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Review Article

On Collective Properties of Dense QCD Matter

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A short review of the two recently analyzed collective effects in dense non-Abelian matter, the photon and dilepton production in nonequilibrium glasma and polarization properties of turbulent Abelian and non-Abelian plasmas, is given.

1. Introduction

Working out a quantitative description of the properties of dense strongly interacting matter produced in ultrarelativistic heavy ion collisions presents one of the most fascinating problems in high energy physics. The main goal of the present review is to expand an analysis of various properties of dense non-Abelian matter presented in [1] by discussing, in particular, several new topics [2–4], enriching our understanding of the early stages of ultrarelativistic heavy ion collisions.

The conceptually simplest way of organizing experimental information obtained at RHIC [5–8] and LHC [9–11] into a more or less coherent framework is to describe the late stages of these collisions in terms of standard relativistic hydrodynamical expansion of primordial quark-gluon matter that, after a short transient period, reaches sufficient level of local isotropization and equilibration allowing the usage of hydrodynamics in its standard form. In particular, the presence of strong elliptic flow suggests the picture of strongly coupled and, therefore, low viscosity matter. A detailed discussion of the corresponding issues can be found in, for example, reviews [12, 13] devoted to RHIC and LHC results, respectively. The collective effects can also become pronounced in high-multiplicity proton-proton collisions [14].

The actual physical picture is, most likely, much more complicated. In the absence of realistic mechanisms leading to extremely fast isotropization needed for describing the experimental data within the framework of standard hydro [15], the recent discussion [16–19] focused on building a generalization of hydrodynamical approach on systems with anisotropic pressure that naturally arise in the glasma-based description of the physics of the early stages of heavy ion collisions [1, 20, 21]. Of particular interest are results of [18, 19] showing, within the AdS-CFT duality paradigm, that hydrodynamic description can be valid for unexpectedly large pressure gradients. The new paradigm of anisotropic hydrodynamics is, in our opinion, one of the most promising new approaches to the physics of high energy nuclear collisions.

At the most fundamental level a description of early stages of high energy nuclear collisions in the weak coupling regime is based on the idea that large gluon density and, correspondingly, large occupation numbers of low energy gluon modes make it natural to use tree-level Yang-Mills equations with sources in the strong field regime as a major building block for the theoretical description of ultrarelativistic nuclear collisions. At the early stage a strongly nonisotropic tree-level gluon field configuration arising immediately after collision, the glasma [20, 21] is formed. A very recent development [22] suggests that temporal evolution of glasma involves formation of a transient coherent object, the gluon condensate. This can have interesting experimental consequences, in particular for photon and dilepton production [2] discussed in Section 2.

The glasma is, however, unstable with respect to boost-noninvariant quantum fluctuations [23–25]. At later stages of its evolution these instabilities were shown to drive a system towards a state characterized by the turbulent Kolmogorov momentum spectrum of its modes [26]. The same

Kolmogorov spectrum was earlier discovered in a simplified scalar model of multiparticle production in heavy ion collisions [27, 28]. A possible relation between these instabilities and low effective viscosity in expanding geometry was recently discussed in [29].

The origin of the initial glasma instabilities and the physical picture underlying the turbulent-like glasma at later stages of its evolution, however, do still remain unclear. One possible scenario is that of the Weibel-type instabilities of soft field modes present both in QED and QCD plasma having their origin in the momentum anisotropy of hard sources [30–36] that eventually lead to the formation of the turbulent Kolmogorov cascade [37–40].

Of major importance to the physics of turbulent quantum field theory that provide another important benchmark for the physics of heavy ion collisions are also the fixed-box studies in the framework of classical statistical lattice gauge theory [41–43] and a study of the turbulent cascade in the isotropic QCD matter in [44]. Let us also note that there is no doubt that the genuinely stochastic nature of the classical Yang-Mills equation [45] should by itself play an important role in the physics of turbulent non-Abelian matter. The precise relation is however still to be studied.

The importance of turbulent effects, discussed in Section 3, makes it natural to study their effects on physically important quantities like shear viscosity. The corresponding calculation, discussed in Section 3.1 below was made in [46–48] in a setting generalizing the one used in the earlier studies of turbulent QED plasma [49, 50], in which turbulent plasma is described as a system of hard thermal modes and the stochastic turbulent fields characterized by some spatial and temporal correlation lengths. It was shown that plasma turbulence can serve as a natural source of the above-mentioned anomalous smallness of viscosity of strongly interacting matter created in high energy heavy ion collisions. The relation between the anomalously small viscosity of turbulent plasmas and anomalously large jet quenching in them [51] is described in Section 3.2.

The physics of turbulence, both in liquids [52–58] and plasma [59], is essentially that of space-time structures that appear at the event-by-event level and, after averaging, give rise to Kolmogorov (or, more generally, multifractal) scaling of the structure functions. The event-by-event stochastic inhomogeneity of turbulent plasma can therefore play an important role in forming its physical properties. In Section 3.3 we consider, following [3, 4], the turbulent contributions to the most fundamental physical characteristics of plasma, the properties of its collective modes, and plasmons for ultrarelativistic Abelian plasmas. The generalization of these results to the non-Abelian case is considered in Section 3.4. These results are new. The effects in question can broadly be described as nonlinear Landau damping [50]. One of the most interesting effects we see is a nonlinear Landau instability for transverse plasmons at large turbulent fields, that is, a phenomenon similar to nonlinear Landau damping. The origin of the phenomena considered in the paper is in the stochastic inhomogeneity of the turbulent electromagnetic fields in QED plasma; in this respect they are similar to the phenomenon of the stochastic transition radiation [60–62]. In particular, similarly to the stochastic transition radiation, the turbulent contributions to plasmon properties discussed in this paper vanish in the limit of vanishing correlation length of the stochastic turbulent fields. Finally, in Section 3.1, we consider the anomalous turbulent contributions to plasma viscosity [46–48].

2. Photons and Dileptons from Glasma

In this paragraph we will present, following [2], the main results on photon and dilepton production in glasma.

2.1. Thermalizing Glasma: Basic Facts. Let us first discuss the kinetic framework for glasma evolution as developed in [22]. Evolution of primordial non-Abelian matter produced in ultrarelativistic heavy ion collisions proceeds through several stages. A natural separation of scales is provided by the saturation momentum $Q_{\rm sat}$. At earliest times $0 \leq t \sim 1/Q_{\rm sat}$ the system can be described in terms of coherent chromoelectric and chromomagnetic flux tubes. The physics of this stage is that of gluons, so in terms of electromagnetic signals this stage is of no special interest. The density of quarks becomes substantial at times closer to the thermalization time $t_{\rm therm}$, so in studying the nonequilibrium contributions to photon and dilepton production we will focus at the time interval $1/Q_{\rm sat} \ll t \ll t_{\rm therm}$.

Let us assume that the gluon momentum density f_g can be written in the following form:

$$f_g = \frac{\Lambda_s}{\alpha_s p} F_g\left(\frac{p}{\Lambda}\right), \tag{1}$$

where the infrared and ultraviolet momentum cutoffs Λ_s and Λ are defined as follows. Initially, at $t_0 \sim 1/Q_{\rm sat}$, we have $\Lambda(t_0) = \Lambda_s(t_0) \sim Q_{\rm sat}$, whereas at thermalization time $\Lambda_s(t_{\rm therm}) \sim \alpha_s T_i$ and $\Lambda(t_{\rm therm}) \sim T_i$. In estimating the physical cross-sections one can use the following simple parametrization of f_g as a function of the gluon energy E_g:

$$\begin{aligned} f_g\left(E_g\right) &= \text{const.}, \quad E_g < \Lambda_s, \\ f_g\left(E_g\right) &= \text{const.} \frac{\Lambda_s}{E_g}, \quad \Lambda_s < E_g < \Lambda, \\ f_g\left(E_g\right) &= 0, \quad E_g > \Lambda. \end{aligned} \tag{2}$$

A key physical point of primary importance for the physics of the early stage of heavy ion collisions is that the phase space for the gluons is initially overoccupied so that the number density of gluons n_g and their energy density ϵ_g are related by

$$\frac{n_g}{\epsilon_g^{3/4}} \sim \frac{1}{\alpha_s^{1/4}}, \tag{3}$$

while for a thermally equilibrated Bose system it is necessary that this ratio should be less than a number of the order 1. This fact is a basis for the hypothesis [22] that the "extra" gluonic degrees of freedom are hidden in the highly coherent color singlet and spin singlet configuration that can be (approximately) described as a transient Bose condensate with a density

$$f_{\text{cond}} = n_{\text{cond}}\delta^3(p). \tag{4}$$

It is natural to think that the condensate is formed by gluons with masses of order of the natural infrared cutoff of the problem, the Debye mass:

$$M^2_{\text{Debye}} \sim \Lambda\Lambda_s. \tag{5}$$

One of the key features arising in many problems related to physical properties of primordial strongly interacting matter in heavy ion collisions is the natural asymmetry between longitudinal and transverse degrees of freedom which in the problem under consideration is parametrized by the fixed asymmetry between the typical transverse and longitudinal pressures δ that can conveniently be parametrized by the relation between the longitudinal pressure P_L and energy density ϵ:

$$P_L = \delta\epsilon, \tag{6}$$

where $0 \le \delta \le 1/3$, with $\delta = 0$ and $\delta = 1/3$ corresponding to the free-streaming (thus maximal anisotropy between the longitudinal and transverse pressure) and the isotropic expansion, respectively. The pressure anisotropy does of course reflect the difference of characteristic scales of transverse and longitudinal momenta.

The time evolution of the scales Λ_s and Λ wes found to be [22]

$$\begin{aligned}\Lambda_s &\sim Q_s\left(\frac{t_0}{t}\right)^{(4+\delta)/7},\\ \Lambda &\sim Q_s\left(\frac{t_0}{t}\right)^{(1+2\delta)/7}\end{aligned} \tag{7}$$

which, in turn, leads to the following temporal evolution of the gluon density and Debye mass:

$$\begin{aligned}n_g &\sim \frac{Q_s^3}{\alpha_s}\left(\frac{t_0}{t}\right)^{(6+5\delta)/7},\\ M^2_{\text{Debye}} &\sim \Lambda\Lambda_s \sim Q^2_{\text{sat}}\left(\frac{t_0}{t}\right)^{(5+3\delta)/7}\end{aligned} \tag{8}$$

and, finally, the thermalization time:

$$t_{\text{therm}} \sim t_0\left(\frac{1}{\alpha_s}\right)^{7/(3-\delta)}. \tag{9}$$

The description of the model is completed by introducing the quark distribution function:

$$f_q = F_q\left(\frac{p}{\Lambda}\right) \tag{10}$$

and assuming, for simplicity, the proportionality between the condensate density and that of gluons [22]:

$$n_{\text{cond}} = \kappa n_{\text{gluon}}, \tag{11}$$

where κ is a constant of order 1.

2.2. Electromagnetic Particle Production from the Glasma. Let us start with deriving a rate of photon production from glasma. The standard expression for the fixed-box photon production rate from the Compton channel $gq \to \gamma q$ reads

$$\begin{aligned}E\frac{dN}{d^4x d^3p} \propto{} & F_q\left(\frac{E}{\Lambda}\right)\frac{1}{E}\int_{\mu^2}^{\infty} ds\left(s-\mu^2\right)\sigma_{gq\to\gamma q}(s)\\ & \times\int_{s/4E}^{\infty} dE_g f_g\left(E_g\right)\left[1-F_q\left(\frac{E_g}{\Lambda}\right)\right],\end{aligned} \tag{12}$$

where E is the photon energy and $\mathbf{p}$ its three-momentum. The lower limit for integration over gluon energy E_g follows from kinematics; μ^2 is an infrared cutoff needed to regularize the $t(u)$ channel singularity for diagrams with massless particle exchange which in our case is the Debye mass $\mu^2 = \Lambda\Lambda_s$, and $\sigma_{gq\to\gamma q}(s)$ is the cross-section for gluon Compton effect $gq \to \gamma q$.

In the high energy limit and for small quark densities F_q, (12) simplifies to

$$\begin{aligned}&E\frac{dN}{d^4x d^3p}\\ &\quad\propto F_q\left(\frac{E}{\Lambda}\right)\frac{\Lambda_s\Lambda}{E}\int_1^{\infty} dy\,\ln y\int_{(y\Lambda_s\Lambda)/4E}^{\infty} dE_g f_g\left(E_g\right).\end{aligned} \tag{13}$$

Using the explicit parametrization of gluon density equation (2), it is straightforward to obtain

$$E\frac{dN}{d^4x d^3p} \propto F_q\left(\frac{E}{\Lambda}\right)\Lambda\Lambda_s\phi\left(\frac{E}{\Lambda}\right), \tag{14}$$

where $\phi(E/\Lambda)$ is some analytically calculable function. Let us thus assume that the photon production rate from glasma is given by

$$\frac{dN}{d^4x dy d^2k_T} = \frac{\alpha}{\pi}\Lambda_s\Lambda g\left(\frac{E}{\Lambda}\right), \tag{15}$$

where y is the photon rapidity.

To obtain the overall rate, we need to integrate over longitudinal coordinates. We assume that the early time expansion is purely longitudinal and that, in the integration, the space-time rapidity is strongly correlated with that of the momentum space-rapidity. We then have that

$$\frac{dN}{d^2r_T dy d^2k_T} \sim \alpha\int t\,dt\Lambda_s\Lambda g\left(\frac{k_T}{\Lambda}\right). \tag{16}$$

Using the result of the previous section for the time dependence of the scales Λ and Λ_s, we get

$$t dt = \kappa'\frac{d\Lambda}{\Lambda}\frac{1}{Q^2_{\text{sat}}}\left(\frac{Q_{\text{sat}}}{\Lambda}\right)^{14/(1+2\delta)}. \tag{17}$$

The constant κ' is of order 1.

Doing the integration over Λ in (16), we find that

$$\frac{dN}{d^2r_T dy d^2k_T} \sim \alpha\left(\frac{Q_{\text{sat}}}{k_T}\right)^{(9-3\delta)/(1+2\delta)}. \tag{18}$$

Now integrating over d^2r_T and identifying the overlap cross-section as proportional to the number of participants, we finally obtain

$$\frac{dN_\gamma}{dyd^2k_T} = \alpha R_0^2 N_{\text{part}}^{2/3}\left(\frac{Q_{\text{sat}}}{k_T}\right)^\eta, \tag{19}$$

where $\eta = (9-3\delta)/(1+2\delta)$. The factor of $N_{\text{part}}^{2/3}$ arises because the number of participants in a collision is proportional to the nuclear volume $R^3 \sim N_{\text{part}}$, and R_0 is a constant with dimensions of a length. A detailed analysis [2] shows that the formula (19) should be valid for the transverse momenta in the interval $1\,\text{GeV} \le k_T \le 10\,\text{GeV}$. The analytical results are compared to RHIC data in Figure 1 (for details of the comparison see [2]).

From Figure 1 we see that our simple model provides a good description of the experimental data.

The analysis of dilepton production is more complicated because there are two sources of dileptons. The first is due to annihilation of quarks in the glasma. The expression for the static rate of production of dilepton pairs with invariant mass M for massless quarks and leptons reads

$$\begin{aligned}\frac{dN^{l^+l^-}}{d^4xdM^2} &\sim M^2\sigma_{q\hat{q}\to l^+l^-}\left(M^2\right)\int_0^\infty dE_qF_q\left(\frac{E_q}{\Lambda}\right)\\ &\quad\times\int_{M^2/4E_q}^\infty dE_{\overline{q}}F_{\overline{q}}\left(\frac{E_{\overline{q}}}{\Lambda}\right)\\ &= M^2\sigma_{q\hat{q}\to l^+l^-}\left(M^2\right)\Lambda^2\\ &\quad\times\int_0^\infty dyF_q\left(y\right)\int_{M^2/4\Lambda^2y}dxF_q\left(x\right).\end{aligned} \tag{20}$$

Taking into account that $M^2\sigma_{q\hat{q}\to l^+l^-}(M^2) \sim$ const., we can already see the scaling behavior of the static dilepton production rate (24), that is,

$$\frac{dN^{l^+l^-}}{d^4xdM^2} \sim \Lambda^2\Phi\left(\frac{M}{\Lambda}\right) \tag{21}$$

with $\Phi(M/\Lambda) \equiv \int_0^\infty dyF_q(y)\int_{(M/\Lambda)^2/4y}dxF_q(x)$. To further explicitly demonstrate the scaling behavior of the static dilepton production rate (24), let us consider two examples with explicit forms of quark distribution function.

First let us consider a simple hard-cutoff quark distribution function $F_q = \theta(\Lambda - E)$. In this case we can easily obtain

$$\frac{dN^{l^+l^-}}{d^4xdM^2} \sim \Lambda^2\left[1-\frac{(M/\Lambda)^2}{4}+\frac{(M/\Lambda)^2}{4}\ln\frac{(M/\Lambda)^2}{4}\right]. \tag{22}$$

Second let us consider an exponential quark distribution function $F_q = \exp(-E/\Lambda)$. In this case we get the following result:

$$\begin{aligned}\frac{dN^{l^+l^-}}{d^4xdM^2} &\sim \Lambda^2\sqrt{\frac{M^2}{\Lambda^2}}K_1\left(\frac{M}{\Lambda}\right)\\ &= M\Lambda K_1\left(\frac{M}{\Lambda}\right) \equiv \Lambda^2\left[\frac{M}{\Lambda}K_1\left(\frac{M}{\Lambda}\right)\right],\end{aligned} \tag{23}$$

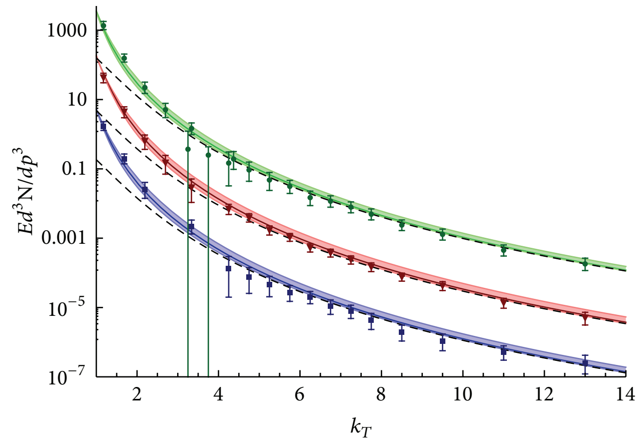

Figure 1: Comparison between the PHENIX photon data and the present model for three centrality bins [2]. The best fit corresponds to $\delta = 0.144 \pm 0.0045$. The parameter λ controls the x dependence of the saturation momentum with the best fit value $\lambda = 0.29 \pm 0.05$. For each of three color bands, the η (or δ) varies as $\eta = 6.65 \mp 0.60$ ($\delta \simeq 0.144 \pm 0.045$) at $\lambda = 0.29$.

where $K_1(M/\Lambda)$ is a Bessel function. This leads to the simple conjecture for the dilepton rate due to the annihilation mechanism:

$$\frac{dN_{DY}}{d^4xdM^2} = \alpha^2\Lambda^2 g'\left(\frac{M}{\Lambda}\right) \tag{24}$$

which, in the direct analogy with the above-described calculation for photons, leads to

$$\frac{dN_{DY}}{dydM^2} \sim \alpha^2 R_0'^2 N_{\text{part}}^{2/3}\left(\frac{Q_{\text{sat}}}{M}\right)^\eta \tag{25}$$

with $\eta = 4(3-\delta)/(1+2\delta)$.

The second possible source of dileptons is the annihilation of gluons into a quark loop from which the quarks then subsequently decay into a virtual photon and eventually the dilepton: see the illustration in Figure 2. Such a virtual process is naively suppressed by factors of α_s. Here, however, the gluons arise from a highly coherent condensate, and the corresponding factors of α_s are compensated by inverse factors $1/\alpha_s$ from the coherence of the condensate. In other words, the usual power counting for diagrams in terms of α_s has to be changed when the coherent condensate with high occupation is present.

Here we estimate the rate for the three-gluon decay of the condensate into a dilepton. On dimensional grounds, we expect that

$$\frac{dN_{C\to DY}}{d^4xdydM^2} = \alpha^2\frac{\left(\alpha_s n_{\text{gluon}}\right)^3}{M_{\text{Debye}}^7}g''\left(\frac{M}{M_{\text{Debye}}}\right), \tag{26}$$

where we have assumed that the condensate density is of the order of the gluon number density as in (17) and that

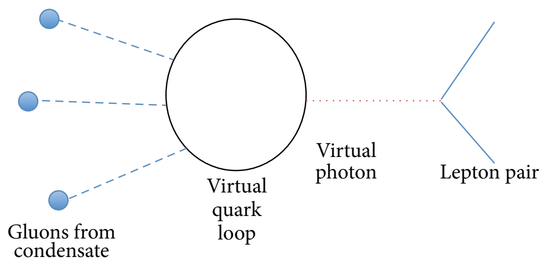

FIGURE 2: Three gluons from the condensate annihilate into a virtual quark loop, that subsequently decays into a virtual photon and then into a dilepton.

the typical scale for the energy of gluons in the condensate is of order of the Debye mass. Integration over time leads to

$$\frac{dN_{C\to DY}}{dydM^2} \sim \alpha^2 R_0'^2 N_{\text{part}}^{2/3}\left(\frac{Q_{\text{sat}}}{M}\right)^{\eta'}, \tag{27}$$

where

$$\eta'_{\text{perturbative}} = \frac{9(3-\delta)}{5+3\delta}. \tag{28}$$

The analytical results are compared to RHIC data in Figure 3 (for details of the comparison see [2]).

From Figure 3 we see that our simple model provides a good description of the experimental data.

3. Turbulent Plasma

It is well known that experimentally observed physical properties of usual electromagnetic plasmas are dramatically different from predictions based on textbook equilibrium Vlasov plasma [49, 50]. A picture accommodating these observations is that of turbulent plasma, in which in addition to hard particles and soft self-consistent mean field there exist random electromagnetic fields leading, for example, to anomalously small viscosity and conductivity of turbulent plasma as compared to the equilibrium one. In this section we consider the key physical characteristics of turbulent ultrarelativistic plasma, its anomalously small viscosity [46–48], and anomalously large jet quenching [51] as well its polarization properties in the Abelian [3, 4] and non-Abelian cases.

3.1. Turbulent Anomalous Viscosity. One of the most important requirements for any scenario describing the relevant physics of the early stage of nuclear collisions is its ability of explaining an (effectively) small viscosity characterizing collective expansion of dense matter created in these collisions. Let us recall the standard kinetic theory expression for shear viscosity:

$$\eta = \frac{1}{3} n \langle p\rangle_T \lambda_f, \tag{29}$$

where n is the density of the medium, $\langle p\rangle_T$ is the thermal momentum of particles, and λ_f is the mean free path.

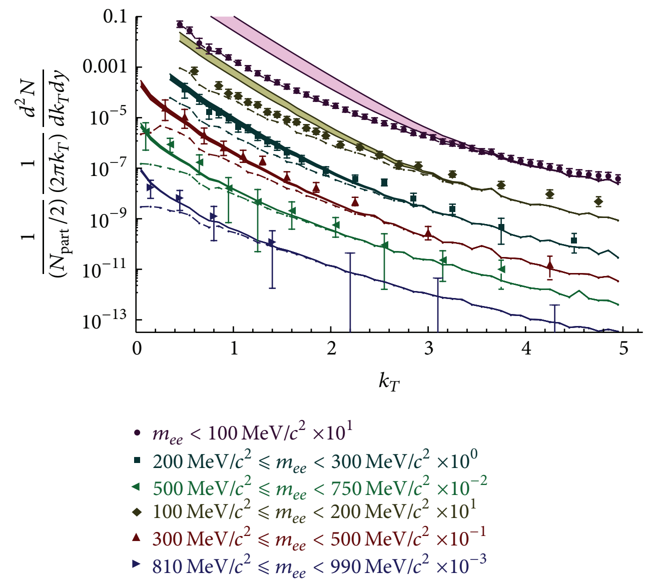

FIGURE 3: (Color online) The comparison between the PHENIX dilepton data and the model [2]. Min. Bias Au + Au $\sqrt{S_{\text{NN}}} = 200$ GeV. For each color band, the η' (or δ) varies as $\eta' \simeq 4.73 \pm 0.20$ ($\delta = 0.144 \mp 0.045$) at $\lambda = 0.29$.

In the standard perturbative case the source of λ_f is perturbative scattering which is parametrically weak and, therefore, leads to very large values of λ_f and, consequently, shear viscosity η. The situation is dramatically different in turbulent plasmas [46–48], where particle scatters on strong turbulent fields. The physical picture is here that of plasma particles scattering on domains of coherent turbulent fields, the size of the domains being controlled by the corresponding correlation length l, so that

$$\lambda_f = \frac{\langle p\rangle_T^2}{g^2 Q^2 \langle B^2\rangle l}, \tag{30}$$

leading to the anomalously small turbulent shear viscosity [46–48]

$$\eta_A \approx \frac{(9/4)\, sT^3}{g^2 Q^2 \langle B^2\rangle l} \tag{31}$$

valid for the nearly equilibrated plasma.

3.2. Turbulent Jet Quenching. Another very attractive feature of the turbulent plasma scenario is its natural ability to describe the observed strong jet quenching, that is, large energy loss experienced by fast particles propagating through early QCD matter [51]. Indeed, it is easy to calculate transverse broadening $\langle(\Delta p_\perp)^2\rangle$ induced by the same process of particle scattering on turbulent field domains and the corresponding (anomalous) jet quenching parameter $\hat{q}_A$:

$$\hat{q}_A = g^2 Q^2 \left\langle B^2\right\rangle l. \tag{32}$$

Leading, in particular, to an interesting relation between the two above-discussed anomalous quantities [51]:

$$\frac{\eta_A}{s} \propto \frac{T^3}{\hat{q}_A}, \qquad (33)$$

that is, the smaller is the anomalous turbulent viscosity, the large is the anomalous turbulent jet quenching.

3.3. Turbulent Polarization: QED Plasma. Let us first consider polarization properties of the turbulent ultrarelativistic QED plasma [3, 4]. A weakly turbulent plasma is described as perturbation of an equilibrated system of (quasi-)particles by weak turbulent fields $F^T_{\mu\nu}$. In the collisionless Vlasov approximation we employ, the plasma properties are defined by the following system of equations ($F^R_{\mu\nu}$ is a regular nonturbulent field):

$$\begin{aligned} p^\mu \left[\partial_\mu - eq\left(F^R_{\mu\nu} + F^T_{\mu\nu}\right)\frac{\partial}{\partial p_\nu}\right] f(p,x,q) &= 0, \\ \partial^\mu \left(F^R_{\mu\nu} + F^T_{\mu\nu}\right) = j_\nu(x) &= e\sum_{q,s}\int dp p_\nu q f(p,x,q). \end{aligned} \qquad (34)$$

The stochastic ensemble of turbulent fields is assumed to be Gaussian and characterized by the following correlators:

$$\left\langle F^T_{\mu\nu}\right\rangle = 0, \qquad \left\langle F^{T\mu\nu}(x) F^{T\mu'\nu'}(y)\right\rangle = K^{\mu\nu\mu'\nu'}(x,y). \qquad (35)$$

Following [46–48] we use the parametrization of $K^{\mu\nu\mu'\nu'}(x,y)$ of the form

$$K^{\mu\nu\mu'\nu'}(x) = K_0^{\mu\nu\mu'\nu'} \exp\left[-\frac{t^2}{2\tau^2} - \frac{r^2}{2a^2}\right]. \qquad (36)$$

By definition, turbulent polarization is defined as a response to a regular perturbation that depends on turbulent fields. In the linear response approximation it is fully described by the polarization tensor $\Pi^{\mu\nu}(k)$ which can be computed by taking a variational derivative of the averaged induced current $\langle j^\mu(k \mid F^R, F^T)\rangle_{F^T}$ over the regular gauge potential A^R_ν:

$$\Pi^{\mu\nu}(k) = \frac{\delta\left\langle j^\mu\left(k \mid F^R, F^T\right)\right\rangle_{F^T}}{\delta A^R_\nu}, \qquad (37)$$

$$\left\langle j^\mu\left(k \mid F^R, F^T\right)\right\rangle_{F^T} = e\sum_{q,s}\int dP p_\nu q \left\langle \delta f\left(p,k,q \mid F^R, F^T\right)\right\rangle_{F^T}. \qquad (38)$$

To organize the calculation in the efficient way it is useful to rewrite (34) as follows:

$$f = f^{eq} + Gp^\mu F_{\mu\nu}\partial^\mu_p f, \quad G \equiv \frac{eq}{\imath((pk) + \imath\epsilon)}, \qquad (39)$$

where f^{eq} is a distribution function characterizing the original nonturbulent plasma and introduces the following systematic expansion in the turbulent and regular fields:

$$\begin{aligned} \delta f &= \sum_{m=0}\sum_{n=0}\rho^m\tau^n\delta f_{mn}, \\ F^{\mu\nu} &= \sum_{m=0}\sum_{n=0}\rho^m\tau^n F^{\mu\nu}_{mn}, \end{aligned} \qquad (40)$$

where powers of ρ count those of F^R and powers of τ count those of F^T. Turbulent polarization is described by contributions of the first order in the regular and the second in the turbulent fields. The lowest nontrivial contribution to the induced current (38) is thus given by δf_{12}:

$$\delta f \simeq \delta f_{\mathrm{HTL}} + \left\langle \delta f_{12}\right\rangle_{\mathrm{I}} + \left\langle \delta f_{12}\right\rangle_{\mathrm{II}}, \qquad (41)$$

where

$$\begin{aligned} \delta f_{\mathrm{HTL}} &= Gp_\mu F^{\mu\nu}_{10}\partial_{\mu,p} f^{eq}, \\ \left\langle \delta f_{12}\right\rangle_{\mathrm{I}} &= Gp_\mu \left\langle F^{\mu\nu}_{01}\partial_{\nu,p} Gp_{\mu'} F^{\mu'\nu'}_{10}\partial_{\nu',p} Gp_\rho F^{\rho\sigma}_{01}\right\rangle \partial_{\sigma,p} f^{eq}, \\ \left\langle \delta f_{12}\right\rangle_{\mathrm{II}} &= Gp_\mu \left\langle F^{\mu\nu}_{01}\partial_{\nu,p} Gp_{\mu'} F^{\mu'\nu'}_{01}\partial_{\nu',p} Gp_\rho F^{\rho\sigma}_{10}\right\rangle \partial_{\sigma,p} f^{eq}. \end{aligned} \qquad (42)$$

Generically one has the following decomposition of the polarization tensor:

$$\begin{aligned} &\Pi_{ij}(\omega, \mathbf{k}; l) \\ &\quad = \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\Pi_T(\omega, |\mathbf{k}|; l) + \frac{k_i k_j}{k^2}\Pi_L(\omega, |\mathbf{k}|; l), \end{aligned} \qquad (43)$$

where $l \equiv \sqrt{2}(\tau a)/\sqrt{\tau^2 + a^2}$. In the leading order in turbulent fields its components can be rewritten as sums of the leading hard thermal loops (HTL) contributions and the gradient expansion in the scale of turbulent fluctuations l:

$$\begin{aligned} &\Pi_{L(T)}(\omega, \mathbf{k}; l) = \Pi^{\mathrm{HTL}}_{L(T)}(\omega, \mathbf{k}) + \Pi^{\mathrm{turb}}_{L(T)}(\omega, \mathbf{k} \mid l), \\ &\Pi^{\mathrm{turb}}_{L(T)}(\omega, |\mathbf{k}|; l) \\ &\quad = \sum_{n=1}^{\infty}\frac{(|\mathbf{k}|\, l)^n}{\mathbf{k}^2}\left[\phi^{(n)}_{L(T)}(x)\left\langle E^2_{\mathrm{turb}}\right\rangle + \chi^{(n)}_{L(T)}(x)\left\langle B^2_{\mathrm{turb}}\right\rangle\right], \end{aligned} \qquad (44)$$

where $x = \omega/|\mathbf{k}|$, and the HTL contributions to the polarization tensor read

$$\begin{aligned} \Pi^{\mathrm{HTL}}_L(\omega, |\mathbf{k}|) &= -m_D^2 x^2\left[1 - \frac{x}{2}L(x)\right], \\ \Pi^{\mathrm{HTL}}_T(\omega, |\mathbf{k}|) &= m_D^2\frac{x^2}{2}\left[1 + \frac{1}{2x}\left(1 - x^2\right)L(x)\right], \\ L(x) &\equiv \ln\left|\frac{1+x}{1-x}\right| - \imath\pi\theta(1-x); \qquad m_D^2 = \frac{e^2T^2}{3}. \end{aligned} \qquad (45)$$

The computation of turbulent polarization was carried out to second order in the gradient expansion [3, 4]. To the leading order in the gradient expansion one gets

$$\begin{aligned}
&\phi_{I\,T}^{(1)}(x)\\
&\quad= \frac{\imath e^4}{6\pi\sqrt{\pi}}\, 2x\left[\frac{4+10x^2-6x^4}{3\left(1-x^2\right)} + x\left(1-x^2\right)L(x)\right],\\
&\phi_{I\,L}^{(1)}(x) = -\frac{\imath e^4}{6\pi\sqrt{\pi}}\,\frac{8x^3}{3\left(1-x^2\right)^2},\\
&\chi_{I\,T}^{(1)}(x) = \frac{\imath e^4}{6\pi\sqrt{\pi}}\, 4x\left[\frac{-2+6x^2}{3\left(1-x^2\right)} + xL(x)\right],\\
&\chi_{I\,L}^{(1)}(x) = -\frac{\imath e^4}{6\pi\sqrt{\pi}}\,\frac{8x^3}{3\left(1-x^2\right)^2}.
\end{aligned} \tag{46}$$

Let us first discuss the turbulent contributions to the imaginary part of the polarization tensor [3, 4]. These can be summarized as follows. The sign of the imaginary part of the turbulent contribution to the polarization operator in the time-like domain $x > 1$ is negative and corresponds to turbulent damping of time-like collective excitations. This refers to both transverse and longitudinal modes. As the HTL contribution in this domain is absent, this turbulent damping is a universal phenomenon present for all ω, k such that $\omega > k$ and all values of the parameters involved $(l, \langle B^2\rangle, \langle E^2\rangle)$. The turbulent damping leads to an attenuation of the propagation of collective excitations at some characteristic distance. The situation in the space-like domain $x < 1$ is more diverse. In contrast with the time-like domain the gradient expansion for the imaginary part of the polarization tensor starts from the negative HTL contribution corresponding to Landau damping. The imaginary parts of turbulent contributions to the longitudinal polarization tensor are negative and are thus amplifying the Landau damping. The most interesting contributions come from the turbulent contributions to the transverse polarization tensor. The electric contribution $\mathrm{Im}[\phi_T^{(1)}(x)]$ in the space-like domain is positive at all x while the magnetic contribution $\mathrm{Im}[\chi_T^{(1)}(x)]$ is negative for $x < x^* \approx 0.43$ and positive for $x > x^*$. This means that the turbulent plasma becomes unstable for sufficiently strong turbulent fields.

It is also of interest to analyze the effects of turbulence on the properties of collective excitations of QED plasma, the plasmons [4]. The plasmons are characterized by dispersion relations $\omega_{T(L)}(|\mathbf{k}|)$ that are read from the solutions of dispersion equation for the corresponding components of dielectric permittivity, which are just a real part of zeroes of inverse transverse and longitudinal wave propagators:

$$\begin{aligned}
&\mathrm{Re}\left[\mathbf{k}^2\left(1-\frac{\Pi_L(k^0,|\mathbf{k}|)}{\omega^2}\right)\Bigg|_{k^0=\omega_L(|\mathbf{k}|)}\right] = 0,\\
&\mathrm{Re}\left[\mathbf{k}^2-\left(k^0\right)^2+\Pi_T\left(k^0,|\mathbf{k}|\right)|_{k^0=\omega_T(|\mathbf{k}|)}\right] = 0.
\end{aligned} \tag{47}$$

Thus, real part of polarization tensor corresponds to propagation of plasmons in a medium, while its imaginary part defines plasmon smearing.

Let us focus first on a shift of plasmons dispersion relations in turbulent medium. In general dispersion equations can be solved only numerically. Analytical expressions can be obtained in certain limits. Let us focus on the deeply time-like regime of $x \gg 1$. In nonturbulent HTL Vlasov plasma the time-like plasmon modes do not decay, since imaginary part of polarization tensor in that limit is zero. For frequencies $(k/\omega_{\mathrm{pl}}) \ll 1$ the corresponding solutions of dispersion equations may be expanded as powers of $|\mathbf{k}|/\omega_{\mathrm{pl}}$:

$$\begin{aligned}
&\omega_L^2(|\mathbf{k}|)_{\mathrm{HTL}} = \omega_{\mathrm{pl}}^2\left(1+\frac{3}{5}\left(\frac{|\mathbf{k}|}{\omega_{\mathrm{pl}}}\right)^2 + O\left(\left(\frac{|\mathbf{k}|}{\omega_{\mathrm{pl}}}\right)^4\right)\right),\\
&\omega_T^2(|\mathbf{k}|)_{\mathrm{HTL}} = \omega_{\mathrm{pl}}^2\left(1+\frac{6}{5}\left(\frac{|\mathbf{k}|}{\omega_{\mathrm{pl}}}\right)^2 + O\left(\left(\frac{|\mathbf{k}|}{\omega_{\mathrm{pl}}}\right)^4\right)\right),
\end{aligned} \tag{48}$$

where we have used a standard definition for the plasma frequency $\omega_{\mathrm{pl}}^2 = m_D^2/3$.

In a turbulent plasma plasmons decay even in a Vlasov limit since polarization tensor has imaginary part. As to the turbulent modifications of the HTL dispersion relation (48), it can be conveniently written as

$$\begin{aligned}
&\omega_L^2(|\mathbf{k}|)_{\mathrm{turb}}\\
&\quad= \left(\omega_{\mathrm{pl}\,L}^{\mathrm{turb}}\right)^2\left(1+\frac{3}{5}y_L^2\right) - \frac{e^4l^2}{6\pi^2}\left(\frac{24}{5}\left\langle E^2\right\rangle + \frac{64}{15}\left\langle B^2\right\rangle\right)y_L^2\\
&\qquad+ O\left(y_L^4\right),\\
&\omega_T^2(|\mathbf{k}|)_{\mathrm{turb}}\\
&\quad= \left(\omega_{\mathrm{pl}\,T}^{\mathrm{turb}}\right)^2\left(1+\frac{3}{5}y_T^2\right) - \frac{e^4l^2}{6\pi^2}\left(\frac{24}{7}\langle E^2\rangle + \frac{32}{15}\langle B^2\rangle\right)y_T^2\\
&\qquad+ O\left(y_T^4\right),
\end{aligned} \tag{49}$$

where

$$\begin{aligned}
&y_L = \frac{|\mathbf{k}|}{\omega_{\mathrm{pl}\,L}^{\mathrm{turb}}}, \qquad y_T = \frac{|\mathbf{k}|}{\omega_{\mathrm{pl}\,T}^{\mathrm{turb}}},\\
&\left(\omega_{\mathrm{pl}\,L}^{\mathrm{turb}}\right)^2 = \omega_{\mathrm{pl}\,L}^2 - \frac{e^4l^2}{6\pi^2}\left(\frac{16}{3}\left\langle E^2\right\rangle + \frac{8}{3}\left\langle B^2\right\rangle\right),\\
&\left(\omega_{\mathrm{pl}\,T}^{\mathrm{turb}}\right)^2 = \omega_{\mathrm{pl}\,T}^2 - \frac{e^4l^2}{6\pi^2}\left(\frac{128}{15}\left\langle E^2\right\rangle + \frac{8}{3}\left\langle B^2\right\rangle\right).
\end{aligned} \tag{50}$$

Now let us consider the plasmon smearing. It can be easily seen that a rate of decay for plasmons is connected to the imaginary part of polarization tensor by a formula:

$$\Gamma_{T(L)} = \sqrt{-\,\mathrm{Im}\left(\Pi_{T(L)}\right)}. \tag{51}$$

In the time-like region considered previously the imaginary parts of both transverse and longitudinal components of polarization tensor are negative so that there is no instability for time-like modes. It should also be noted that turbulent smearing is a leading order effect in (kl) as compared to the turbulent modification of plasmon dispersion relations.

3.4. Turbulent Polarization: QCD Plasma. Let us now discuss a generalization of the results of [3, 4] on the polarization properties on the non-Abelian QCD plasma.

The generalization of (34) to the non-Abelian case reads

$$p^{\mu}\left[\partial_{\mu}-gf_{abc}A_{\mu}^{b}Q^{c}\frac{\partial}{\partial_{Q^{a}}}-gQ_{a}F_{\mu\nu}^{a}\frac{\partial}{\partial_{p_{\nu}}}\right]=0, \quad (52)$$

where the fields $F_{\mu\nu}$ satisfy the Yang-Mills equations:

$$D^{\mu}F_{\mu\nu}^{a}=j_{\nu}^{a}. \quad (53)$$

The main distinction from the Abelian case is the dependence of the distribution function on the color spin Q, where for $SU(3)Q = (Q^1, Q^2, \ldots, Q^8)$, so that $f(x,p,Q)$. The components of color spin $Q = (Q^1, Q^2, \ldots, Q^8)$ are dynamic variables satisfying the Wong equation:

$$\frac{dQ^{a}}{d\tau}=-gf^{abc}p^{\mu}A_{\mu}^{b}Q^{c} \quad (54)$$

which, with (52), (53), completes the dynamical description of QCD plasma.

The description of the properties of turbulent QCD plasma is based on separating the regular and turbulent contributions of the distribution functions and gauge potentials A_{a}^{μ}:

$$f=f^{R}+f^{T},\qquad A_{\mu}^{a}=A_{\mu}^{Ra}+A_{\mu}^{Ta}, \quad (55)$$

where we assume that $\langle A_{\mu}^{a}\rangle = A_{\mu}^{Ra}$ and $\langle A_{\mu}^{Ta}\rangle = 0$.

It is possible to define gauge transformations of regular and turbulent gauge potentials in such a way that

$$\delta A_{\mu}^{Ra}=\partial_{\mu}\alpha^{a}+gf^{abc}A_{\mu}^{Rb}\alpha^{c},$$
$$\delta A_{\mu}^{Tb}=gf^{abc}A_{\mu}^{Tb}\alpha^{c} \quad (56)$$

which is technically equivalent to choosing the background field gauge. A very useful property following from this choice is that the basic property of turbulent fields $\langle A_{\mu}^{Ta}\rangle = 0$ is gauge invariant. The corresponding decomposition of gauge field strength reads

$$F_{\mu\nu}^{a}=F_{\mu\nu}^{Ra}+\mathbf{F}_{\mu\nu}^{Ta}+\mathscr{F}_{\mu\nu}^{Ta}, \quad (57)$$

where

$$F_{\mu\nu}^{Ra}=\partial_{\mu}A_{\nu}^{Ra}-\partial_{\nu}A_{\mu}^{Ra}+gf^{abc}A_{\mu}^{Rb}A_{\nu}^{Rc},$$
$$\mathscr{F}_{\mu\nu}^{Ta}=\partial_{\mu}A_{\nu}^{Ta}-\partial_{\nu}A_{\mu}^{Ta}+gf^{abc}A_{\mu}^{Tb}A_{\nu}^{Tc}, \quad (58)$$
$$\mathbf{F}_{\mu\nu}^{Ta}=gf^{abc}\left(A_{\mu}^{Tb}A_{\nu}^{Rc}+A_{\mu}^{Rb}A_{\nu}^{Tc}\right).$$

The computation of polarization properties proceeds through expanding f^R, f^T in a series in the regular potential $(f^{(0)} \sim (A^R)^0, f^{(1)} \sim (A^R)^1, \ldots)$. Assuming $f^{R(0)}(x,p,Q) = f^{eq}(p)$, we have

$$\left(p^{\mu}\partial_{\mu}\right)f^{T(0)}=gp^{\mu}Q_{a}\mathscr{F}_{\mu\nu}^{Ta}\frac{\partial}{\partial p^{\nu}}f^{R(0)}. \quad (59)$$

The corresponding equations for the first-order turbulent contributions to the distribution function read

$$\begin{aligned}\left(p^{\mu}\partial_{\mu}\right)f^{T(1)} &= gp^{\mu}f^{abc}A_{\mu}^{Tb}Q^{c}\frac{\partial}{\partial Q^{a}}f^{R(1)}\\ &+g^{2}p^{\mu}f^{abc}A_{\mu}^{Rb}Q^{c}\frac{1}{(p\partial)}p^{\mu'}\mathscr{F}_{\mu'\nu}^{Ta}\frac{\partial}{\partial p_{\nu}}f^{R(0)}\\ &+gp^{\mu}Q_{a}\mathbf{F}_{\mu\nu}^{Ta}\frac{\partial}{\partial_{\nu}}f^{R(0)}+gp^{\mu}Q_{a}\mathscr{F}_{\mu\nu}^{Ta}\frac{\partial}{\partial p_{\nu}}f^{R(1)}\\ &+g^{2}p^{\mu}p^{\mu}p'Q^{a}F_{\mu\nu}^{Ra}\frac{\partial}{\partial p_{\nu}}\frac{1}{\left(p^{\mu}\partial_{\mu}\right)}Q_{c}\mathscr{F}_{\mu'\nu'}^{Tc}\frac{\partial}{\partial p_{\nu'}}f^{R(0)}\\ &+g^{2}p^{\mu}f^{abc}A_{\mu}^{Rb}Q^{c}\frac{\partial}{\partial Q^{a}}\frac{1}{\left(p^{\mu}\partial_{\mu}\right)}p_{d}^{\mu'}\mathscr{F}_{\mu'\nu}^{Td}\frac{\partial}{\partial p_{nu}}f^{R(0)},\end{aligned} \quad (60)$$

$$\begin{aligned}\left(p^{\mu}\partial_{\mu}\right)f^{R(1)} &= gp^{\mu}f^{abc}\left\langle A_{\mu}^{Tb}Q^{c}\frac{\partial}{Q^{a}}f^{T(1)}\right\rangle\\ &+gp^{\mu}Q_{a}\left\langle\mathscr{F}_{\mu\nu}^{Ta}\frac{\partial}{\partial p_{\nu}}f^{T(1)}\right\rangle+gp^{\mu}Q_{a}\left\langle\mathbf{F}_{\mu\nu}^{R}\frac{\partial}{\partial p_{\nu}}f^{T(0)}\right\rangle\\ &+gp^{\mu}Q_{a}F_{\mu\nu}^{Ra}\frac{\partial}{\partial p_{\nu}}f^{R(0)}.\end{aligned} \quad (61)$$

Substituting (59) and (60) to (61) one arrives at the final expression for the first-order regular correction to the distribution function. A detailed analysis shows that in the relevant long wavelength limit we are left with only two contributions:

$$f^{R(1)}=\mathrm{HTL}+I_{1}+I_{2}, \quad (62)$$

where

$$\begin{aligned}I_{1}&=g^{3}p^{\mu}f^{abc}\\ &\times\left\langle A_{\mu}^{Tb}Q^{c}\frac{\partial}{\partial Q^{a}}p^{\mu'}\frac{1}{p^{\mu}\partial_{\mu}}f^{def}A_{\mu'}^{Re}Q^{f}\right.\\ &\qquad\left.\times\frac{\partial}{\partial Q^{d}}\frac{1}{p^{\mu}\partial_{\mu}}p^{\mu'}Q_{g}\mathscr{F}_{\mu''\nu}\right\rangle\\ &\times\frac{\partial}{\partial p_{\nu}}f^{R(0)},\end{aligned}$$

$$I_2 = g^2 p^\mu f^{abc} \times \left\langle A_\mu^{Tb} Q^c \frac{\partial}{\partial Q^a} \frac{1}{\left(p^\mu \partial_\mu\right)} p^{\mu'} Q_d \mathbf{F}_{\mu'\nu}^{Td} \right\rangle \frac{\partial}{\partial p_\nu} f^{R(0)}. \tag{63}$$

Averaging over the stochastic color fields involves two two-point correlators:

$$\begin{aligned} \left\langle A_\mu^{Ta}(x) A_\nu^{Tb}(y) \right\rangle &= G_{\mu\nu}^{ab}(x,y), \\ \left\langle \mathscr{F}_{\mu\nu}^{Ta}(x) U^{ab}(x,y) \mathscr{F}_{\mu'\nu'}^{Tb}(y) \right\rangle &= K_{\mu\nu\mu'\nu'}^{ab}(x,y). \end{aligned} \tag{64}$$

We will restrict our consideration to the plasma which is on average homogeneous, $K_{\mu\nu\mu'\nu'}^{Ta}(x,y) = K_{\mu\nu\mu'\nu'}^{Ta}(x-y)$ (same for $G_{\mu\nu}^{ab}$), and assume that the stochastic correlators are symmetric under permutations of both color and Lorentz indices.

Let us choose the following explicit parametrization for the non-Abelian correlation function $G_{\mu\nu}^{ab}$:

$$G_{\mu\nu}^{ab} = \delta_{ab}\left[g_{\mu\nu}g_{\nu 0}\left\langle A_0^2\right\rangle + \frac{1}{3}\widehat{\delta}_{\mu\nu}\left\langle \mathbf{A}^2\right\rangle\right] \exp\left[-\frac{r^2}{2a^2} - \frac{t^2}{2\tau^2}\right]. \tag{65}$$

Defining $f^{Ra(1)}(x,p) = \int Q^a dQ f^{R(1)}(x,p,Q)$, we get

$$\left[\left(p^\mu \partial_\mu\right) + p\gamma\right] f^{Rl(1)} = \int Q^l dQ \left(\mathrm{HTL} + I_1 + I_2\right), \tag{66}$$

where

$$\gamma = g^2 \frac{N^2-1}{4N}\sqrt{\pi} l \left[\left\langle A_0^2\right\rangle + \left\langle \frac{1}{3}\mathbf{A}^2\right\rangle\right] \tag{67}$$

and $l = 1/\sqrt{(1/2a^2) + (1/2\tau^2)}$. Let us stress that only the sum of contributions from HTL, I_1, and I_2, is gauge invariant.

Detailed calculations show that to the leading order there is no contribution from the non-Abelian correlator $G_{\mu\nu}^{ab}$, so that the only modification distinguishing QCD plasma from the QED one is an overall normalization so that

$$\begin{aligned} \phi_{T,L}(x) &\longrightarrow C_{q(g)}\phi_{T,L}(x), \\ \chi_{T,L}(x) &\longrightarrow C_{q(g)}\chi_{T,L}(x), \end{aligned} \tag{68}$$

where for quarks

$$C_q = g^4 N_q \frac{N^2-1}{4N} \tag{69}$$

and for gluons

$$C_g = \frac{2g^3N^2}{N + \left(N_q/2\right)}, \tag{70}$$

where we have taken into account a necessity of introducing an infrared cutoff at m_D when computing the integral $\int dp(df/dp)$ for massless bosons.

From the above analysis we can conclude that, apart from trivial color factors, in the considered approximation, the properties of the turbulent non-Abelian plasma are equivalent to those of the Abelian one.

4. Conclusions

In the present paper we have reviewed several results related to the important role played by collective effects in the physics of heavy ion collisions and, more broadly, dense non-Abelian matter. The results on anomalous viscosity [46–48] and anomalous jet quenching [51] are not new and are included for making the discussion of the properties of turbulent ultrarelativistic plasma self-contained. The main emphasis is made on new results on photon and dilepton production in glasma [2], polarization properties of ultrarelativistic Abelian [3, 4] and non-Abelian turbulent plasmas.

Discussing electromagnetic signals originating from glasma and, possibly, gluon condensate we have demonstrated that the unusual properties of photon and dilepton spectra observed at RHIC can naturally be described by taking into account collective gluon degrees of freedom.

The analysis of polarization properties of Abelian [3, 4] and non-Abelian ultrarelativistic turbulent plasmas revealed interesting structure of turbulent contributions to both imaginary and real parts of the polarization tensor and, correspondingly, to the plasmon properties. In particular, we have described an interesting instability of turbulent plasma for strong enough turbulent fields and have shown that the polarization properties of turbulent non-Abelian plasmas are, up to the trivial color factors, in the considered approximation equivalent to those of the Abelian one.

With a wealth of new experimental data and theoretical ideas the physics of high energy heavy ion collisions remains a truly exciting domain of research. We are sure that in the near future we will see rapid progress both in the quality of experimental data and that of theoretical research.

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