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REVIEW

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Uplink resource allocation in cooperative OFDMA with multiplexing mobile relays

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Abstract

Cooperative relaying is an important feature for the fourth generation wireless system to upgrade system performance. Mobile relays can offer better results than fixed relays without any additional infrastructure cost. However, efficient cooperation decision as well as resource allocation are critical to satisfy model constraints as required quality of service (QoS). In this work, simple mobile users with advantageous channels can act as potential relays for cell edge users for an uplink transmission. They multiplex, in the frequency domain, their own data to that of the relayed sources, with the objective for both relay and sources to reach a target data rate. An optimal joint resource blocks (RB) allocation and power allocation scheme under a required data rate constraint per user is proposed. The optimization problem is formulated to minimize the total system power. Dual decomposition and subgradient method are used to solve the optimization problem after dividing it into independent subproblems with less complexity to find the optimal solution. The cooperation decision and the sources-relays association is either performed as a first step of resource allocation, or jointly optimized with RB and power allocation. Simulation results show that these proposed algorithms both reduce system's power consumption while ensuring the required QoS. Joint optimization of relay selection, RB and power allocation provides a higher power consumption decrease, but requires higher complexity and overhead.

Keywords: Multiplexing mobile relays, Resource allocation, OFDMA, Required QoS, Optimization problem, Uplink

1 Introduction

Replying to quality of service requirement with always greedy data application is still an important challenge for wireless cellular networks. Technical constraints push researchers and operators to provide solutions allowing users to acquire high performances independently of their geographical distance from Base Station (BS). In addition to the orthogonal frequency division multiple access (OFDMA) technology, relays are among the principal features of the fourth generation (4G) wireless systems. Relaying technologies, inspired from ad hoc multihop networks, are currently receiving much attention to improve cellular network's performance where bandwidth and power are limited. Instead of deploying BS, relay stations become a solution to reduce high deployment cost and can provide capacity and coverage comparable to small cells. Relaying data aims to upgrade user's performance especially in cell border where users suffer from large signal attenuation. Relaying topology and behavior are standardized in both long term evolution (LTE) Advanced [1] and International Mobile Telecommunication Advanced (IMT-Advanced). In these standards, relays have to be fixed in positions beforehand planned by the operators and become a part of the fixed access network. Each relay is then attached to a designated BS in a static topology. Moreover, relaying data can be considered for a single hop or for multihop using one or multiple relays to transmit information from source to destination. In this context, the LTE Advanced standard allows only two hops when the IEEE.802.s standard offers a multihop relaying scheme [2].

1.1 Literature overview

Many relay transmission schemes are proposed to relay information from source to destination in two time intervals [3, 4]. A relay can use the decode and forward scheme

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(DF) where it decodes the received signal in the first transmit time interval (TTI), re-encodes and then forwards it to destination in the second TTI [5]. A relay may also use amplify and forward scheme (AF) where it just forwards the received signal with an amplification factor. It is proven in [3] that DF scheme can achieve better performance than AF scheme but it is more complex. Several solutions using relays are proposed in the litterature. We can differentiate relays used as virtual multiple input multiple output (MIMO) to exploit spatial diversity [5, 6] which need combining techniques at the destination and relays used as repeaters where source has no direct link to destination [7].

While only fixed relays' architecture is optimized in the standards [8], mobile relays are studied to offer dynamic relaying topology. Mobile relaying has been investigated in the Wireless World Initiative New Radio (WINNER) project [9] contributing in the development and the assessment of 3GPP LTE and IEEE 802.16 (WiMAX) [10] standards and in the Advanced Radio Interface Technologies for 4G Systems (ARTIST4G) project providing innovative concepts to cellular mobile radio communications [11]. Mobile relays can be considered as a serious candidate for the 5G wireless systems. A mobile relay can have the same technical characteristics as a fixed relay but its location dynamically changes. In [12], relaying use cases are studied to prove the relaying improvement for mobile relays. Some examples for this type of mobile relays are relays placed on transportation vehicles such as buses or trains. These relays can be placed to serve users traveling in theses vehicles or to serve users in the street. Another type of mobile relays is to use simple user terminals as relays. Users can have advantageous location and channel conditions to relay some cell border users. This type of mobile relay can upgrade system performance without any additional infrastructure cost. An unpredictable dynamic topology is offered depending on sources and relays mobility [13].

Resource allocation for cooperative networks has been actively studied in the literature for both downlink and uplink. The principal features to discuss are relay selection, subcarriers' allocation and power allocation that can be treated separately or jointly. The selection of relay partners is an important element to successful cooperative strategy [4]. The pairing step may be realized as a centralized process where the BS collects necessary channel and location information from users and relays and decides then to attach users to appropriate relays. Relays selection may also be established in a distributed manner where users or relays decide to make cooperative pairs [3]. It can be made before transmission with the objective to achieve some required level of performance [4]. It can also occur during the transmission time as a proactive selection or as an on-demand relay selection when the direct link's channel quality to the destination decreases. We note that for multihop relaying, an initial path selection from the source to the BS can be initially defined, involving all potential relays [13, 14].

Depending on the system objective and the constraints to respect, resource allocation for a system with relays is generally formulated as an optimization problem. The resource allocation problems are then solved via mathematical tools or heuristics to find the optimal or suboptimal solutions. In [7, 15], the authors formulate an optimization problem to maximize the total system throughput with one source, one destination and a set of fixed AF and DF relays, respectively, where the source may use one or multiple relays to transmit data to destination. In [16], resource allocation considering an uplink relaying system with one destination, several sources, and several fixed relays are studied to maximize system throughput using AF and DF schemes with a minimum data rate constraint per user. In [17], joint power allocation, relay selection, and subcarrier assignment with a minimal data rate per user is discussed for a downlink system model with fixed relays. Downlink energy-efficiency maximization under proportional rate constraint is investigated in [18]. Resource allocation for the multiple access relay channel, with successive interference cancellation at the relay, is studied in [19]. In [20], joint resource allocation is considered for uplink system where relays are fixed. It is solved via an iterative algorithm based on dual decomposition theory. Dual resolution method is adopted after problem adaptations to solve optimization problems in [7, 13, 16]. Dual decomposition [21] consists in dividing the global problem into subproblems to be solved independently. It is a resolution method for convex problems [22] and can be adopted for non-convex problems [23] with some adaptations in the initial problem.

1.2 Contributions

In this work, we propose a new resource allocation algorithm for an uplink multiuser OFDMA relay network in the context of green communications where we aim to save battery life by minimizing the consumed transmit power. We consider a relaying system model where DF relays are simple users with advantageous positions to relay cell-edge users. The main novelty of this work is that relays forward relayed data to the BS and multiplex the relayed data with their own data in different RB. Multiplexing in the frequency domain allows all mobile users to fulfill their QoS constraints, even though some users help others through relaying. In the literature, fixed relays are generally investigated. In addition, relays have no data to transmit to the BS. The major contribution of this work is that relays are mobile users and that have their own information to transmit. To the best of our knowledge, this is the first work studying this system model where mobile relays multiplex their own data to the relayed data.

Two different strategies are studied regarding relay selection: it is either performed before resource allocation, depending on average channel gains. In this case, relayed sources are cell edge users, and a relayed source chooses only one relay. In the second method, relay selection is dynamically performed in each RB, depending on its channel gain. Then, any user may become a relayed source or a relay, and a relayed source may choose different relays on different RB. The RB and power allocation problem is formulated as an optimization problem that aims to minimize the total consumed power, while achieving a target data rate for all users, whether they are relayed source, relays of non relayed sources. Dual Lagrange decomposition is adopted for theoretical resolution and an iterative algorithm is proposed to find the optimal solution.

To summarize, the main contributions of this paper are as follows:

- A cooperative relaying model is proposed, where mobile users may serve as relays to other users, while still transmitting their own data to the BS.
- The corresponding RB and power allocation algorithm, aiming at minimizing the total consumed power, is determined using Lagrange dual decomposition.
- Two relay selection algorithms are proposed: a fixed relay selection strategy, where a source uses the same relay on all RB, and an adaptive strategy where relay selection is jointly optimized with resource allocation. In this case, a source may use different relays on different RB, and may also directly transmit to the BS on some other RB.
- The complexity and overhead of the two algorithm's variants are evaluated, and several simulation results are provided to assess their performance.

This paper is organized as follows. Section 2 describes the adopted system model and the constraints to respect, formulates the associated optimization problem, and provides the proposed resolution algorithm. Section 3 details the resolution steps of the optimization problem. Section 4 presents simulation results. Finally, Section 5 concludes the paper.

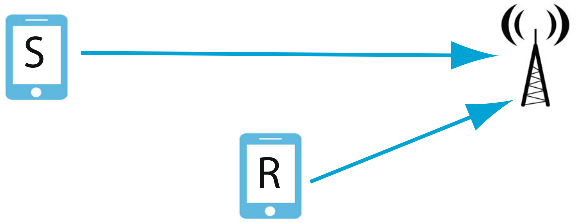
2 System model and problem formulation

In this section, we present the adopted system model and assumptions. Then, we formulate the optimization problem and the associated constraints. We finally enumerate the resolution steps in the proposed resolution algorithm.

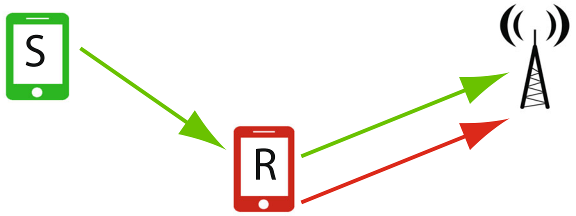
2.1 System model

Relaying is used in this work to improve uplink system performance from users to BS. Simple mobile are used as relays and transmission can be made according to two possible schemes: direct transmission where each user directly transmits to the BS (Fig. 1a) or cooperative scheme where a user R can relay a source S in addition to its own data (Fig. 1b) thanks to its position approximately in the halfway between S and BS. We consider a single cell uplink OFDMA transmission system with one BS with an omnidirectional antenna, K users and N RBs. The channel is assumed a frequency-selective Rayleigh fading channel with slow fading and the noise is Additive White Gaussian (AWGN). The users are uniformly distributed in the cell and experience pathloss and log-normal shadowing.

Our model is a cooperative system where some users can be relays for other users while still transmitting their own data. Source's relayed data and relay's data are then multiplexed by the relay and transmitted to the BS. Users are divided into three groups: *Not Relayed Sources (NRS)*, *Relays (R)*, and *Relayed Sources (RS)* (Fig. 2). These groups are either defined in a first step, if relay selection is fixed, or determined by the joint relay selection and resource allocation algorithm. These strategies are detailed in Sections 2.3.1 and 2.3.2, respectively.



(a) Case 1: Not cooperative model



(b) Case 2: Cooperative model

Fig. 1 Example 1: transmission schemes

Mobile users are assumed half-duplex and thus cannot transmit and receive during the same TTI. Full duplex transmission would require that received and transmit data would use distant RBs, to avoid inter-RB interferences. We did not consider that case in this work. The transmission process takes then two phases: In the first TTI, NRS transmit to the BS and RS transmit to their relays while relays are listening. In the second TTI, RS are silent, NRS and R transmit to the BS. R transmit at the same time their own data and the data of their RS thanks to multi-carrier transmission. For relayed data, the DF method is adopted at the relay.

The objective of our model is to outperform the system without cooperation in minimizing the whole system transmit power subject to a constraint of minimal rate per user. The objective has been chosen as optimizing energy consumption to reduce the overall environmental effects.

We consider the average data rate and power per TTI. The user rate for user k and RB j, with DF if relaying is used, can be expressed as follows:

$$R_k^{(j)} = \log_2\left(1 + P_k^{(j)}\gamma_{k,k}^{(j)}\right), \text{ if } k \text{ is a not relayed source} \tag{1a}$$

$$R_k^{(j)} = \frac{1}{2}\ \min\left\{\log_2\left(1 + P_k^{(j)}\gamma_{k,r}^{(j)}\right); \log_2\left(1+ P_r^{(j)}\gamma_{r,r}^{(j)}\right)\right\}, \text{ if } k \text{ is a relayed source with relay } r \tag{1b}$$

$$R_k^{(j)} = \frac{1}{2}\ \log_2\left(1 + P_k^{(j)}\gamma_{k,k}^{(j)}\right), \text{ if } k \text{ is a relay} \tag{1c}$$

where $P_k^{(j)}$ is the transmit power of user k in RB j and $\gamma_{k,k'}^{(j)}$ is the channel coefficient gain expressed as:

$$\gamma_{k,k'}^{(j)} = \frac{g_{k,k'}^{(j)}}{L_{k,k'}\ S_{k,k'}\ N_{rb}} \tag{2}$$

$g_{k,k'}^{(j)}$ is the square Rayleigh fading in RB j between user k and user k' if $k \neq k'$, or between user k and the BS if $k = k'$. $L_{k,k'}$ and $S_{k,k'}$ are respectively the pathloss and the shadowing experienced by user k considering their direct links when $k = k'$ and considering the indirect links via user k' when $k' \neq k$. N_{rb} is the noise power per RB.

2.2 Problem formulation

Our objective is to minimize the whole system transmit power subject to several constraints. If we consider one NRS, one RS and one R having RBs j, j' and j'', respectively, Table 1 details the consumed transmit power per user per TTI:

Table 1 Power expended per user per TTI

	NRS	RS	R
TTI 1	$P_{NRS}^{(j)}$	$P_{RS}^{(j')}$	0
TTI 2	$P_{NRS}^{(j)}$	0	$P_R^{(j')} + P_R^{(j'')}$
Average Power per TTI	$P_{NRS}^{(j)}$	$\frac{1}{2}P_{RS}^{(j')}$	$\frac{1}{2}\left(P_R^{(j')} + P_R^{(j'')}\right)$

Let $\mathcal{S}_K = \{1, .., K\}$ be the set of K users and $\mathcal{S}_N = \{1, .., N\}$ be the set of N RBs. The general optimization problem is expressed as:

$$\underset{\mathbf{a},\mathbf{b},\mathbf{P}}{\text{minimize}} \quad \sum_{k=1}^{K}\sum_{j=1}^{N}\left(1 - \frac{b_k}{2}\right) a_{k,k}^{(j)} P_k^{(j)} + \frac{1}{2}\sum_{k=1}^{K}\sum_{r\neq k}\sum_{j=1}^{N} b_k a_{k,r}^{(j)}\left(P_k^{(j)} + P_r^{(j)}\right) \tag{3a}$$

subject to

$$\sum_{k=1}^{K}\sum_{r=1}^{K} a_{k,r}^{(j)} \leq 1\ \forall j \in \mathcal{S}_N \tag{3b}$$

$$\sum_{r=1}^{K}\sum_{j=1}^{N} a_{k,r}^{(j)} R_k^{(j)} \geq R_t\ \forall k \in \mathcal{S}_K \tag{3c}$$

$$a_{k,r}^{(j)} \in \{0, 1\}\ \forall \left(k, r, j\right) \in \mathcal{S}_K \times \mathcal{S}_K \times \mathcal{S}_N \tag{3d}$$

$$b_k \in \{0, 1\}\ \forall k \in \mathcal{S}_K \tag{3e}$$

$$P_k^{(j)} \geq 0\ \forall k, j \in \mathcal{S}_K \times \mathcal{S}_N \tag{3f}$$

where

- $\mathbf{b} = [b_1, b_2, ..., b_K]^T$ is the vector of users decisions of cooperation. $b_k = 1$ is is k is a R or a RS, and $b_k = 0$ otherwise. Please note that in the joint relay selection strategy, a user is considered a RS if its data is relayed in at least one RB. Similarly, a user is considered a R if it relays some data in at least one RB.
- $\mathbf{P}$ is the power matrix per user in each RB:

$$\mathbf{P} = \begin{pmatrix} P_1^{(1)} & P_1^{(2)} & .... & P_1^{(N)} \\ P_2^{(1)} & P_2^{(2)} & .... & P_2^{(N)} \\ . & . & . & . \\ . & . & . & . \\ P_K^{(1)} & P_K^{(2)} & .... & P_K^{(N)} \end{pmatrix} \tag{4}$$

- $\mathbf{a}$ is the RB allocation matrix per couple of (source, relay) and each RB j:

$$\mathbf{a} = \begin{pmatrix} a_{1,1}^{(1)} & .. & a_{1,K}^{(1)} & a_{1,1}^{(2)} & .. & a_{1,K}^{(N)} \\ a_{2,1}^{(1)} & .. & a_{2,K}^{(1)} & a_{2,1}^{(2)} & .. & a_{2,K}^{(N)} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ a_{K,1}^{(1)} & .. & a_{K,K}^{(1)} & a_{K,1}^{(2)} & .. & a_{K,K}^{(N)} \end{pmatrix} \tag{5}$$

- Constraint (3e) represents the cooperative decision for user k, $b_k = 1$ if user k is involved in a cooperative manner (k is a RS or a R), $b_k = 0$ otherwise.
- Constraints (3b) and (3d) represent the RB allocation constraints, $a_{k,k}^{(j)} = 1$ means that RB j is assigned to

the transmission of user k towards the BS. $a_{k,r}^{(j)} = 1$ with $k \neq r$ means that RB j is assigned to the transmission of user k towards relay r in the first TTI and transmission of relayed data from r to BS in the second TTI. If there exists at least one subcarrier j such that $a_{k,r}^{(j)} = 1$, then $b_k = 1$ and $b_r = 1$.
- Constraint (3c) indicates the required target data rate per user R_t.
- Constraint (3f) ensures that all powers are positive.
- The first item of the optimization problem (3a) represents both the transmit power for a NRS in two TTIs and the transmit power for a relay for its proper data for only one TTI (expressed by the $\frac{1}{2}$ factor). The second item of the optimization problem represents the transmit power consumed to transmit relayed data.

The different natures of the constraints makes the problem difficult to solve. Having both continous and boolean variables makes the problem a combinatorial optimization problem with excessive computational complexity to find the global optimal solution. To put our problem in a resolvable form, we relax the boolean variable $a_{k,r}^{j}$ to be continous in $[0, 1]$ based on the time sharing process. A RB is then shared by several users that can have the same RB j but not at the same moment. It is proved that relaxing the optimization problem leads to an upper bound solution of the primal optimization problem [24]. It is also proved in [23, 25] that the duality gap of an optimization problem is considered insignificant if the number of subcarrier is high[1].

To solve our optimization problem, we propose a sub-optimal heuristic based on the dual method [23] that consists to find iteratively the optimal solution for the two following subproblems:

1. The optimal power allocation subproblem
2. The optimal resource block allocation subproblem (and relay selection if relay selection is not fixed)

The Algorithm 1 presents the proposed iterative algorithm, the details of each step will be detailed in Sections 3.1 and 3.2.

Algorithm 1 Proposed Sub-Optimal algorithm for Resource Block and Power Allocation (Problem (3))

```
1: Initialize the RB Allocation
2: Optimal Power Allocation for the given RB Allocation (subproblem (14a))
3: Optimal RB Allocation (and relay selection if relay selection is not fixed) using resulting powers from step 2 (subproblem (21))
4: Update Lagrangian variable (Eq. (28))
5: if condition of convergence is verified (Eq. (29)) then
6:     The sub-optimal solution is the RB Allocation resulting from step 3 with power values resulting from step 2.
7: else
8:     return to step 2.
9: end if
```

2.3 Relay selection strategy

Two different relay selection strategies are proposed: a sub-optimal heuristic, and a relay selection that is jointly performed with resource allocation.

2.3.1 Fixed relay selection

The fixed relay selection strategies aims at decreasing the computational complexity of the resource allocation algorithm, and at decreasing the overhead due to information exchange between RS and R as well. With this strategy, potential RS are paired with potential R in a first step, before resource allocation. In this case, the value of b_k is fixed in the resource allocation algorithm. In order to simplify relay search, considering d_k the distance of user k to the BS, it has been decided the following:

- Users with distance $d_k < \frac{R}{3}$ will not have any advantage of being relayed because of their low distance to BS. Furthermore, they are far from cell border users so they are not seen as potential relays. Users with $d_k < \frac{R}{3}$ will be thus non relayed sources and will not act as potential relays.
- Users with distance $d_k > \frac{2R}{3}$ are in the cell border and will take advantage of being relayed if a user at mid distance from them and the BS exists. Users with $d_k > \frac{2R}{3}$ are thus potential relayed sources.
- Users with distance $\frac{R}{3} < d_k < \frac{2R}{3}$ can act as potential relays for users with $d_k > \frac{2R}{3}$. Because of their relative low distance from the BS, these users will not be relayed.
- A mobile user with $d_k > \frac{2R}{3}$ can have only one associated relay in order to lower signaling.

First, each user in the border finds its potential best relay and compares the data rate that it can achieve with the indirect link using this relay to the data rate with the direct link to the BS. It then decides between direct or indirect links. If the user chooses the direct link, it becomes a NRS even if it is in the cell border. A potential Relay not used by any RS becomes also a NRS. At the end of this first step, we have initialized sets of NRS, R and RS depending on the users cooperation decision (see example 1 in Fig. 2). We assume that a relay can support one or more RS but a RS can have only one relay.

The relaying decision consists in comparing direct link to the BS to the best indirect available link. For this, a

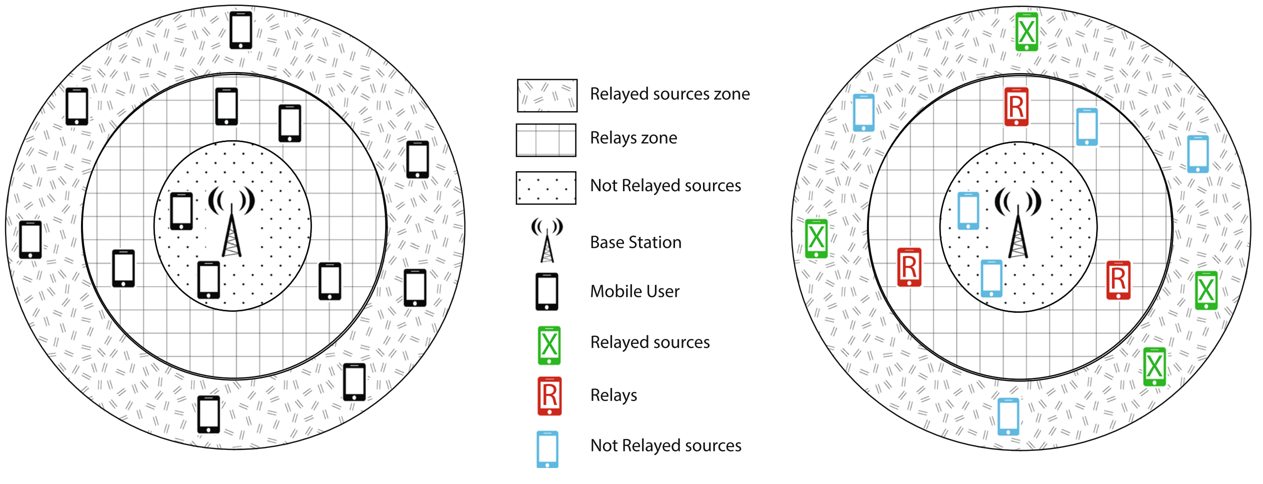


Fig. 2 System model - initialized example, fixed relay selection

potential RS s chooses first the best relay r^* for it as follows:

$$r^* = \max_{r} \min\left(\tilde{\gamma}_{s,r}, \tilde{\gamma}_{r,r}\right) \tag{6}$$

with $\tilde{\gamma}_{s,r}$ the average channel coefficient gain between s and r defined as follows:

$$\tilde{\gamma}_{k,k'} = \frac{1}{L_{k,k'}\, S_{k,k'}\, N_{rb}} \tag{7}$$

Once r^* is found, s compares it with its direct link to the BS. If $\tilde{\gamma}_{s,s} < \min\left(\tilde{\gamma}_{s,r^*}, \tilde{\gamma}_{r^*,r^*}\right)$, then relaying will be advantageous for s, relaying scheme via r^* is then adopted. Else, relaying is considered not advantageous and s will be a NRS.

2.3.2 Joint relay selection, RB, and power allocation

The second proposed relay selection strategy includes relay selection in the resource allocation algorithm. Then the optimization variables are **a**, **P**, and **b**. Users can transmit directly to the BS, or via relay cooperation. A relay can support one or more RS, and a RS can be relayed by one or more relays, but in different RB. In a specific RB, only one relay is assigned for cooperation.

This implies that in Algorithm 1, users nature (R, RS, or NRS) is updated after RB allocation has been optimized, in step 3. This provides a higher flexibility since a RS is not compelled to transmit all its data through the relay, and can choose several relays. Besides, frequency diversity is exploited in the relay selection and in the RB allocation, which cannot be performed with fixed relay selection. Consequently, higher power consumption decrease are expected. They will, however, be achieved at the expense of additional computational complexity and signalling overhead. These additional costs are detailed in Section 3.4.

3 Problem resolution

The dual method is adopted to resolve theoretically the optimization problem (3). Solving the hard primal problem in the dual domain begins by decomposing it into subproblems easier to solve. The master problem distributes to each subproblem the resources it can use and the price to pay. In turn, each subproblem returns to the master problem its solution with the amount of the resources it uses [21].

The Lagrangian function of problem (3) is written as:

$$\begin{aligned} L\left(\mathbf{a},\mathbf{b},\mathbf{P},\boldsymbol{\lambda}\right) = &\sum_{k=1}^{K}\sum_{j=1}^{N}\left(1-\frac{b_k}{2}\right)a_{k,k}^{(j)}P_k^{(j)} \\ &+\frac{1}{2}\sum_{k=1}^{K}\sum_{r\neq k}\sum_{j=1}^{N} b_k a_{k,r}^{(j)}\left(P_k^{(j)}+P_r^{(j)}\right) \\ &-\sum_{k=1}^{K}\sum_{r=1}^{K}\sum_{j=1}^{N}\lambda_k\, a_{k,r}^{(j)}\, R_k^{(j)} + \sum_{k=1}^{K}\lambda_k R_t \end{aligned} \tag{8}$$

where $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, ..., \lambda_K]^T$ is the vector of dual variables associated to the required data rate constraint.

The Lagrangian dual function is then expressed as:

$$g\left(\boldsymbol{\lambda}\right) = \begin{cases} \min\limits_{\mathbf{a},\,\mathbf{b},\,\mathbf{P}} L\left(\mathbf{a},\mathbf{b},\mathbf{P},\boldsymbol{\lambda}\right) \\ \text{subject to} \\ \sum_{k=1}^{K}\sum_{r=1}^{K} a_{k,r}^{(j)} \leq 1 \;\forall j \in \mathcal{S}_N \\ a_{k,r}^{(j)} \in [0..1] \;\; \forall k,r,j \in \mathcal{S}_K \times \mathcal{S}_K \times \mathcal{S}_N \\ b_k \in [0..1] \;\; \forall k \in \mathcal{S}_K \\ P_k^{(j)} \geq 0 \;\forall k,j \in \mathcal{S}_K \times \mathcal{S}_N \end{cases} \tag{9}$$

The problem can be solved by solving its dual problem as follows:

$$\begin{aligned} &\underset{\boldsymbol{\lambda}}{\text{maximize}} \;\; g(\boldsymbol{\lambda}) \\ &\text{subject to} \;\; \lambda_k \geq 0 \;\forall k \in \mathcal{S}_K \end{aligned} \tag{10}$$

The dual problem is solved with two levels of optimization. At the lower level, the Lagrangian (8) is decomposed into N subproblems with Lagrangian $L^{(j)}(\mathbf{a}, \mathbf{b}, \mathbf{P})$ at each RB that can be solved independently. They are solved with a fixed λ. Then, the obtained subproblems solutions are used to update λ. This step is detailed in Section 3.3.

The subproblem for each RB j can be expressed as:

$$\begin{aligned}\underset{\mathbf{a},\,\mathbf{b},\,\mathbf{P}}{\text{minimize}} = &\sum_{k=1}^{K}\left(1-\frac{b_k}{2}\right)a_{k,k}^{(j)}P_k^{(j)}\\ &+\frac{1}{2}\sum_{k=1}^{K}\sum_{r\neq k}b_k a_{k,r}^{(j)}\left(P_k^{(j)}+P_r^{(j)}\right)\\ &-\sum_{k=1}^{K}\sum_{r=1}^{K}\lambda_k\, a_{k,r}^{(j)}\,R_k^{(j)}\end{aligned} \tag{11}$$

Subject to:

$$\sum_{k=1}^{K}\sum_{r=1}^{K}a_{k,r}^{(j)} \le 1\ \forall j \in \mathcal{S}_N$$

$$a_{k,r}^{(j)} \in [0..1]\ \ \forall k,r,j \in \mathcal{S}_K \times \mathcal{S}_K \times \mathcal{S}_N \tag{12}$$

$$b_k \in [0..1]\ \ \forall k \in \mathcal{S}_K \tag{13}$$

$$P_k^{(j)} \ge 0\ \forall k,j \in \mathcal{S}_K \times \mathcal{S}_N$$

To solve problem (11), a second decomposition is necessary to solve independently the two subproblems: optimal power allocation and optimal RB allocation (and relay selection in the second relay selection strategy).

3.1 Optimal power allocation for a given resource block allocation and relay selection

For a given RB allocation, we aim in this section to find the optimal power allocation. Assuming b_k and $a_{k,r}^{(j)}$ fixed for all k, r and j, only the positive power's constraint remains (Eq. (3f)) and the optimization problem can be expressed as:

$$\begin{aligned}\underset{\mathbf{P}}{\text{minimize}} &\sum_{k=1}^{K}\left(1-\frac{b_k}{2}\right)a_{k,k}^{(j)}P_k^{(j)}\\ &+\frac{1}{2}\sum_{k=1}^{K}\sum_{r\neq k}b_k a_{k,r}^{(j)}\left(P_k^{(j)}+P_r^{(j)}\right)\\ &-\sum_{k=1}^{K}\sum_{r=1}^{K}\lambda_k\, a_{k,r}^{(j)}\,R_k^{(j)}\end{aligned} \tag{14a}$$

$$\text{subject to } P_k^{(j)} \ge 0\ \forall k,j \in \mathcal{S}_K \times \mathcal{S}_N \tag{14b}$$

Since only $\mathbf{P}$ is a variable, the optimization Lagrangian of this problem is convex by definition and can be written as:

$$\begin{aligned}L_{bis}^{(j)}(\mathbf{P},\boldsymbol{\lambda}) = &\sum_{k=1}^{K}\left(1-\frac{b_k}{2}\right)a_{k,k}^{(j)}P_k^{(j)}\\ &+\frac{1}{2}\sum_{k=1}^{K}\sum_{r\neq k}b_k a_{k,r}^{(j)}\left(P_k^{(j)}+P_r^{(j)}\right)\\ &-\sum_{k=1}^{K}\sum_{r=1}^{K}\lambda_k\, a_{k,r}^{(j)}\,R_k^{(j)}-\sum_{k=1}^{K}\nu_k^{(j)}\,P_k^{(j)}\end{aligned} \tag{15}$$

where $\nu_k^{(j)}$ is the Langrangian variable associated to the power constraint. Since the optimization problem (15) is convex, the Karush-Kuhn-Tucker (KKT) conditions are used to find its global optimum:

$$\Delta L_{bis}^{(j)} = 0 \tag{16a}$$

$$\nu_k^{(j)}\,P_k^{(j)} = 0\ \forall k \in \mathcal{S}_K \tag{16b}$$

$$\nu_k^{(j)} \ge 0\ \forall k \in \mathcal{S}_K \tag{16c}$$

Considering the different types of users, we evaluate the optimal transmit power for each user in each RB. This is done by differentiating $L_{bis}^{(j)}$ with respect to $\mathbf{P}$, substituting Eq. (1) into Eq. (15) and applying the KKT conditions. Depending on the user's nature, the theoretical optimal power expressions are calculated as follows:

- k is a not relayed source or a relay transmitting its own data in RB j:

  $$P_k^{(j)} = \left[\frac{\lambda_k}{\ln(2)} - \frac{1}{\gamma_{k,k}^{(j)}}\right]^{+} \tag{17}$$

  with $[x]^{+} = \max\{0, x\}$.
- k is a relayed source with relay r

  Let us first remind the throughput expression (1b):

  $$R_k^{(j)} = \frac{1}{2}\min\left\{\log_2\left(1+P_k^{(j)}\gamma_{k,r}^{(j)}\right);\log_2\left(1+P_r^{(j)}\gamma_{r,r}^{(j)}\right)\right\}$$

  In cooperative mode, the total transmit power is minimized when the source and the relay forward the same amount of data. Consequently, the rate is the minimum of the rates on the two links (see Eq. (1b)). To achieve this, we assume that:

  $$P_k^{(j)}\,\gamma_{k,r}^{(j)} = P_r^{(j)}\,\gamma_{r,r}^{(j)} \tag{18}$$

  - Solving problem (15) leads to the following expression for the power of the RS k in RB j:

    $$P_k^{(j)} = \left[\frac{\lambda_k\,\gamma_{r,r}^{(j)}}{\ln(2)\left(\gamma_{k,r}^{(j)}+\gamma_{r,r}^{(j)}\right)} - \frac{1}{\gamma_{k,r}^{(j)}}\right]^{+} \tag{19}$$

- From Eq. (18), we obtain that the power of the relay r for the relayed data of RS k is: :

$$P_r^{(j)} = \left[\frac{\lambda_k \, \gamma_{k,r}^{(j)}}{\ln(2)\left(\gamma_{k,r}^{(j)} + \gamma_{r,r}^{(j)}\right)} - \frac{1}{\gamma_{r,r}^{(j)}} \right]^+ \tag{20}$$

Corresponding to user's nature, optimal power expressions are calculated. We can notice that a relay has different power expressions for its own data (Eq. (17)) and for the data it relays (Eq. (20)). If relay selection is fixed, users nature is known. But in the joint relay selection and resource allocation strategy, for each user, K power values must be computed (one for each RB and for each potential source-relay pair, as well as the power is k is a NRS), although eventually only one of them will be chosen.

3.2 Optimal resource block allocation

The second subproblem to solve is the optimal RB allocation using the optimal power allocation studied above. The Lagrangian per RB j can be written as:

$$\begin{aligned} L^{(j)}\left(\mathbf{a}, \boldsymbol{\lambda}\right) = & \sum_{k=1}^{K} \left(1 - \frac{b_k}{2}\right) a_{k,k}^{(j)} P_k^{(j)} \\ & + \frac{1}{2} \sum_{k=1}^{K} \sum_{r \neq k} b_k a_{k,r}^{(j)} \left(P_k^{(j)} + P_r^{(j)}\right) \\ & - \sum_{k=1}^{K} \sum_{r=1}^{K} \lambda_k \, a_{k,r}^{(j)} \, R_k^{(j)} + \sum_{k=1}^{K} \lambda_k \, R_t \end{aligned} \tag{21}$$

The objective is to minimize $L^{(j)}$, subject to constraints (3b), (12), and (13).

The Lagrangian dual function is written as follows:

$$g(\boldsymbol{\lambda}) = \min_{\mathbf{a}} \left(L^{(j)}\right) = \max_{\mathbf{a}} \left(-L^{(j)}\right) \tag{22}$$

$g(\boldsymbol{\lambda})$ can be written as:

$$g(\boldsymbol{\lambda}) = \underset{\mathbf{a}}{\text{maximize}} \sum_{k=1}^{K} \sum_{r=1}^{K} a_{k,r}^{(j)} \, G_{k,r}^{(j)} - \sum_{k=1}^{K} \lambda_k \, R_t \tag{23}$$

where $\mathbf{G} = [\, G_{k,r}^{(j)}]$ is a $K \times K \times N$ matrix representing the potential gain of couple (k, r) if it earns RB j. The gain function is expressed according to users nature as:

- if $k = r$ and k is a not relayed source:

$$G_{k,r}^{(j)} = \lambda_k \, \log_2\left(1 + P_k^{(j)} \gamma_{k,k}^{(j)}\right) - P_k^{(j)} \tag{24}$$

- if $k = r$ and k is a relay transmitting its own data in RB j:

$$G_{k,r}^{(j)} = \frac{\lambda_k}{2} \, \log_2\left(1 + P_k^{(j)} \gamma_{k,k}^{(j)}\right) - \frac{1}{2} P_k^{(j)} \tag{25}$$

- if k is a relayed source and $k \neq r$:

$$G_{k,r}^{(j)} = \frac{\lambda_k}{2} \log_2\left(1 + P_{k,r}^{(j)} \gamma_{k,r}^{(j)}\right) - \frac{1}{2}\left(P_{k,r}^{(j)} + P_{r,r}^{(j)}\right) \tag{26}$$

The gains are calculated for each RB j, then, j is allocated to couple (k, r) maximizing its gain on it:

$$a_{k,r}^{(j)} = \begin{cases} 1\text{for } (k,r)^* = \arg\max\limits_{(k,r)} G_{k,r}^{(j)} \\ 0\text{otherwise} \end{cases} \tag{27}$$

In the joint relay selection, RB and power allocation strategy, if $k = r$, then user k is a NRS, and b_k is set to 0. Otherwise, if $k \neq r$, then k and r are cooperating, which implies that $b_k = 1$ and $b_r = 1$. If there exists at least one j such that $a_{k,r}^{(j)} = 1$, then user k becomes a RS, and user r a R. Please note that a relay cannot itself be relayed by another mobile user.

3.3 Lagrangian variable update

The last step in our algorithm is to update dual variables and to test the convergence condition for solving problem (10). Using results of current iteration t, $\boldsymbol{\lambda}$ for iteration $t+1$ is updated for each user as follows:

$$\lambda_k(t+1) = \left[\lambda_k(t) + \eta_k(t) \left(R_t - \sum_{r=1}^{K} \sum_{j=1}^{N} a_{k,r}^{(j)}(t) R_k^{(j)}(t) \right) \right]^+ \tag{28}$$

where η is the diminishing step size as the update of dual variable is performed according to the diminishing step approach [26] for each user k. Equation (28) shows that if user k has a data rate higher than R_t, it has to reduce its λ_k and then to reduce its power consumed to achieve the required data rate. On the other hand, if user k has a lower data rate than R_t, the dual variable update allows it to increase its λ_k and so its powers' value, it can then reach R_t by earning more RB or by raising its consumed power amount.

The algorithm is considered to converge when the variation of λ_k is negligible for all k as follows:

$$\left| \frac{\lambda_k(t+1) - \lambda_k(t)}{\lambda_k(t+1)} \right| < \epsilon \; \forall k \tag{29}$$

where ϵ is set close to zero.

3.4 Complexity and overhead comparison of the relay strategies

The complexity of Algorithm 1 depends on the number of iterations until convergence. In each iteration, step 2 requires to compute $N \times K$ power values $P_k^{(j)}$ per user and

RB if the relay selection is fixed. In the joint relay selection and resource allocation strategy, $N^2 \times K$ power values must be computed, as explained in Section 3.1.

Similary, step 3 of the algorithm also requires $N \times K$ computations of $G_{k,r}^{(j)}$ if the pairs (k, r) are already known, and $N^2 \times K$ is they are not. Finally, the determination of b_k value at the end of step 3 in the joint relay selection and resource allocation strategy does not incur any additional complexity. We can conclude that the additional complexity of the second relay selection strategy may become an issue only if the number of users is high.

The second relay selection strategy also increases the overhead, since the channel gains between any two pairs of users must be known by the BS. In the fixed relay strategy, only the channel gains between fixed source-relay pairs must be known. Once the BS has determined the values of $\mathbf{P}, \mathbf{a}$, and $\mathbf{b}$, one signalling message must be sent to any user, indicating which RB it must use for its own data transmission, and if the user is a relay, which RB it should listen to perform decode and forward. Relays do not need to know which sources they are relaying, and sources omnidirectionnally transmit, so they do not need to know their relays.

4 Performance evaluations

Simulations are presented in this section to analyze the proposed approach's performance. We consider a single circular cell with radius $R = 1$ Km, K users and N RBs that we vary along simulations. We assume a total bandwidth[2] $B = 20$ MHz equitably divided between the RBs. Rayleigh channels with slow fading are considered and the power density for AWGN noise is $N_0 = -174$ dBm/Hz. Users are uniformly distributed in the cell and suffer from log-normal shadowing with standard deviation equal to 6 dB and from pathloss according to the LTE model with frequency $F = 2.6$ GHz: $L_{dB}(d_{k,k'}) = 128.1 + 37.6 \log_{10}(d_{k,k'})$ where $d_{k,k'}$ is the distance in Km from user k to user k'. If $k = k'$, $d_{k,k'}$ is the distance of user k to the BS.

The step size for λ_k is set to $\eta_k = \frac{\lambda_k}{\sqrt{t}}$ for $t < 2000$ where t is the iteration index. When t exceeds 2000, η_k becomes invariant. ϵ from Eq. (29) is set to 0.001. For classical mobile cellular networks, the transmit power of a mobile user is generally of the order of 21 dBm. Considering such emitted power and for cell radius of 1 Km, expected data rates for cell border users are lower than 2 bits/s/Hz. Based on this observation, R_t is varied in the simulations in the range $[0.5..1.5]$ bits/s/Hz. Results are averaged over 1000 simulations to get realistic results.

In the following, the proposed solution is compared to the optimal exhaustive solution for a special case with low users and RB number for evaluation. Then, convergence of the proposed solution is studied and the achieved performances are presented.

4.1 Performance results with fixed relay selection strategy

4.1.1 Optimality Evaluation

To find the optimal solution, exhaustive search is necessary for both RB allocation and power allocation. The best solution minimizing the system transmit power is then equal to the optimal solution. The complexity of this search is high and grows with the number of users and RBs. For a given number of users, all possible combinations of RB allocations have to be studied. Then, for each RB allocation, optimal power allocation for all users is established ensuring required target data rate. The optimal solution offering the lowest total system power is finally identified. All possible source-relay pairs must be considered which increases again the complexity of the optimal solution search.

With 2 users where user 1 is relay and user 2 is relayed source, the number of RB allocation's possible combinations is

$$M = \sum_{i=1}^{N-1} C_N^i = 2^N - 2 \tag{30}$$

where N is the number of RB. Having $N = 8$ RBs, we have $M = 254$, for $N = 16$ RBs, $M = 65\,534$ and for $N = 32$ RBs, M exceeds 10^9 possible RB allocation's combinations.

If we consider three users when one is a not relayed source, one is a relayed source and one is a relay, the number of RB allocation's possible combinations is

$$L = \sum_{i=1}^{N+2} C_N^i \sum_{j=1}^{N-1-i} C_{N-i}^j = 3^N - 2^N - 2^{N+1} + 3 \tag{31}$$

For $N = 8$ RBs, $L = 5\,796$, for $N = 16$ RBs, $L = 42\,850\,116$, and for $N = 32$ RBs, L exceeds 10^{15} possible RB allocation.

The optimal power allocation via waterfilling method is then performed for each possible RB allocation respecting the required QoS.

Finding the optimal solution requires high computational cost and high time period that can not be realized in realistic cellular networks. Suboptimal solutions are therefore involved to approach the optimal solution. Table 2 compares system transmit powers with our proposed solution and with the optimal solution for two users and eight RBs. Both system models with and without relaying are considered. We can remark that the proposed solution approaches the optimal solution. The difference of proposed model applied to system without relaying is only 1 % comparing to the optimal solution. For the model with

Table 2 System transmit power (dBm)

Proposed with relay	Exhaustive with relay	Proposed without relay	Exhaustive without relay
7.16	6.24	9.39	9.34

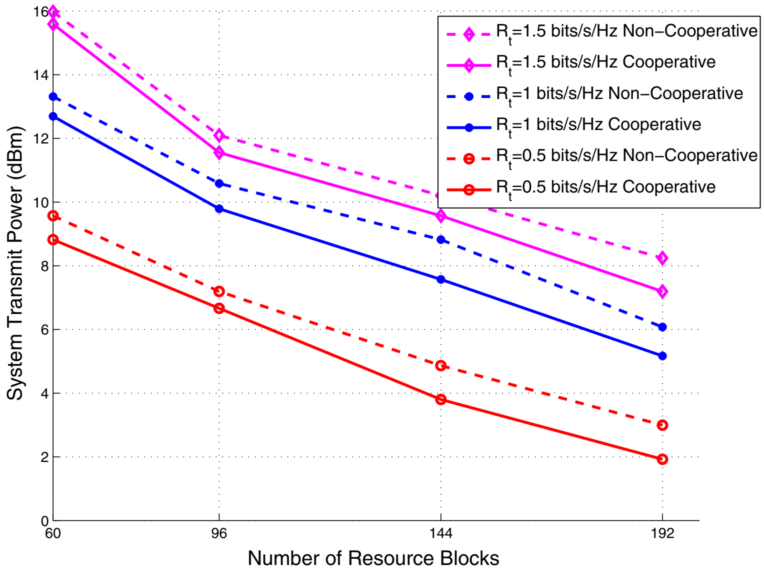


Fig. 3 System transmit power for 18 users, first relay selection strategy

relay, the difference is 17 %. The proposed solution can reduce the system transmit power by 39 % comparing to the optimal solution without relaying. Applying the proposed solution to a system model with a higher number of users especially in cell edge can be very interesting in order to decrease system energy consumption[3].

4.1.2 Convergence analysis

In this section, the convergence rate of the proposed algorithm is studied. A simulation is considered convergent if it respects the Lagrangian variable variation constraints (Eq. (29)) and the required data rate per user constraint as $R_k = R_t \pm 0.1\ R_t\ \forall k$. The convergence rate is studied for 18 users and different RBs numbers and R_t values. The minimal convergence rate is 30 % for 60 RBs and $R_t = 1.5$ bits/s/Hz and it can reach 65 % for 192 RBs for the same R_t. The convergence rates can be justified by the hard convergence constraints. If we relaxed these constraints by expanding the R_t admissible variation range for example, convergence rates would be improved. Then, we can observe that the convergence rate increases when the number of RBs grows, thanks to the increase in frequency

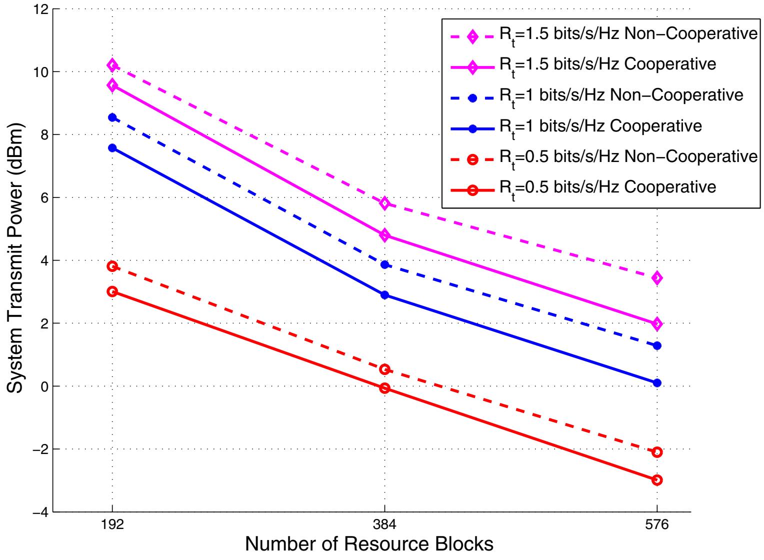


Fig. 4 System transmit power for 30 users, first relay selection strategy

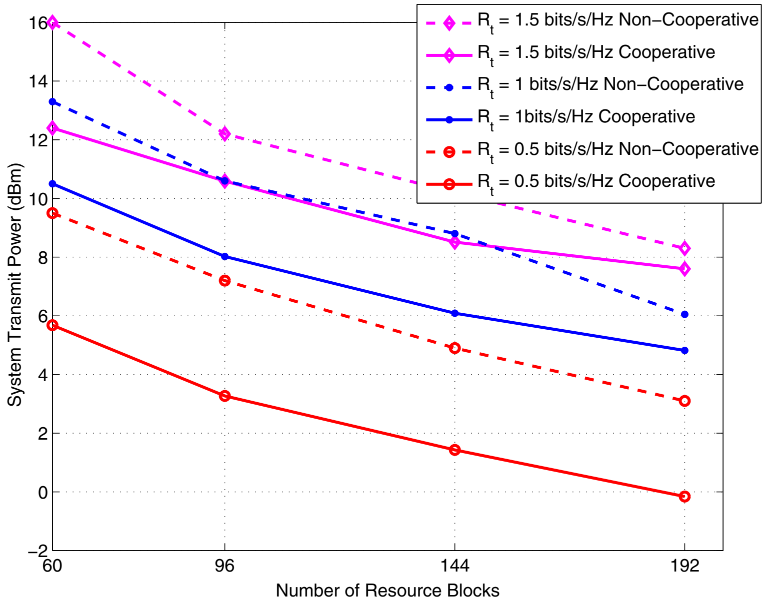


Fig. 5 System transmit power for 18 users, second relay selection strategy

diversity. Indeed, users are more likely to find RB with good channel gains, and thus to achieve their required data rate.

4.1.3 Achieved performances

The system transmit power for different users and RB numbers and various R_t values is presented in this section. Figures 3 and 4 show the system transmit power for, respectively, 18 and 30 users. The gain offered by the proposed algorithm reaches 21 % for 18 users, 192 RBs and $R_t = 1.5$ bits/s/Hz as global gain, and achieves up to 28 % with 576 RB and 30 users. Higher gains are obtained when the number of users increases, since it is then more likely that a mobile user will find another mobile user with adequate location to efficiently serve as a relay. We can also observe that the system transmit power decreases when the number of RB number grows, this is obtained thanks to the frequency diversity. When a high number of RB is available, the RB allocation step can be established more efficiently and the transmit power is then saved.

From simulation results, it is shown that the proposed algorithm offers better performance comparing to the model without relaying. The transmit power can be saved especially in the cell edge. This result can be exploited to reduce interference level in a multicell system model.

4.2 Performance results with joint relay selection, RB, and power allocation

4.2.1 Achieved performances

When joint relay selection, RB and power allocation is used, the system transmit power is even more decreased, as shown by Fig. 5 for 18 users. The power gain is up to 47 % when $R_t = 1$ bit/s/Hz, 50 % when $R_t = 1.5$ bits/s/Hz and 59 % when $R_t = 0.5$ bits/s/Hz.

The average percentage of RS, R and NRS as well as the average distance between each user and the BS, depending on its nature, are gathered in Table 3. It is averaged on all RB values (from 60 to 192), with $R_t = 1$ bit/s/Hz. Relayed sources are mainly located at the border of the cell, whereas relays are in the second ring in the cell. This is consistent with the areas that are chosen in the fixed relay selection strategy, and thus justifies this choice. The main difference with the fixed relay selection strategy is that NRS can be located anywhere in the cell when relay selection is optimized. Besides, since source-relay pairs may be located anywhere in the cell with this relay selection strategies, the ratio of R and RS is high, and few users remain NRS. The average ratio per users nature also shows that relays help several RS in their transmission.

Finally, Figs. 6 and 7 represent the average transmit power per user depending on its nature, as well as the average transmit power among all users when $R_t = 1$ bit/s/Hz and $R_t = 0.5$ bits/s/Hz, respectively. Relays consume more power than RS, because they have to transmit their data as well as the relayed data and are inactive half of the time. Nevertheless, the average power per relay remains lower than the average power per user if

Table 3 Average ratio per user type and distance to the BS, when $R_t = 1$ bit/s/Hz

	NRS	RS	R
Average ratio (%)	11.3	53.1	35.6
Average distance to the BS (m)	689	782	517

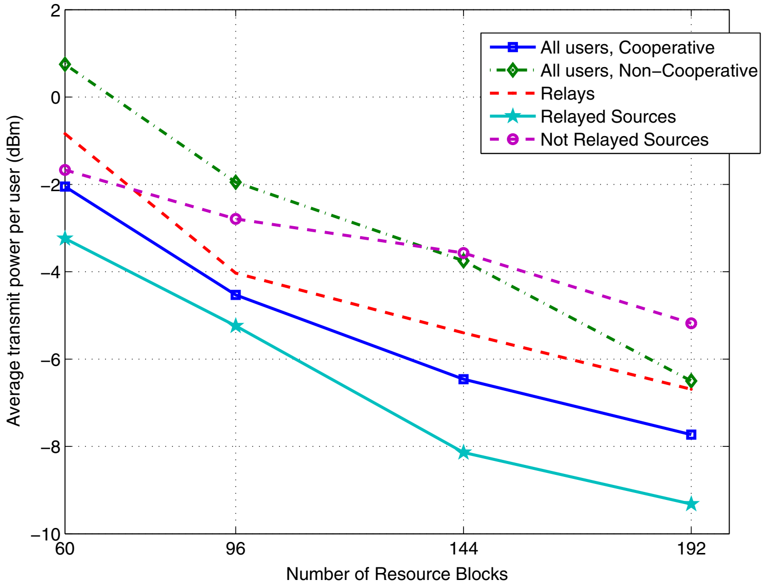


Fig. 6 Average transmit power per user for 18 users, with $R_t = 1$ bit/s/Hz

no relaying is allowed. Consequently, using relays remains beneficial, when considering all users. Besides, RS have low transmit power, even though they are located at the border of the cell. The sum transmit power decrease is only due to the RS power decrease, and the R power increase remains limited enough to achieve a global gain. We can notice that NRS have high transmit power values when $R_t = 1$ bit/s/Hz. These users may be located anywhere in the cell (as shown in Table 3), and some of them may be located at the border of cell, with no potential helpful relay. The average transmit power is high because of some NRS users with very high power values. This tendency is less important when the target data rate is low, as shown on Fig. 7.

Besides, since mobile users are moving in the cell, they are relays at some location, but will become relayed sources whenever they move towards the cell edge. The proposed cooperative scenario is based on the assumption that some mobile users accept to relay some other mobile users at some point, knowing that they will be helped

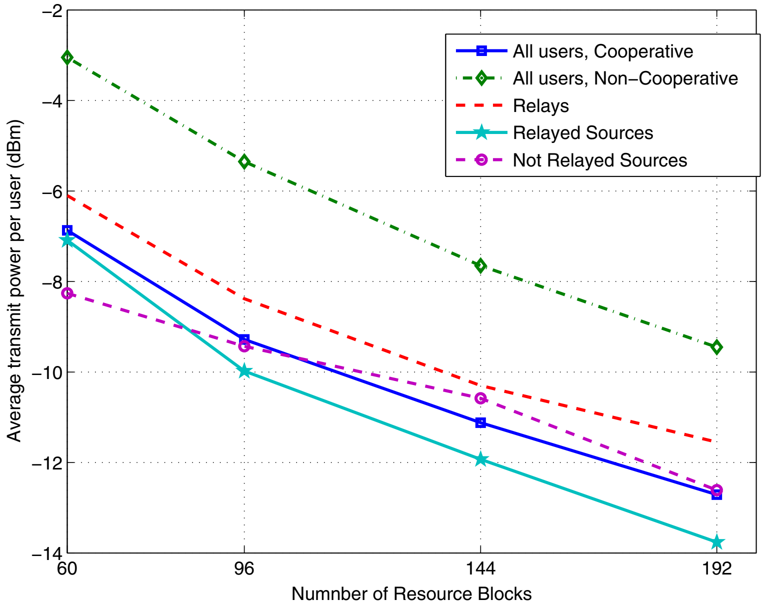


Fig. 7 Average transmit power per user for 18 users, with $R_t = 0.5$ bit/s/Hz

through relaying by other mobile users later. The local power consumption increase when a mobile user acts as a relay is compensated for by an important power consumption decrease when the same mobile user becomes a relayed source.

5 Conclusions

In this paper, we have studied resource allocation for relayed uplink transmission in OFDMA system. Compared to previous published results, our system model considers mobile relays that have to multiplex their own data to the relayed data, so that the relay as well as the relayed sources all achieve the same target data rate. Two strategies have been proposed for relay selection: it is either performed as an initialization phase by the BS, based on average channel gains, or it is jointly optimized with RB and power allocation. An iterative algorithm solving the optimization problem that aims at minimizing the total system transmit power under the target data rate constraint has been determined. The primal optimization problem has been decomposed into subproblems where resource allocation and power allocation are solved in an iterative manner. Dual decomposition and subgradient methods have been used for this purpose.

Simulation results show that the proposed algorithm is very close to optimal solutions found by exhaustive search, with low number of users and RB. When the number of users and RBs is growing, the proposed algorithm gives valuable performances enhancement compared to solutions without relay with the fixed relay selection strategies. With the joint relay selection strategy, power consumption is even lower. This strategy is more flexible and thus better benefits from frequency and multi-user diversity. However, its complexity is higher, and it incurs additional overhead. Comparing the average power per user type (relay, relayed source and non-relayed source) and their location in the cell allows to conclude that the sub-optimal fixed relay strategy achieves a good compromise between transmit power decrease and complexity.

Endnotes

[1] We must note that in the final step of problem resolution, $a_{k,r}^{(j)}$ are converted to boolean variables (Eq. (27))

[2] Please note that we do not use RB number compliant with the LTE standard and that the total bandwidth is fixed and does not vary for all simulations.

[3] We note that gain values consider power values in mW and not in dBm.

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