# New Delay-Range-Dependent Robust Exponential Stability Criteria of Uncertain Impulsive Switched Linear Systems with Mixed Interval Nondifferentiable Time-Varying Delays and Nonlinear Perturbations 

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#### Abstract

We investigate the problem of robust exponential stability analysis for uncertain impulsive switched linear systems with timevarying delays and nonlinear perturbations. The time delays are continuous functions belonging to the given interval delays, which mean that the lower and upper bounds for the time-varying delays are available, but the delay functions are not necessary to be differentiable. The uncertainties under consideration are nonlinear time-varying parameter uncertainties and norm-bounded uncertainties, respectively. Based on the combination of mixed model transformation, Halanay inequality, utilization of zero equations, decomposition technique of coefficient matrices, and a common Lyapunov functional, new delay-range-dependent robust exponential stability criteria are established for the systems in terms of linear matrix inequalities (LMIs). A numerical example is presented to illustrate the effectiveness of the proposed method.


## 1. Introduction

The problem of stability analysis for dynamical systems with time delays and uncertainties has been intensively studied since these systems often occur in many industrial systems such as chemical processes, biological systems, population dynamics, neural networks, large-scale systems, and network control systems. The occurrence of the time delays and uncertainties may cause frequently the source of instability or poor performances in various systems. Thus, there has been growing interest in stability analysis and controller design for time-delay systems. However, authors investigated the robust synchronization of coupled fuzzy cellular neural networks with differentiable time-varying delay in $[1,2]$. Stability criteria for time-delay systems are generally divided into two classes: delay-independent one and delaydependent one. Delay-independent stability criteria tend
to be more conservative, especially for small size delay; such criteria do not give any information on the size of the delay. On the other hand, delay-dependent stability criteria are concerned with the size of the delay and usually provide a maximal delay size. Most of the existing delaydependent stability criteria are presented by using LyapunovKrasovskii approach or Lyapunov-Razumikhin approach. In recent years, much attention has been paid to stability analysis of the uncertain linear systems with interval timevarying delay [3-6]. In [5], the authors studied the delaydependent stability problem for uncertain linear systems with interval time-varying delay. The restriction on the derivative of the interval time-varying delay was removed. Moreover, robust stability analysis of uncertain linear systems with time-varying delays and nonlinear perturbations has received the attention of a lot of theoreticians and engineers in this field over the last few decades [7-14]. Furthermore, authors
studied the delay-dependent robust stability criteria for linear systems with discrete interval time-varying delay, discrete constant delay, and nonlinear perturbations in [15]. However, a descriptor model transformation and a corresponding Lyapunov-Krasovskii functional have been introduced for stability analysis of systems with delays in [16]. In [17], the authors studied the problem of stability for linear switching system with time-varying delays.

Over the past decades, the problem of stability analysis for dynamic systems with impulsive effects and switching has arisen in a wide range of disciplines, such as physics, chemical engineering, and biology [18-28]. These systems are usually called impulsive switched systems. In [24], the authors studied the asymptotic stability problem for a class of impulsive switched systems with time-invariant delays based on LMI approach. Stability criteria of uncertain impulsive switched systems with time-invariant delays are introduced in [25]. Most of the existing delay-dependent stability criteria for time-delay systems are obtained as the upper bounds on the derivative time-varying delays by using LyapunovKrasovskii functional. However, it appears that few results are available for stability analysis for impulsive switched systems with time-varying delays. In consequence, it is important and interesting to study the problem of robust stability analysis for uncertain impulsive switched systems with interval nondifferentiable time-varying delays and nonlinear perturbations by using a common Lyapunov functional and Halanay lemma.

In this paper, we present the delay-range-dependent robust exponential stability criteria for uncertain impulsive switched linear systems with mixed interval nondifferentiable time-varying delays and nonlinear perturbations. Based on Halanay inequality, mixed model transformation, utilization of zero equations, decomposition technique of coefficient matrices, and a common Lyapunov functional, some new delay-range-dependent robust exponential stability criteria are derived in terms of LMIs for the systems. In order to reduce the complexity of stability criteria for calculation and finding solutions, mixed model transformation $[13,16]$ and Halanay inequality [29-31] are used. Finally, an illustrative example is given to show the effectiveness and advantages of the developed method.

## 2. Problem Formulation and Preliminaries

The following notations will be used in this paper: $N$ denotes the set of all natural numbers; $R^{+}$denotes the set of all real nonnegative numbers; $R^{n}$ denotes the $n$-dimensional Euclidean space equipped with the Euclidean norm $\|\cdot\| ; R^{n \times r}$ denotes the space of all matrices of ( $n \times r$ )-dimensions; $A^{T}$ denotes the transpose of the matrix $A ; A$ is symmetric if $A=A^{T} ; I$ denotes the identity matrix; $\lambda(A)$ denotes the set of all eigenvalues of $A ; \lambda_{\max }(A)=\max \{\operatorname{Re} \lambda: \lambda \in \lambda(A)\}$; $\lambda_{\text {min }}(A)=\min \{\operatorname{Re} \lambda: \lambda \in \lambda(A)\} ;$ matrix $A$ is called semipositive definite $(A \geq 0)$ if $x^{T} A x \geq 0$, for all $x \in R^{n}$; $A$ is positive definite $(A>0)$ if $x^{T} A x>0$ for all $x \in R^{n}-\{0\}$; matrix $B$ is called seminegative definite $(B \leq 0)$ if $x^{T} B x \leq 0$, for all $x \in R^{n} ; B$ is negative definite $(B<0)$ if $x^{T} B x<0$ for
all $x \in R^{n}-\{0\} ; A>B$ means $A-B>0(B-A<0) ; A \geq B$ means $A-B \geq 0(B-A \leq 0) ; \bar{h}=\max \left\{h_{2}, r_{2}\right\}, h_{2}, r_{2} \in R^{+}$; $x_{t}=x(t+s), s \in[-\bar{h}, 0]$.

Consider the following uncertain impulsive switched linear system with time delays:

$$
\begin{aligned}
\dot{x}(t)= & A_{i_{k}}(t) x(t)+B_{i_{k}}(t) x\left(t-h_{i_{k}}(t)\right) \\
& +C_{i_{k}}(t) x\left(t-r_{i_{k}}(t)\right)+f_{i_{k}}(t, x(t)) \\
& +g_{i_{k}}\left(t, x\left(t-h_{i_{k}}(t)\right)\right) \\
& +w_{i_{k}}\left(t, x\left(t-r_{i_{k}}(t)\right)\right), \quad t \neq t_{k}, \\
\Delta x(t)= & x(t)-x\left(t^{-}\right)=G_{k} x\left(t^{-}-h_{i_{k}}\left(t^{-}\right)\right), \\
x\left(t_{0}+s\right)= & \phi(s), \quad \forall s \in[-\bar{h}, 0], \\
A_{i_{k}}(t)= & A_{i_{k}}+\Delta A_{i_{k}}(t), \\
B_{i_{k}}(t)= & B_{i_{k}}+\Delta B_{i_{k}}(t), \\
C_{i_{k}}(t)= & C_{i_{k}}+\Delta C_{i_{k}}(t),
\end{aligned}
$$

where $x(t) \in R^{n}$ denotes the state variable and $i_{k} \in\{1,2$, $\ldots, m\}, k, m \in N . A_{i_{k}}, B_{i_{k}}, C_{i_{k}}$, and $G_{k}$ are given constant matrices of appropriate dimensions. The delays $h_{i_{k}}(t)$ and $r_{i_{k}}(t)$ are interval time-varying bounded continuous functions satisfying

$$
\begin{align*}
& 0 \leq h_{1} \leq h_{i_{k}}(t) \leq h_{2}  \tag{2}\\
& 0 \leq r_{1} \leq r_{i_{k}}(t) \leq r_{2}
\end{align*}
$$

where $h_{1}, h_{2}, r_{1}$, and $r_{2}$ are given positive real constants. The uncertainties $f_{i_{k}}(\cdot), g_{i_{k}}(\cdot)$, and $w_{i_{k}}(\cdot)$ represent the nonlinear parameter perturbations with respect to the current state $x(t)$, the delayed state $x\left(t-h_{i_{k}}(t)\right)$, and $x\left(t-r_{i_{k}}(t)\right)$, respectively. They satisfy that $f_{i_{k}}(t, 0) \stackrel{ }{=} 0, g_{i_{k}}(t, 0)=0, w_{i_{k}}(t, 0)=0$, and

$$
\begin{align*}
& f_{i_{k}}^{T}(t, x(t)) f_{i_{k}}(t, x(t)) \leq \eta^{2} x^{T}(t) x(t), \\
& g_{i_{k}}^{T}\left(t, x\left(t-h_{i_{k}}(t)\right)\right) g_{i_{k}}\left(t, x\left(t-h_{i_{k}}(t)\right)\right) \\
& \quad \leq \rho^{2} x^{T}\left(t-h_{i_{k}}(t)\right) x\left(t-h_{i_{k}}(t)\right),  \tag{3}\\
& w_{i_{k}}^{T}\left(t, x\left(t-r_{i_{k}}(t)\right)\right) w_{i_{k}}\left(t, x\left(t-r_{i_{k}}(t)\right)\right) \\
& \quad \leq \zeta^{2} x^{T}\left(t-r_{i_{k}}(t)\right) x\left(t-r_{i_{k}}(t)\right),
\end{align*}
$$

where $\eta, \rho$, and $\zeta$ are given positive real constants. The uncertain matrices $\Delta A_{i_{k}}(t), \Delta B_{i_{k}}(t)$, and $\Delta C_{i_{k}}(t)$ are norm bounded and can be described as

$$
\begin{align*}
& {\left[\begin{array}{lll}
\Delta A_{i_{k}}(t) & \Delta B_{i_{k}}(t) & \Delta C_{i_{k}}(t)
\end{array}\right]} \\
& \quad=K_{i_{k}} \Delta_{i_{k}}(t)\left[\begin{array}{lll}
L_{i_{k}}^{1} & L_{i_{k}}^{2} & L_{i_{k}}^{3}
\end{array}\right], \tag{4}
\end{align*}
$$

where $K_{i_{k}}, L_{i_{k}}^{1}, L_{i_{k}}^{2}$, and $L_{i_{k}}^{3}$ are given constant matrices of appropriate dimensions. The class of parametric uncertainties $\Delta_{i_{k}}(t)$, which satisfies

$$
\begin{equation*}
\Delta_{i_{k}}(t)=F_{i_{k}}(t)\left[I-J F_{i_{k}}(t)\right]^{-1} \tag{5}
\end{equation*}
$$

is said to be admissible where $J$ is a known matrix satisfying

$$
\begin{equation*}
I-J J^{T}>0 \tag{6}
\end{equation*}
$$

and $F_{i_{k}}(t)$ is uncertain matrix satisfying

$$
\begin{equation*}
F_{i_{k}}^{T}(t) F_{i_{k}}(t) \leq I \tag{7}
\end{equation*}
$$

Hence, $\Delta x(t)=x\left(t_{k}^{+}\right)-x\left(t_{k}^{-}\right), \lim _{v \rightarrow 0^{+}} x\left(t_{k}+v\right)=x\left(t_{k}^{+}\right)$, and $x\left(t_{k}^{-}\right)=\lim _{v \rightarrow 0^{+}} x(t-v) . \phi(t)$ is the initial function with the norm $\|\phi\|=\sup _{\theta \in[-\bar{h}, 0]}\|\phi(\theta)\|$. We assume that the solution of the impulsive switched system (1) is right continuous; that is, $x\left(t_{k}^{+}\right)=x\left(t_{k}\right) \cdot t_{k}$ is an impulsive switching time point and $t_{0}<t_{1}<t_{2}<\cdots<t_{k}<\cdots, t_{k} \rightarrow+\infty$ as $k \rightarrow+\infty$, and we introduce the quantity

$$
\begin{equation*}
\tau=\inf \left\{t_{i+1}-t_{i}: i=1,2,3, \ldots\right\} \tag{8}
\end{equation*}
$$

This $\tau$ is called the dwell time of system (1). Under the switching law of system (1), at the time $t_{k}$, the system switches to the $i_{k}$ subsystem from the $i_{k-1}$ subsystem.

Definition 1. Given $\beta>0$, system (1) is robustly exponentially stable, if there exist switching function $i_{k}$ and positive real constant $K$ such that any solution $x(t, \phi)$ of the system satisfies

$$
\begin{equation*}
\|x(t, \phi)\| \leq K\|\phi\| e^{-\beta t}, \quad \forall t \in R^{+} . \tag{9}
\end{equation*}
$$

Lemma 2 (see [29] (Halanay lemma)). Let $m(t)$ be a positive scalar function and assume that the following condition holds:

$$
\begin{equation*}
D^{+} m(t) \leq-a m(t)+b \bar{m}(t), \quad t \geq t_{0} \tag{10}
\end{equation*}
$$

where $D^{+} m(t)=\lim \sup _{\Delta t \rightarrow 0^{+}}((m(t+\Delta t)-m(t)) / \Delta t), 0<$ $b<a$. Then, there exists $\beta>0$ such that, for all $t \geq t_{0}$,

$$
\begin{equation*}
m(t) \leq \bar{m}\left(t_{0}\right) e^{-\beta\left(t-t_{0}\right)} \tag{11}
\end{equation*}
$$

Here, $\bar{m}(t)=\sup _{t-\bar{h} \leq s \leq t}\{m(s)\}$ and $\beta$ satisfies $\beta-a+b e^{\beta \bar{h}}=0$.
Lemma 3 (see [32] (Schur complement lemma)). Given constant symmetric matrices $X, Y, Z$ where $Y>0$, then $X+$ $Z^{T} Y^{-1} Z<0$ if and only if

$$
\begin{align*}
\left(\begin{array}{cc}
X & Z^{T} \\
Z & -Y
\end{array}\right)<0  \tag{12}\\
o r\left(\begin{array}{cc}
-Y & Z \\
Z^{T} & X
\end{array}\right)<0 .
\end{align*}
$$

Lemma 4 (see [33]). For given matrices $Q=Q^{T}, H, E, R=$ $R^{T}>0$ of appropriate dimension, then

$$
\begin{equation*}
Q+H F E+E^{T} F^{T} H^{T}<0 \tag{13}
\end{equation*}
$$

for all $F$ satisfies $F^{T} F \leq R$, if and only if there exists a positive number $\epsilon>0$, such that

$$
\begin{equation*}
Q+\epsilon^{-1} H H^{T}+\epsilon E^{T} R E<0 \tag{14}
\end{equation*}
$$

Lemma 5 (see [34]). Suppose that $\Delta(t)$ is given by (5)-(7). Let $M, S$, and $N$ be real matrices of appropriate dimensions with $M=M^{T}$. Then, the inequality

$$
\begin{equation*}
M+S \Delta(t) N+N^{T} \Delta^{T}(t) S^{T}<0 \tag{15}
\end{equation*}
$$

holds if and only if, for any scalar $\delta>0$,

$$
\left(\begin{array}{ccc}
M & S & \delta N^{T}  \tag{16}\\
S^{T} & -\delta I & \delta J^{T} \\
\delta N & \delta J & -\delta I
\end{array}\right)<0
$$

Lemma 6. Let $G_{k}$ be given matrices as in (1). Let $P$ be symmetric positive definite matrix. Then,

$$
\left(\begin{array}{cc}
P & P G_{k}  \tag{17}\\
G_{k}^{T} P & G_{k}^{T} P G_{k}
\end{array}\right) \leq \delta_{k} I
$$

if and only if

$$
\left(\begin{array}{ccc}
-\delta_{k} I & 0 & P  \tag{18}\\
0 & -\delta_{k} I & G_{k}^{T} P \\
P & P G_{k} & -P
\end{array}\right) \leq 0
$$

for $\delta_{k}$ are positive real constants, $k \in N$.
Proof. Consider inequality (17); we have

$$
\left(\begin{array}{cc}
P & P G_{k}  \tag{19}\\
G_{k}^{T} P & G_{k}^{T} P G_{k}
\end{array}\right) \leq \delta_{k} I
$$

Equivalently,

$$
\left(\begin{array}{cc}
-\delta_{k} I & 0  \tag{20}\\
0 & -\delta_{k} I
\end{array}\right)+\binom{I}{G_{k}^{T}} P\left(\begin{array}{ll}
I & G_{k}
\end{array}\right) \leq 0
$$

By using Lemma 3 (Schur complement lemma) in the above inequality, we get

$$
\left(\begin{array}{ccc}
-\delta_{k} I & 0 & I  \tag{21}\\
0 & -\delta_{k} I & G_{k}^{T} \\
I & G_{k} & -P^{-1}
\end{array}\right) \leq 0
$$

Premultiplying (21) by $\operatorname{diag}\{I, I, P\}$ and postmultiplying by $\operatorname{diag}\{I, I, P\}$, we obtain the result. The proof of the lemma is complete.

Remark 7. Conditions (6) and (7) guarantee that $I-J F_{i_{k}}(t)$ is invertible. It is easy to show that when $J=0$, the parametric uncertainty of linear fractional form reduces to a normbounded one.

The objectives of this paper are (i) to establish new delay-range-dependent sufficient conditions for exponential stability of nominal system (1) and (ii) to establish new delay-range-dependent sufficient conditions for robust exponential stability of system (1).

## 3. Main Results

In this section, we first present the exponential stability criteria with delays dependence for nominal system (1) via LMI approach. Rewrite the nominal system (1) in the following descriptor system:

$$
\begin{align*}
\dot{x}(t)= & y(t) \\
y(t)= & A_{i_{k}} x(t)+B_{i_{k}} x\left(t-h_{i_{k}}(t)\right)+C_{i_{k}} x\left(t-r_{i_{k}}(t)\right) \\
& +f_{i_{k}}(t, x(t))+g_{i_{k}}\left(t, x\left(t-h_{i_{k}}(t)\right)\right)  \tag{22}\\
& +w_{i_{k}}\left(t, x\left(t-r_{i_{k}}(t)\right)\right) .
\end{align*}
$$

Let us decompose the constant matrices $B_{i_{k}}$ and $C_{i_{k}}$ as

$$
\begin{align*}
& B_{i_{k}}=B_{i_{k}}^{1}+B_{i_{k}}^{2}  \tag{23}\\
& C_{i_{k}}=C_{i_{k}}^{1}+C_{i_{k}}^{2},
\end{align*}
$$

where $B_{i_{k}}^{1}, B_{i_{k}}^{2}, C_{i_{k}}^{1}$, and $C_{i_{k}}^{2}$ are given real constant matrices with appropriate dimensions. By Leibniz-Newton formula, we have

$$
\begin{align*}
& 0=x(t)-x\left(t-h_{i_{k}}(t)\right)-\int_{t-h_{i_{k}}(t)}^{t} \dot{x}(s) d s,  \tag{24}\\
& 0=x(t)-x\left(t-r_{i_{k}}(t)\right)-\int_{t-r_{i_{k}}(t)}^{t} \dot{x}(s) d s
\end{align*}
$$

By utilizing the following zero equations, we get

$$
\begin{align*}
& 0=N_{1} x(t)-N_{1} x\left(t-h_{i_{k}}(t)\right)-N_{1} \int_{t-h_{i_{k}}(t)}^{t} \dot{x}(s) d s \\
& 0=N_{2} x(t)-N_{2} x\left(t-r_{i_{k}}(t)\right)-N_{2} \int_{t-r_{i_{k}}(t)}^{t} \dot{x}(s) d s \tag{25}
\end{align*}
$$

where $N_{1}$ and $N_{2}$ are real constant matrices with appropriate dimensions which will be chosen to guarantee the exponential stability of the nominal system (1). By (23)-(25), system (22) can be represented by the form

$$
\begin{align*}
\dot{x}(t)= & y(t)+\left(N_{1}+N_{2}\right) x(t)-N_{1} x\left(t-h_{i_{k}}(t)\right) \\
& -N_{2} x\left(t-r_{i_{k}}(t)\right)-N_{1} \int_{t-h_{i_{k}}(t)}^{t} y(s) d s \\
& -N_{2} \int_{t-r_{i_{k}}(t)}^{t} y(s) d s, \\
y(t)= & \left(A_{i_{k}}+B_{i_{k}}^{1}+C_{i_{k}}^{1}\right) x(t)+B_{i_{k}}^{2} x\left(t-h_{i_{k}}(t)\right)  \tag{26}\\
& +C_{i_{k}}^{2} x\left(t-r_{i_{k}}(t)\right)+f_{i_{k}}(t, x(t)) \\
& +g_{i_{k}}\left(t, x\left(t-h_{i_{k}}(t)\right)\right) \\
& +w_{i_{k}}\left(t, x\left(t-r_{i_{k}}(t)\right)\right) \\
& -B_{i_{k}}^{1} \int_{t-h_{i_{k}}(t)}^{t} y(s) d s-C_{i_{k}}^{1} \int_{t-r_{i_{k}}(t)}^{t} y(s) d s .
\end{align*}
$$

We now introduce the following notations for later use:

$$
\begin{equation*}
\sum_{i_{k}}=\left(\Sigma_{i, j}^{i_{k}}\right)_{9 \times 9} \tag{27}
\end{equation*}
$$

where $\Sigma_{i, j}^{i_{k}}=\Sigma_{j, i}^{i_{k} T}, i, j=1,2,3, \ldots, 9$,

$$
\begin{aligned}
U= & P N_{1}, \\
W= & P N_{2}, \\
\Sigma_{1,1}^{i_{k}}= & U+W+U^{T}+W^{T}+Q_{1}^{T}\left(A_{i_{k}}+B_{i_{k}}^{1}+C_{i_{k}}^{1}\right) \\
& +\left(A_{i_{k}}+B_{i_{k}}^{1}+C_{i_{k}}^{1}\right)^{T} Q_{1}+\epsilon_{1} \eta^{2} I+a P, \\
\Sigma_{1,2}^{i_{k}}= & P-Q_{1}^{T}+\left(A_{i_{k}}+B_{i_{k}}^{1}+C_{i_{k}}^{1}\right)^{T} Q_{2}, \\
\Sigma_{1,3}^{i_{k}}= & -U+Q_{1}^{T} B_{i_{k}}^{2}+\left(h_{2}-h_{1}\right) Q_{3}, \\
\Sigma_{1,4}^{i_{k}}= & -W+Q_{1}^{T} C_{i_{k}}^{2}+\left(r_{2}-r_{1}\right) Q_{4}, \\
\Sigma_{1,5}^{i_{k}}= & Q_{1}^{T} \\
\Sigma_{1,6}^{i_{k}}= & Q_{1}^{T} \\
\Sigma_{1,7}^{i_{k}}= & Q_{1}^{T}, \\
\Sigma_{1,8}^{i_{k}}= & -U-Q_{1}^{T} B_{i_{k}}^{1}+\left(h_{2}-h_{1}\right) Q_{5},
\end{aligned}
$$

$\Sigma_{1,9}^{i_{k}}=-W-Q_{1}^{T} C_{i_{k}}^{1}+\left(r_{2}-r_{1}\right) Q_{6}$,
$\Sigma_{2,2}^{i_{k}}=-Q_{2}^{T}-Q_{2}$,
$\Sigma_{2,3}^{i_{k}}=Q_{2}^{T} B_{i_{k}}^{2}$,
$\Sigma_{2,4}^{i_{k}}=Q_{2}^{T} C_{i_{k}}^{2}$,
$\sum_{2,5}^{i_{k}}=Q_{2}^{T}$,
$\Sigma_{2,6}^{i_{k}}=Q_{2}^{T}$,
$\Sigma_{2,7}^{i_{k}}=Q_{2}^{T}$,
$\sum_{2,8}^{i_{k}}=-Q_{2}^{T} B_{i_{k}}^{1}$,
$\sum_{2,9}^{i_{k}}=-Q_{2}^{T} C_{i_{k}}^{1}$,
$\Sigma_{3,3}^{i_{k}}=-\left(h_{2}-h_{1}\right) Q_{3}-\left(h_{2}-h_{1}\right) Q_{3}^{T}+\epsilon_{2} \rho^{2} I-b P$,
$\Sigma_{3,4}^{i_{k}}=\Sigma_{3,5}^{i_{k}}=\Sigma_{3,6}^{i_{k}}=\Sigma_{3,7}^{i_{k}}=0$,
$\Sigma_{3,8}^{i_{k}}=-\left(h_{2}-h_{1}\right) Q_{3}^{T}-\left(h_{2}-h_{1}\right) Q_{5}$,
$\Sigma_{3,9}^{i_{k}}=0$,
$\sum_{4,4}^{i_{k}}=-\left(r_{2}-r_{1}\right) Q_{4}-\left(r_{2}-r_{1}\right) Q_{4}^{T}+\epsilon_{3} \zeta^{2} I-c P$,
$\Sigma_{4,5}^{i_{k}}=\Sigma_{4,6}^{i_{k}}=\Sigma_{4,7}^{i_{k}}=\Sigma_{4,8}^{i_{k}}=0$,
$\Sigma_{4,9}^{i_{k}}=-\left(r_{2}-r_{1}\right) Q_{4}^{T}-\left(r_{2}-r_{1}\right) Q_{6}$,
$\sum_{5,5}^{i_{k}}=-\epsilon_{1} I$,
$\Sigma_{5,6}^{i_{k}}=\Sigma_{5,7}^{i_{k}}=\Sigma_{5,8}^{i_{k}}=\sum_{5,9}^{i_{k}}=0$,
$\Sigma_{6,6}^{i_{k}}=-\epsilon_{2} I$,
$\Sigma_{6,7}^{i_{k}}=\Sigma_{6,8}^{i_{k}}=\Sigma_{6,9}^{i_{k}}=0$,
$\Sigma_{7,7}^{i_{k}}=-\epsilon_{3} I$,
$\Sigma_{7,8}^{i_{k}}=\Sigma_{7,9}^{i_{k}}=0$,
$\Sigma_{8,8}^{i_{k}}=-\left(h_{2}-h_{1}\right) Q_{5}-\left(h_{2}-h_{1}\right) Q_{5}^{T}$,
$\sum_{8,9}^{i_{k}}=0$,
$\Sigma_{9,9}^{i_{k}}=-\left(r_{2}-r_{1}\right) Q_{6}-\left(r_{2}-r_{1}\right) Q_{6}^{T}$.

Theorem 8. The nominal system (1) is exponentially stable if there exist symmetric positive definite matrix $P$, any appropriate dimensional matrices $N_{1}, N_{2}$, and $Q_{i}, i=1,2, \ldots, 6$, and
positive real constants $\mu, \lambda, \eta, \rho, \zeta, \epsilon_{1}, \epsilon_{2}, \epsilon_{3}, a, b$, and $c$ with $a>b+c$ and $\delta_{k}>0$ for all $k \in N$ such that the following LMIs hold:

$$
\begin{align*}
& \sum_{i_{k}}<0,  \tag{29}\\
& \left(\begin{array}{ccc}
-\delta_{k} I & 0 & P \\
0 & -\delta_{k} I & G_{k}^{T} P \\
P & P G_{k} & -P
\end{array}\right) \leq 0,  \tag{30}\\
& \mu \bar{h} \leq \inf _{k \in N}\left\{t_{k}-t_{k-1}\right\},  \tag{31}\\
& \max \left\{\bar{\delta}_{k}+\bar{\delta}_{k} e^{\lambda \bar{h}}\right\} \leq M<e^{\lambda_{\mu} \bar{h}}, \tag{32}
\end{align*}
$$

where $\bar{\delta}_{k}=\delta_{k} / \lambda_{\text {min }}(P), k \in N$, and $\lambda$ is the unique positive root of the equation $\lambda-a+(b+c) e^{\lambda \bar{h}}=0$.

Proof. Consider a common Lyapunov functional

$$
\begin{equation*}
V(x(t))=x^{T}(t) P x(t) \tag{33}
\end{equation*}
$$

for $t \in\left[t_{k-1}, t_{k}\right)$ and a symmetric positive definite matrix $P$. It is easy to see that

$$
\begin{equation*}
\lambda_{1}\|x\|^{2} \leq V(x(t)) \leq \lambda_{2}\|x\|^{2} \tag{34}
\end{equation*}
$$

where $\lambda_{1}=\lambda_{\text {min }}(P)$ and $\lambda_{2}=\lambda_{\max }(P)$. The Dini derivative of $V(x(t))$ along the trajectories of system (26) is given by

$$
\begin{aligned}
& D^{+} V(x(t))=2 x^{T}(t) P\left[y(t)+\left(N_{1}+N_{2}\right) x(t)\right. \\
& \quad-N_{1} x\left(t-h_{i_{k}}(t)\right)-N_{2} x\left(t-r_{i_{k}}(t)\right) \\
& \left.\quad-N_{1} \int_{t-h_{i_{k}}(t)}^{t} y(s) d s-N_{2} \int_{t-r_{i_{k}}(t)}^{t} y(s) d s\right] \\
& \quad+2 x^{T}(t) Q_{1}^{T}\left[-y(t)+\left(A_{i_{k}}+B_{i_{k}}^{1}+C_{i_{k}}^{1}\right) x(t)\right. \\
& \quad+B_{i_{k}}^{2} x\left(t-h_{i_{k}}(t)\right)+C_{i_{k}}^{2} x\left(t-r_{i_{k}}(t)\right) \\
& \quad+f_{i_{k}}(t, x(t))+g_{i_{k}}\left(t, x\left(t-h_{i_{k}}(t)\right)\right) \\
& \quad+w_{i_{k}}\left(t, x\left(t-r_{i_{k}}(t)\right)\right)-B_{i_{k}}^{1} \int_{t-h_{i_{k}}(t)}^{t} y(s) d s
\end{aligned}
$$

$$
\begin{align*}
& \left.-C_{i_{k}}^{1} \int_{t-r_{i_{k}}(t)}^{t} y(s) d s\right]+2 y^{T}(t) Q_{2}^{T}[-y(t) \\
& +\left(A_{i_{k}}+B_{i_{k}}^{1}+C_{i_{k}}^{1}\right) x(t)+B_{i_{k}}^{2} x\left(t-h_{i_{k}}(t)\right) \\
& +C_{i_{k}}^{2} x\left(t-r_{i_{k}}(t)\right)+f_{i_{k}}(t, x(t)) \\
& +g_{i_{k}}\left(t, x\left(t-h_{i_{k}}(t)\right)\right)+w_{i_{k}}\left(t, x\left(t-r_{i_{k}}(t)\right)\right) \\
& \left.-B_{i_{k}}^{1} \int_{t-h_{i_{k}}(t)}^{t} y(s) d s-C_{i_{k}}^{1} \int_{t-r_{i_{k}}(t)}^{t} y(s) d s\right]+2\left(h_{2}\right. \\
& \left.-h_{1}\right) x^{T}\left(t-h_{i_{k}}(t)\right) Q_{3}^{T}\left[x(t)-x\left(t-h_{i_{k}}(t)\right)\right. \\
& \left.-\int_{t-h_{i_{k}}(t)}^{t} y(s) d s\right]+2\left(r_{2}-r_{1}\right) x^{T}\left(t-r_{i_{k}}(t)\right) \\
& \text { - } Q_{4}^{T}[x(t) \\
& \left.-x\left(t-r_{i_{k}}(t)\right)-\int_{t-r_{i_{k}}(t)}^{t} y(s) d s\right]+2\left(h_{2}-h_{1}\right) \\
& \cdot \int_{t-h_{i_{k}}(t)}^{t} y^{T}(s) d s Q_{5}^{T}\left[x(t)-x\left(t-h_{i_{k}}(t)\right)\right. \\
& \left.-\int_{t-h_{i_{k}}(t)}^{t} y(s) d s\right]+2\left(r_{2}-r_{1}\right) \int_{t-r_{i_{k}}(t)}^{t} y^{T}(s) d s \\
& \text { - } Q_{6}^{T}[x(t) \\
& \left.-x\left(t-r_{i_{k}}(t)\right)-\int_{t-r_{i_{k}}(t)}^{t} y(s) d s\right], \tag{35}
\end{align*}
$$

for any appropriate dimensional matrices $Q_{i}, i=1,2, \ldots, 6$. Since

$$
\begin{align*}
0 \leq & \epsilon_{1} \eta^{2} x^{T}(t) x(t)-\epsilon_{1} f_{i_{k}}^{T}(\cdot) f_{i_{k}}(\cdot), \\
0 \leq & \epsilon_{2} \rho^{2} x^{T}\left(t-h_{i_{k}}(t)\right) x\left(t-h_{i_{k}}(t)\right) \\
& \quad-\epsilon_{2} g_{i_{k}}^{T}(\cdot) g_{i_{k}}(\cdot),  \tag{36}\\
0 \leq & \epsilon_{3} \zeta^{2} x^{T}\left(t-r_{i_{k}}(t)\right) x\left(t-r_{i_{k}}(t)\right)-\epsilon_{3} h_{i_{k}}^{T}(\cdot) h_{i_{k}}(\cdot),
\end{align*}
$$

for positive real constants $\epsilon_{1}, \epsilon_{2}$, and $\epsilon_{3}$, we obtain

$$
\begin{align*}
D^{+} V(x(t)) \leq & \omega^{T}(t) \sum_{i_{k}} \omega(t)-a V(x(t))  \tag{37}\\
& +(b+c) \bar{V}(x(t))
\end{align*}
$$

where

$$
\begin{align*}
& \omega^{T}(t)=\left[x^{T}(t), y^{T}(t), x^{T}\left(t-h_{i_{k}}(t)\right)\right. \\
& \quad x^{T}\left(t-r_{i_{k}}(t)\right), f_{i_{k}}^{T}(t, x(t)), g_{i_{k}}^{T}\left(t, x\left(t-h_{i_{k}}(t)\right)\right), \\
& \quad w_{i_{k}}^{T}\left(t, x\left(t-r_{i_{k}}(t)\right)\right), \int_{t-h_{i_{k}}(t)}^{t} y^{T}(s) d s  \tag{38}\\
& \left.\quad \int_{t-r_{i_{k}}(t)}^{t} y^{T}(s) d s\right] \\
& \bar{V}(x(t))=\sup _{t-\bar{h} \leq s \leq t}\{V(x(s))\} .
\end{align*}
$$

From (29) and (37), we obtain

$$
\begin{equation*}
D^{+} V(x(t)) \leq-a V(x(t))+(b+c) \bar{V}(x(t)) \tag{39}
\end{equation*}
$$

By (39) and Lemma 2 with $a>b+c$ for $a, b, c \in R^{+}$, we obtain that there exists $\lambda>0$ such that, for all $t \in\left[t_{k-1}, t_{k}\right), k \in N$,

$$
\begin{equation*}
V(x(t)) \leq \bar{V}\left(x\left(t_{k-1}\right)\right) e^{-\lambda\left(t-t_{k-1}\right)} \tag{40}
\end{equation*}
$$

where $\bar{V}\left(x\left(t_{k-1}\right)\right)=\sup _{t_{k-1}-\bar{h} \leq s \leq t_{k-1}}\{V(x(s))\}$. Consider the case when $t=t_{k}$. In this case, we have

$$
\begin{align*}
& V\left(x\left(t_{k}\right)\right)=x^{T}\left(t_{k}\right) P x\left(t_{k}\right) \\
& \quad=\left[x\left(t_{k}^{-}\right)+G_{k} x\left(t_{k}^{-}-h_{i_{k}}\left(t_{k}^{-}\right)\right)\right]^{T} \\
& \quad \cdot P\left[x\left(t_{k}^{-}\right)+G_{k} x\left(t_{k}^{-}-h_{i_{k}}\left(t_{k}^{-}\right)\right)\right]=x^{T}\left(t_{k}^{-}\right) P x\left(t_{k}^{-}\right) \\
& \\
& \quad+2 x^{T}\left(t_{k}^{-}\right) P G_{k} x\left(t_{k}^{-}-h_{i_{k}}\left(t_{k}^{-}\right)\right)  \tag{41}\\
& \\
& \quad+x^{T}\left(t_{k}^{-}-h_{i_{k}}\left(t_{k}^{-}\right)\right) G_{k}^{T} P G_{k} x\left(t_{k}^{-}-h_{i_{k}}\left(t_{k}^{-}\right)\right) \\
& \\
& \quad=\binom{x\left(t_{k}^{-}\right)}{x\left(t_{k}^{-}-h_{i_{k}}\left(t_{k}^{-}\right)\right)}^{T}
\end{align*}
$$

$$
\cdot\left(\begin{array}{cc}
P & P G_{k} \\
G_{k}^{T} P & G_{k}^{T} P G_{k}
\end{array}\right)\binom{x\left(t_{k}^{-}\right)}{x\left(t_{k}^{-}-h_{i_{k}}\left(t_{k}^{-}\right)\right)}
$$

By (30) and (34) and Lemma 6, we get

$$
\begin{equation*}
V\left(x\left(t_{k}\right)\right) \leq \bar{\delta}_{k} V\left(x\left(t_{k}^{-}\right)\right)+\bar{\delta}_{k} V\left(x\left(t_{k}^{-}-h_{i_{k}}\left(t_{k}^{-}\right)\right)\right) \tag{42}
\end{equation*}
$$

where $\bar{\delta}_{k}=\delta_{k} / \lambda_{1}$. For $x(t)=\phi(t)$, with $t \in\left[t_{0}-\bar{h}, t_{0}\right]$, we will show that

$$
\begin{align*}
& V(x(t)) \leq \lambda_{2} M^{k-1}\|\phi\|^{2} e^{-\lambda\left(t-t_{0}\right)}, \\
&  \tag{43}\\
& t \in\left[t_{k-1}, t_{k}\right), k \in N
\end{align*}
$$

where $\|\phi\|=\sup _{t_{0}-\bar{h} \leq t \leq t_{0}}\|\phi(t)\|$. We can prove inequality (43) by mathematical induction. Indeed, when $k=1$, we have

$$
\begin{align*}
& V(x(t)) \leq \lambda_{2}\|x(t)\|^{2}=\lambda_{2}\|\phi(t)\|^{2},  \tag{44}\\
& \\
& t \in\left[t_{0}-\bar{h}, t_{0}\right] .
\end{align*}
$$

Since $\|\phi\|^{2}=\sup _{t_{0}-\bar{h} \leq t \leq t_{0}}\|\phi(t)\|^{2}$, we have

$$
\begin{equation*}
\bar{V}\left(x\left(t_{0}\right)\right) \leq \lambda_{2}\|\phi\|^{2} . \tag{45}
\end{equation*}
$$

From (40) and (45), we obtain

$$
\begin{align*}
V(x(t)) & \leq \bar{V}\left(x\left(t_{0}\right)\right) e^{-\lambda\left(t-t_{0}\right)} \leq \lambda_{2}\|\phi\|^{2} e^{-\lambda\left(t-t_{0}\right)} \\
& \leq \lambda_{2} M^{0}\|\phi\|^{2} e^{-\lambda\left(t-t_{0}\right)}, \quad t \in\left[t_{0}, t_{1}\right) \tag{46}
\end{align*}
$$

Therefore, (43) holds for $k=1$.
Next, we assume that (43) holds for $k \leq m, m \geq 1$. Then, we need to show that (43) holds when $k=m+1$. By the above induction assumption, (32), (40), and (43), we have

$$
\begin{align*}
V\left(x\left(t_{m}\right)\right) \leq & \bar{\delta}_{m} V\left(x\left(t_{m}^{-}\right)\right)+\bar{\delta}_{m} V\left(x\left(t_{m}^{-}-h\left(t_{m}^{-}\right)\right)\right) \\
\leq & \lambda_{2} M^{m-1} \bar{\delta}_{m}\|\phi\|^{2} e^{-\lambda\left(t_{m}-t_{0}\right)} \\
& +\lambda_{2} M^{m-1} \bar{\delta}_{m}\|\phi\|^{2} e^{-\lambda\left(t_{m}-h\left(t_{m}\right)-t_{0}\right)}  \tag{47}\\
\leq & \lambda_{2} M^{m-1}\left(\bar{\delta}_{m}+\bar{\delta}_{m} e^{\lambda \bar{h}}\right)\|\phi\|^{2} e^{-\lambda\left(t_{m}-t_{0}\right)} \\
\leq & \lambda_{2} M^{m}\|\phi\|^{2} e^{-\lambda\left(t_{m}-t_{0}\right)}
\end{align*}
$$

Hence, it follows from (40) and (47) that

$$
\begin{align*}
& V(x(t)) \leq \bar{V}\left(x\left(t_{m}\right)\right) e^{-\lambda\left(t-t_{m}\right)}=\max _{t_{m}-\bar{h} \leq t \leq t_{m}}\{V(x(t))\} \\
& \quad \cdot e^{-\lambda\left(t-t_{m}\right)} \\
& \quad=\max \left\{\sup _{t_{m-1}-\bar{h} \leq s<t_{m}}\{V(x(t))\},\left\{V\left(x\left(t_{m}\right)\right)\right\}\right\} \\
&  \tag{48}\\
& \cdot e^{-\lambda\left(t-t_{m}\right)} \leq \max \left\{\lambda_{2} M^{m-1}\|\phi\|^{2}\right. \\
& \left.\quad \cdot e^{-\lambda\left(t_{m}-\bar{h}-t_{0}\right)}, \lambda_{2} M^{m}\|\phi\|^{2} e^{-\lambda\left(t_{m}-t_{0}\right)}\right\} e^{-\lambda\left(t-t_{m}\right)} \\
& \quad=\max \left\{\lambda_{2} M^{m-1} e^{\lambda \bar{h}}, \lambda_{2} M^{m}\right\}\|\phi\|^{2} \\
& \quad \cdot e^{-\lambda\left(t_{m}-t_{0}\right)} e^{-\lambda\left(t-t_{m}\right)} \leq \lambda_{2} M^{m}\|\phi\|^{2} e^{-\lambda\left(t-t_{0}\right)} .
\end{align*}
$$

Therefore, (43) holds for all $k \in N$. By (31), we get that $k-1 \leq$ $\left(t_{k-1}-t_{0}\right) / \mu \bar{h}$, which implies

$$
\begin{equation*}
M^{k-1} \leq e^{\left(\left(t_{k-1}-t_{0}\right) \ln M\right) / \mu \bar{h}} \leq e^{\left(\left(t-t_{0}\right) \ln M\right) / \mu \bar{h}} \tag{49}
\end{equation*}
$$

for $t \in\left[t_{k-1}, t_{k}\right)$. We get

$$
\begin{align*}
\|x(t)\|^{2} & \leq \frac{V(x(t))}{\lambda_{1}} \leq \frac{\lambda_{2}}{\lambda_{1}}\|\phi\|^{2}(M)^{k-1} e^{-\lambda\left(t-t_{0}\right)}  \tag{50}\\
& \leq \frac{\lambda_{2}}{\lambda_{1}}\|\phi\|^{2} e^{(-\lambda+(\ln M) / \mu \bar{h})\left(t-t_{0}\right)}
\end{align*}
$$

Finally, we conclude that

$$
\begin{equation*}
\|x(t)\| \leq K\|\phi\| e^{-\beta\left(t-t_{0}\right)}, \quad t \geq t_{0} \tag{51}
\end{equation*}
$$

where $\beta=(1 / 2)[\lambda-(\ln M) / \mu \bar{h}]>0, K=\sqrt{\lambda_{2} / \lambda_{1}}>0$. This means that the nominal system (1) is exponentially stable. The proof of the theorem is complete.

Next, we now present the new delay-range-dependent robust exponential stability criteria for system (1). We introduce the following notations for later use:

$$
\begin{align*}
& \Gamma_{i_{k}}^{T} \\
& =\left[\begin{array}{lllllllll}
K_{i_{k}}^{T} Q_{1} & K_{i_{k}}^{T} Q_{2} & 0 & 0 & 0 & 0 & 0 & 0 & \left(v_{2}-v_{1}\right) K_{i_{k}}^{T} Q_{3}
\end{array}\right], \\
& \Upsilon_{i_{k}}=\left[\begin{array}{lllllllll}
L_{i_{k}}^{1} & 0 & L_{i_{k}}^{2} & L_{i_{k}}^{3} & 0 & 0 & 0 & 0 & L_{i_{k}}^{4}
\end{array}\right] \text {, }  \tag{52}\\
& \widetilde{\sum_{i_{k}}}=\left(\begin{array}{ccc}
\sum_{i_{k}} & \Gamma_{i_{k}} & \sigma \Upsilon_{i_{k}}^{T} \\
\Gamma_{i_{k}}^{T} & -\sigma I & \sigma J^{T} \\
\sigma \Upsilon_{i_{k}} & \sigma J & -\sigma I
\end{array}\right) .
\end{align*}
$$

Theorem 9. System (1) is robustly exponentially stable if there exist symmetric positive definite matrix $P$, any appropriate dimensional matrices $N_{1}, N_{2}$, and $Q_{i}, i=1,2, \ldots, 6$, and positive real constants $\sigma, \mu, \lambda, \eta, \rho, \zeta, \epsilon_{1}, \epsilon_{2}, \epsilon_{3}, a, b$, and $c$ with $a>b+c$ and $\delta_{k}>0$ for all $k \in N$ such that the following LMIs hold:

$$
\begin{align*}
& \widetilde{i_{i_{k}}}<0,  \tag{53}\\
& \left(\begin{array}{ccc}
-\delta_{k} I & 0 & P \\
0 & -\delta_{k} I & G_{k}^{T} P \\
P & P G_{k} & -P
\end{array}\right) \leq 0,  \tag{54}\\
& \mu \bar{h} \leq \inf _{k \in N}\left\{t_{k}-t_{k-1}\right\},  \tag{55}\\
& \max \left\{\bar{\delta}_{k}+\bar{\delta}_{k} e^{\bar{h}}\right\} \leq M<e^{\lambda \mu \bar{h}}, \tag{56}
\end{align*}
$$

where $\bar{\delta}_{k}=\delta_{k} / \lambda_{\text {min }}(P), k \in N$, and $\lambda$ is the unique positive root of the equation $\lambda-a+(b+c) e^{\lambda \bar{h}}=0$.

Proof. Replacing $A_{i_{k}}, B_{i_{k}}^{2}$, and $C_{i_{k}}^{2}$ in (29) with $A_{i_{k}}(t)=A_{i_{k}}+$ $K_{i_{k}} \Delta_{i_{k}}(t) L_{i_{k}}^{1}, B_{i_{k}}^{2}(t)=B_{i_{k}}^{2}+K_{i_{k}} \Delta_{i_{k}}(t) L_{i_{k}}^{2}$, and $C_{i_{k}}^{2}(t)=C_{i_{k}}^{2}+$ $K_{i_{k}} \Delta_{i_{k}}(t) L_{i_{k}}^{3}$, respectively, we find that

$$
\begin{equation*}
\sum_{i_{k}}+\Gamma_{i_{k}} \Delta_{i_{k}}(t) \Upsilon_{i_{k}}+\Upsilon_{i_{k}}^{T} \Delta_{i_{k}}(t)^{T} \Gamma_{i_{k}}^{T}<0 \tag{57}
\end{equation*}
$$

By Lemma 5, we can find that (53) is equivalent to (57) where $\sigma$ is positive real constant. The proof of the theorem is complete.

## 4. Numerical Example

Example 1. Consider the following uncertain impulsive switched linear system with mixed interval time-varying
delays and nonlinear perturbations (1) under a given switching law. That is, the switching status alternates as $i_{1} \rightarrow i_{2} \rightarrow$ $i_{1} \rightarrow i_{2} \rightarrow \cdots$. We consider robust exponential stability performance of system (1) by using Theorem 9. System (1) is specified as follows:

$$
\begin{aligned}
& A_{1}=\left(\begin{array}{cc}
-4 & 0 \\
-1 & -3
\end{array}\right), \\
& A_{2}=\left(\begin{array}{cc}
-3 & 1 \\
0 & -2
\end{array}\right) \text {, } \\
& B_{1}=\left(\begin{array}{cc}
-1 & 0 \\
-1 & -1
\end{array}\right) \text {, } \\
& B_{2}=\left(\begin{array}{cc}
-1 & 0 \\
-1 & -1
\end{array}\right) \text {, } \\
& C_{1}=\left(\begin{array}{cc}
-1 & 0 \\
-1 & -1
\end{array}\right) \text {, } \\
& C_{2}=\left(\begin{array}{cc}
-1 & 0 \\
-1 & -1
\end{array}\right), \\
& K_{1}=\left(\begin{array}{cc}
0.3 & -0.1 \\
0.2 & 0.5
\end{array}\right), \\
& K_{2}=\left(\begin{array}{cc}
0.2 & 0.2 \\
-0.1 & 0.4
\end{array}\right), \\
& L_{1}^{1}=\left(\begin{array}{cc}
0.4 & 0.1 \\
-0.2 & -0.2
\end{array}\right) \text {, } \\
& L_{2}^{1}=\left(\begin{array}{cc}
0.3 & -0.1 \\
-0.1 & -0.2
\end{array}\right) \text {, } \\
& L_{1}^{2}=\left(\begin{array}{cc}
-0.1 & 0.2 \\
0 & -0.2
\end{array}\right) \text {, } \\
& L_{2}^{2}=\left(\begin{array}{cc}
0.2 & 0.1 \\
0 & -0.2
\end{array}\right) \text {, } \\
& L_{1}^{3}=\left(\begin{array}{cc}
-0.2 & 0 \\
-0.3 & 0.3
\end{array}\right) \text {, } \\
& L_{2}^{3}=\left(\begin{array}{cc}
-0.3 & 0 \\
-0.3 & 0.3
\end{array}\right) \text {, } \\
& G_{k}=\left(\begin{array}{cc}
-0.5 & 0 \\
0 & -0.5
\end{array}\right) \text {, } \\
& J=\left(\begin{array}{cc}
0.1 & 0 \\
0 & 0.1
\end{array}\right) \text {, } \\
& f_{1}(t, x(t))=\binom{0.1 \cos (t) x_{1}(t)}{0.1 \cos (t) x_{2}(t)} \text {, }
\end{aligned}
$$

$$
\begin{gather*}
f_{2}(t, x(t))=\binom{0.1 \sin (t) x_{1}(t)}{0.1 \cos (t) x_{2}(t)}, \\
g_{1}\left(t, x\left(t-h_{1}(t)\right)\right)=\binom{0.1 \sin (t) x_{1}\left(t-h_{1}(t)\right)}{0.1 \sin (t) x_{2}\left(t-h_{1}(t)\right)}, \\
g_{2}\left(t, x\left(t-h_{2}(t)\right)\right)=\binom{0.1 \cos (t) x_{1}\left(t-h_{2}(t)\right)}{0.1 \sin (t) x_{2}\left(t-h_{2}(t)\right)}, \\
h_{1}\left(t, x\left(t-r_{1}(t)\right)\right)=\binom{0.1 \sin (t) x_{1}\left(t-r_{1}(t)\right)}{0.1 \sin (t) x_{2}\left(t-r_{1}(t)\right)}, \\
h_{2}\left(t, x\left(t-r_{2}(t)\right)\right)=\binom{0.1 \cos (t) x_{1}\left(t-r_{2}(t)\right)}{0.1 \cos (t) x_{2}\left(t-r_{2}(t)\right)} \tag{58}
\end{gather*}
$$

Decompose the matrices $B_{1}=B_{1}^{1}+B_{1}^{2}, B_{2}=B_{2}^{1}+B_{2}^{2}, C_{1}=$ $C_{1}^{1}+C_{1}^{2}$, and $C_{2}=C_{2}^{1}+C_{2}^{2}$, where

$$
\begin{aligned}
B_{1}^{1} & =\left(\begin{array}{cc}
-0.8 & 0 \\
-0.5 & -0.7
\end{array}\right), \\
B_{1}^{2} & =\left(\begin{array}{cc}
-0.2 & 0 \\
-0.5 & -0.3
\end{array}\right), \\
B_{2}^{1} & =\left(\begin{array}{cc}
-0.7 & 0 \\
-0.4 & -0.8
\end{array}\right), \\
B_{2}^{2} & =\left(\begin{array}{cc}
-0.3 & 0 \\
-0.6 & -0.2
\end{array}\right), \\
C_{1}^{1} & =\left(\begin{array}{cc}
-0.6 & 0 \\
-0.5 & -0.8
\end{array}\right), \\
C_{1}^{2} & =\left(\begin{array}{cc}
-0.4 & 0 \\
-0.5 & -0.2
\end{array}\right), \\
C_{2}^{1} & =\left(\begin{array}{cc}
-0.5 & 0 \\
-0.5 & -0.5
\end{array}\right), \\
C_{2}^{2} & =\left(\begin{array}{cc}
-0.5 & 0 \\
-0.5 & -0.5
\end{array}\right), \\
h_{1}(t) & =0.5+|\sin (t)|, \\
h_{2}(t) & =0.6+0.8|\cos (t)|, \\
r_{1}(t) & =0.1+|\cos (t)|, \\
r_{2}(t) & =0.1+1.2|\sin (t)|, \\
a & =2.3, \\
b & =1, \\
c & =1 .
\end{aligned}
$$



Figure 1: The numerical solutions $x_{1}(t)$ and $x_{2}(t)$ are presented for system (1) in Example 1.

It is easy to see that $\eta=0.1, \rho=0.1, \zeta=0.1, h_{1}=0.5$, $h_{2}=1.5, r_{1}=0.1$, and $r_{2}=1.3$. By using LMI Toolbox in MATLAB, we use (29)-(32) in Theorem 9. This example shows that the solutions of LMIs are given as follows:

$$
\begin{align*}
& P=\left(\begin{array}{cc}
2.52 & -0.17 \\
-0.17 & 0.64
\end{array}\right), \\
& Q 1=\left(\begin{array}{cc}
1.40 & -0.12 \\
-0.12 & 0.56
\end{array}\right), \\
& Q 2=\left(\begin{array}{cc}
0.41 & -0.10 \\
-0.10 & 0.16
\end{array}\right), \\
& Q 3=\left(\begin{array}{cc}
0.24 & 0.01 \\
0.01 & 0.45
\end{array}\right), \\
& Q 4=\left(\begin{array}{cc}
0.24 & 0.04 \\
0.04 & 0.43
\end{array}\right),  \tag{60}\\
& Q 5=\left(\begin{array}{cc}
0.70 & -0.002 \\
-0.002 & 0.56
\end{array}\right), \\
& Q 6=\left(\begin{array}{ll}
0.57 & 0.01 \\
0.01 & 0.46
\end{array}\right), \\
& N_{1}=\left(\begin{array}{ll}
0.33 & -0.01 \\
0.13 & -0.33
\end{array}\right), \\
& N_{2}=\left(\begin{array}{ll}
0.23 & 0.003 \\
0.14 & -0.24
\end{array}\right),
\end{align*}
$$

$\delta_{k}=4.62, \mu=4, \epsilon_{1}=4.62, \epsilon_{2}=4.46, \epsilon_{3}=4.36$, and $\sigma=$ 1.53. The numerical solutions $x_{1}(t)$ and $x_{2}(t)$ of system (1) with $\phi^{T}(t)=\left[\begin{array}{ll}-3 & 5\end{array}\right],-3 \leq t \leq 0$, are plotted in Figure 1. This shows that those solutions converge to zero.

## 5. Conclusions

We have presented the problem of robust exponential stability criteria for uncertain impulsive switched linear systems with mixed interval nondifferentiable time-varying delays and nonlinear perturbations. By using a common Lyapunov functional, mixed model transformation, Halanay inequality, utilization of zero equations, and LMI approach, new delay-range-dependent robust exponential stability criteria for the systems are established in terms of LMIs. Finally, the theoretical result is illustrated well with a simulation example.

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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