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Alternate control delayed systems

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Abstract

In the previous paper (Feng *et al.* in Adv. Differ. Equ. 2014:305, 2014), we have already used sandwich control to control a system. But when we considered the influence of delay, can sandwich control also be applied in the delayed system? In order to answer this question, we first introduce the alternate delayed system, then we study the exponential stability of delayed chaotic neural networks by means of alternate control. Some sufficient conditions are given in terms of a set of linear matrix inequalities to ensure the exponential stability of the system. Numerical simulations are presented to verify the correction of the obtained results.

Keywords: alternate control delayed system; globally exponential stabilization; Lu oscillator

1 Introduction

Alternate control [1] is a special case of switching control [2] and is a generalization of intermittent control [3, 4]. In an alternate control system, two different controls are applied alternately. So there is not *rest time* for the control. This system is suitable for the case in which the time is precious.

In [1] Feng *et al.* studied the alternate control system without delay. They have obtained some conditions in terms of LIMs to ensure the stability of the non-delayed system. For delayed systems [5–8], we know that the methods used are different from the ones without delay. There are many papers about delayed system [9–11]. A delayed system is much more difficult to study than the non-delayed one, we are trying to get some conditions to ensure the stability of the delayed system in the theory of control [12–14].

In this paper, we consider the influence of the delay of the system by means of alternate control, that is to say, we study the delayed system by means of alternate control. First of all, we introduce an alternate delayed system. Then we investigate the stability of it by constructing a Lyapunov function, and we obtain stability conditions in terms of LMIs. Lastly we study the stability of Lu oscillator by using the results obtained in the paper.

2 Problem formulation and preliminaries

Consider a class of delayed nonlinear systems described by

$$\begin{cases} \dot{x}(t) = Ax(t) + f(x(t)) + g(x(t-\tau)) + u(t), & t > 0, \\ x(t) = \phi(t), & t \in [-\tau, 0], \end{cases} \tag{1}$$

where $x \in R^n$ presents state vector, f and g are continuous nonlinear functions of $R^n \to R^n$ with $f(0) = g(0) = 0$ and there exist two diagonal matrices $L_1 = \operatorname{diag}(l_1^{(1)}, l_2^{(1)}, \ldots, l_n^{(1)}) \geq 0$

and $L_2 = \mathrm{diag}(l_1^{(2)}, l_2^{(2)}, \ldots, l_n^{(2)}) \geq 0$ such that $\|f(x)\|^2 \leq x^T L_1 x$ and $\|g(x)\|^2 \leq x^T L_2 x$ for any $x \in R^n$, $A \in R^{n\times n}$ is a constant matrix, ϕ is a function of $R^n \to R^n$, $u(t)$ denotes the external input of system (1).

For stabilizing the origin of system (1) by means of periodically alternate control, we assume that the control imposed on the system is of the following form:

$$u(t) = \begin{cases} K_1 x(t), & mT < t \leq mT + \theta, \\ K_2 x(t), & mT + \theta < t \leq (m+1)T, \end{cases} \tag{2}$$

where $K_1, K_2 \in R^{n\times n}$ are constant matrices, $T > 0$ denotes the control period, $\theta \in (0, T)$ is a constant.

Our target is to design suitable T, θ, K_1 and K_2 such that system (1) can be stabilized at the origin.

By the control law (2), system (1) can be rewritten as follows with $m = 0, 1, 2, \ldots$:

$$\begin{cases} \dot{x}(t) = Ax(t) + f(x(t)) + g(x(t-\tau)) + K_1 x(t), & mT < t \leq mT + \theta, \\ \dot{x}(t) = Ax(t) + f(x(t)) + g(x(t-\tau)) + K_2 x(t), & mT + \theta < t \leq (m+1)T, \\ x(t) = \phi(t), & t \in [-\tau, 0]. \end{cases} \tag{3}$$

It is obvious that system (3) is a classical switched system where the switching rule only depends on the time. Specifically, the switching rule of system (3) depends on T and θ.

In the sequel, we will use the following definitions and lemmas.

Lemma 1 (Sanchez and Perez [15]) *Given any real matrices* Σ_1, Σ_2, Σ_3 *of appropriate dimensions and a scalar* $\epsilon \geq 0$ *such that* $0 < \Sigma_3 = \Sigma_3^T$, *the following inequality holds*:

$$\Sigma_1^T \Sigma_2 + \Sigma_2^T \Sigma_1 \leq \epsilon \Sigma_1^T \Sigma_3 \Sigma_1 + \epsilon^{-1} \Sigma_2^T \Sigma_3^{-1} \Sigma_2. \tag{4}$$

Lemma 2 (Boyd *et al.* [16], Horn and Johnson [17]) *The LMI*

$$\begin{bmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{bmatrix} > 0,$$

where $Q(x) = Q^T(x)$, $R(x) = R^T(x)$ *and* $S(x)$ *depend affinely on* x, *is equivalent to*

$$R(x) > 0, \quad Q(x) - S(x)R^{-1}(x)S^T(x) > 0.$$

Definition 1 The zero solution of (1) is said to be globally exponentially stable if there are two constants $M(|\phi|) > 0$, $\gamma > 0$ such that

$$\|x(t)\| \leq M(|\phi|) \exp(-\gamma t), \quad t > 0,$$

where $|\phi| = \sup_{-\tau \leq t \leq 0} \|\phi(t)\|$.

Definition 2 Right-upper Dini's derivative of a function $V : R^+ \times R^n \to R^+$ is defined by

$$D^+ V(t, x(t)) = \limsup \frac{1}{h}\left[V(t+h, x(t+h)) - V(t, x(t))\right].$$

Note that $V(x(t))$ or $V(x)$ is short for $V(t, x(t))$.

Lemma 3 (Halany inequality [18]) *Assume that* $\tau > 0$ *and* $\omega : [\mu - \tau, \infty) \to [0, \infty)$ *is a continuous function such that*

$$\dot{\omega}(t) \leq -a\omega(t) + b \sup_{t-\tau \leq \theta \leq t} \omega(\theta)$$

is satisfied for all $t \geq \mu$. *If* $a > b > 0$, *then*

$$\omega(t) \leq \overline{\omega}(\mu) \exp\bigl(-\gamma(t-\mu)\bigr), \quad t \geq \mu,$$

where $\overline{\omega}(t) = \sup_{t-\tau \leq \theta \leq t} \omega(\theta)$ *and* $\gamma > 0$ *is the smallest real root of the equation*

$$a - b\exp(\gamma\tau) = \gamma.$$

Lemma 4 ([3]) *Assume that* $\tau > 0$ *and* $\omega : [\mu - \tau, \infty) \to [0, \infty)$ *is a continuous function such that*

$$\dot{\omega}(t) \leq a\omega(t) + b\omega(t-\tau)$$

is satisfied for all $t \geq \mu$. *If* $a > 0$ *and* $b > 0$, *then*

$$\omega(t) \leq \overline{\omega}(\mu) \exp\bigl(\eta(t-\mu+\tau)\bigr), \quad t \geq \mu,$$

where $\overline{\omega}(t) = \sup_{t-\tau \leq \theta \leq t} \omega(\theta)$ *and* $\eta > 0$ *is the unique root of the equation*

$$a + b\exp(-\eta\tau) = \eta.$$

Throughout this paper, we use P^T, $\lambda_M(P)$ and $\lambda_m(P)$ to denote the transpose, the maximum eigenvalue and the minimum eigenvalue of a square matrix P, respectively. $\|x\|$ is used to denote the Euclidean norm of the vector x. The matrix norm $\|\cdot\|$ is also referred to as the Euclidean norm. We use $P > 0$ (< 0, ≤ 0, ≥ 0) to denote a symmetrical positive (negative, semi-negative, semi-positive) definite matrix P. $f(x(t_1^-))$ is defined by $f(x(t_1^-)) = \lim_{t \to t_1^-} f(x(t))$.

3 Main results

Theorem 1 *If* $\theta > \tau$ *and there exist a symmetric and positive definite matrix* $P \in R^{n\times n}$, *positive scalar constants* $g_1 > 0$, $g_2 > 0$, $q_1 > 0$, $q_2 > 0$, $\epsilon_1 > 0$, $\epsilon_2 > 0$, $\eta_1 > 0$ *and* $\eta_2 > 0$ *such that the following hold*:

(1) $PA + A^TP + PK_1 + K_1^TP + (\epsilon_1 + \eta_1)P^2 + \epsilon_1^{-1}L_1 + g_1P \leq 0$,
(2) $PA + A^TP + PK_2 + K_2^TP + (\epsilon_2 + \eta_2)P^2 + \epsilon_2^{-1}L_1 - g_2P \leq 0$,
(3) $\eta_1^{-1}L_2 - q_1P \leq 0$,
(4) $\eta_2^{-1}L_2 - q_2P \leq 0$,
(5) $g_1 > q_1$ *and* $\gamma(\theta - \tau) - \eta(T - \theta + \tau) > 0$,

where $\gamma > 0$ *is the smallest real root of the equation* $g_1 - q_1\exp(\gamma\tau) = \gamma$ *and* $\eta > 0$ *is the unique root of the equation* $g_2 + q_2\exp(-\eta\tau) = \eta$, *then the origin of system* (3) *is globally* *exponentially stable, and*

$$\|x(t)\| < \sqrt{\frac{\lambda_M(P)}{\lambda_m(P)}}|\phi| \exp\left(-\left(\gamma(\theta-\tau)-\eta(T-\theta+\tau)\right)\frac{t-\theta}{2T}\right), \quad t>0,$$

where $|\phi| = \sup_{-\tau\leq t\leq 0}\|\phi(t)\|$.

Proof Let us construct the following Lyapunov function:

$$V\bigl(x(t)\bigr) = x^T(x)Px(t), \tag{5}$$

from which we obtain that

$$\lambda_m(P)\bigl\|x(t)\bigr\|^2 \leq V\bigl(x(t)\bigr) \leq \lambda_M(P)\bigl\|x(t)\bigr\|. \tag{6}$$

If $mT < t \leq mT + \theta$, then by (3), (4) and (5) we have that

$$\begin{aligned}
\dot{V}\bigl(x(t)\bigr) &= 2x^T(t)P\dot{x}(t) \\
&= 2x^T(t)P\bigl[Ax(t) + f\bigl(x(t)\bigr) + g\bigl(x(t-\tau)\bigr) + K_1x(t)\bigr] \\
&= 2x^T(t)PAx(t) + 2x^T(t)Pf(x) + 2x^TPg\bigl(x(t-\tau)\bigr) + 2x^TPK_1x(t) \\
&= x^T\bigl[PA + A^TP + PK_1 + K_1^TP\bigr]x + 2x^TPf(x) + 2x^TPg\bigl(x(t-\tau)\bigr) \\
&\leq x^T\bigl[PA + A^TP + PK_1 + K_1^TP\bigr]x \\
&\quad + \epsilon_1x^T(t)P^2x(t) + \epsilon_1^{-1}x^T(t)L_1x(t) \\
&\quad + \eta_1x^T(t)P^2x(t) + \eta_1^{-1}x^T(t-\tau)L_2x(t-\tau) \\
&= -g_1V\bigl(x(t)\bigr) + x^T\bigl[PA + A^TP + PK_1 + K_1^TP \\
&\quad + (\epsilon_1+\eta_1)P^2 + \epsilon_1^{-1}L_1 + g_1P\bigr]x + q_1V\bigl(x(t-\tau)\bigr) \\
&\quad + x^T(t-\tau)\bigl(\eta_1^{-1}L_2 - q_1P\bigr)x(t-\tau) \\
&\leq -g_1V\bigl(x(t)\bigr) + q_1V\bigl(x(t-\tau)\bigr),
\end{aligned}$$

which implies that

$$V\bigl(x(t)\bigr) \leq \overline{V}\bigl(x(mT)\bigr)\exp\bigl(-\gamma(t-mT)\bigr), \tag{7}$$

where $\gamma > 0$ is the smallest real root of the equation $g_1 - q_1\exp(\gamma\tau) = \gamma$.

Similarly, if $mT + \theta < t \leq (m+1)T$, then we have that

$$\begin{aligned}
\dot{V}(x) &= 2x^TP\dot{x} \\
&\leq g_2V\bigl(x(t)\bigr) + x^T\bigl[PA + A^TP + PK_2 + K_2^TP \\
&\quad + (\epsilon_2+\eta_2)P^2 + \epsilon_2^{-1}L_1 - g_2P\bigr]x + q_2V\bigl(x(t-\tau)\bigr) \\
&\quad + x^T(t-\tau)\bigl(\eta_2^{-1}L_2 - q_2P\bigr)x(t-\tau) \\
&\leq g_2V\bigl(x(t)\bigr) + q_2V\bigl(x(t-\tau)\bigr),
\end{aligned}$$

which implies that

$$V\big(x(t)\big) \le \overline{V}\big(x(mT+\theta)\big)\exp\big(\eta(t-mT-\theta+\tau)\big), \tag{8}$$

where $\eta > 0$ is the unique root of the equation $g_2 + q_2\exp(-\eta\tau) = \eta$.

It follows from (7) and (8) that

(1) If $0 < t \le \theta$, then we have that

$$V\big(x(t)\big) \le \overline{V}\big(x(0)\big)\exp(-\gamma t).$$

So

$$\overline{V}\big(x(\theta)\big) = \sup_{\theta-\tau\le t\le\theta} V(t) \le \sup_{\theta-\tau\le t\le\theta}\big(\overline{V}\big(x(0)\big)\exp(-\gamma t)\big) = \overline{V}\big(x(0)\big)\exp\big(-\gamma(\theta-\tau)\big).$$

(2) If $\theta < t \le T$, then we have that

$$\begin{aligned} V\big(x(t)\big) &\le \overline{V}\big(x(\theta)\big)\exp\big(\eta(t-\theta+\tau)\big) \\ &\le \overline{V}\big(x(0)\big)\exp\big(-\gamma(\theta-\tau)+\eta(t-\theta+\tau)\big). \end{aligned}$$

So

$$\overline{V}\big(x(T)\big) \le \overline{V}\big(x(0)\big)\exp\big(-\gamma(\theta-\tau)+\eta(T-\theta+\tau)\big).$$

(3) If $T < t \le T+\theta$, then we have that

$$\begin{aligned} V\big(x(t)\big) &\le \overline{V}\big(x(T)\big)\exp\big(-\gamma(t-T)\big) \\ &\le \overline{V}\big(x(0)\big)\exp\big(-\gamma(t-T)-\gamma(\theta-\tau)+\eta(T-\theta+\tau)\big). \end{aligned}$$

So

$$\overline{V}\big(x(T+\theta)\big) \le \overline{V}\big(x(0)\big)\exp\big(-\gamma(2\theta-2\tau)+\eta(T-\theta+\tau)\big).$$

(4) If $T+\theta < t \le 2T$, then we have that

$$\begin{aligned} V\big(x(t)\big) &\le \overline{V}\big(x(T+\theta)\big)\exp\big(\eta(t-T-\theta+\tau)\big) \\ &\le \overline{V}\big(x(0)\big)\exp\big(\eta(t-T-\theta+\tau)-\gamma(2\theta-2\tau)+\eta(T-\theta+\tau)\big). \end{aligned}$$

So

$$\overline{V}\big(x(2T)\big) \le \overline{V}\big(x(0)\big)\exp\big(2\eta(T-\theta+\tau)-2\gamma(\theta-\tau)\big).$$

(5) If $2T < t \le 2T+\theta$, then we have that

$$\begin{aligned} V\big(x(t)\big) &\le \overline{V}\big(x(2T)\big)\exp\big(-\gamma(t-2T)\big) \\ &\le \overline{V}\big(x(0)\big)\exp\big(-\gamma(t-2T)+2\eta(T-\theta+\tau)-\gamma(2\theta-2\tau)\big). \end{aligned}$$

So

$$\overline{V}(x(2T+\theta) \leq \overline{V}\big(x(0)\big)\exp\big(2\eta(T-\theta+\tau)-3\gamma(\theta-\tau)\big).$$

(6) If $2T+\theta<t\leq 3T$, then we have that

$$\begin{aligned} V\big(x(t)\big) &\leq \overline{V}\big(x(2T+\theta)\big)\exp\big(\eta(t-2T-\theta+\tau)\big) \\ &\leq \overline{V}\big(x(0)\big)\exp\big(\eta(t-2T-\theta+\tau)+2\eta(T-\theta+\tau)-3\gamma(\theta-\tau)\big). \end{aligned}$$

So

$$\overline{V}\big(x(3T)\big) \leq \overline{V}\big(x(0)\big)\exp\big(3\eta(T-\theta+\tau)-3\gamma(\theta-\tau)\big).$$

By induction, we have that

(7) If $mT<t\leq mT+\theta$, *i.e.*, $\frac{t-\theta}{T}<m\leq\frac{t}{T}$, then we have that

$$V\big(x(t)\big) \leq \overline{V}\big(x(0)\big)\exp\big(-\gamma\big(t-m(T-\theta+\tau)\big)+m\eta(T-\theta+\tau)\big). \tag{9}$$

(8) If $mT+\theta<t\leq(m+1)T$, *i.e.*, $\frac{t}{T}<m+1\leq\frac{t+T-\theta}{T}$, then we have that

$$\begin{aligned} V\big(x(t)\big) &\leq \overline{V}\big(x(0)\big)\exp\big(\eta(t-mT-\theta+\tau)+m\eta(T-\theta+\tau)-(m+1)\gamma(\theta-\tau)\big) \\ &= \overline{V}\big(x(0)\big)\exp\big(-\gamma(m+1)(\theta-\tau)+\eta\big(t-(m+1)(\theta-\tau)\big)\big). \end{aligned} \tag{10}$$

From (9) we know that

$$\begin{aligned} V\big(x(t)\big) &\leq \overline{V}\big(x(0)\big)\exp\big(-\gamma\big(t-m(T-\theta+\tau)\big)+m\eta(T-\theta+\tau)\big) \\ &\leq \overline{V}\big(x(0)\big)\exp\big(-\gamma\big(mT-m(T-\theta+\tau)\big)+m\eta(T-\theta+\tau)\big) \\ &= \overline{V}\big(x(0)\big)\exp\big(-\big(\gamma(\theta-\tau)-\eta(T-\theta+\tau)\big)m\big) \\ &< \overline{V}\big(x(0)\big)\exp\left(-\big(\gamma(\theta-\tau)-\eta(T-\theta+\tau)\big)\frac{t-\theta}{T}\right), \end{aligned} \tag{11}$$

where $mT<t\leq mT+\tau$.

From (10) we know that

$$\begin{aligned} V\big(x(t)\big) &\leq \overline{V}\big(x(0)\big)\exp\big(-\gamma(m+1)(\theta-\tau)+\eta\big(t-(m+1)(\theta-\tau)\big)\big) \\ &\leq \overline{V}\big(x(0)\big)\exp\big(-\gamma(m+1)(\theta-\tau)+\eta\big((m+1)T-(m+1)(\theta-\tau)\big)\big) \\ &= \overline{V}\big(x(0)\big)\exp\big(-\big(\gamma(\theta-\tau)-\eta(T-\theta+\tau)\big)(m+1)\big) \\ &< \overline{V}\big(x(0)\big)\exp\left(-\big(\gamma(\theta-\tau)-\eta(T-\theta+\tau)\big)\frac{t}{T}\right) \\ &\leq \overline{V}\big(x(0)\big)\exp\left(-\big(\gamma(\theta-\tau)-\eta(T-\theta+\tau)\big)\frac{t-\theta}{T}\right), \end{aligned} \tag{12}$$

where $mT+\tau<t\leq(m+1)T$.

It follows from (11) and (12) that, for any $t > 0$,

$$V\bigl(x(t)\bigr) < \overline{V}\bigl(x(0)\bigr)\exp\biggl(-\bigl(\gamma(\theta-\tau)-\eta(T-\theta+\tau)\bigr)\frac{t-\theta}{T}\biggr). \tag{13}$$

By (5), (6) and (13), we conclude that

$$\bigl\|x(t)\bigr\| < \sqrt{\frac{\lambda_M(P)}{\lambda_m(P)}}|\phi|\exp\biggl(-\bigl(\gamma(\theta-\tau)-\eta(T-\theta+\tau)\bigr)\frac{t-\theta}{2T}\biggr), \quad t > 0,$$

where $|\phi| = \sup_{-\tau\le t\le 0}\|\phi(t)\|$.

So we finish the proof. □

From Lemma 2, we know that the two conditions of Theorem 1 are equivalent to the following two LMIs, respectively:

$$\begin{bmatrix} PA + A^TP + PK_1 + K_1^TP + \epsilon_1^{-1}L_1 + g_1P & -P \\ -P & -(\epsilon_1+\eta_1)^{-1}I \end{bmatrix} \le 0, \tag{14}$$

$$\begin{bmatrix} PA + A^TP + PK_2 + K_2^TP + \epsilon_2^{-1}L_2 - g_2P & -P \\ -P & -(\epsilon_2+\eta_2)^{-1}I \end{bmatrix} \le 0. \tag{15}$$

Corollary 1 *If $\theta > \tau$ and there exist a symmetric and positive definite matrix $P \in R^{n\times n}$, positive scalar constants $\epsilon_1 > 0$, $\epsilon_2 > 0$, $\eta_1 > 0$, $\eta_2 > 0$, $q_1 > 0$, $q_2 > 0$ and $\eta > 0$ such that the following hold*:

(1) $PA + A^TP + PK_1 + K_1^TP + (\epsilon_1+\eta_1)P^2 + \epsilon_1^{-1}L_1 + g_1P \le 0$, *where* $g_1 = \gamma + q_1\exp(\gamma\tau)$ *and* $\gamma = \frac{\eta(T-\theta+\tau)}{\theta-\tau} + q_1$,

(2) $PA + A^TP + PK_2 + K_2^TP + (\epsilon_2+\eta_2)P^2 + \epsilon_2^{-1}L_1 - g_2P \le 0$, *where* $g_2 = \eta - q_2\exp(-\eta\tau) > 0$,

(3) $\eta_1^{-1}L_2 - q_1P \le 0$,

(4) $\eta_2^{-1}L_2 - q_2P \le 0$, *then the origin of system* (3) *is globally exponentially stable, and*

$$\bigl\|x(t)\bigr\| < \sqrt{\frac{\lambda_M(P)}{\lambda_m(P)}}|\phi|\exp\biggl(-\bigl(\gamma(\theta-\tau)-\eta(T-\theta+\tau)\bigr)\frac{t-\theta}{2T}\biggr), \quad t > 0,$$

where $|\phi| = \sup_{-\tau\le t\le 0}\|\phi(t)\|$.

Proof In fact, the previous four conditions can imply

$$g_1 > q_1$$

and

$$\gamma(\theta-\tau) - \eta(T-\theta+\tau) > 0.$$

From condition (1) we know

$$g_1 = \gamma + q_1\exp(\gamma\tau) = \frac{\eta(T-\theta+\tau)}{\theta-\tau} + q_1 + q_1\exp(\gamma\tau) > q_1$$

and

$$\gamma = \frac{\eta(T-\theta+\tau)}{\theta-\tau} + q_1 > \frac{\eta(T-\theta+\tau)}{\theta-\tau},$$

which implies

$$\gamma(\theta-\tau) - \eta(T-\theta+\tau) > 0.$$

Thus, the fifth condition of Theorem 1 is valid. So the proof is completed. □

Remark 1 In order to judge the global exponential stability of system (3), Corollary 1 needs to determine the existence of a symmetric and positive definite matrix $P \in R^{n\times n}$ and seven positive scalar constants ϵ_1, ϵ_2, η_1, η_2, q_1, q_2 and η by the four linear matrix inequalities listed in it, while Theorem 1 has to determine the existence of a symmetric and positive definite matrix $P \in R^{n\times n}$ and eight positive scalar constants ϵ_1, ϵ_2, η_1, η_2, q_1, q_2, g_1 and g_2 by the five conditions of it. From this view of point, Corollary 1 is more applicative than Theorem 1.

4 Numerical example

Consider the neural oscillator model described by the following delayed differential equation:

$$\dot{x}(t) = Ax(t) + f\bigl(x(t)\bigr) + g\bigl(x(t-1)\bigr), \tag{16}$$

where

$$A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad f\bigl(x(t)\bigr) = \begin{pmatrix} 2 & -0.1 \\ -5 & 3 \end{pmatrix} \begin{pmatrix} \tanh x_1(t) \\ \tanh x_2(t) \end{pmatrix}$$

and

$$g(x(t-1)) = \begin{pmatrix} -1.5 & -0.1 \\ -0.2 & -2.5 \end{pmatrix} \begin{pmatrix} \tanh x_1(t-1) \\ \tanh x_2(t-1) \end{pmatrix}.$$

This model was named *Lu oscillator* [19] and it is shown to be chaotic as in Figure 1. The time response curves are shown in Figure 2.

It is easy to obtain that

$$\begin{aligned} \bigl\|f\bigl(x(t)\bigr)\bigr\|^2 &= f^T\bigl(x(t)\bigr)f\bigl(x(t)\bigr) \\ &= \begin{pmatrix} \tanh x_1(t) \\ \tanh x_2(t) \end{pmatrix}^T \begin{pmatrix} 2 & -0.1 \\ -5 & 3 \end{pmatrix}^T \begin{pmatrix} 2 & -0.1 \\ -5 & 3 \end{pmatrix} \begin{pmatrix} \tanh x_1(t) \\ \tanh x_2(t) \end{pmatrix} \\ &\le x^T L_1 x, \end{aligned}$$

where $L_1 = \begin{pmatrix} 29 & -15.2 \\ -15.2 & 9.01 \end{pmatrix}$.

Similarly, we can get that $\|g(x(t))\|^2 \le x^T L_2 x$, where $L_2 = \begin{pmatrix} 2.29 & 0.65 \\ 0.65 & 6.26 \end{pmatrix}$.

Next, we will use Theorem 1 to judge the global exponential stability of system (16).

Figure 1 Chaotic behavior of Lu oscillator determined by system (16) with the initial values $x(\theta) = (2, -1)^T$, where $\theta \in [-1, 0]$.

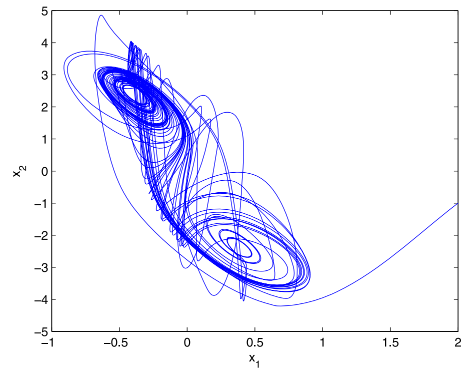
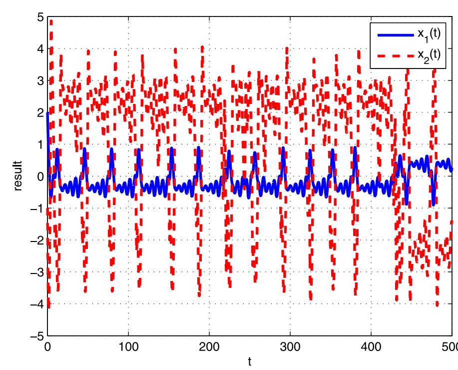


Figure 2 Time response curves of Lu oscillator without control when the initial values are $x(\theta) = (2, -1)^T$, where $\theta \in [-1, 0]$.



Choosing

$$K_1 = \operatorname{diag}(-25, -25),$$
$$K_2 = \operatorname{diag}(-10, -10),$$

with $T = 3$ and $\theta = 1.5$, solving LMIs (14), (15), $\eta_1^{-1} L_2 - q_1 P \le 0$, $\eta_2^{-1} L_2 - q_2 P \le 0$ and inequalities $g_1 > q_1$, $\gamma(\theta - 1) - \eta(T - \theta + 1) > 0$, where $\gamma > 0$ is the smallest real root of the equation $g_1 - q_1 \exp(\gamma) = \gamma$ and $\eta > 0$ is the unique root of the equation $g_2 + q_2 \exp(-\eta) = \eta$, we obtain a feasible solution:

$$\epsilon_1 = 8.6, \qquad \epsilon_2 = 0.6, \qquad \eta_1 = 9, \qquad \eta_2 = 11, \qquad g_1 = 28.2703, \qquad g_2 = 0.3211,$$
$$q_1 = 0.72, \qquad q_2 = 0.63,$$

and

$$P = \begin{bmatrix} 0.9498 & 0.0049 \\ 0.0049 & 1.0066 \end{bmatrix}.$$

Thus by Theorem 1 we obtain that the origin of system (3) is globally exponentially stable. The time response curves of Lu oscillator with alternate control are shown in Figure 3, while Figure 4 shows the corresponding control signal.

In the following, we will apply Corollary 1 to determine the global exponential stability of system (16).

Figure 3 Time response curves of Lu oscillator under alternate control with $T = 3$, $\theta = 1.5$, $K_1 = \mathrm{diag}(-10, -10)$, $K_2 = \mathrm{diag}(-6, -6)$ and the initial values $x(\theta) = (2, -1)^T$, where $\theta \in [-1, 0]$.

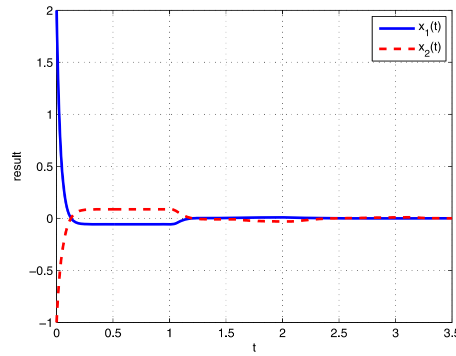
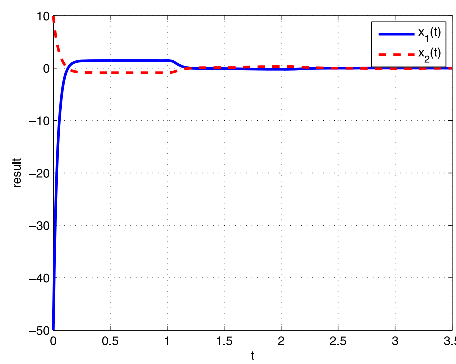


Figure 4 Control signal for Lu oscillator of $T = 3$, $\theta = 1.5$, $K_1 = \mathrm{diag}(-10, -10)$ and $K_2 = \mathrm{diag}(-6, -6)$.



Choosing

$$K_1 = \mathrm{diag}(-15, -15),$$
$$K_2 = \mathrm{diag}(-9, -9),$$

with $T = 3$ and $\theta = 1.5$, solving LMIs (14), (15), $\eta_1^{-1} L_2 - q_1 P \leq 0$ and $\eta_2^{-1} L_2 - q_2 P \leq 0$, where $g_1 = \gamma + q_1 \exp(\gamma\tau)$ and $\gamma = \frac{\eta(T-\theta+\tau)}{\theta-\tau} + q_1$ and $g_2 = \eta - q_2 \exp(-\eta\tau) > 0$, we obtain a feasible solution:

$$\epsilon_1 = 4.6, \qquad \epsilon_2 = 0.6, \qquad \eta_1 = 9, \qquad \eta_2 = 11,$$
$$q_1 = 0.52, \qquad q_2 = 0.43, \qquad \eta = 0.45$$

and

$$P = \begin{bmatrix} 1.2212 & 0.0532 \\ 0.0532 & 1.3590 \end{bmatrix}.$$

Thus by Corollary 1 we obtain that the origin of system (16) is globally exponentially stable. The time response curves of Lu oscillator with alternate control are shown in Figure 5. The control signal is shown in Figure 6.

Figure 5 Time response curves of Lu oscillator under alternate control with $T = 3$, $\theta = 1.5$, $K_1 = \text{diag}(-15, -15)$, $K_2 = \text{diag}(-11, -11)$ and the initial values $x(\theta) = (2, -1)^T$, where $\theta \in [-1, 0]$.

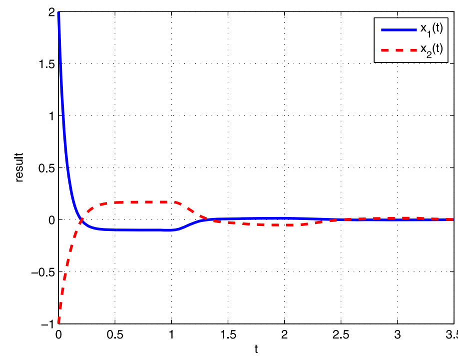
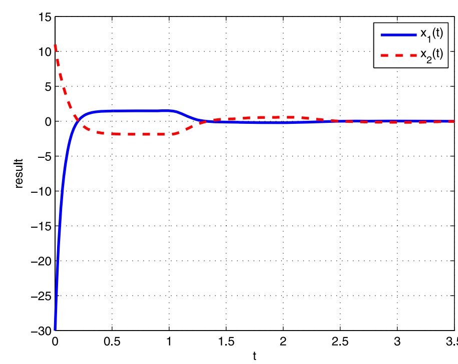


Figure 6 Control signal for Lu oscillator of $T = 3$, $\theta = 1.5$, $K_1 = \text{diag}(-15, -15)$ and $K_2 = \text{diag}(-11, -11)$.



5 Conclusions

This paper studies the delayed system by using alternate control method. Some conditions to ensure the stability of the system are given in terms of linear matrix inequalities. By the results obtained, the Lu oscillate is controlled.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

The ideal of *alternate control delayed system* was proposed by CL and YF. The main theory was proved by YF and DT. The paper was typed by YF and TH and all the figures were provided by TH. All authors read and approved the final manuscript.

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