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Non-Newtonian polytropic filtration systems with nonlinear boundary conditions

Wanjuan Du* and Zhongping Li

* Correspondence:
duwanjuan28@163.com
College of Mathematic and Information, China West Normal University, Nanchong 637002, PR China

Abstract

This article deals with the global existence and the blow-up of non-Newtonian polytropic filtration systems with nonlinear boundary conditions. Necessary and sufficient conditions on the global existence of all positive (weak) solutions are obtained by constructing various upper and lower solutions.

Mathematics Subject Classification (2000)
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Introduction

In this article, we study the global existence and the blow-up of non-Newtonian polytropic filtration systems with nonlinear boundary conditions

$$\begin{aligned}
&(u_i^{k_i})_t = \Delta_{m_i} u_i \quad (i = 1, \dots, n), && x \in \Omega,\ t > 0,\\
&\nabla_{m_i} u_i \cdot \nu = \prod_{j=1}^{n} u_j^{m_{ij}} \quad (i = 1, \dots, n), && x \in \partial\Omega,\ t > 0,\\
&u_i(x, 0) = u_{i0}(x) > 0 \quad (i = 1, \dots, n), && x \in \bar{\Omega},
\end{aligned} \tag{1.1}$$

where

$$\Delta_{m_i} u_i = \mathrm{div}(|\nabla u_i|^{m_i-1}\nabla u_i) = \sum_{j=1}^{N} (|\nabla u_i|^{m_i-1} u_{ix_j})_{x_j},\ \ \nabla_{m_i} u_i = (|\nabla u_i|^{m_i-1} u_{ix_1}, \dots, |\nabla u_i|^{m_i-1} u_{ix_N}),$$

$\Omega \subset \mathbb{R}^N$ is a bounded domain with smooth boundary $\partial\Omega$, ν is the outward normal vector on the boundary $\partial\Omega$, and the constants k_i, $m_i > 0$, $m_{ij} \geq 0$, $i, j = 1,\dots, n$; $u_{i0}(x)$ ($i = 1,\dots, n$) are positive C^1 functions, satisfying the compatibility conditions.

The particular feature of the equations in (1.1) is their power- and gradient-dependent diffusibility. Such equations arise in some physical models, such as population dynamics, chemical reactions, heat transfer, and so on. In particular, equations in (1.1) may be used to describe the nonstationary flows in a porous medium of fluids with a power dependence of the tangential stress on the velocity of displacement under polytropic conditions. In this case, the equations in (1.1) are called the non-Newtonian polytropic filtration equations which have been intensively studied (see [1-4] and the references therein). For the Neuman problem (1.1), the local existence of solutions in time have been established; see the monograph [4].

We note that most previous works deal with special cases of (1.1) (see [5-13]). For example, Sun and Wang [7] studied system (1.1) with $n = 1$ (the single-equation case) and showed that all positive (weak) solutions of (1.1) exist globally if and only if $m_{11} \le k_1$ when $k_1 \le m_1$; and exist globally if and only if $m_{11} \le \frac{m_1(k_1+1)}{m_1+1}$ when $k_1 > m_1$. In [13], Wang studied the case $n = 2$ of (1.1) in one dimension. Recently, Li et al. [5] extended the results of [13] into more general N-dimensional domain.

On the other hand, for systems involving more than two equations when $m_i = 1 (i = 1, \ldots, n)$, the special case $k_i = 1 (i = 1, \ldots, n)$ (heat equations) is concerned by Wang and Wang [9], and the case $k_i \le 1 (i = 1, \ldots, n)$ (porous medium equations) is discussed in [12]. In both studies, they obtained the necessary and sufficient conditions to the global existence of solutions. The fast-slow diffusion equations (there exists $i (i = 1, \ldots, n)$ such that $k_i > 1$) is studied by Qi et al. [6], and they obtained the necessary and sufficient blow up conditions for the special case $\Omega = B_R(0)$ (the ball centered at the origin in $\mathbb{R}^N$ with radius R). However, for the general domain Ω, they only gave some sufficient conditions to the global existence and the blow-up of solutions.

The aim of this article is to study the long-time behavior of solutions to systems (1.1) and provide a simple criterion of the classification of global existence and nonexistence of solutions for general powers k_i m_i, indices m_{ij}, and number n.

Define

$$b_i = \min\{k_i, \tfrac{m_i(k_i+1)}{m_i+1}\}, \quad b_{ij} = b_i \delta_{ij}, \quad i, j = 1, .., n,$$
$$B = (b_{ij})_{n\times n}, \quad M = (m_{ij})_{n\times n}, \quad A = B - M.$$

Our main result is

Theorem. *All positive solutions of (1.1) exist globally if and only if all of the principal minor determinants of A are non-negative.*

Remark. The conclusion of Theorem covers the results of [5-13]. Moreover, this article provides the necessary and sufficient conditions to the global existence and the blow-up of solutions in the general domain Ω. Therefore, this article improves the results of [6].

The rest of this article is organized as follows. Some preliminaries will be given in next section. The above theorem will be proved in Section 3.

Preliminaries

As is well known that degenerate and singular equations need not possess classical solutions, we give a precise definition of a weak solution to (1.1).

Definition. *Let $T > 0$ and $Q_T = \Omega \times (0, T]$. A vector function $(u_1(x, t), .., u_n(x, t))$ is called a weak upper (or lower) solution to (1.1) in Q_T if*

(i). $u_i(x,t)\,(i = 1, \ldots, n) \in L^\infty(0, T; W^{1,\infty}(\Omega)) \cap W^{1,2}(0, T; L^2(\Omega)) \cap C(\overline{Q_T})$;

(ii). $(u_1(x, 0), \ldots, u_n(x, 0)) \ge (\le) (u_{10}(x), \ldots, u_{n0}(x))$;

(iii). for any positive functions $\psi_i (i = 1, \ldots, n) \in L^1(0, T; W^{1,2}(\Omega)) \cap L^2(Q_T)$, *we have*

$$\int\int_{Q_T} [(u_i^{k_i})_t \psi_i + \nabla_{m_i} u_i \cdot \nabla \psi_i]\, dxdt \ge (\le) \int_0^T \int_{\partial\Omega} \prod_{j=1}^n u_j^{m_{ij}} \psi_i dsdt \quad (i = 1, \ldots, n).$$

In particular, $(u_1(x, t), \ldots, u_n(x, t))$ *is called a weak solution of (1.1) if it is both a weak upper and a lower solution. For every $T < \infty$, if $(u_1(x, t), \ldots, u_n(x, t))$ is a solution of (1.1) in Q_T, then we say that $(u_1(x, t), \ldots, u_n(x, t))$ is global.*

Lemma 2.1 (Comparison Principle.) *Assume that* $u_{i0}(i = 1,..., n)$ *are positive* $C^1(\bar{\Omega})$*functions and* $(u_1,..., u_n)$ *is any weak solution of (1.1). Also assume that* $(\underline{u}_1,..., \underline{u}_n) \geq (\delta,..., \delta) > 0$ *and* $(\bar{u}_1, \ldots, \bar{u}_n)$*are the lower and upper solutions of (1.1) in* Q_T*, respectively, with nonlinear boundary flux* $(\underline{\lambda} \prod_{j=1}^n \underline{u}_j^{m_{1j}}, \ldots, \underline{\lambda} \prod_{j=1}^n \underline{u}_j^{m_{nj}})$*and* $(\bar{\lambda} \prod_{j=1}^n \bar{u}_j^{m_{1j}}, \ldots, \bar{\lambda} \prod_{j=1}^n \bar{u}_j^{m_{nj}})$*, where* $0 < \underline{\lambda} < 1 < \bar{\lambda}$*. Then we have* $(\bar{u}_1, \ldots, \bar{u}_n) \geq (u_1, \ldots, u_n) \geq (\underline{u}_1, \ldots, \underline{u}_n)$*in* Q_T*.*

When $n = 2$, the proof of Lemma 2.1 is given in [5]. When $n > 2$, the proof is similar.

For convenience, we denote $0 < \underline{\lambda} < 1 < \bar{\lambda}$, which are fixed constants, and let $\delta = \min_{1 \leq i \leq n}\{\min_{\bar{\Omega}} u_{i0}(x)\} > 0$.

In the following, we describe three lemmas, which can be obtained directly from Lemmas 2.7-2.9 in [6].

Lemma 2.2 *Suppose all the principal minor determinants of A are non-negative. If A is irreducible, then for any positive constant c, there exists* $\alpha = (\alpha_1,..., \alpha_n)^T$ *such that A* $\alpha \geq 0$ *and* $\alpha_i > c$ $(i = 1,..., n)$*.*

Lemma 2.3 *Suppose that all the lower-order principal minor determinants of A are non-negative and A is irreducible. For any positive constant C, there exist large positive constants* $L_i(i = 1,..., n)$ *such that*

$$\prod_{j=1}^{n} L_j^{a_{ij}} \geq C \quad (i = 1, \ldots, n).$$

Lemma 2.4 *Suppose that all the lower-order principal minor determinants of A are non-negative and* $|A| < 0$*. Then, A is irreducible and, for any positive constant C, there exists* $\alpha = (\alpha_1,..., \alpha_n)^T$*, with* $\alpha_i > 0$ $(i = 1,..., n)$ *such that*

$$\min\{k_i, \quad \tfrac{m_i(k_i+1)}{m_i+1}\}\alpha_i - \sum_{j=1}^{n} m_{ij}\alpha_j < -C \quad (i = 1, \ldots, n).$$

Proof of Theorem

First, we note that if A is reducible, then the full system (1.1) can be reduced to several sub-systems, independent of each other. Therefore, in the following, we assume that A is irreducible. In addition, we suppose that $k_1 - m_1 \leq k_2 - m_2 \leq \cdots k_n - m_n$.

Let $\varphi_{m_i}(x)(i = 1, \ldots, n)$ be the first eigenfunction of

$$-\Delta_{m_i}\varphi_{m_i} = \lambda\varphi_{m_i}^{m_i}(x) \quad \text{in} \quad \Omega, \quad \varphi_{m_i}(x) = 0 \quad \text{on} \quad \partial\Omega \tag{3.1}$$

with the first eigenvalue λ_{m_i}, normalized by $||\varphi_{m_i}(x)||_\infty = 1$, then $\lambda_{m_i} > 0$, $\varphi_{m_i}(x) > 0$ in Ω and $\varphi_{m_i}(x) \in W_0^{1,m_i+1} \cap C^1(\Omega)$ and $\frac{\partial\varphi_{m_i}(x)}{\partial\nu} < 0$ on $\partial\Omega$ (see [14-16]).

Thus, there exist some positive constants A_{m_i}, B_{m_i}, C_{m_i}, and D_{m_i} such that

$$A_{m_i} \leq -\frac{\partial\varphi_{m_i}(x)}{\partial\nu} \leq B_{m_i}, \quad |\nabla\varphi_{m_i}(x)| \geq C_{m_i}, \quad x \in \partial\Omega; \quad |\nabla\varphi_{m_i}(x)| \leq D_{m_i}, \quad x \in \bar{\Omega}. \tag{3.2}$$

We also have $\left|\nabla\varphi_{m_i}(x)\right| \geq E_{m_i}$ provided $x \in \{x \in \Omega : dist(x, \partial\Omega) \leq \varepsilon_{m_i}\}$ with $E_{m_i} = \frac{C_{m_i}}{2}$ and some positive constant ε_{m_i}. For the fixed ε_{m_i}, there exists a positive constant F_{m_i} such that $\varphi_{m_i}(x) \geq F_{m_i}$ if $x \in \{x \in \Omega : dist(x, \partial\Omega) > \varepsilon_{m_i}\}$.

Proof of the sufficiency. We divide this proof into three different cases.

Case 1. ($k_i < m_i$ (i = 1,..., n)). Let

$$\bar{u}_i(x,t) = P_i e^{\alpha_i t} \log\left((1-\varphi_{m_i}(x))e^{\frac{(k_i-m_i)\alpha_i t}{m_i}} + Q_i\right) \quad (i = 1,\ldots,n), \tag{3.3}$$

where Q_i satisfies $Q_i \log Q_i \geq \frac{2(m_i-k_i)}{m_i}$ $\quad (i = 1,\ldots,n)$, and constants P_i, α_i (i = 1,..., n) remain to be determined. Since $Q_i \log Q_i \geq \frac{2(m_i-k_i)}{m_i}$, by performing direct calculations, we have

$$\begin{aligned}
(\bar{u}_i^{k_i})_t \geq{}& k_i\alpha_i P_i^{k_i} e^{k_i\alpha_i t}\left(\log((1-\varphi_{m_i}(x))e^{\frac{(k_i-m_i)\alpha_i t}{m_i}} + Q_i)\right)^{k_i} \\
&+ k_i\alpha_i P_i^{k_i} e^{k_i\alpha_i t}\left(\log((1-\varphi_{m_i}(x))e^{\frac{(k_i-m_i)\alpha_i t}{m_i}} + Q_i)\right)^{k_i-1} \\
&\times \frac{\frac{k_i-m_i}{m_i}(1-\varphi_{m_i}(x))e^{\frac{(k_i-m_i)\alpha_i t}{m_i}}}{(1-\varphi_{m_i}(x))e^{\frac{(k_i-m_i)\alpha_i t}{m_i}} + Q_i} \\
\geq{}& \frac{k_i\alpha_i}{2} P_i^{k_i} e^{k_i\alpha_i t}\left(\log((1-\varphi_{m_i}(x))e^{\frac{(k_i-m_i)\alpha_i t}{m_i}} + Q_i)\right)^{k_i} \\
\geq{}& \frac{k_i\alpha_i}{2} P_i^{k_i} e^{k_i\alpha_i t}(\log Q_i)^{k_i}, \\
\Delta_{m_i}\bar{u}_i ={}& \sum_{j=1}^{N}\left(\frac{P_i^{m_i} e^{k_i\alpha_i t}(-|\nabla\varphi_{m_i}(x)|^{m_i-1}(\varphi_{m_i})_{x_j})}{((1-\varphi_{m_i}(x))e^{\frac{(k_i-m_i)\alpha_i t}{m_i}} + Q_i)^{m_i}}\right)_{x_j} \leq \frac{\lambda_{m_i} P_i^{m_i} e^{k_i\alpha_i t}}{Q_i^{m_i}}
\end{aligned}$$

in $\Omega \times \mathbb{R}^+$. By setting $c_{m_i} = C_{m_i}$ if $m_i \geq 1$, $c_{m_i} = D_{m_i}$ if $m_i < 1$, we have one the boundary that

$$\begin{aligned}
&\nabla_{m_i}\overline{u_i}\cdot\nu \geq \frac{P_i^{m_i} c_{m_i}^{m_i-1} A_{m_i}}{(1+Q_i)^{m_i}} e^{k_i\alpha_i t} \quad (i = 1,\ldots,n), \\
&\prod_{j=1}^{n}\bar{u}_j^{m_{ij}} \leq \prod_{j=1}^{n}(P_j\log(1+Q_j))^{m_{ij}} e^{\sum_{j=1}^{n} m_{ij}\alpha_j t} \quad (i = 1,\ldots,n).
\end{aligned}$$

we have

$$\nabla_{m_i}\bar{u}_i\cdot\nu \geq \bar{\lambda}\prod_{j=1}^{n}\bar{u}_j^{m_{ij}} \quad (i = 1,\ldots,n)$$

if

$$\frac{P_i^{m_i} c_{m_i}^{m_i-1} A_{m_i}}{(1+Q_i)^{m_i}} \geq \bar{\lambda}\prod_{j=1}^{n}(P_j\log(1+Q_j))^{m_{ij}} \;\; (i = 1,\ldots,n) \tag{3.4}$$

and

$$k_i\alpha_i \geq \sum_{j=1}^{n} m_{ij}\alpha_j \ \ (i = 1, \ldots, n). \tag{3.5}$$

Note that $k_i < m_i (i = 1,..., n)$. From Lemmas 2.2 and 2.3, we know that inequalities (3.4) and (3.5) hold for suitable choices of P_i, α_i $(i = 1,..., n)$. Moreover, if we choose P_i, α_i to be large enough such that

$$P_i \log Q_i \geq ||u_{i0}||_\infty, \quad \alpha_i \geq \frac{2\lambda_{m_i} P_i^{m_i - k_i}}{k_i Q_i^{m_i} (\log Q_i)^{k_i}},$$

then $\overline{u_i}(x, 0) \geq u_{i0}$, $(\bar{u}_i^{k_i})_t \geq \Delta_{m_i} \bar{u}_i \quad (i = 1, \ldots, n)$. Therefore, we have proved that $(\bar{u}_1, \ldots, \bar{u}_n)$ is a global upper solution of the system (1.1). The global existence of solutions to the problem (1.1) follows from the comparison principle.

Case 2. ($k_i \geq m_i$ $(i = 1,..., n)$). Let

$$\bar{u}_i(x,t) = e^{\alpha_i t}\left(M + \bar{\lambda}^{\frac{1}{m_i}} e^{-L_i\varphi_{m_i} e^{\frac{(k_i-m_i)\alpha_i t}{m_i+1}}} (2M)^{\frac{\sum_{j=1}^{n} m_{ij}}{m_i}} L_i^{-1} A_i^{-\frac{1}{m_i}}\right) \ \ (i = 1, \ldots, n), \tag{3.6}$$

where $A_i = A_{m_i} C_{m_i}^{m_i - 1}$ if $m_i \geq 1$, $A_i = A_{m_i} D_{m_i}^{m_i - 1}$ if $m_i < 1$, φ_{m_i}, A_{m_i}, B_{m_i}, C_{m_i} are defined in (3.1) and (3.2), $\alpha_i (i = 1,..., n)$ are positive constants that remain to be determined, and

$$M = \max_{1 \leq i \leq n}\{1, ||u_{i0}||_\infty\}, \quad L_i = \bar{\lambda}^{\frac{1}{m_i}} 2^{\frac{\sum_{j=1}^{n} m_{ij}}{m_i}} M^{\frac{\sum_{j=1}^{n} m_{ij} - m_i}{m_i}} A_i^{-\frac{1}{m_i}} \max\left\{1, \quad \frac{2(k_i - m_i)}{m_i + 1}\right\}.$$

Since $-ye^{-y} \geq -e^{-1}$ for any $y > 0$, we know that $-L_i\varphi_{m_i} e^{\frac{(k_i-m_i)\alpha_i t}{m_i+1}} e^{-L_i\varphi_{m_i} e^{\frac{(k_i-m_i)\alpha_i t}{m_i+1}}} \geq -e^{-1}$. Thus, for $(x, t) \in \Omega \times \mathbb{R}^+$, a simple computation shows that

$$\begin{aligned}
(\bar{u}_i^{k_i})_t &= k_i\alpha_i e^{k_i\alpha_i t}\left(M + \bar{\lambda}^{\frac{1}{m_i}} e^{-L_i\varphi_{m_i} e^{\frac{(k_i-m_i)\alpha_i t}{m_i+1}}} (2M)^{\frac{\sum_{j=1}^{n} m_{ij}}{m_i}} L_i^{-1} A_i^{-\frac{1}{m_i}}\right)^{k_i} \\
&\quad + k_i e^{k_i\alpha_i t}\left(M + \bar{\lambda}^{\frac{1}{m_i}} e^{-L_i\varphi_{m_i} e^{\frac{(k_i-m_i)\alpha_i t}{m_i+1}}} (2M)^{\frac{\sum_{j=1}^{n} m_{ij}}{m_i}} L_i^{-1} A_i^{-\frac{1}{m_i}}\right)^{k_i - 1} \\
&\quad \times \bar{\lambda}^{\frac{1}{m_i}} (2M)^{\frac{\sum_{j=1}^{n} m_{ij}}{m_i}} L_i^{-1} A_i^{-\frac{1}{m_i}} \frac{(k_i - m_i)\alpha_i}{m_i + 1} (-L_i\varphi_{m_i}) e^{\frac{(k_i-m_i)\alpha_i t}{m_i+1}} e^{-L_i\varphi_{m_i} e^{\frac{(k_i-m_i)\alpha_i t}{m_i+1}}} \\
&\geq \frac{1}{2} k_i\alpha_i e^{k_i\alpha_i t}.
\end{aligned}$$

In addition, we have

$$\begin{aligned}
\Delta_{m_i}\bar{u}_i &\leq \bar{\lambda}\lambda_{m_i}(2M)^{\sum_{j=1}^{n} m_{ij}} A_i^{-1} \varphi_{m_i}^{m_i} e^{m_i\alpha_i t} e^{\frac{m_i(k_i-m_i)\alpha_i t}{m_i+1}} e^{-L_i m_i\varphi_{m_i} e^{\frac{(k_i-m_i)\alpha_i t}{m_i+1}}} \\
&\quad + \bar{\lambda} L_i m_i (2M)^{\sum_{j=1}^{n} m_{ij}} A_i^{-1} e^{k_i\alpha_i t} e^{-L_i m_i\varphi_{m_i} e^{\frac{(k_i-m_i)\alpha_i t}{m_i+1}}} \left|\nabla\varphi_{m_i}\right|^{m_i+1} \\
&\leq \bar{\lambda}(\lambda_{m_i} + L_i m_i D_{m_i}^{m_i+1})(2M)^{\sum_{j=1}^{n} m_{ij}} A_i^{-1} e^{k_i\alpha_i t}.
\end{aligned}$$

Noting $\varphi_{m_i} = 0$ $(i = 1, 2, \dots, n)$ on $\partial\Omega$, we have on the boundary that

$$\nabla_{m_i}\bar{u}_i \cdot \nu \geq \bar{\lambda}(2M)^{\sum_{j=1}^{n} m_{ij}} e^{\frac{m_i(k_i-m_i)\alpha_i t}{m_i+1}},$$
$$\prod_{j=1}^{n} \bar{u}_j^{m_{ij}} \leq (2M)^{\sum_{j=1}^{n} m_{ij}} e^{\sum_{j=1}^{n} m_{ij}\alpha_j t}.$$

Then, we have

$$\nabla_{m_i}\bar{u}_i \cdot \nu \geq \bar{\lambda} \prod_{j=1}^{n} \bar{u}_j^{m_{ij}} \ (i = 1, \dots, n)$$

if

$$\frac{m_i(k_i-1)\alpha_i}{m_i+1} \geq \sum_{j=1}^{n} m_{ij}\alpha_j \ (i = 1, \dots, n). \tag{3.7}$$

From Lemma 2.2, we know that inequalities (3.7) hold for suitable choices of $\alpha_i(i = 1,..., n)$. Moreover, if we choose ∞_i to be large enough such that

$$\alpha_i \geq 2\bar{\lambda}(\lambda_{m_i} + L_i m_i D_{m_i}^{m_i+1})(2M)^{\sum_{j=1}^{n} m_{ij}}(k_i A_i)^{-1},$$

then $(\bar{u}_i^{k_i})_t \geq \Delta_{m_i}\bar{u}_i$ $(i = 1, \dots, n)$. Therefore, we have shown that $(\bar{u}_1, \dots, \bar{u}_n)$ is an upper solution of (1.1) and exists globally. Therefore, $(u_1, \dots, u_n) \leq (\bar{u}_1, \dots, \bar{u}_n)$, and hence the solution $(u_1,..., u_n)$ of (1.1) exists globally.

Case 3. ($k_i < m_i$ $(i = 1,..., s)$; $k_i \geq m_i$ $(i = s + 1,..., n)$). Let $\bar{u}_i(x, t)$ $(i = 1, \dots, s)$ be as in (3.3) and

$$\bar{u}_i(x,t) = e^{\alpha_i t}\left(M_i + \bar{\lambda}^{\frac{1}{m_i}} e^{-L_i\varphi_{m_i} e^{\frac{(k_i-m_i)\alpha_i t}{m_i+1}}} (2M_i)^{\frac{k_i+1}{m_i+1}} L_i^{-1} A_i^{-\frac{1}{m_i}}\right) \ (i = s+1, \dots, n),$$

where φ_{m_i}, and A_i are as in case 2. By Lemma 2.3, we choose $P_i \geq (\log Q_i)^{-1}||u_{i0}||_\infty$ $(i = 1,..., s)$ and $M_i \geq \max\{1, ||u_{i0}||_\infty\}$ $(i = s + 1,..., n)$ such that

$$\begin{aligned} \frac{P_i^{m_i} c_{m_i}^{m_i-1} A_{m_i}}{(1+Q_i)^{m_i}} &\geq \prod_{j=1}^{s} (P_j \log(1+Q_j))^{m_{ij}} \prod_{j=s+1}^{n} (2M_j)^{m_{ij}} \quad (i = 1, \dots, s), \\ \bar{\lambda}(2M_i)^{\frac{m_i(k_i+1)}{m_i+1}} &\geq \prod_{j=1}^{s} (P_j \log(1+Q_j))^{m_{ij}} \prod_{j=s+1}^{n} (2M_j)^{m_{ij}} \quad (i = s+1, \dots, n). \end{aligned} \tag{3.8}$$

Set

$$L_i = \bar{\lambda}^{\frac{1}{m_i}} 2^{\frac{k_i+1}{m_i+1}} M_i^{\frac{k_i-m_i}{m_i+1}} A_i^{-\frac{1}{m_i}} \max\left\{1, \frac{2(k_i-m_i)}{m_i+1}\right\} \quad (i = s+1, \dots, n).$$

By similar arguments, in cases 1 and 2, we have on the boundary that

$$\nabla_{m_i}\bar{u}_i\cdot\nu \ge \frac{P_i^{m_i}c_{m_i}^{m_i-1}A_{m_i}}{(1+Q_i)^{m_i}}e^{k_i\alpha_i t}\quad(i=1,\dots,s),$$
$$\nabla_{m_i}\bar{u}_i\cdot\nu \ge \bar{\lambda}(2M_i)^{\frac{m_i(k_i+1)}{m_i+1}}e^{\frac{m_i(k_i-1)\alpha_i t}{m_i+1}}\quad(i=s+1,\dots,n),$$
$$\prod_{j=1}^{n}\bar{u}_j^{m_{ij}}\le\prod_{j=1}^{s}(P_j\log(1+Q_j))^{m_{ij}}\prod_{j=s+1}^{n}(2M_j)^{m_{ij}}e^{\sum_{j=1}^{n}m_{ij}\alpha_j t}\quad(i=1,\dots,n).$$

Therefore employing (3.8), we see that

$$\nabla_{m_i}\bar{u}_i\cdot\nu\ge\bar{\lambda}\prod_{j=1}^{n}\bar{u}_j^{m_{ij}}\ (i=1,\dots,n)$$

if we knew

$$k_i\alpha_i\ge\sum_{j=1}^{n}m_{ij}\alpha_j(i=1,\dots,s),\quad \frac{m_i(k_i-1)\alpha_i}{m_i+1}\ge\sum_{j=1}^{n}m_{ij}\alpha_j\ (i=s+1,,\dots,n).\tag{3.9}$$

We deduce from Lemma 2.2 that (3.9) holds for suitable choices of α_i (i = 1,..., n). Moreover, we can choose α_i large enough to assure that

$$\alpha_i\ge\frac{2\lambda_{m_i}P_i^{m_i-k_i}}{k_iQ_i^{m_i}(\log Q_i)^{k_i}},\ (i=1,\dots,s),$$
$$\alpha_i\ge 2\bar{\lambda}(\lambda_{m_i}+L_im_iD_{m_i}^{m_i+1})(2M_i)^{\frac{m_i(k_i+1)}{m_i+1}}(k_iA_i)^{-1}\ (i=s+1,\dots,n),$$

Then, as in the calculations of cases 1 and 2, we have $(\bar{u}_i^{k_i})_t\ge\Delta_{m_i}\bar{u}_i\quad(i=1,\dots,n)$. We prove that $(\bar{u}_1,\dots,\bar{u}_n)$ is an upper solution of (1.1), so (u_1,..., u_n) exists globally.

Proof of the necessity.

Without loss of generality, we first assume that all the lower-order principal minor determinants of A are non-negative, and $|A|<0$, for, if not, there exists some lth-order ($1\le l<n$) principal minor determinant det$A_{l\times l}$ of $A=(a_{ij})_{n\times n}$ which is negative. Without loss of generality, we may consider that

$$A_{l\times l}=\begin{pmatrix}a_{11}&\dots&a_{1l}\\a_{12}&\dots&a_{2l}\\\dots&\dots&\dots\\a_{l1}&\dots&a_{ll}\end{pmatrix}$$

and all of the sth-order ($1\le s\le l-1$) principal minor determinants det$A_{s\times s}$ of $A_{l\times l}$ are non-negative. Then, we consider the following problem:

$$\begin{aligned}&(w_i^{k_i})_t=\Delta_{m_i}w_i\ (i=1,\dots,l), && x\in\Omega,\ t>0,\\&\nabla_{m_i}w_i\cdot\nu=\delta^{\sum_{j=l+1}^{n}m_{ij}}\prod_{j=1}^{n}w_j^{m_{ij}}\ (i=1,\dots,l), && x\in\partial\Omega,\ t>0,\\&w_i(x,0)=u_{i0}(x)\ (i=1,\dots,l), && x\in\bar{\Omega}.\end{aligned}\tag{3.10}$$

Note that $\delta=\min_{1\le i\le n}\{\min_{\bar{\Omega}}u_{i0}(x)\}>0$. If we can prove that the solution (w_1,..., w_l) of (3.10) blows up in finite time, then (w_1,... w_l, δ,..., δ) is a lower solution of (1.1) that blows up in finite time. Therefore, the solution of (1.1) blows up in finite time.

We will complete the proof of the necessity of our theorem in three different cases.

Case 1. ($k_i < m_i$ (i = 1,..., n)). Let

$$\underline{u}_i = Y_i^{\rho_i} \quad \text{and} \quad Y_i = ah^{1+\frac{1}{m_i}}(x) + (b-ct)^{-\gamma_i} \; (i = 1, \ldots, n), \tag{3.11}$$

where $h(x) = \sum_{i=1}^N x_i + Nd + 1$, $d = \max\{|x| \,|\, x \in \bar{\Omega}\}$, $\rho_i = \frac{m_i + \frac{1}{\gamma_i}}{m_i - k_i}$, $\gamma_i = \frac{(m_i - k_i)\alpha_i - 1}{m_i}$, the α_i are as given in Lemma 2.4 and satisfy $\alpha_i > \frac{1}{m_i - k_i}$,

$$b = \max_{1\le i\le n}\{1, (\tfrac{1}{2}\delta^{\frac{1}{\rho_i}})^{-\frac{1}{\gamma_i}}\},\ a = \min_{1\le i\le n}\left\{b^{-\gamma_i}(2Nd+1)^{-\frac{1+m_i}{m_i}},\ \left(\underline{\lambda}^{-1}\left[\frac{(1+m_i)\rho_i N^{\frac{1}{2}} 2^{\rho_i - 1}}{m_i}\right]^{m_i}(2Nd+1)\right)^{-\frac{1}{m_i}} b^{-\frac{\sum_{j=1}^n m_{ij}\alpha_j}{m_i}}\right\} \tag{3.12}$$

$$c = \min_{1\le i\le n}\left\{\frac{a^{m_i}\rho_i^{m_i-1}(1+\frac{1}{m_i})^{m_i} N^{\frac{m_i+1}{2}}}{k_i\gamma_i}\right\}.$$

By direct computation for $(x, t) \in \Omega \times (0, \frac{b}{c})$, we have

$$(\underline{u}_i^{k_i})_t = ck_i\rho_i\gamma_i Y_i^{k_i\rho_i - 1}(b-ct)^{-(\gamma_i+1)}, \quad \nabla \underline{u}_i = a\rho_i(1+\tfrac{1}{m_i})Y_i^{\rho_i-1}h^{\frac{1}{m_i}}(x)(1,\ldots,1),$$

$$\begin{aligned}
\Delta_{m_i}\underline{u}_i &= \sum_{j=1}^N\left((a\rho_i(1+\tfrac{1}{m_i}))^{m_i}N^{\frac{m_i-1}{2}}Y_i^{m_i(\rho_i-1)}h(x)\right)_{x_j} \\
&= (a\rho_i(1+\tfrac{1}{m_i}))^{m_i}N^{\frac{m_i+1}{2}}Y_i^{m_i(\rho_i-1)} \\
&\quad + m_i(\rho_i-1)\rho_i^{m_i}(a(1+\tfrac{1}{m_i}))^{m_i+1}N^{\frac{m_i+1}{2}}h^{1+\frac{1}{m_i}}(x)Y_i^{m_i(\rho_i-1)-1} \\
&\ge (a\rho_i(1+\tfrac{1}{m_i}))^{m_i}N^{\frac{m_i+1}{2}}Y_i^{k_i\rho_i-1}Y_i^{m_i(\rho_i-1)-k_i\rho_i+1} \\
&\ge (\underline{u}_i^{k_i})_t \; (i = 1, \ldots, n).
\end{aligned}$$

For $(x, t) \in \partial\Omega \times (0, \frac{b}{c})$, we have

$$\begin{aligned}
\nabla_{m_i}\underline{u}_i \cdot \nu &\le (a\rho_i(1+\tfrac{1}{m_i}))^{m_i}N^{\frac{m_i}{2}}(2Nd+1)2^{m_i(\rho_i-1)}(b-ct)^{-m_i(\rho_i-1)\gamma_i} \\
&= (a\rho_i(1+\tfrac{1}{m_i}))^{m_i}N^{\frac{m_i}{2}}(2Nd+1)2^{m_i(\rho_i-1)}(b-ct)^{-(k_i\alpha_i+1)} \; (i = 1, \ldots, n),
\end{aligned}$$

$$\prod_{j=1}^n \underline{u}_j^{m_{ij}} = \prod_{j=1}^n Y_j^{m_{ij}\rho_j} \ge \prod_{j=1}^n (b-ct)^{-\sum_{j=1}^n m_{ij}\alpha_j} (i = 1, \ldots, n).$$

Thus, by (3.12) and Lemma 2.4, we have

$$\nabla_{m_i}\underline{u}_i \cdot \nu \le \underline{\lambda}\prod_{j=1}^n \underline{u}_j^{m_{ij}} \; (i = 1, \ldots, n).$$

We confirm that ($\underline{u}_1$,..., $\underline{u}_n$) is a lower solution of (1.1), which blows up in finite time. We know by the comparison principle that the solution (u_1,..., u_n) blows up in finite time.

Case 2. ($k_i \ge m_i$ (i = 1,..., n)). Let $d_{m_i} = C_{m_i}$ if $m_i < 1$, $d_{m_i} = D_{m_i}$ if $m_i \ge 1$. for $k_i \ge m_i$ (i = 1,..., n), set

$$\underline{u}_i = \frac{1}{(b-ct)^{\alpha_i}} e^{\frac{-a\varphi_{m_i}(x)}{(b-ct)^{\beta_i}}}, \tag{3.13}$$

where $\alpha_i (i = 1,..., n)$ are to determined later and

$$\beta_i = \frac{(k_i-m_i)\alpha_i+1}{m_i+1},\ b = \max_{1\le i\le n}\{1,\quad \delta^{-\frac{1}{\alpha_i}}\}, \tag{3.14}$$

$$a = \min_{1\le i\le n}\left\{1,\ \underline{\lambda}^{\frac{1}{m_i}}(B_{m_i}d_{m_i}^{m_i-1})^{-\frac{1}{m_i}} b^{\frac{-\sum_{j=1}^n m_{ij}\alpha_j}{m_i}}\right\}, \tag{3.15}$$

$$c = \min_{1\le i\le n}\left\{\frac{m_i a^{m_i+1}E_{m_i}^{m_i+1}}{k_i\alpha_i},\quad \frac{\lambda_{m_i}(k_i-m_i)a^{m_i+1}F_{m_i}^{m_i+1}}{k_i\alpha_i}\right\}. \tag{3.16}$$

By a direct computation, for $x \in \Omega$, $0 < t < c/b$, we obtain that

$$\begin{aligned}(\underline{u}_i^{k_i})_t &= k_i\alpha_i c e^{\frac{-ak_i\varphi_{m_i}(x)}{(b-ct)^{\beta_i}}}(b-ct)^{-(k_i\alpha_i+1)} - \frac{e^{\frac{-ak_i\varphi_{m_i}(x)}{(b-ct)^{\beta_i}}} ak_i\beta_i c\varphi_{m_i}(x)}{(b-ct)^{k_i\alpha_i}(b-ct)^{\beta_i+1}}\\ &\le k_i\alpha_i c e^{\frac{-ak_i\varphi_{m_i}(x)}{(b-ct)^{\beta_i}}}(b-ct)^{-(k_i\alpha_i+1)},\\ \Delta_{m_i}\underline{u}_i &= \frac{\lambda_{m_i}a^{m_i}\varphi_{m_i}^{m_i} e^{\frac{-am_i\varphi_{m_i}(x)}{(b-ct)^{\beta_i}}}}{(b-ct)^{m_i(\alpha_i+\beta_i)}} + \frac{m_i a^{m_i+1} e^{\frac{-am_i\varphi_{m_i}(x)}{(b-ct)^{\beta_i}}}|\nabla\varphi_{m_i}|^{m_i+1}}{(b-ct)^{m_i(\alpha_i+\beta_i)+\beta_i}}.\end{aligned} \tag{3.17}$$

If $x \in \{x \in \Omega:\ dist(x, \partial\Omega) > \varepsilon_{m_i}\}$, we have $\varphi_{m_i} \ge F_{m_i}$, and thus

$$\Delta_{m_i}\underline{u}_i \ge \frac{\lambda_{m_i}a^{m_i}F_{m_i}^{m_i} e^{\frac{-a_i m_i\varphi_{m_i}(x)}{(b-ct)^{\beta_i}}}}{(b-ct)^{m_i(\alpha_i+\beta_i)}}. \tag{3.18}$$

On the other hand, since $-ye^{-y} \ge -e^{-1}$ for any $y > 0$, we have

$$(\underline{u}_i^{k_i})_t \le k_i\alpha_i c e^{\frac{-ak_i\varphi_{m_i}(x)}{(b-ct)^{\beta_i}}}(b-ct)^{-(k_i\alpha_i+1)} \le \frac{k_i\alpha_i c e^{\frac{-am_i\varphi_{m_i}(x)}{(b-ct)^{\beta_i}}}}{a(k_i-m_i)F_{m_i}e(b-ct)^{m_i(\alpha_i+\beta_i)}}. \tag{3.19}$$

We have by (3.16), (3.18), and (3.19) that $(\underline{u}_i^{k_i})_t \le \Delta_{m_i}\underline{u}_i\ (i = 1,\dots,n)$.

If $x \in \{x \in \Omega:\ dist(x, \partial\Omega) \le \varepsilon_{m_i}\}$, then $|\nabla\varphi_{m_i}| \ge E_{m_i}$, and then

$$\Delta_{m_i}\underline{u}_i \ge \frac{m_i a^{m_i+1}E_{m_i}^{m_i+1} e^{\frac{-ak_i\varphi_{m_i}(x)}{(b-ct)^{\beta_i}}}}{(b-ct)^{m_i(\alpha_i+\beta_i)+\beta_i}} = \frac{m_i a^{m_i+1}E_{m_i}^{m_i+1} e^{\frac{-ak_i\varphi_{m_i}(x)}{(b-ct)^{\beta_i}}}}{(b-ct)^{k_i\alpha_i+1}}. \tag{3.20}$$

It follows from (3.16), (3.17), and (3.20) that $(\underline{u}_i^{k_i})_t \le \Delta_{m_i}\underline{u}_i\ (i = 1,\dots,n)$.

We have on the boundary that

$$\begin{aligned}\nabla_{m_i}\underline{u}_i\cdot\nu &= \frac{a^{m_i}|\nabla\varphi_{m_i}|^{m_i-1}e^{\frac{-am_i\varphi_{m_i}(x)}{(b-ct)^{\beta_i}}}(-\frac{\partial\varphi_{m_i}}{\partial\nu})}{(b-ct)^{m_i(\alpha_i+\beta_i)}} \le \frac{a^{m_i}B_{m_i}d_{m_i}^{m_i-1}}{(b-ct)^{m_i(\alpha_i+\beta_i)}}\ (i = 1,\dots,n),\\ \prod_{j=1}^n \underline{u}_j^{m_{ij}} &= \frac{1}{(b-ct)^{\sum_{j=1}^n m_{ij}\alpha_j}}\ (i = 1,2,\dots,n).\end{aligned} \tag{3.21}$$

Moreover, by (3.14) and Lemma 2.4, we have that

$$m_i(\alpha_i + \beta_i) \le \sum_{j=1}^{n} m_{ij}\alpha_j \ \ (i = 1, \ldots, n). \tag{3.22}$$

(3.15), (3.21), and (3.22) imply that $\nabla_{m_i} \underline{u}_i \cdot \nu \le \underline{\lambda} \prod_{j=1}^{n} \underline{u}_j^{m_{ij}} (i = 1, \ldots, n)$. Therefore, $(\underline{u}_1,\ldots, \underline{u}_1)$ is a lower solution of (1.1).

For $k_i = m_i (i = 1,\ldots, n)$, let

$$\underline{u}_i = \frac{1}{(b-ct)^{\alpha_i}} e^{\frac{-a\varphi_{m_i}(x)}{(b-ct)^{\frac{1}{m_i}}}}. \tag{3.23}$$

For $k_i = m_i$ $(i = 1,\ldots, s)$ and $k_i > m_i$ $(i = s + 1,\ldots, n)$, let $\bar{u}_i(x, t)$ as in (3.13) and (3.23). Using similar arguments as above, we can prove that $(\underline{u}_1,\ldots, \underline{u}_n)$ is a lower solution of (1.1). Therefore, $(\underline{u}_1,\ldots, \underline{u}_n) \le (u_1,\ldots, u_n)$. Consequently, $(u_1,\ldots, u_n)$ blows up in finite time.

Case 3. ($k_i < m_i$ $(i = 1,\ldots, s)$; $k_i \ge m_i$ $(i = s + 1,\ldots, n)$). Let $\bar{u}_i(x, t)$ $(i = 1, \ldots, s)$ be as in (3.11) and

$$\underline{u}_i = \frac{1}{(b-ct)^{\alpha_i}} e^{\frac{-a\varphi_{m_i}(x)}{(b-ct)^{\beta_i}}} \ \ (i = s + 1, \ldots, n),$$

where α_i's are to determined later and

$$\beta_i = \frac{(k_i-m_i)\alpha_i+1}{m_i+1} \ (i = s + 1, \ldots, n), \quad b = \max\{1, \ \max_{1\leqslant i\leqslant s}\{(\tfrac{1}{2}\delta^{\frac{1}{\rho_i}})^{-\frac{1}{\gamma_i}}\}, \quad \max_{s+1\leqslant i\leqslant n}\{\delta^{-\frac{1}{\alpha_i}}\}\},$$

$$a = \min\left\{\min_{s+1\leqslant i\leqslant n}\{\underline{\lambda}^{\frac{1}{m_i}}(B_{m_i}d_{m_i}^{m_i-1})^{-\frac{1}{m_i}} b^{\frac{-\sum_{j=1}^{n} m_{ij}\alpha_j}{m_i}}\}, \min_{1\leqslant i\leqslant s}\{b^{-\gamma_i}(2Nd+1)^{-\frac{1+m_i}{m_i}}, \left(\underline{\lambda}^{-1}[\frac{(1+m_i)\rho_i N^{\frac{1}{2}} 2^{\rho_i-1}}{m_i}]^{m_i}(2Nd+1)\right)^{-\frac{1}{m_i}} b^{-\frac{-\sum_{j=1}^{n} m_{ij}\alpha_j}{m_i}}\}\right\},$$

$$c = \min\left\{\min_{1\leqslant i\leqslant s}\{\frac{a^{m_i}\rho_i^{m_i-1}(1+\frac{1}{m_i})^{m_i}N^{\frac{m_i+1}{2}}}{k_i\gamma_i}\}, \min_{s+1\leqslant i\leqslant n}\left\{\frac{m_i a^{m_i+1}E_{m_i}^{m_i+1}}{k_i\alpha_i}, \frac{\lambda_{m_i}(k_i-m_i)a^{m_i+1}F_{m_i}^{m_i+1}}{k_i\alpha_i}\right\}\right\}.$$

Based on arguments in cases 1 and 2, we have $(\underline{u}_i^{k_i})_t \le \Delta_{m_i}\underline{u}_i$ $(i = 1, \ldots, n)$ for $(x, t) \in \Omega \times (0, \frac{b}{c})$. Furthermore, for $(x, t) \in \partial\Omega \times (0, \frac{b}{c})$, we have

$$\nabla_{m_i}\underline{u}_i \cdot \nu \le (a\rho_i(1 + \tfrac{1}{m_i}))^{m_i} N^{\frac{m_i}{2}} (2Nd + 1)2^{m_i(\rho_i-1)}(b - ct)^{-(k_i\alpha_i+1)} \ (i = 1, \ldots, s),$$

$$\nabla_{m_i}\underline{u}_i \cdot \nu \le a^{m_i} B_{m_i} d_{m_i}^{m_i-1}(b - ct)^{-m_i(\alpha_i+\beta_i)} \ (i = s + 1, \ldots, n),$$

$$\prod_{j=1}^{n} \underline{u}_j^{m_{ij}} \ge (b - ct)^{-\sum_{j=1}^{n} m_{ij}\alpha_j} \ (i = 1, \ldots, n).$$

Thus,

$$\nabla_{m_i}\underline{u}_i \cdot \nu \le \underline{\lambda} \prod_{j=1}^{n} \underline{u}_j^{m_{ij}} \ (i = 1, \ldots, n)$$

holds if

$$\begin{aligned} k_i\alpha_i + 1 &\le \sum_{j=1}^{n} m_{ij}\alpha_j \quad (i = 1, \ldots, s), \\ m_i(\alpha_i + \beta_i) &\le \sum_{j=1}^{n} m_{ij}\alpha_j \quad (i = s+1, , \ldots, n). \end{aligned} \tag{3.24}$$

From Lemma 2.4, we know that inequalities (3.24) hold for suitable choices of α_i (i = 1,..., n). We show that ($\underline{u}_1$,..., $\underline{u}_n$) is a lower solution of (1.1). Since ($\underline{u}_1$,..., $\underline{u}_n$) blows up in finite time, it follows that the solution of (1.1) blows up in finite time.

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Authors' contributions
DW carried out all studies in the paper. LZ participated in the design of the study in the paper.

Competing interests
The authors declare that they have no competing interests.

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