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Letter

Observational aspects of symmetries of the neutral B meson system

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Abstract We revisit various results, which have been obtained by the BABAR and Belle Collaborations over the last 13 years, concerning symmetry properties of the Hamiltonian, which governs the time evolution and the decay of neutral B mesons. We find that those measurements, which established CP-violation in B meson decay, 13 years ago, had as well established T (time-reversal) symmetry violation. They also confirmed CPT symmetry in the decay ($T_{\mathrm{CPT}} = 0$) and symmetry with respect to time-reversal ($\epsilon = 0$) and to CPT ($\delta = 0$) in the $B^0\bar{B}^0$ oscillation.

1 Introduction

A system of neutral mesons such as B^0, $\bar{B}^0$ or K^0, $\bar{K}^0$ is a privileged laboratory for the study of weak-interaction's symmetries. Even though the phenomenological framework has been well understood since a long time [1–4], recent discussions in the physics community [5,6] show that it may be useful to revisit a few points, in order to fully (and correctly) exploit the experimental results. This process is then at the origin of the present note.

We focus on the $B^0\bar{B}^0$ system, and refer to experimental results [7–11] that have been achieved by measurements of the decay products of $B^0\bar{B}^0$ pairs created in the entangled antisymmetric state

$$|\Psi\rangle = (|B^0\rangle|\bar{B}^0\rangle - |\bar{B}^0\rangle|B^0\rangle)/\sqrt{2} \tag{1}$$

where the first B in this notation moves in the direction $\mathbf{p}$ and the second one in the direction $-\mathbf{p}$.

The Weisskopf–Wigner approximation [1] allows one to calculate the time evolution of some neutral, two-dimensional B-meson state as $|B(t)\rangle = \mathrm{e}^{-i\Lambda t}|B(0)\rangle$, and the amplitude $A_{Bf}(t)$ for its subsequent decay into a state f as

$$A_{Bf}(t) = \langle f|T\mathrm{e}^{-i\Lambda t}|B\rangle. \tag{2}$$

T and Λ are represented by the constant, complex 2×2 matrices $T = (T^{ij}) = \langle f^i|T|B^j\rangle$ and $\Lambda = (\Lambda^{ij}) = \langle B^i|\Lambda|B^j\rangle$, $i, j = 1(2)$. We consider experiments with final states $f^i = J/\psi K^i$ or $f^i = l^i\nu(\bar{\nu})X$. Here $B^{1(2)}$, $K^{1(2)}$ and $l^{1(2)}$ stand for the flavor eigenstates B^0 ($\bar{B}^0$), K^0 ($\bar{K}^0$) and $\mu^+(\mu^-)$ or $e^+(e^-)$, respectively.

We recall that a symmetry is a property of the hermitian Hamiltonian ($H = H_0 + H_{\mathrm{weak}}$) of the Schrödinger equation which is defined in a space sufficiently complete to include all the particle states under consideration, also the decay products [1]. Thus the aim of the experiments is to establish properties of the weak interaction Hamiltonian H_{weak} by measuring observable combinations of the elements of Λ and of T, which represent these properties.

As CP-violation implies T and/or CPT-violation, we specifically consider the classical aim posed by the discoverers of T-violation [12] "to express quantitatively the fraction of the observed CP-violation due to T-violation and CPT-violation separately".

In passing, we show that a more recent treatment, which attempts to define T-symmetry violation without reference to the weak interaction Hamiltonian [13], is a special case within our phenomenology.

2 Observables of symmetries

Together with a parametrization of the matrices Λ and T, Eqs. (1) and (2) are a sufficient basis for the description of the symmetry properties of the experimental results [7–11]. Symmetry properties of the Hamiltonian often manifest themselves in an especially simple and direct way in relations between measured quantities. Here, Table 1 gives a summary, with

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Table 1 A symmetry S of H_{weak} implies vanishing values among the observables Λ_T, Λ_{CPT}, T_T, T_{CPT}. Decay modes are assumed to proceed through one single amplitude

S	Requires for Λ matrix	Requires for T matrix
T	$\Lambda_T \equiv \|\Lambda^{21}\|^2 - \|\Lambda^{12}\|^2 = 0$	$T_T \equiv \text{Im}(T^{11*}T^{22}) = 0$
CPT	$\Lambda_{\text{CPT}} \equiv \Lambda^{22} - \Lambda^{11} = 0$	$T_{\text{CPT}} \equiv \|T^{11}\|^2 - \|T^{22}\|^2 = 0$
CP	$\Lambda_T = 0$ and $\Lambda_{\text{CPT}} = 0$	$T_T = 0$ and $T_{\text{CPT}} = 0$

definitions and derivations as found in [1–4], and the phase conventions of [2]. Our approach is analogous to [14].

Let us pose

$$\Lambda^{11} = m - i\gamma/2 - \delta\Delta m, \quad \Lambda^{22} = m - i\gamma/2 + \delta\,\Delta m, \tag{3}$$

$$\Lambda^{12} = (1-2\epsilon)\Delta m/2, \quad \Lambda^{21} = (1+2\epsilon)\,\Delta m/2 \tag{4}$$

with real m, γ, Δm, ϵ, and complex δ. For the observables of the symmetry violations in the matrix Λ, i.e. in the $B^0\bar{B}^0$ oscillation, we deduce from Eqs. (3), (4), and Table 1

$$\Lambda_T = 2\epsilon(\Delta m)^2 + \mathcal{O}(\epsilon^2), \tag{5}$$

$$\Lambda_{\text{CPT}} = 2\delta\Delta m. \tag{6}$$

We note that, with Eqs. (3), (4), and (5), the difference of the widths of the eigenstates of Λ becomes $\Delta\Gamma = 2\ \text{Im}(\Delta m\sqrt{1-4\epsilon^2+4\delta^2})$. This lets us recognize that, if $\Delta\Gamma = 0$, our matrix Λ may still allow for a finite ϵ ($|\epsilon| < 1/2$), in accordance with [15]. This is in contrast to widely repeated affirmations [16] that $\Delta\Gamma = 0$ would imply time-reversal symmetry of Λ, i.e. $\epsilon = \Lambda_T = 0$. On the other hand, the chosen reality of the common factor Δm in $\Lambda^{12}$and Λ^{21}anticipates $\Delta\Gamma \approx 0$ for small symmetry violations $\epsilon^2 \ll 1, |\ \delta\ |^2 \ll 1$. The reality of ϵ is due to the $B^0\bar{B}^0$ phase convention used.

In terms of $\Lambda = M - \frac{i}{2}\Gamma$ ($M = M^\dagger, \Gamma = \Gamma^\dagger$), $\Lambda^{12} = |\ M^{12}\ |\ \mathrm{e}^{i\phi_M} - \frac{i}{2}\ |\ \Gamma^{12}\ |\ \mathrm{e}^{i\phi_\Gamma}$, the relation to Eqs. (3) to (5) is given by $\epsilon = -\frac{1}{4}\ |\ \Gamma^{12}\ |\ /\ |\ M^{12}\ | \times \sin(\phi_\Gamma)$, $\Delta m = 2\ |\ M^{12}\ |$, $\phi_M = 0$ and $\Delta\Gamma \approx -2\ |\ \Gamma^{12}\ |\cos(\phi_\Gamma)$. We admit $|\ \Gamma^{12}\ | \ll |\ M^{12}\ |$.

In order to calculate the amplitude $A_{Bf}(t)$ in Eq. (2), we need to evaluate the exponential in terms of Λ. We do this by summing up the power series (as explained in [14]). Let $U = (U^{ij}) = \mathrm{e}^{-i\Lambda t}$ and find

$$U^{11} = U_0(\cos(\omega t) + i\ 2\delta\sin(\omega t)),$$

$$U^{22} = U_0(\cos(\omega t) - i\ 2\delta\sin(\omega t)), \tag{7}$$

$$U^{12} = U_0(-i\ (1-2\epsilon)\sin(\omega t)),$$

$$U^{21} = U_0(-i\ (1+2\epsilon)\sin(\omega t)), \tag{8}$$

$$|\ U_0\ |^2 = \mathrm{e}^{-\gamma t},$$

$$\omega = \Delta m/2 + \mathcal{O}(|\ \delta\ |^2, \epsilon^2). \tag{9}$$

For the matrix $(T^{ij}) = (\langle J/\psi K^i|T|B^j\rangle)$, we assume

$$T^{12} = T^{21} = 0, \tag{10}$$

corresponding to the "$\Delta B = \Delta S$ rule". From Table 1 and with the (arbitrary) normalization $|\ T^{11}\ |^2 + |\ T^{22}\ |^2 = 2$ we deduce the useful identity among the (diagonal) elements of T,

$$T_T^2 + T_{\text{CPT}}^2/4 + (Re(T^{11*}T^{22}))^2$$
$$\equiv (|\ T^{11}\ |^2 + |\ T^{22}\ |^2)^2/4 = 1. \tag{11}$$

Results based on Eqs. (1) to (11) will turn out to be sensitive to all the four asymmetry parameters in Table 1.

Throughout this work, we assume that all decay modes discussed in this paper proceed through a single amplitude. Two interfering amplitudes may fake non-vanishing values of T_{CPT} or T_T, depending on their weak and strong phases, without the presence of the corresponding symmetry violations in the Hamiltonian.

3 Experiments

3.1 General description

Call $\mathcal{A}_{f_1,f_2}(t)$ the amplitude for the decay of an entangled, antisymmetric $B^0\bar{B}^0$ pair into a final state with the two observed particles f_1 (at time t_0) and f_2 (at a later time $t > t_0$). With specific choices of the two final states f_1, f_2, we can uniquely represent the complete set of results of the CP-, T-, and CPT-symmetry violation studies listed in Table 2 and performed by [7–9] through [11], by making use of Eq. (12) below [2,17], whose derivation we sketch here. We note with [14, section 2.7], that the time evolution acts on the two-particle state $|\Psi\rangle$ of Eq. (1) solely by a multiplicative factor, which is independent of the symmetry violations under consideration, and which does not influence the decay properties of $|\Psi\rangle$. We may thus, without loss of generality, arbitrarily choose $t_0 = 0$, $t > 0$, and apply Eq. (2) to the single-particle components in $|\Psi\rangle$, to obtain

$$\mathcal{A}_{f_1,f_2}(t) = (\ \langle f_1|T|B^0\rangle\langle f_2|T\mathrm{e}^{-i\Lambda t}|\bar{B}^0\rangle$$
$$-\langle f_1|T|\bar{B}^0\rangle\langle f_2|\ T\mathrm{e}^{-i\Lambda t}|B^0\rangle\ )/\sqrt{2}. \tag{12}$$

In rewriting (12), we can explicitly derive the formula for the state $|S_{f_1}\rangle$, which survived the decay to f_1, and its (single particle) time evolution and decay to f_2 as

$$\mathcal{A}_{f_1,f_2}(t) \equiv \mathcal{A}_{S_{f_1},f_2}(t) = \langle f_2|\ T\mathrm{e}^{-i\Lambda t}|S_{f_1}\rangle \tag{13}$$

with

$$|S_{f_1}\rangle = b|B^0\rangle + \bar{b}|\bar{B}^0\rangle$$
$$b = -\langle f_1|T|\bar{B}^0\rangle/\sqrt{2}$$
$$\bar{b} = \quad \langle f_1|T|B^0\rangle/\sqrt{2}. \tag{14}$$

Table 2 The measurements, classified according to Eq. (12). General expressions for the expected decay time distributions in terms of $T_{\rm CPT}, T_T, \epsilon, \delta$. In the limit $\epsilon = \delta = 0$, they are all of the form $(1 \pm \frac{1}{2} T_{\rm CPT} \cos(\Delta m\, t) \pm T_T \sin(\Delta m\, t)) e^{-\gamma t}$. l^- is a shorthand for $\mu^- \bar{\nu}_\mu X$ or $e^- \bar{\nu}_e X$, l^+ for $\mu^+ \nu_\mu X$, etc. By the "$\Delta B = \Delta Q$ rule", a $B^0(\bar{B}^0)$ decays semileptonically always into $l^+ + \cdots$ $(l^- + \cdots)$. $|K_{S(L)}\rangle = (|K^0\rangle \pm |\bar{K^0}\rangle)/\sqrt{2}$ has been used. All 10 measurements have been performed

Name of measurement	First decay f_1	Second decay f_2	$\mid \mathcal{A}_{f_1,f_2}(t) \mid^2 \propto a + b \cos(\Delta m\, t) + c \sin(\Delta m\, t)$		
			a	b	c
$B^0 \to K_S^0$ {1}	l^-	$J/\psi K_S^0$	$1 + p_1$	$+\frac{1}{2} T_{\rm CPT} - p_1$	$+T_T + q_1$
$\bar{B}^0 \to K_S^0$ {2}	l^+	$J/\psi K_S^0$	$1 + p_2$	$-\frac{1}{2} T_{\rm CPT} - p_2$	$-T_T + q_2$
$K_L^0 \to \bar{B}^0$ {3}	$J/\psi K_S^0$	l^-	$1 + p_1$	$+\frac{1}{2} T_{\rm CPT} - p_1$	$-T_T - q_1$
$K_L^0 \to B^0$ {4}	$J/\psi K_S^0$	l^+	$1 + p_2$	$-\frac{1}{2} T_{\rm CPT} - p_2$	$+T_T - q_2$
$B^0 \to K_L^0$ {5}	l^-	$J/\psi K_L^0$	$1 + p_5$	$+\frac{1}{2} T_{\rm CPT} - p_5$	$-T_T + q_5$
$\bar{B}^0 \to K_L^0$ {6}	l^+	$J/\psi K_L^0$	$1 + p_6$	$-\frac{1}{2} T_{\rm CPT} - p_6$	$+T_T + q_6$
$K_S^0 \to \bar{B}^0$ {7}	$J/\psi K_L^0$	l^-	$1 + p_5$	$+\frac{1}{2} T_{\rm CPT} - p_5$	$+T_T - q_5$
$K_S^0 \to B^0${8}	$J/\psi K_L^0$	l^+	$1 + p_6$	$-\frac{1}{2} T_{\rm CPT} - p_6$	$-T_T - q_6$
$\bar{B}^0 \to B^0$ {9}	l^+	l^+	$\frac{1}{2}(1 - 4\epsilon)$	$-\frac{1}{2}(1 - 4\epsilon)$	0
$B^0 \to \bar{B}^0$ {10}	l^-	l^-	$\frac{1}{2}(1 + 4\epsilon)$	$-\frac{1}{2}(1 + 4\epsilon)$	0

The terms with ϵ and δ (upper signs for p_1, p_5, q_1, q_5).

$p_1(p_2) = \epsilon\,(\pm 2 - T_{\rm CPT}) \mp 2Re(\delta) \cdot Re(T^{11*}T^{22}) - 2{\rm Im}(\delta) T_T$

$p_5(p_6) = \epsilon\,(\pm 2 - T_{\rm CPT}) \pm 2Re(\delta) \cdot Re(T^{11*}T^{22}) + 2{\rm Im}(\delta) T_T$

$q_1(q_2) = \epsilon \cdot 2\, T_T - {\rm Im}(\delta)(\pm 2 + T_{\rm CPT})$

$q_5(q_6) = -\epsilon \cdot 2\, T_T - {\rm Im}(\delta)(\pm 2 + T_{\rm CPT})$

Identity: $q_1 + q_6 - (q_2 + q_5) = 0$

The variety of expected decay time distributions $\mid \mathcal{A}_{f_1,f_2}(t) \mid^2$ is displayed in Table 2. We find that the parameters of the data analysis are the T and CPT violation parameters of the T matrix, T_T and $T_{\rm CPT}$, concerning the decay, and those, p_i, q_i ($i = 1, 2, 5, 6$), concerning mainly the $B^0\bar{B}^0$ oscillation matrix Λ. In the limit of CP symmetry of Λ the p_i, q_i all vanish. Then T_T and $T_{\rm CPT}$ are exactly associated each with its own proper time dependence: T_T with $\pm\sin(\Delta m t)$, $T_{\rm CPT}$ with $\pm\cos(\Delta m t)$. Table 2 also allows one to read off the relations of the measured distributions to the symmetry violating parameters of Λ and T, as demonstrated below, and also to construct combinations of data which are true signatures for specific violations.

3.2 The earlier results

The experiments [7–10] have measured in 2001/2 all the data sets listed in Table 2, and thereby discovered CP-violation in the matrix T. We show now that these data furthermore establish time-reversal symmetry violation in $H_{\rm weak}$ and are compatible as well with CPT-symmetry of the T matrix as with $\epsilon = 0$, $\delta = 0$, i.e. we have CP symmetry of Λ. To this purpose we consult Table 2 and calculate

$$\begin{aligned}\{1\} - \{2\} = (p_1 - p_2) \\ + (T_{\rm CPT} - (p_1 - p_2)) \cos(\Delta m t) \\ + (2T_T + (q_1 - q_2)) \sin(\Delta m t).\end{aligned}$$

Similarly, we calculate {5}–{6} and summarize the results as follows:

$$\begin{aligned}{\rm CP}_{S(L)} &\equiv \mid \mathcal{A}_{l^-, J/\psi K^0_{S(L)}}(t) \mid^2 - \mid \mathcal{A}_{l^+, J/\psi K^0_{S(L)}}(t) \mid^2 \\ &\propto 4\epsilon \mp 4Re(\delta) \cdot Re(T^{11*}T^{22}) \\ &\quad + \{T_{\rm CPT} - 4\epsilon \pm 4Re(\delta) \\ &\quad \cdot Re(T^{11*}T^{22})\} \cos(\Delta m t) \\ &\quad + \{\pm 2T_T - 4{\rm Im}(\delta)\} \sin(\Delta m t). \qquad (15)\end{aligned}$$

The experimental results for ${\rm CP}_S$ and ${\rm CP}_L$ show no time independent terms, $4\epsilon \mp 4Re(\delta) \cdot Re(T^{11*}T^{22}) \approx 0$, and no $\cos(\Delta m\, t)$ signals, $\{T_{\rm CPT} - 4\epsilon \pm 4Re(\delta) \cdot Re(T^{11*}T^{22})\} \approx 0$. From this we conclude $\epsilon \approx 0$, $4Re(\delta) \cdot Re(T^{11*}T^{22}) \approx 0$, and $T_{\rm CPT} \approx 0$. The $\sin(\Delta m\, t)$ amplitudes are equal but with opposite signs, and, in absolute value, <2, implying ${\rm Im}(\delta) \approx 0$ and $\mid T_T \mid^2 < 1$. From (11) now follows $Re(T^{11*}T^{22}) \neq 0$ and thus $Re(\delta) \approx 0$. The p_i and q_i defined in Table 2 are thus all compatible with zero.

Quantitative results for T_T and $T_{\rm CPT}$ may be read off from [7–9], who analyze their data also with two free parameters [7,9], corresponding to T_T and $T_{\rm CPT}$.

The experiment [10] has set a stringent limit on T-symmetry violation in the Λ matrix of the $B^0\bar{B}^0$ system with a direct measurement of ϵ. See Table 2 (entries {9} and {10}) and Table 3. The method is analogous to the one of the CPLEAR experiment [18,19] for the $K^0\bar{K}^0$ system, where also a signature for T-violation ("Kabir asymme-

Table 3 A selection of expectations for the experiment of Ref. [11]. Due to the presence of $T_{\rm CPT}$, of the p_i and q_i, our results contradict the attempt [13,21] to define the differences $\{2a\}$ to $\{2d\}$, each as a signature for T-violation. In the lower part, signatures for T- and CPT-symmetry violations are indicated

Display in [11]	Rates compared	Expected $\propto a + b\cos(\Delta m\, t) + c\sin(\Delta m\, t)$		
		a	b	c
Figure 2a	$\{2\} - \{7\} \equiv \{2a\}$	$p_2 - p_5$	$-\,T_{\rm CPT} - (p_2 - p_5)$	$-2\,T_T \; + q_2 + q_5$
Figure 2b	$\{4\} - \{5\} \equiv \{2b\}$	$p_2 - p_5$	$-\,T_{\rm CPT} - (p_2 - p_5)$	$+2\,T_T \; - q_2 - q_5$
Figure 2c	$\{6\} - \{3\} \equiv \{2c\}$	$p_6 - p_1$	$-\,T_{\rm CPT} - (p_6 - p_1)$	$+2\,T_T \; + q_1 + q_6$
Figure 2d	$\{8\} - \{1\} \equiv \{2d\}$	$p_6 - p_1$	$-\,T_{\rm CPT} - (p_6 - p_1)$	$-2\,T_T \; - q_1 - q_6$
Signatures are	For T_T	$-8\,T_T\sin(\Delta m\, t)$	$\propto\ \{2a\} - \{2b\} - \{2c\} + \{2d\}$	
	For $T_{\rm CPT}$	$-4\,T_{\rm CPT}$	$\propto\ \{2a\} + \{2b\} + \{2c\} + \{2d\} \quad (t = 0)$	
	For Λ_T	4ϵ	$\approx (\{10\} - \{9\})/(\{10\} + \{9\})$	

try") has been directly measured. The experiments make use of the general identity, valid in two dimensions (see [14]), $\Lambda^{21}/\Lambda^{12} \equiv (\mathrm{e}^{-i\Lambda t})^{21}(\mathrm{e}^{-i\Lambda t})^{12} = U^{21}/U^{12}$ from which

$$\epsilon \approx \frac{1}{4}\frac{|\,\Lambda^{21}\,|^2 - |\,\Lambda^{12}\,|^2}{|\,\Lambda^{21}\,|^2 + |\,\Lambda^{12}\,|^2} \equiv \frac{1}{4}\frac{|\,U^{21}\,|^2 - |\,U^{12}\,|^2}{|\,U^{21}\,|^2 + |\,U^{12}\,|^2} = \frac{1}{4}\frac{|\,\mathcal{A}_{l^-l^-}\,|^2 - |\,\mathcal{A}_{l^+l^+}\,|^2}{|\,\mathcal{A}_{l^-l^-}\,|^2 + |\,\mathcal{A}_{l^+l^+}\,|^2}\,, \qquad (16)$$

the connection from the data to the T-symmetry violation signal, ϵ, follows—without any assumptions on CPT-symmetry or on the value of $\Delta\Gamma$ of the Λ matrix.

A reanalysis of the results in 2007 of the BABAR and Belle Collaborations by [20] has shown that the data contradict *motion-reversal symmetry* (see [5,6]) in the $B^0\bar{B}^0$ system.

In summary, the discovered CP-violation in the $B^0\bar{B}^0$ system is T-symmetry violation in the decay-amplitude matrix T, $T_T \neq 0$ with $T_{\rm CPT} \approx 0$.

In the $K^0\bar{K}^0$ system, however, the CP-violation is T-symmetry violation in oscillations, $\Lambda_T \neq 0$ with $\Lambda_{\rm CPT} \approx 0$.

3.3 Recent results

The analysis by [11] is based on [13] with novel notions of CPT-, CP-, and T-symmetry, which, in contrast to the classical definitions [1], are not related to properties of the weak interaction Hamiltonian, but to comparisons of surviving states $|S_{f_1}\rangle$ with suitably motion-reversal transformed ones of type $|S_{f_1'}\rangle$. The novel definitions are less general than the classical ones as they need the assumption of $T_{\rm CPT} = 0$. This new analysis then becomes a special case of our present work, and in turn loses the possibility to address the "classical aim", mentioned in our Sect. 1 (details below).

To prove that the phenomenology of [13] uses $T_{\rm CPT} = 0$, it is sufficient to express their eq. (A.5 of [13]) in terms of the elements of the matrix T, T^{11} and T^{22}, to find

$$\alpha\beta^* = -1 = -\,|\,T^{11}\,|^2\,/\,|\,T^{22}\,|^2 \quad \text{or} \quad T_{\rm CPT} = 0.$$

The work of [13] specifies three sets of four pairs of measurements, whose comparisons are supposed to indicate the violations of the three symmetries mentioned above. (See Tables 1, 2, 3 of [13].) Each of the 24 measurements is completely determined by the products of the first and the second decay of the antisymmetric, entangled $B^0\bar{B}^0$ pair. Their amplitudes are thus uniquely given by our Eq. (12). The corresponding rates are listed in our Table 2, labeled $\{1\}$ to $\{8\}$.

The envisaged T-violating comparisons, labeled $\{2a\}$ to $\{2d\}$ in Table 3, depend also on $T_{\rm CPT}$, and thus contradict the affirmation in [11] that "Any difference in these two rates is evidence for T-symmetry violation", since a T-symmetric, CPT-violating Hamiltonian $H_{\rm weak}$ ($T_T = 0, T_{\rm CPT} \neq 0$) would just also create such rate differences.

The CP-violating comparisons in Table 2 of [13] also depend on $T_{\rm CPT}\cos(\Delta m t)$ and on $T_T\sin(\Delta m t)$. This confirms that T- and/or CPT-violation imply CP-violation. T-violation in the (2 by 2 dimensional) $B^0\bar{B}^0$ system is thus never independent of CP-violation. See also [15].

The CPT-violating comparisons in Table 3 of [13] neither depend on $T_{\rm CPT}$ nor on T_T, and they are thus, contrary to the authors' intentions, unable to detect CPT-symmetry violation in the matrix T.

Nevertheless, the measured decay time distributions $\{2a\}$ to $\{2d\}$ show a dominant $\sin(\Delta m\, t)$ time dependence, meaning, for this reason, that $T_{\rm CPT} \approx 0$, and with the previous knowledge about the vanishing of the q_i that $T_T \neq 0$, i.e. T-symmetry violation is confirmed. (More combinations are discussed in [5,6].) In the lower part of Table 3, we indicate rate combinations which are true signatures of T- or CPT-symmetry violations.

4 Conclusion

The experiments [7,8] have discovered CP violation in the $B^0\bar{B}^0$ system. Our analysis shows that this CP violation is dominantly T-violation, with the same statistical significance. Furthermore, their data sets contain the information which allows for the estimation of all symmetry-violating parameters indicated in Table 1. CP symmetry of the matrix Λ, which governs the $B^0\bar{B}^0$ oscillation, is confirmed.

The novel definitions of the symmetries (CP, T, CPT) used by [13,21] are more restrictive than the classical ones [1].

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References

1. T.D. Lee, R. Oehme, C.N. Yang, Phys. Rev. **106**, 340 (1957)
2. C.P. Enz, R.R. Lewis, Helv. Phys. Acta **38**, 860 (1965)
3. J.S. Bell, J. Steinberger, in *Weak Interactions of Kaons*, ed. by R.G. Moorhouse et al. Proceedings of Oxford International Conference on Elementary Particles (Chilton, RAL, 1965), pp. 195–222
4. P.K. Kabir, Nature **220**, 1310 (1968)
5. K.R. Schubert, Mainz Workshop (2013). https://indico.cern.ch/event/244839. arXiv:1401.6938 [hep-ex]
6. K.R. Schubert, T violation and CPT tests in neutral-meson systems, review. arXiv:1409.5998 [hep-ex]
7. B. Aubert et al., BABAR Collaboration, Phys. Rev. Lett. **87**, 091801 (2001)
8. K. Abe et al., Belle Collaboration, Phys. Rev. Lett. **87**, 091802 (2001)
9. K. Abe et al., Belle Collaboration, Phys. Rev. D **66**, 071102(R) (2002)
10. B. Aubert et al., BABAR Collaboration, Phys. Rev. Lett. **88**, 231801 (2002)
11. J.P. Lees et al., BABAR Collaboration, Phys. Rev. Lett. **109**, 211801 (2012)
12. K.R. Schubert et al., Phys. Lett. B **31**, 662 (1970)
13. J. Bernabeu, F. Martinez-Vidal, P. Villanueva Pérez, JHEP **08**, 064 (2012)
14. M. Fidecaro, H.-J. Gerber, Rep. Prog. Phys. **69**, 1713 (2006)
15. H.-J. Gerber, Eur. Phys. J. C **35**, 195 (2004)
16. L. Wolfenstein, Int. J. Mod. Phys. E **8**, 501 (1999)
17. T.B. Day, Phys. Rev. **121**, 1204 (1961)
18. A. Angelopoulos et al., CPLEAR Collaboration, Phys. Rep. **374**, 175 (2003)
19. P. Bloch, L. Tauscher, Annu. Rev. Nucl. Part. Sci **53**, 123 (2003)
20. E. Alvarez, A. Szynkman, Mod. Phys Lett. A **23**, 2085 (2008). arXiv:hep-ph/0611370v2
21. J. Bernabeu, F.J. Botella, M. Nebot, Phys. Lett. B **728**, 95 (2014)