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On a q-analog of some numbers and polynomials

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Abstract

In this paper, we introduce new q-analogs of the Changhee numbers and polynomials of the first kind and of the second kind. We also derive some new interesting identities related to the Stirling numbers of the first kind and of the second kind, the Euler polynomials of higher order and the q-analogs of Euler polynomials by applying the p-adic integrals method and some summation transform techniques. It turns out that some well-known results are derived as special cases.

MSC: 05A19; 11B68; 11B83

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1 Introduction

In mathematics, special functions (or special polynomials) are known as 'useful functions'. Because of their remarkable properties, special functions have been used for centuries. For instance, since they have numerous applications in astronomy, trigonometric functions have been studied for over a thousand years. Since then, the theory of special functions has been continuously developed with contributions by a host of mathematicians, including Euler, Legendre, Laplace, Gauss, Kummer, Eisenstein, Riemann, Ramanujan, and so on.

In the past years, the development of new special functions and of applications of special functions to new areas of mathematics have initiated a resurgence of interest in the p-adic analysis, q-analysis, analytic number theory, combinatorics, and so on. Moreover, in recent years, the various generalizations of the familiar special polynomials have been defined by using p-adic q-integral on $\mathbb{Z}_p$ and p-adic fermionic q-deformed integrals on $\mathbb{Z}_p$ introduced and investigated by Kim [1–4]. Srivastava and Todorov [5] derived an interesting extension of a representation for the generalized Bernoulli numbers in order to obtain interesting special cases considered earlier by Gould [6]. For more on these issues, *e.g.*, see [6–21].

Let p be chosen as a fixed odd prime number. Throughout this paper, we make use of the following notations. $\mathbb{Z}_p$ denotes the ring of integers, $\mathbb{Q}_p$ denotes the field of p-adic numbers, and $\mathbb{C}_p$ denotes the completion of the algebraic closure of $\mathbb{Q}_p$. The p-adic norm $|\cdot|_p$ is normalized by

$$|p|_p = p^{-1}.$$

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Let $C(\mathbb{Z}_p)$ be the space of continuous functions on $\mathbb{Z}_p$. For $f \in C(\mathbb{Z}_p)$, the fermionic p-adic q-invariant integral on $\mathbb{Z}_p$ is defined by Kim [1, 2], as follows:

$$I_{-q}(f) = \int_{\mathbb{Z}_p} f(x)\, d\mu_{-q}(x) = \lim_{n\to\infty} \sum_{x=0}^{p^n-1} f(x) \frac{(-q)^x}{[p^n]_{-q}}. \tag{1.1}$$

It follows from (1.1) that

$$qI_{-q}(f_1) + I_{-q}(f) = [2]_q f(0), \tag{1.2}$$

where $f_1(x) := f(x+1)$. Obviously

$$\lim_{q\to 1} I_{-q}(f) = I_{-1}(f) = \lim_{n\to\infty} \sum_{x=0}^{p^n-1} f(x)(-1)^x \quad \textit{cf.}\ [22, 23]. \tag{1.3}$$

In [23], the Changhee polynomials are defined by substituting $f(x) = (1+t)^x$ into (1.3) with the case $|t|_p < p^{-\frac{1}{p-1}}$, as follows:

$$\begin{aligned} \int_{\mathbb{Z}_p} (1+t)^{x+y}\, d\mu_{-1}(y) &= \sum_{n=0}^{\infty} \left(\int_{\mathbb{Z}_p} (x+y)_n \, d\mu_{-1}(y) \right) \frac{t^n}{n!} \\ &= \sum_{n=0}^{\infty} Ch_n(x) \frac{t^n}{n!} \\ &= \frac{2}{2+t}(1+t)^x, \end{aligned} \tag{1.4}$$

where $(x)_n$ is known as the *Pochhammer symbol* (or *decreasing factorial*) defined by

$$\begin{aligned} (x)_n &= x(x-1)\cdots(x-n+1) \\ &= \sum_{k=0}^{n} S_1(n,k) x^k \end{aligned} \tag{1.5}$$

and here $S_1(n,k)$ is the Stirling number of the first kind (see [23–25]).

In [23], Kim *et al.* introduced using p-adic integral techniques the idea that the Changee numbers are closely related to the Euler numbers as follows:

$$E_m = \sum_{n=0}^{m} Ch_n S_2(n,m),$$

where $S_2(n,m)$ is the Stirling number of the second kind defined by the following generating series:

$$\sum_{n=m}^{\infty} S_2(n,m) \frac{t^n}{n!} = \frac{(e^t-1)^m}{m!} \quad \textit{cf.}\ [23, 24]. \tag{1.6}$$

In [15], Srivastava extended the Stirling numbers of the second kind to λ-Stirling numbers of the second kind as follows:

$$\frac{(\lambda e^t - 1)^m}{m!} = \sum_{n=0}^{\infty} \mathcal{S}(n,m;\lambda)\frac{t^n}{n!} \quad \bigl(m \in \mathbb{N}^* \text{ and } \lambda \in \mathbb{C}\bigr)$$

with, of course,

$$\mathcal{S}(n,m;1) := S_2(n,m).$$

In [2], the Euler polynomials of (real or complex) order α

$$E_n^{(\alpha)}(x)$$

(sometimes called the Euler polynomials of higher order) are introduced by the following generating function:

$$\left(\frac{2}{e^t+1}\right)^{\alpha} e^{xt} = \sum_{n=0}^{\infty} E_n^{(\alpha)}(x)\frac{t^n}{n!} \quad \bigl(|t| < \pi\bigr) \tag{1.7}$$

with, of course,

$$E_n^{(1)}(x) := E_n(x) \quad \text{and} \quad E_n^{(\alpha)}(0) := E_n^{(\alpha)},$$

where $E_n(x)$ and $E_n^{(\alpha)}$ are the nth Euler polynomials and the nth Euler numbers of order α.

Recently, Kim *et al.* have studied the various generalizations of Changhee polynomials *cf.* [24, 26, 27]. Our q-analogs of the Changhee numbers and polynomials in the present paper are different from Kim *et al.*'s q-analogs of the Changhee numbers and polynomials. In this paper, we introduce a q-analog of the Changhee polynomials and derive some new interesting identities.

2 On a q-analog of Changhee numbers and polynomials

Let us now consider the following p-adic q-integral representation in accordance with the Pochhammer symbol:

$$\int_{\mathbb{Z}_p} q^{-y}(x+y)_n \, d\mu_{-q}(y) \quad \bigl(n \in \mathbb{Z}_+ = \mathbb{N} \cup \{0\}\bigr). \tag{2.1}$$

From (2.1), we have

$$\begin{aligned} \sum_{n=0}^{\infty}\left(\int_{\mathbb{Z}_p} q^{-y}(x+y)_n \, d\mu_{-q}(y)\right)\frac{t^n}{n!} &= \int_{\mathbb{Z}_p} q^{-y}\left(\sum_{n=0}^{\infty}\binom{x+y}{n}t^n\right) d\mu_{-q}(y) \\ &= \int_{\mathbb{Z}_p} q^{-y}(1+t)^{x+y} \, d\mu_{-q}(y), \end{aligned} \tag{2.2}$$

where $t \in \mathbb{C}_p$ with $|t|_p < p^{-\frac{1}{p-1}}$. Applying (1.2) to (2.2) gives

$$\int_{\mathbb{Z}_p} q^{-y}(1+t)^{x+y} \, d\mu_{-q}(y) = \frac{1+q}{1+(1+t)^{-1}}(1+t)^{x-1}. \tag{2.3}$$

Let

$$F_q(x,t) = \frac{1+q}{1+(1+t)^{-1}}(1+t)^{x-1}.$$

Then

$$\lim_{q\to 1} F_q(x,t) = \frac{2}{2+t}(1+t)^x.$$

Notice that $F_q(x,t)$ seems to be a new q-extension of the generating function for aforementioned Changhee polynomials of the first kind. Therefore, from (1.4) and (2.3), we obtain the following definition.

Definition 1 Let $F_q(x,t) = \sum_{n=0}^{\infty} Ch_n(x \mid q)\frac{t^n}{n!}$, where $Ch_n(x \mid q)$ is called a q-analog of the nth Changhee polynomials of the first kind. Then we have for $n \geq 0$

$$\sum_{n=0}^{\infty} Ch_n(x \mid q)\frac{t^n}{n!} = \frac{1+q}{1+(1+t)^{-1}}(1+t)^{x-1}.$$

Moreover,

$$Ch_n(x \mid q) = \int_{\mathbb{Z}_p} q^{-y}(x+y)_n \, d\mu_{-q}(y).$$

In the case $x = 0$ in Definition 1, we have $Ch_n(0 \mid q) := Ch_n(q)$, which stands for the q-analog of the nth Changhee numbers of the first kind. It follows from (1.4) and Definition 1 that

$$\frac{1+q}{2} Ch_n(x) = Ch_n(x \mid q). \tag{2.4}$$

Equation (2.4) shows that our q-analog of the Changhee polynomials of the first kind is closely related to the Changhee polynomials. From (1.5) we have

$$Ch_n(q) = \sum_{k=0}^{n} S_1(n,k)E_k(q), \tag{2.5}$$

where $E_k(q)$ are the q-Euler polynomials derived from

$$E_k(q) = \int_{\mathbb{Z}_p} q^{-y}y^k \, d\mu_{-q}(y).$$

From (1.5), we have

$$\begin{aligned} Ch_n(x \mid q) &= \int_{\mathbb{Z}_p} q^{-y}(x+y)_n \, d\mu_{-q}(y) \\ &= \sum_{k=0}^{n} S_1(n,k)E_k(x \mid q), \end{aligned} \tag{2.6}$$

where $E_k(x \mid q)$ are the q-Euler polynomials introduced by

$$E_k(x \mid q) = \int_{\mathbb{Z}_p} q^{-y}(x+y)^k \, d\mu_{-q}(y).$$

From Definition 1, we have

$$\begin{aligned}
\frac{1+q}{qe^t+1} &= \sum_{n=0}^{\infty} Ch_n(q) \frac{1}{n!} \left(\frac{e^t}{q} - 1 \right)^n \\
&= \sum_{n=0}^{\infty} Ch_n(q) \frac{1}{n!} n! \sum_{m=n}^{\infty} S_2(m,n) \frac{(t-\log q)^m}{m!} \\
&= \sum_{m=0}^{\infty} \left(\sum_{n=0}^{m} Ch_n(q) S_2(m,n) \right) \frac{(t-\log q)^m}{m!}.
\end{aligned} \tag{2.7}$$

It follows from (2.7) that

$$\sum_{m=0}^{\infty} \left(\frac{1+q}{1+q^2} E_m(q^2) \right) \frac{t^m}{m!} = \sum_{m=0}^{\infty} \left(\sum_{n=0}^{m} Ch_n(q) S_2(m,n) \right) \frac{t^m}{m!}.$$

When we compare the coefficients $\frac{t^n}{n!}$ of both sides of the above we have

$$\frac{1+q}{1+q^2} E_m(q^2) = \sum_{n=0}^{m} Ch_n(q) S_2(m,n).$$

Therefore, we obtain the following theorem.

Theorem 1 *For $m \geq 0$, we have*

$$\frac{1+q}{1+q^2} E_m(q^2) = \sum_{n=0}^{m} Ch_n(q) S_2(m,n).$$

The increasing factorial sequence is known as

$$x^{(n)} = x(x+1)(x+2)\cdots(x+n-1) \quad (n \in \mathbb{N}^*).$$

For more information as regards this sequence, see [11, 23, 24, 26, 27].

Let us define the q-analog of the Changhee numbers of the second kind as follows:

$$\widehat{Ch}_n(q) = \int_{\mathbb{Z}_p} q^{-y}(-y)_n \, d\mu_{-q}(y) \quad (n \in \mathbb{N}^*). \tag{2.8}$$

It is easy to observe that

$$x^{(n)} = (-1)^n(-x)_n = \sum_{k=0}^{n} S_1(n,k)(-1)^{n-k} x^k. \tag{2.9}$$

By virtue of (2.8) and (2.9), it leads to

$$\begin{aligned}\widehat{Ch}_n(q) &= \int_{\mathbb{Z}_p} q^{-y}(-y)_n\,d\mu_{-q}(y)\\ &= \int_{\mathbb{Z}_p} q^{-y}y^{(n)}(-1)^n\,d\mu_{-q}(y)\\ &= \sum_{k=0}^{n} S_1(n,k)(-1)^k E_k(q).\end{aligned}\tag{2.10}$$

Thus, we state the following theorem.

Theorem 2 *The following holds true*:

$$\widehat{Ch}_n(q) = \sum_{k=0}^{n} S_1(n,k)(-1)^k E_k(q).$$

Let us now consider the generating function of the q-Changhee numbers of the second kind as follows:

$$\begin{aligned}\sum_{n=0}^{\infty}\widehat{Ch}_n(q)\frac{t^n}{n!} &= \sum_{n=0}^{\infty}\left(\int_{\mathbb{Z}_p} q^{-y}(-y)_n\,d\mu_{-q}(y)\right)\frac{t^n}{n!}\\ &= \int_{\mathbb{Z}_p} q^{-y}\left(\sum_{n=0}^{\infty}\binom{-y}{n}t^n\right)d\mu_{-q}(y)\\ &= \int_{\mathbb{Z}_p} q^{-y}(1+t)^{-y}\,d\mu_{-q}(y),\end{aligned}\tag{2.11}$$

in which $\int_{\mathbb{Z}_p} q^{-y}(1+t)^{-y}\,d\mu_{-q}(y)$ equals

$$\frac{(1+q)}{1+(1+t)^{-1}}.\tag{2.12}$$

Then, combining (2.11) with (2.12), we state the following definition.

Definition 2 For $n \geq 0$, we have

$$\begin{aligned}\sum_{n=0}^{\infty}\widehat{Ch}_n(q)\frac{t^n}{n!} &= \int_{\mathbb{Z}_p} q^{-y}(1+t)^{-y}\,d\mu_{-q}(y)\\ &= \frac{(1+q)}{1+(1+t)^{-1}}.\end{aligned}$$

Let us consider the q-Changhee polynomials of the second kind as follows:

$$\frac{1+q}{2+t}(1+t)^{1-x} = \sum_{n=0}^{\infty}\widehat{Ch}_n(x \mid q)\frac{t^n}{n!}.\tag{2.13}$$

Combining (2.4) with (2.13) at the value $x = 1$, we have

$$\widehat{Ch}_n(1 \mid q) = \frac{1+q}{2}Ch_n = Ch_n(q).$$

It follows from (2.13) that

$$\int_{\mathbb{Z}_p} q^{-y}(1+t)^{-x-y}\,d\mu_{-q}(y) = \sum_{n=0}^{\infty} \widehat{Ch}_n(x \mid q)\frac{t^n}{n!}. \tag{2.14}$$

From (2.14) gives

$$\begin{aligned}\widehat{Ch}_n(x \mid q) &= \int_{\mathbb{Z}_p} q^{-y}(-x-y)_n\,d\mu_{-q}(y) \\ &= \sum_{k=0}^{n}(-1)^k S_1(n,k)E_k(x \mid q) \quad (n \ge 0).\end{aligned} \tag{2.15}$$

Then, by (2.15), we have the following theorem.

Theorem 3 *The following holds true*:

$$\widehat{Ch}_n(x \mid q) = \sum_{k=0}^{n}(-1)^k S_1(n,k)E_k(x \mid q) \quad (n \ge 0).$$

From (2.13) and (2.14), we have

$$\begin{aligned}q^{1-x}\left(\frac{1+q}{qe^t+1}\right)e^{(1-x)t} &= \sum_{n=0}^{\infty} \widehat{Ch}_n(x \mid q)\frac{1}{n!}\left(qe^t-1\right)^n \\ &= \sum_{n=0}^{\infty} \widehat{Ch}_n(x \mid q)\sum_{m=n}^{\infty} \mathcal{S}(m,n;q)\frac{t^m}{m!} \\ &= \sum_{m=0}^{\infty}\left(\sum_{n=0}^{m} \widehat{Ch}_n(x \mid q)\mathcal{S}(m,n;q)\right)\frac{t^m}{m!}.\end{aligned} \tag{2.16}$$

Further

$$\begin{aligned}q^{1-x}\int_{\mathbb{Z}_p} e^{(1-x+y)t}\,d\mu_{-q}(y) &= \sum_{n=0}^{\infty} \widehat{Ch}_n(x \mid q)\frac{(qe^t-1)^n}{n!} \\ &= \sum_{m=0}^{\infty}\left(\sum_{n=0}^{m} \widehat{Ch}_n(x \mid q)\mathcal{S}(m,n;q)\right)\frac{t^m}{m!}.\end{aligned} \tag{2.17}$$

Therefore, by (2.16) and (2.17), we obtain the following theorem.

Theorem 4 *The following equality holds true*:

$$q^{1-x}\mathcal{E}_m(1-x \mid q) = (-1)^n q^{1-x}\mathcal{E}_m\left(x \mid q^{-1}\right) = \sum_{n=0}^{m} \widehat{Ch}_n(x \mid q)\mathcal{S}(m,n;q),$$

where $\mathcal{E}_m(x \mid q)$ may be called the mth q-Euler polynomials

$$\mathcal{E}_m(x \mid q) = \int_{\mathbb{Z}_p} (x+y)^m\,d\mu_{-q}(y)$$

because

$$\lim_{q\to 1}\mathcal{E}_m(x\mid q):=E_m(x).$$

From Definition 1 and (2.15) we have

$$\begin{aligned}
(-1)^n\frac{Ch_n(q)}{n!} &= (-1)^n\int_{\mathbb{Z}_p} q^{-y}\binom{y}{n}\,d\mu_{-q}(y)\\
&= \int_{\mathbb{Z}_p} q^{-y}\binom{-y+n-1}{n}\,d\mu_{-q}(y)\\
&= \sum_{m=0}^{n}\binom{n-1}{n-m}\int_{\mathbb{Z}_p} q^{-y}\binom{-y}{m}\,d\mu_{-q}(y)\\
&= \sum_{m=1}^{n}\binom{n-1}{m-1}\frac{\widehat{Ch}_m(q)}{m!}
\end{aligned}\tag{2.18}$$

and

$$\begin{aligned}
(-1)^n\frac{\widehat{Ch}_n(q)}{n!} &= (-1)^n\int_{\mathbb{Z}_p} q^{-y}\binom{-y}{n}\,d\mu_{-q}(y)\\
&= \int_{\mathbb{Z}_p} q^{-y}\binom{y+n-1}{n}\,d\mu_{-q}(y)\\
&= \sum_{m=0}^{n}\binom{n-1}{n-m}\int_{\mathbb{Z}_p} q^{-y}\binom{y}{m}\,d\mu_{-q}(x)\\
&= \sum_{m=1}^{n}\binom{n-1}{m-1}\frac{\widehat{Ch}_m(q)}{m!}.
\end{aligned}\tag{2.19}$$

Therefore, we get the following theorem.

Theorem 5 *The following holds*:

$$(-1)^n\frac{Ch_n(q)}{n!}=\sum_{m=1}^{n}\binom{n-1}{m-1}\frac{\widehat{Ch}_m(q)}{m!}$$

and

$$(-1)^n\frac{\widehat{Ch}_n(q)}{n!}=\sum_{m=1}^{n}\binom{n-1}{m-1}\frac{\widehat{Ch}_m(q)}{m!}.$$

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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