A FUZZY CAPACITY ACQUISITION AND ALLOCATION MODEL FOR BANDWIDTH BROKERS UNDER UNCERTAINTY

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ABSTRACT

In this paper, bandwidth acquisition and allocation problem of a telecommunications Bandwidth Broker (BB) is analyzed under uncertain end-user capacity requests. Furthermore, related objective function coefficients such as revenue and costs are modeled as fuzzy numbers in order to cope with vague market conditions. By using fuzzy mathematical programming solution approach suggested by Zhang *et al.* (2003) deterministic equivalent of single objective profit maximization problem of BB is obtained. Two performance statistics namely fuzzy EVPI and fuzzy VSS are defined to demostrate the efficiency of proposed methodology compared to deterministic approach. Numerical experiments showed that fuzzy stochastic method provides approximately over 10% more profit depending upon problem size in comparison with deterministic strategy.

KEYWORDS

Telecommunications market, Bandwidth Broker, fuzzy mathematical programming, fuzzy VSS and EVPI.

1 INTRODUCTION AND RELATED WORKS

Telecommunication network problems are subject to uncertainties like other real world problems. The sources of the uncertainties in the telecommunications may be environmental or originated from the system itself. In all circumstances, neglected uncertainties in the models may result in inaccurate solutions (Turan, 2012). Hence, we model and solve bandwidth acquisition and allocation problem of a telecommunications Bandwidth Broker (BB), which is inherently complex and structurally uncertain.

We consider BB's profit maximization problem in an environment in which the firm can lease network capacity at competitive prices from different backbone providers (BPs) with different service quality (QoS). In order to gain profit, BB has to first acquire (lease) the capacity (bandwidth) from BPs then lease bandwidth to end-users. After leasing decision, BB has to allocate leased bandwidth into acquired capacities by meeting QoS requirements of each customer.

The solution of the proposed model provide essential strategic planning information to the decision makers of BB such as how much bandwidth for how long should be leased from each BP, which service providers should be chosen by considering QoS levels, after realization of customers' demands which customer's bandwidth request should be accepted and which of them should be rejected and finally how accepted bandwidth demands should be allocated into leased capacity by considering quality of service parameters such as delay and jitter.

There are some studies related to modeling different parameters as fuzzy numbers in the telecom literature. The existing studies concern both design and implementation of network infrastructure and operating policies including QoS issues. Turan *et al.* (2012) investigate bandwidth allocation problem of firms that use bandwidth to complete their operations. They model delay and jitter amounts guaranteed by network providers and maximum delay and jitter levels tolerable as fuzzy numbers. Carlson *et al.* (1998a) discuss lower level telecommunications problem such as instillation of new equipments and routing of data flow in the network. In their model, only cost parameters are modeled as fuzzy numbers. The demands of customers are assumed to be deterministic in the discussed model. Carlson *et al.* (1998b) examine again a lower level telecommunications network problem of the backbone provider under uncertain customers demands. The uncertain demands of customers are integrated into model as triangular fuzzy numbers. The researchers assume that rest of the parameters such as costs terms and link capacities are known in advance and stay fixed through planning horizon.

Alminana *et al.*(2007) use mixed 0-1 two-stage stochastic programming methodology to solve rerouting problem under uncertainty. Riis *et al.* (2005) design internet protocol network of largest network provider of Denmark (TDC) by utilizing two-stage linear programming algorithm. Authors assume demand for capacity is uncertain. Terblanche *et al.* (2011) not only investigate problems of routing but also investigate choosing and installation of network equipments in order to find the most efficient configuration and design. They claim that amount of data traffic can not be known in advance. Furthermore, they use stochastic and robust optimization techniques in their research.

The rest of the paper organized as follows: Section 2 provides a brief summary of fuzzy mathematical programming afterwards discuss proposed methodology and fuzzy stochastic profit maximization problem of BB. Extensive computational study and definition of new performance measures are given in Section 3. Finally, Section 4 draws conclusions about study.

2 FUZZY STOCHASTIC LINEAR PROGRAMMING MODEL

A fuzzy linear programming (FLP) model can be written as in Eq.(1) by using definition and notations introduced by Zhang *et al.* (2003).

$$\begin{aligned} \text{Maximize:} \quad & <\tilde{c}, x>_F = \sum_{i=1}^{n} \tilde{c}_i x_i, \\ \text{Subject to:} \quad & Ax \leqslant b,\ x \geqslant 0, \end{aligned} \tag{1}$$

where <> operator denoting the scalar multiplication of vectors. $\tilde{c} = (\tilde{c}_1, \tilde{c}_2, \ldots, \tilde{c}_n)$ represents array of fuzzy numbers. A is a matrix with dimensions $m \times n$ and $b \in R^m$ is a vector.

We assume that all fuzzy numbers used in mathematical model have triangular membership function. In other words, any objective function coefficient can be redefined as $\tilde{c}_i = (c_i^L, c_i^M, c_i^U)$. The linear multi-objective programming equivalent of model (1) is given in Eq.(2).

$$\begin{aligned} \text{Maximize:} \quad & (<c^L, x>,\ <c^M, x>,\ <c^U, x>)^T, \\ \text{Subject to:} \quad & Ax \leqslant b,\ x \geqslant 0, \end{aligned} \tag{2}$$

Following is the single objective counterpart of the model (2):

$$\begin{aligned} \text{Maximize} \quad & <w, \tilde{c}, x> = (w_1 < c^L, x > + \\ & w_2 < c^M, x > + w_3 < c^U, x >), \\ \text{Subject to:} \quad & Ax \leqslant b,\ x \geqslant 0, \end{aligned} \tag{3}$$

In model (3), w is defined as $w = (w_1, w_2, w_3) \geq 0$. Model turns into parametric linear programming problem, if w is chosen as $w_1 + w_2 + w_3 = 1$ and set to $w_3 - w_1 = \varepsilon$. Here, ε, has to be altered between -1 and 1 (Zhang *et al*, 2003).

All of the sets, parameters and decision variables used in mathematical formulation are presented in Table 1. The objective function of volume based pricing policy with fuzzy coefficients is presented in Eq. (4). Fuzzy revenues and fuzzy opportunity costs depend on scenarios therefore, the expected values for these terms have to be calculated. The fuzzy expected costs/revenues are obtained after mentioned terms are multiplied by realization probability of each scenarios p_s and summed all over possible scenario set. The problem of BB is to maximize fuzzy expected profit under volume based pricing scheme. New model is a fuzzy stochastic linear programming model (FSLP) with fuzzy objective parameters.

$$\begin{aligned} \text{Maximize} \quad & \underbrace{\sum_{s\in\Omega}\sum_{j\in J_s} p_s \tilde{\vartheta}_{js} y_{js}}_{\text{expected fuzzy revenue}} - \overbrace{\sum_{i\in I} \beta_i \tilde{c}_i}^{\text{fuzzy leasing cost}} \\ & - \underbrace{\sum_{s\in\Omega}\sum_{j\in J_s} p_s \tilde{\upsilon}_{js}(1 - y_{js})}_{\text{expexcted fuzzy oppurtunity cost}} \end{aligned} \tag{4}$$

Eq.(5) explains how function Ξ is evaluated in terms of problem parameters.

$$\begin{aligned} \Xi\left(w_1, \{\vartheta^L, c^L, \upsilon^L\}\right) = w_1 \Bigg(& \sum_{s\in\Omega}\sum_{j\in J_s} p_s \vartheta_{js}^L y_{js} - \\ & \sum_{i\in I} \beta_i c_i^L - \sum_{s\in\Omega}\sum_{j\in J_s} p_s \upsilon_{js}^L (1 - y_{js}) \Bigg) \end{aligned} \tag{5}$$

After applying the methodology suggested by Zhang *et al.*(2003) on objective function of BB in Eq. (4), transformed linear objective equation is showed in Eq.(6). Where unit revenues $\left(\tilde{\vartheta}_{js} = (\vartheta_{js}^L, \vartheta_{js}^M, \vartheta_{js}^U)\right)$ and unit costs $\left(\tilde{\upsilon}_{js} = (\upsilon_{js}^L, \upsilon_{js}^M, \upsilon_{js}^U\right)$ and $\tilde{c}_i = (c_i^L, c_i^M, c_i^U))$ are modeled as triangular fuzzy number (TFN). In computational experiments, different w values are selected and run to provide useful statics to decision makers of BB.

$$\begin{aligned} & \text{Maximize}\ \Xi\left(w_1, \{\vartheta^L, c^L, \upsilon^L\}\right) + \\ & \Xi\left(w_2, \{\vartheta^M, c^M, \upsilon^M\}\right) + \Xi\left(w_3, \{\vartheta^U, c^U, \upsilon^U\}\right) \end{aligned} \tag{6}$$

Eqs.(7-16) presents FSLP model constraints. Constraint set (7) guarantees that it is not possible to allocate total customer demand that is more than the purchased capacity from corresponding BP under each scenario. Total capacity allocated to a particular backbone has to smaller than or equal to the acquired bandwidth from corresponding supplier minus capacity losses. Eq.(8) ensures that amount of bandwidth purchased from any

Table 1: Mathematical model notations.

Index Sets	
I	Set of telecommunications BPs
Ω	Set of scenarios that may occur
J_s	Set of end-users under scenario $s (s \in \Omega)$
Parameters	
d_{js}:	Amount of bandwidth request for j.end-user under scenario s
$\tilde{\vartheta}_{js}$:	Fuzzy revenue earned by meeting j.end-user's demand under scenario s
$\tilde{\upsilon}_{js}$:	Fuzzy opportunity cost for not meeting j.end-user's demand under scenario s
p_s:	Realization probability of scenario s
α_i:	Capacity loss ratio for BP i ($\alpha_i \in [0,1]$)
$\tilde{c}_i$:	Fuzzy unit bandwidth cost for leasing capacity from BP i
U_i:	Maximum amount of bandwidth that can be leased from BP i
Φ_i, Ψ_i:	Random variables to denote delay and jitter distribution of BP i, respectively
$\mu_i^\delta, \sigma_i^\delta$:	Mean and standard deviation values corresponding to delay random variable of BP i, respectively
$\mu_i^\rho, \sigma_i^\rho$:	Mean and standard deviation values corresponding to jitter random variable of BP i, respectively
$\Theta_{js}^\delta, \Theta_{js}^\rho$:	Minimum required service level probabilities for delay and jitter measures in order to fulfill j.end-user demand under scenario s, respectively
δ_{js}, ρ_{js}:	Maximum tolerable delay and jitter amounts for end-user j under scenario s, respectively
π:	Minimum required demand fulfillment ratio
$\Pi_{\vartheta_{js}}, \Pi_{\upsilon_{js}}$	Membership functions for revenue and opportunity, respectively
Π_{c_i}:	Membership functions for leasing costs
$\Xi(\cdot,\{\cdot\})$:	function to evaluate objective value
Decision variables	
β_i:	Amount of bandwidth leased from BP i
y_{ijs}:	The Proportion of bandwidth demand of j.end-user allocated into BP i under scenario s
y_{js}:	Total proportion of bandwidth demand of j.end-user meet under scenario s

BP can not be more than BPs' capacity that is sold at the market.

$$\sum_{j \in J_s} y_{ijs} d_{js} \leqslant (1-\alpha_i)\beta_i, \ \forall i \in I \ \forall s \in \Omega \quad (7)$$

$$\beta_i \leqslant U_i, \ \forall i \in I \quad (8)$$

Eq.(9) guarantees that bandwidth request of any end-user under any scenario can only be assigned to a purchased capacity that satisfies delay requirement of user above some predefined probability.

$$\mathbb{P}\{y_{ijs}(\Phi_i - \delta_{js}) \leqslant 0\} \geqslant \Theta_{js}^\delta, \forall s \in \Omega, \forall i \in I, \forall j \in J_s \quad (9)$$

Similarly, Eq.(10) ensures that bandwidth demand of any end-user under any scenario can only be allocated to a particular purchased capacity that meets jitter requirement of customer above predefined probability.

$$\mathbb{P}\{y_{ijs}(\Psi_i - \rho_{js}) \leqslant 0\} \geqslant \Theta_{js}^\rho, \forall s \in \Omega, \forall i \in I, \forall j \in J_s \quad (10)$$

It is generally assumed that QoS parameters are distributed normally. Based on this assumption, deterministic counter parts of Eqs.(9) and (10) can be rewritten as in Eqs.(11) and (12), respectively. $\Gamma_{\Theta_{js}^\delta}$ and $\Gamma_{\Theta_{js}^\rho}$ denote quartile function values for standard normal distribution.

$$y_{ijs}\left(\frac{\delta_{js} - \mu_i^\delta}{\sigma_i^\delta} - \Gamma_{\Theta_{js}^\delta}\right) \geqslant 0 \ \forall s \in \Omega, \forall j \in J_s, \forall i \in I \quad (11)$$

Both inequalities are in linear form, so FSLP of BB can be solved via traditional linear programming texhniques such as simplex algorithm.

$$y_{ijs}\left(\frac{\rho_{js} - \mu_i^\rho}{\sigma_i^\rho} - \Gamma_{\Theta_{js}^\rho}\right) \geqslant 0 \ \forall s \in \Omega, \forall j \in J_s, \forall i \in I \quad (12)$$

Eq.(13) is used for ensuring the minimum demand satisfaction level met under each scenario considered. Eq.(14) ensures that total amount of allocated (satisfied) bandwidth portion of each end-user can not be more than their requests under all scenarios. The constraint set (15) guarantees that total amount of satisfied bandwidth demand of a customer has to be equal to sum of allocated bandwidth portions of that demand into BPs.

$$\sum_{j \in J_s} y_{js} d_{js} - \pi \sum_{j \in J_s} d_{js} \geqslant 0, \forall s \in \Omega \quad (13)$$

$$\sum_{i \in I} y_{ijs} \leqslant 1, \ \forall s \in \Omega, \forall j \in J_s \quad (14)$$

$$y_{js} = \sum_{i \in I} y_{ijs}, \ \forall s \in \Omega, \ \forall j \in J_s \quad (15)$$

$$\beta_i, y_{ijs}, y_{js} \geqslant 0, \forall s \in \Omega, \forall j \in J_s, \forall i \in I \quad (16)$$

3 Computational Experiments

Three different types of problem setting with varying number of BPs ($|I|$), end-users ($|J|$) and scenarios ($|S|$) are chosen for numerical experimentations. Five different and independent problem instances are generated for each type of problem setting. For example, problem set I15J50S10 in Table 2 indicates a problem instance in which there exists 15 BPs, 50 end-users at telecom market and there are total of 10 possible future bandwidth demand scenarios that BB may encounter.

All of the generated problem sets are tested by using different w_1, w_2, w_3 values. Changing values of (w_1, w_2, w_3) are chosen from set $\{(1,0,0),(0,1,0),(0,0,1)\}$ in order to find the most pessimistic, the most expected and the most optimistic values of net profit.

In each test instance and under all scenarios, each end-user's demand d_{js} is generated from normal distribution. In addition, to prevent negative bandwidth request for normal distribution, truncated normal distribution is used. The Realization probabilities of each scenario are generated from uniform distribution between $[0, 1]$ afterwards these probabilities are normalized.

The objective function value of FSLP model discussed model between Eqs.(6)-(16) is denoted as $\tilde{z}^{RP}$. In order to test applicability of proposed model, two new models which are called as WS and EEV have to set up and solved. WS model defines wait-and-see solution of the model and its optimal objective function value is $\tilde{z}^{WS}$. The expected value of perfect information (EVPI) measures the maximum amount that a decision maker is willing to pay in order to know the value of a random variable before making his/her decision (Birge *et al.*, 1997). In this paper, fuzzy EVPI is calculated as following,

$$\widetilde{\text{EVPI}} = \tilde{z}^{WS} \ominus \tilde{z}^{RP}$$

$\ominus$ does not indicate fuzzy subracation operation, due to the fact that decision maker of BB may choose same w values while solving both RP and WS models. The suggested fuzzy EVPI algorithm is presented in Figure 1. Just solving RP and WS models do not provide

for	$s = 1, \ldots, \lvert\Omega\rvert$
	Fix $(w_1, w_2, w_3) = (1,0,0)$
	Solve Eqs. (5-16)
	Save z_s^L
	Fix $(w_1, w_2, w_3) = (0,1,0)$
	Solve Eqs. (5-16)
	Save z_s^M
	Fix $(w_1, w_2, w_3) = (0,0,1)$
	Solve Eqs. (5-16)
	Save z_s^U
end for	
Calculate and Assign	$z_L^{WS} \leftarrow \sum_{s=1}^{\lvert\Omega\rvert} p_s z_s^L$
	$z_M^{WS} \leftarrow \sum_{s=1}^{\lvert\Omega\rvert} p_s z_s^M$
	$z_U^{WS} \leftarrow \sum_{s=1}^{\lvert\Omega\rvert} p_s z_s^U$
	$\tilde{z}^{WS} = (z_L^{WS}, z_M^{WS}, z_U^{WS})$
Calculate and Assign	$\text{EVPI}^L \leftarrow z_L^{WS} - z_L^{RP}$
	$\text{EVPI}^M \leftarrow z_M^{WS} - z_M^{RP}$
	$\text{EVPI}^U \leftarrow z_U^{WS} - z_U^{RP}$

Figure 1: The proposed $\widetilde{\text{EVPI}}$ algorithm.

too much information to decision makers therefore, expected value problem (EEV) also has to be solved. EEV model is obtained by replacing each random variable by their corresponding expected value. The optimal fuzzy objective value of EEV model is denoted as $\tilde{z}^{EEV}$. The difference between $\tilde{z}^{RP}$ and $\tilde{z}^{EEV}$is called as value of stochastic solution(VSS) which is also a fuzzy number. VSS measures the cost of ignoring uncertainty while making a decision (Turan, 2012). Eq. 17 shows how $\widetilde{VSS}$ is calculated.

$$\widetilde{VSS} = \tilde{z}^{RP} \ominus \tilde{z}^{EEV} \tag{17}$$

Here, again $\ominus$ does not indicate fuzzy subtraction operation, due to the fact that decision maker of BB may choose same w values while solving both RP and EEV models. The detailed discussion about fuzzy VSS algorithm can be found in Turan (2012).

Table 2 summarizes results gathered from running fuzzy VSS and fuzzy EVPI algorithms for each problem set on RP, EEV and WS models. All of the results are presented in TFN format. Problem set I15J50S10_3 does not have a feasible solution, which is indicated via ***. As problem size gets larger, $\widetilde{VSS}$ value increases due to the increasing uncertainty added by increasing number of scenarios, end-users and BPs. In addition, $\widetilde{VSS}$ values always greater than zero ($\succ 0$) which implies the fact that solving RP model rather than EEV model leads to more profit in fuzzy sense. In other words, ignoring uncertainty in the bandwidth demand and treating BB's problem as deterministic model result in profit loss. It should be also noted that for all of the test instances solved optimally $\widetilde{VSS} \succ \widetilde{EVPI}$ inequality holds.

Table 2: Fuzzy VSS and EVPI values for each problem set.

Problem Set	$\widetilde{\text{VSS}}$	$\widetilde{\text{EVPI}}$
I15J50S10_0	(134.37, 168.39, 208.35)	(55.72, 59.61, 63.32)
I15J50S10_1	(512.87, 663.86, 814.63)	(164.48, 174.91, 186.66)
I15J50S10_2	((951.67, 1068.25, 1183.52)	(100.94, 118.40, 139.06)
I15J50S10_3	(***, ***, ***)	(520.52, 607.66, 692.04)
I15J50S10_4	(1499.31, 1673.50, 1850.60)	(207.18, 253.38, 294.84)
Average	(776.96, 903.76, 1033.48)	(209.77, 242.79, 275.18)
Problem Set	$\widetilde{\text{VSS}}$	$\widetilde{\text{EVPI}}$
I30J100S50_0	(1394.51, 1755.50, 2116.63)	(276.13, 327.98, 371.44)
I30J100S50_1	(1221.35, 1542.25, 1854.57)	(247.08, 295.03, 344.69)
I30J100S50_2	(927.40, 1190.52, 1452.70)	(420.09, 492.81, 556.95)
I30J100S50_3	(2431.03, 3051.87, 3690.88)	(646.67, 763.28, 872.95)
I30J100S50_4	(1827.09, 2297.66, 2777.53)	(560.96, 674.12, 786.42)
Average	(1560.28, 1967.56, 2378.46)	(430.19, 510.64, 586.49)
Problem Set	$\widetilde{\text{VSS}}$	$\widetilde{\text{EVPI}}$
I50J100S100_0	(1728.44, 2150.82, 2599.03)	(357.88, 433.65, 506.07)
I50J100S100_1	(2243.64, 2834.07, 3421.93)	(469.44, 559.06, 643.20)
I50J100S100_2	(1817.50, 2306.58, 2790.38)	(509.34, 602.77, 694.18)
I50J100S100_3	(1190.82, 1504.82, 1799.91)	(325.80, 395.06, 460.78)
I50J100S100_4	(1996.56, 2496.65, 2997.80)	(217.10, 253.60, 293.26)
Average	(1795.39, 2258.59, 2721.81)	(375.91, 448.83, 519.50)

By using data presented in Table 2, the improvement ratios attained by RP model over EEV model can be calculated. In the same manner, the improvement ratios that can be achieved by knowing the future bandwidth demands and prices may also be calculated from presented data. Besides, decision maker of BB may set up a lower threshold ratio based on his experiences to determine which model to solve (RP or EEV). For mentioned reasons two new statistics are defined as follows:

$$\tilde{\zeta} = \widetilde{VSS} \odot \tilde{z}^{EEV} = \left(\frac{VSS^L}{L_{z}EEV}, \frac{VSS^M}{M_{z}EEV}, \frac{VSS^U}{U_{z}EEV} \right) \tag{18}$$

$$\tilde{\xi} = \widetilde{EVPI} \odot \tilde{z}^{RP} = \left(\frac{EVPI^L}{L_{z}RP}, \frac{EVPI^M}{M_{z}RP}, \frac{EVPI^U}{U_{z}RP} \right) \tag{19}$$

$\tilde{\zeta}$ measures the advantage of fuzzy stochastic approach (RP) over deterministic approach (RP), and $\tilde{\xi}$ measures how much more profit can be obtained by knowing the future bandwidth demands, prices and market conditions in advance. Table 3 presents calculated $\tilde{\zeta}$ and $\tilde{\xi}$ values.

Table 3: Calculated $\tilde{\zeta}$ and $\tilde{\xi}$ values for each problem instances.

Problem Set	ζ	ξ
I15J50S10_0	(0.018, 0.022, 0.026)	(0.007, 0.008, 0.008)
I15J50S10_1	(0.076, 0.098, 0.119)	(0.023, 0.023, 0.024)
I15J50S10_2	(0.142, 0.157, 0.172)	(0.013, 0.015, 0.017)
I15J50S10_3	(****, ****, ****)	(0.069, 0.078, 0.087)
I15J50S10_4	(0.253, 0.284, 0.315)	(0.028, 0.033, 0.038)
Average	(0.115, 0.133, 0.150)	(0.028, 0.032, 0.035)
Problem Set	ζ	ξ
I30J100S50_0	(0.089, 0.109, 0.128)	(0.016, 0.018, 0.020)
I30J100S50_1	(0.087, 0.108, 0.129)	(0.016, 0.019, 0.021)
I30J100S50_2	(0.065, 0.082, 0.098)	(0.027, 0.031, 0.034)
I30J100S50_3	(0.201, 0.259, 0.323)	(0.045, 0.051, 0.058)
I30J100S50_4	(0.134, 0.168, 0.203)	(0.036, 0.042, 0.048)
Average	(0.112, 0.140, 0.168)	(0.028, 0.032, 0.035)
Problem Set	ζ	ξ
I50J100S100_0	(0.125, 0.155, 0.186)	(0.023, 0.027, 0.030)
I50J100S100_1	(0.160, 0.200, 0.239)	(0.029, 0.033, 0.036)
I50J100S100_2	(0.123, 0.153, 0.181)	(0.031, 0.035, 0.038)
I50J100S100_3	(0.076, 0.092, 0.107)	(0.019, 0.022, 0.025)
I50J100S100_4	(0.133, 0.163, 0.191)	(0.013, 0.014, 0.016)
Average	(0.122, 0.151, 0.178)	(0.023, 0.026, 0.029)

One of the most important results is the statistic $\tilde{\zeta}$ is always positive and bigger than the $\tilde{\xi}$ value, which indicates superiority of RP (fuzzy stochastic) modeling over EEV. The close investigation of Table 3 reveals that averages of mentioned statistics gets larger as increasing problem size in other words increasing uncertainty in both market prices and demand is correlated with $\tilde{\zeta}$ value.

Another important issue for decision makers is to test the stability of solution methodology under dynamic settings such as changing fuzzy revenues and costs. In this direction, the largest problem set I50J100S100 is tested under varying fuzzy objective parameters.

As a first part of sensitivity analysis on fuzzy objective parameters, fuzzy unit revenues are altered between (ϑ_{js}) ±10% from their original values. RP, WS and EEV models are run under these new settings and main performance indicators ($\tilde{\zeta}$ and $\tilde{\xi}$) are evaluated and compared to original cases.

Figure 2 and 3 depict results of sensitivity runs on revenue coefficients. The increasing revenues lead to

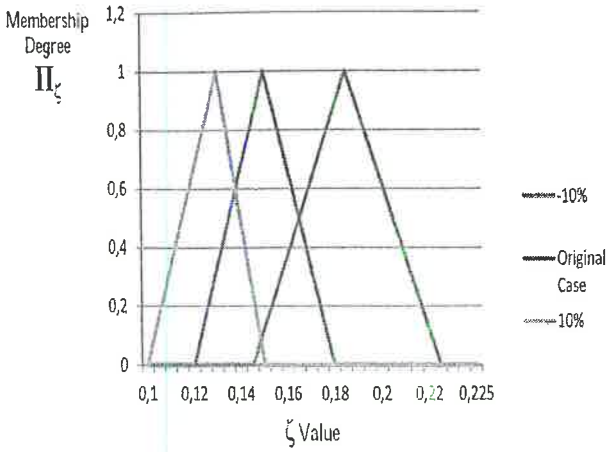

Figure 2: Effect of change in fuzzy unit revenues by ±10% on $\tilde{\zeta}$.

decrease in $\tilde{\zeta}$ values proving that randomness should be taken into account due to the fact that very high competition in telecom market and limited profit margins. The fuzzy unit cost namely leasing and opportunity costs (υ_{js}, c_i) are changed between ±10% from their original settings simultaneously in order to conduct sensitivity analysis on objective parameters. Figure 4 and 5 depict results of sensitivity runs on cost coefficients.

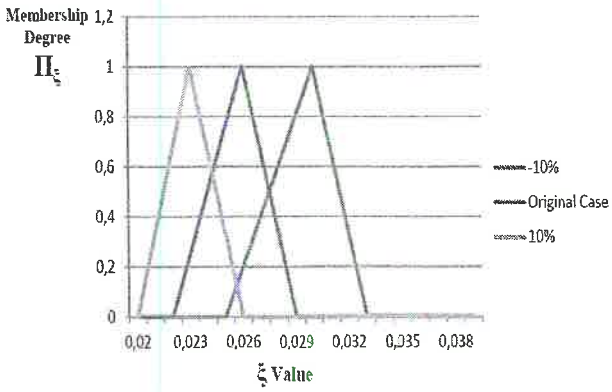

Figure 3: Effect of change in fuzzy unit revenues by ±10% on $\tilde{\xi}$.

The rising costs cause $\tilde{\zeta}$ to rise as well, which verifies that RP model better to be chosen and used by BB under pessimistic market conditions.

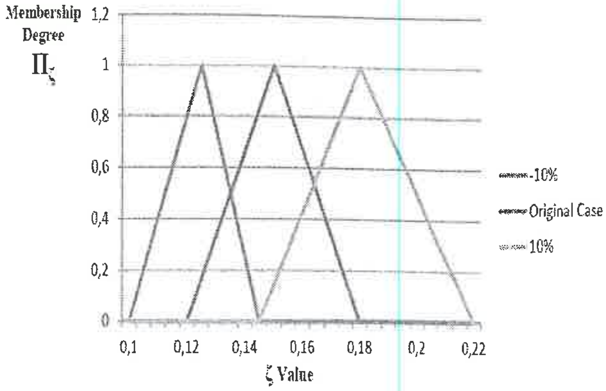

Figure 4: Effect of change in fuzzy unit costs by $\pm 10\%$ on $\tilde{\zeta}$.

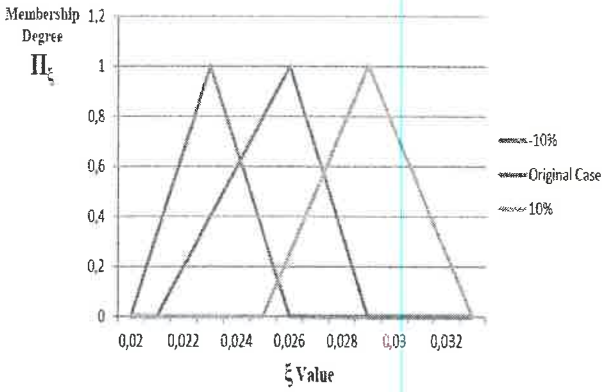

Figure 5: Effect of change in fuzzy unit costs by $\pm 10\%$ on $\tilde{\xi}$.

4 Conclusions

In this research, an optimization problem of BB is analyzed when acquiring and allocating bandwidth from a market in which demands of end-users, prices and QoS levels are not know in advance. In order to handle randomness and vagueness, an integration of stochastic and fuzzy linear programming techniques are employed. Extensive computational study on randomly generated test instances showed that proposed methodology provides more than 10% more profit even in worst case scenarios. Moreover, it is also concluded that increasing problem size makes suggested approach more compelling than deterministic methodology for BB's decision makers. Finally, carried out sensitivity analysis on fuzzy objective parameters demonstrated the robustness of procedure.

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