which must be deducted from gross performance. Third, unless constrained in some fashion,

between delegating to active and passive managers, and that both types of managers must exist in equilibrium.

<sup>&</sup>lt;sup>5</sup> Hiring an active manager is similar to buying information in the framework of Grossman and Stiglitz [1980]. Their framework extended to the current context would imply that the marginal investor is just indifferent

active managers who are compensated by performance relative to a benchmark will choose a portfolio riskier than the benchmark.<sup>6</sup>

If the active manager is risk neutral, (which we assume for simplicity), he will choose an active portfolio that maximizes expected revenue. Note, however, that the nature of the typical asymmetric compensation contract induces a risk neutral manager to take risk into account, because expected compensation depends on tracking error volatility. This essentially induces an asymmetry into active manager revenues, in accordance with reality, which is one justification for the compensation schedule given by (1).

The appendix shows that the market equilibrium looks somewhat like the CAPM, but with an additional cross-sectional term. The relation between individual asset j's expected return and its beta on the market is

$$E(R_{j}) = E(R_{0}) + \beta_{j|M}E(R_{M}-R_{0}) + K[\beta_{j|M}\beta_{M|B}-\beta_{j|B}],$$
(2)

where  $\beta_{i|P}$  is a beta for asset or portfolio i computed against portfolio P, subscripts  $_M$  and  $_B$  indicate the market portfolio and the benchmark, respectively,  $R_0$  is the return on the global minimum variance portfolio, K is a positive constant, and  $\beta_{M|B}>1$ .

The third term on the right of (2) will be positive for some stocks and negative for others. The beta of j on M will generally be smaller than the beta of j on B, but this is offset by the beta of M on B, which is always greater than 1.0. As a general rule, this third term will be positive for assets that are more highly correlated with the market portfolio M than with the benchmark B because the active manager would favor such assets and weigh them more heavily. There is no reason such favor would be bestowed on any particular industry

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<sup>&</sup>lt;sup>6</sup> This is proved in Roll [1992], who shows that the actively managed portfolio will have higher total volatility than the benchmark and a beta against the benchmark greater than 1.0

sector or asset type, though it could be. It could also be stock specific and perhaps depend on the fundamental analysis conducted by the active manager.

#### 3. B. Multiple managers, but with common beliefs.

The results in the previous sub-section can be extended rather easily to allow multiple managers and benchmarks, provided that all active managers share common beliefs. An assumption of homogeneous beliefs is not much of a defect relative to most other asset-pricing models, which usually make the same assumption without even mentioning it. Tractability for this extension is provided by a curious fact about active management relative to a benchmark; viz., every active manager will select a portfolio whose composition differs from the benchmark by a vector that is completely independent of the particular benchmark being employed.<sup>7</sup>

The appendix proves that the above-mentioned result delivers a cross-sectional asset pricing relation identical in form to equation (2), but the unique benchmark must be replaced by a weighted-average benchmark whose weights are proportional to the funds invested in each constituent manager's benchmark. The constant K in (2) remains positive, but is also a weighted average over individual client's allocations and target levels of performance. See the appendix for the precise algebraic form of K.

Heterogeneous beliefs would complicate the result considerably. As shown in the appendix, a simple cross-sectional relation between expected returns and betas depends critically on being able to express those quantities in terms of first and higher moments of a particular probability distribution. With heterogeneous beliefs, it is not even obvious whose expected return should be written down in the asset pricing formula nor to whose beta the expected return should be related. Those elements of asset pricing formulae would

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<sup>&</sup>lt;sup>7</sup> Roll (1992) provides a proof, which is described in the appendix here.

undoubtedly be complicated weighted averages but their exact form will have to be determined by future research.

#### 3. C. Some empirical implications.

The results above have some interesting empirical implications. In particular, the mean-variance delegated agent model can possibly explain some of the well-documented deviations from the CAPM. As an example, suppose one were to compute the classic Fama/MacBeth [1973] cross-sectional regression between average returns and betas,

$$E(R_i) = \gamma_0 + \gamma_1 \beta_{i|M}, (\forall j).$$

Ignoring any problems in estimating expected returns and betas and also ignoring misspecification in the cross-sectional regression,<sup>8</sup> one would probably find, contrary to the CAPM, that  $(\gamma_0 + \gamma) \neq E(R_M)$ , because  $\gamma_1 = E(R_M - R_0) + K\beta_{M|B}$  and  $\gamma_0 = E(R_0) - K\beta_{e|B}$  where the subscript "e" denotes an equally-weighted portfolio (assuming assets are weighted equally in the regression.) The CAPM result would hold only if  $\beta_{M|B} = \beta_{e|B}$ , which seems unlikely since equal-weighted portfolios are usually more volatile than value-weighted portfolios such as M.

Perhaps a more straightforward test, which finesses some econometric difficulties, would first fit the simple market model regression,

$$R_{i,t} = a_i + b_i R_{M,t}. \tag{6}$$

Under a delegated agent equilibrium, the intercept in (6) would be a linear function of  $b_j$  (= $\beta_{j|M}$ ) and  $\beta_{j|B}$  both of which can be readily estimated from the observable data (provided that proxies are available for M and B.) The following cross-sectional relation would hold for the intercept:

$$\mathbf{a}_{i} = \delta_{0} + \delta_{1} \mathbf{b}_{i} + \delta_{2} \beta_{i|B} \tag{7}$$

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 $<sup>^{8}</sup>$  Because the residual contains  $\beta_{j|B}\text{,}$  which seems likely to be correlated with the explanatory variable.

with 
$$\delta_0 + \delta_1 > 0$$
 and  $\delta_2 < 0$ .

Again, this contrasts with the standard CAPM, which implies  $\delta_0 = -\delta_1$  and  $\delta_2 = 0$ .

Another test could be based on a time series fit of (6) with non-stationary parameters. Notice that  $\delta_2$  in (7) is equal to -K, the cross-sectional constant in (5). This constant depends inversely on  $\varphi$ , the fraction of all funds actively managed. Since active management by institutions has been growing dramatically over time, we should observe a secular downward trend in the absolute value of  $\delta_2$ .

We emphasize that the model derived in this section could be generalized. The setup here might be too simplistic in its specification of the delegated agent compensation contract and it faces the unreality of homogeneous beliefs by all active investment managers. We hope, however, that even this simple model can serve as a catalyst for the development of more satisfactory models of delegated management. The next section outlines a few suggestions for further research extensions. We are confident that such research will ultimately produce an asset pricing theory that performs well under empirical scrutiny.

#### 4. Extensions of the basic model

The delegated agent asset-pricing model developed in the previous section clearly is only a first step. Here we suggest a few ways to extend the basic analysis and ask a few questions worthy of further investigation.

A direct extension of the basic model is to recognize the multi-period nature of manager evaluations. Typically, the impact of one quarter's performance depends on what has gone before. Failure to meet a proposed target, say 200 basis points over the S&P 500, may lead to mild questioning at worst. However, several quarters of <u>negative</u> performance relative to the benchmark is often grounds for termination. The obvious extension is to make

manager compensation depend on a <u>sequence</u> of results and to reduce compensation permanently to zero when performance over several recent periods falls below some minimally acceptable threshold. In other words, the probability of termination could be "kinked" at zero excess performance.

One particularly interesting path for future research would be to attempt to "reverse engineer" the delegation process. This paper has stressed certain characteristics of the delegation process. Namely, agents are chosen and compensated based on their periodic performance compared against a benchmark portfolio. In this context, one could investigate what types of preferences, beliefs, and agency and information costs are likely to cause delegation to proceed in this fashion. Our preliminary review indicates that active managers primarily market their skills in the area of fundamental security analysis. What does that imply about investor beliefs?

Another issue and a puzzle concerns compensation, which is most often a fixed percentage of the market value of assets under management. Why is this so? Why aren't performance-based fees more common?

Of course, a starting point for any such reverse engineering should be a detailed understanding about how the delegation process operates in practice. The literature is virtually devoid of work in this area.

A related issue is the sheer growth of delegated investing since the 1950s. Why has it happened? Given the concurrent development of information technology and the ease with which individuals today can obtain information and engage in trading, one might have predicted a decline in delegated investing rather than a dramatic increase. What other forces are responsible for the pattern actually observed?

<sup>&</sup>lt;sup>9</sup> We thank Eric DeBodt for this suggestion.

If delegated investing dominates the capital markets, it may well be that the hedging variables suggested by the ICAPM will not be important for asset pricing even in more sophisticated delegated agent models than the one considered here. In fact, a sophisticated delegated agent model may stand the relation between the CAPM and the ICAPM on its head. Campbell (1996) argues that the CAPM works because it turns out to be a close empirical approximation of the ICAPM. He states, "Cross-sectional variation in covariances with the stock market dwarfs the cross-sectional variation in covariances with any of the other factors, and in this limited sense the CAPM is a good approximate model of stock and bond pricing." (p. 340.) It may be, however, that the CAPM is a reasonably good approximation to the process of delegated investing using benchmarks as proxies for the market. The ICAPM, on the other hand, introduces spurious factors that are not considered in the delegation negotiations and, therefore, do not enter the manager's objective function. The ICAPM works in practice precisely because covariances with these spurious factors are small so that the model is empirically similar to the CAPM.

The delegated agent model implies that stochastic discount factors will not properly price stocks in the cross section, because the utility function of investors has been replaced by the objective function of managers. Benchmark portfolios are brought into play in a manner not considered by models that maximize expected utility. As noted earlier, to the extent that investors retain decision-making regarding allocation of wealth across broad classes of assets, the stochastic discount factor and the ICAPM may be appropriate models for analyzing questions that involve the relative pricing of these asset classes, such as the equity premium puzzle. With respect to the cross-sectional pricing of common stock, however, the models might work only to the extent that they approximate the delegated agent model.

Both the CAPM and ICAPM are designed as stationary models. For this reason, long-

term data, often 50 years or more, is appropriate for testing. The delegated agent model, on the other hand, by its nature implies fundamental nonstationarities. First, as noted previously, the impact of delegated agents has been growing dramatically over time. This alone could cause the pricing relations to change. Second, the principal-agent contracting structure evolves over time in response to new information and innovations in the institutional environment. Ironically, one source of new information is research in finance. For instance, one of the motivating forces for the development of benchmark portfolios as tool for evaluating money managers was the extensive academic research on the performance of fund managers beginning with the work of Jensen (1967).<sup>10</sup> More recently, another layer of delegation has arisen to take advantage of specialized knowledge in finance. Rather than assessing the performance of money managers directly, client investors are relying increasingly on specialized investment consultants who evaluate money managers. The impact of the added layer of delegation is an interesting area for future research.

Our simple delegated agent model assumes a purely quantitative objective function. In practice, there is probably a subjective element as well. Before terminating a manager, the client will often give the manager an opportunity to explain underperformance. In this context, certain types of failure may be more costly than other. Specifically, some managers allege that investing in distressed securities is particularly hazardous. They claim that if underperformance in a given quarter is due to holding companies that were clearly distressed at the beginning of the quarter, the probability of being terminated is higher than if the shortfall was due to large holdings of highly regarded companies. If this be true, managers will require greater returns, everything else held constant, for holding the stock of distressed

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<sup>10</sup> This work and other research on market efficiency was also a key factor in the explosive growth of index funds.

firms while the reverse would be true for well-regarded companies.<sup>11</sup> The net result would be a distress effect in the cross-sectional pricing of common stocks. While this idea is only a conjecture, it illustrates the potential insights that could possibly be derived from an in depth analysis of the delegation process.

#### 5. Conclusions

Despite the enormous research effort devoted to extending the basic capital asset-pricing model, little attention has been focused on the actual investment process. Whereas much work has been done investigating the impact of different preferences or distributional assumptions, the delegation process through which an increasing majority of investment activity occurs has not been integrated into asset pricing theory. This paper takes a step toward addressing that issue. We show that even a simple delegated asset-pricing model produces empirically interesting predictions.

The main contribution of the paper, however, is not our specific model, but its focus on the delegation process. Understanding how delegation works is critical for the development of realistic asset-pricing models for two fundamental reasons. First, delegated investing means that relative asset prices depend on the objective functions of agents, not the utility functions of investors. Because of the information, aggregation and incentive problems that are unavoidable with delegated decision-making, the objective functions of agents are likely to differ from consumer utility functions in significant respects. One respect that we

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<sup>&</sup>lt;sup>11</sup>Distress in this context is probably correlated with high book to market ratios, but the two are not identical. Technology firms can also be distressed. The theory appears to imply that Fortune's most respected companies should have lower average returns, however initial empirical evidence presented Antunovich and Laster (1998) is inconsistent with such a prediction.

identify is the central role of benchmark portfolios in the objective functions of money managers.

Second, a truly complete asset-pricing model should explain the delegation process itself. To date, asset-pricing models have been judged almost exclusively by the predictions they make regarding asset returns and related macroeconomic variables. Unfortunately, the noisiness of asset returns makes it extraordinarily difficult to choose among competing models, particularly among the highly sophisticated versions. The delegation process provides an entirely new dimension along which competing models can be compared. If a model predicts that a variable is central to asset pricing, but if that variable is not involved in the negotiations between investors and managers or incorporated into managers' objective functions, then the theory is probably wrong.

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### Appendix

# A mean/variance asset-pricing model with delegated investment management

This appendix derives equation (2) of the text, the cross-sectional asset pricing relation between expected returns and betas under the assumption that investors delegate their decisions to representative active and passive managers. The first section derives the basic result and the second section extends this result to the case of multiple managers and many investors.

#### A.1. Delegated investing to one active and one passive manager.

#### Assumptions:

- The passive manager invests everything in portfolio, B, which is also the benchmark for the active manager.
- A fraction  $\varphi$  of all funds is invested with the active manager.
- The active manager is risk neutral and maximizes expected revenue, E(S), over a compensation schedule that is specified by investors as E(S)=vG-qT, where v,q>0 are compensation parameters,  $G=E(R_A-R_B)$  is expected "performance,"  $T=Var(R_A-R_B)$  is tracking error variance,  $R_B$  is the total return on the passive benchmark and  $R_A$  is the return on the actively managed portfolio, "A."
- V and R are, respectively, the covariance matrix of returns and the vector of expected returns 12 corresponding to the beliefs of the active manager.

Roll [1992] shows that an active manager will select a portfolio whose investment proportions in individual assets consist of the benchmark's proportions plus the investment

<sup>12</sup> Matrices and vectors are indicated in bold face.

weightings of a particular hedge portfolio. Specifically,

$$\mathbf{w}_{\mathbf{A}} = \mathbf{w}_{\mathbf{B}} + \mathbf{D}[\mathbf{w}_{1} - \mathbf{w}_{0}] \tag{A-1}$$

where the boldface  $\mathbf{w}$ 's denote vectors of investment proportions. As the subscripts indicate,  $\mathbf{w}_A$  is the vector of investment proportions for the actively managed portfolio,  $\mathbf{w}_B$  is for the benchmark, and  $\mathbf{w}_1$  and  $\mathbf{w}_0$  are the investment proportions of the two specific portfolios located on what the active manager believes to be the efficient frontier. Portfolio "0" is his global minimum variance portfolio. Portfolio "1" is a portfolio on the positively sloped segment of his efficient frontier whose corresponding "zero-beta" portfolio has an expected return of zero; hence,  $Var(R_1) > Var(R_0)$  and  $E(R_1) > E(R_0)$ . The constant D is given by  $D = G/E(R_1 - R_0)$ .

It is well known (e.g., Roll [1977, appendix]) that the investment proportions  $\mathbf{w}_1$  and  $\mathbf{w}_0$  have simple forms; specifically

$$\mathbf{w}_1 = \mathbf{V}^{-1} \mathbf{R} / \mathbf{b} \text{ and } \mathbf{w}_0 = \mathbf{V}^{-1} \mathbf{1} / \mathbf{c},$$
 (A-2)

where **1** is the unit vector and b and c are scalar constants,  $b=1^{\circ}V^{-1}R$  and  $c=1^{\circ}V^{-1}1$ , which assure that the investment proportion vectors sum to unity. A third efficient set constant is the quadratic form  $a=R^{\circ}V^{-1}R$ . It is easy to prove and it will be useful to know that  $E(R_0)=b/c$ ,  $Var(R_0)=1/c$ ,  $E(R_1)=a/b$  and  $Var(R_1)=a/b^2$ . All of these vectors and efficient set parameters depend on the beliefs of the active manager as encapsulated in **V** and **R**.

The variance of tracking error is  $T = (\mathbf{w}_A - \mathbf{w}_B)^* \mathbf{V}(\mathbf{w}_A - \mathbf{w}_B) = D^2(\mathbf{w}_1 - \mathbf{w}_0)^* \mathbf{V}(\mathbf{w}_1 - \mathbf{w}_0)$ . Substituting from (A-2) and simplifying using the definitions of a, b, and c, we obtain

$$T = \gamma G^{2}, \tag{A-3}$$
where  $\gamma = \frac{Var(R_{1}) - Var(R_{0})}{\left[E(R_{1} - R_{0})\right]^{2}} > 0.$ 

The assumed compensation schedule for the active manager, equation (1) from the text, is

$$E(S) = vG-qT, (1)$$

where S is the manager's revenue. Substituting the for tracking error volatility, (i.e., T from (A-3)), it is straightforward to show that the active manager will choose a portfolio whose expected performance is given by

$$G^* = \frac{v}{2\gamma q}$$
.

Once this target performance is determined, the active portfolio composition is a foregone conclusion because there exists a unique portfolio that minimizes tracking error volatility while delivering G\* in expected performance.

By the well-known "two fund" theorem of efficient set algebra, which says that the investment proportions of any mean/variance efficient portfolio can be expressed as linear combinations of any two other mean/variance efficient portfolio, we see from (A-1) that the managed portfolio A will be mean/variance efficient if and only if the benchmark B is efficient. If the benchmark is <u>not</u> efficient, the actively managed portfolio will not be efficient either (although it will have the lowest possible volatility of tracking error for the targeted expected performance.)

Logically, investors must have some doubt about whether the benchmark <u>is</u> efficient; otherwise, why would they hire an active manager? If B were efficient, the passive manager could not be beaten, except by taking on more risk, and active management would always be inferior because of its larger fees.

Since by assumption there is only one passive and one active representative manager in the economy, the market equilibrium is rather simple. The market portfolio of all assets consists of a weighted average of the actively managed portfolio and the passively managed benchmark with weights proportional to the fractions actively and passively managed. In other words, if  $\varphi$  is the fraction of all investment funds managed actively, the market portfolio M will have investment proportions

$$\mathbf{w}_{\mathrm{M}} = \varphi \mathbf{w}_{\mathrm{A}} + (1 - \varphi) \mathbf{w}_{\mathrm{B}} = \mathbf{w}_{\mathrm{B}} + \varphi \mathbf{D} [\mathbf{w}_{1} - \mathbf{w}_{0}]. \tag{A-4}$$

Since the market portfolio will <u>not</u> be mean/variance efficient (when the benchmark B is not), the simple capital asset-pricing model, a linear relation between individual expected returns and "betas" on the market portfolio, will not hold in equilibrium. As we will now show, however, there will be a more complex relation between betas and expected returns.

Using (A-4), the vector of covariances between the market portfolio and individual assets is

$$\mathbf{V}\mathbf{w}_{M} = \mathbf{V}\mathbf{w}_{B} + \varphi \mathbf{D}\mathbf{V}[\mathbf{w}_{1} - \mathbf{w}_{0}] = \mathbf{V}\mathbf{w}_{B} + (\varphi \mathbf{D}/b)[\mathbf{R} - \mathbf{E}(\mathbf{R}_{0})\mathbf{1}].$$
 (A-5)

The variance of the market's return can thus be computed as

$$Var(R_M) = \mathbf{w}_M \mathbf{V} \mathbf{w}_M = Cov(R_M, R_B) + (\varphi D/b)E(R_M - R_0). \tag{A-6}$$

Now let the bold face symbol  $\beta_{|P}$  denote the vector of individual asset "betas" on portfolio P; e.g., the  $j^{th}$  element of  $\beta_{|M}$  is  $Cov(R_j,R_M)/Var(R_M)$  for individual asset j. Let  $\beta_{j|P}$ , a scalar, denote the beta of asset or portfolio j computed against portfolio P; i.e.,  $\beta_{j|P} = Cov(R_j,R_P)/Var(R_P)$ .

Equation (A-5) is equivalent to

$$\beta_{\text{IM}} \text{Var}(R_{\text{M}}) = \mathbf{V} \mathbf{w}_{\text{B}} + (\varphi D/b) [\mathbf{R} - E(R_0) \mathbf{1}]. \tag{A-7}$$

Substituting for Var(R<sub>M</sub>) in (A-7) from (A-6) and simplifying, we obtain the vector equation describing the cross-sectional relation between betas and expected returns,

$$\beta_{\text{IM}}E(R_{\text{M}}-R_0) + [b\text{Var}(R_{\text{B}})/(\varphi D)][\beta_{\text{IM}}\beta_{\text{MIB}}-\beta_{\text{IB}}] = \mathbf{R} - E(R_0)\mathbf{1}. \tag{A-8}$$

The j<sup>th</sup> entry in the system of equations (A-8) is

$$E(R_{j}) = E(R_{0}) + \beta_{j|M}E(R_{M}-R_{0}) + [bVar(R_{B})/(\varphi D)][\beta_{j|M}\beta_{M|B}-\beta_{j|B}], \tag{A-9}$$

This is equation (2) of the text because  $E(R_0)/Var(R_0) = (b/c)/(1/c) = b$  with the constant K in (2) defined as  $K = \frac{E(R_0)}{\phi D} \frac{Var(R_B)}{Var(R_0)}$ .

# A.2. Multiple Managers with common beliefs.

From equation (A-1) of the previous section, the vector of <u>differences</u> in investment proportions between any actively managed portfolio, A, and its associated benchmark, B, is given by  $\mathbf{w}_A$ - $\mathbf{w}_B$ , which is equal to  $D[\mathbf{w}_1$ - $\mathbf{w}_0]$ . This last vector does not depend in any way on the benchmark but is completely determined by the active manager's beliefs about expected returns and covariances (that jointly determine the mean/variance efficient portfolio vectors  $\mathbf{w}_1$  and  $\mathbf{w}_0$ ) and by the target level of expected performance. Consequently, for all managers who share common beliefs,  $\mathbf{w}_1$ - $\mathbf{w}_0$  is the same, so they will <u>all</u> select an active portfolio whose investment proportions differ from the benchmark's proportions by a scalar multiple of this same vector.

We now use "B" as a subscript/indicator of diverse benchmarks selected by individual investor/principals for their managers. In particular, let  $\Phi_B$  represent the fraction of all funds devoted by an investor to benchmark B and let  $\phi_B$  denote the fraction of those funds this investor allocates to an active manager, (with 1- $\phi_B$  allocated to the passive manager who invests in B.)<sup>13</sup> Finally, let  $D_B$  indicate the scaled performance target chosen by this active manager to maximize his expected revenue; i.e.,  $D_B = G_B^*/E(R_1 - R_0)$ , where  $G_B^*$  is the optimal target level of expected performance over the benchmark B. Active managers can have different compensation schedules and objective functions so long as they all result in a targeted level of expected

 $^{13}$  There is nothing to prevent the same individual from devoting funds to several different benchmarks.

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performance. Thus, the investment proportions are  $\{\mathbf{w}_B + \phi_B D_B[\mathbf{w}_1 - \mathbf{w}_0]\}$  for the <u>entire</u> portfolio of funds, both active and passive, the investor/principal associates with benchmark B.

Aggregating over all benchmarks, the market portfolio is

$$\mathbf{w}_{\mathrm{M}} = \Sigma_{\mathrm{B}}(\Phi_{\mathrm{B}}\mathbf{w}_{\mathrm{B}}) + [\mathbf{w}_{1} - \mathbf{w}_{0}]\Sigma_{\mathrm{B}}(\Phi_{\mathrm{B}}\Phi_{\mathrm{B}}D_{\mathrm{B}}),$$

with the weighted summations extending over all benchmarks/investor/principals.

This result indicates that the market portfolio will consist of two components; the first is simply a weighted average of the various benchmarks employed by investors with weights,  $\Phi_B$ , proportional to B's allocation of wealth. The second component is also relatively simple; it is the difference between two particular mean/variance efficient portfolio vectors (0 and 1) multiplied by a constant that itself is a weighted average of individual decisions about wealth allocation to active management ( $\phi_B$ ) combined with active manager determinations of target performance, ( $D_B$ ), so as to maximize compensation relative to their respective objective functions.

In other words, if we define  $\mathbf{w}_B^{\dagger} \equiv \Sigma_B(\Phi_B \mathbf{w}_B)$  and  $(\phi D)^{\dagger} \equiv \Sigma_B(\Phi_B \phi_B D_B)$ , then the market equilibrium will again be (2) with B everywhere replaced by  $B^{\dagger}$  and  $\phi D$  replaced by  $(\phi D)^{\dagger}$ . The form of the result is the same, but now both the benchmark and the scaled target performance are weighted averages.

Figure 1: The institutional structure of traditional and delegated agent asset pricing models

