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Existence of nonoscillatory solutions of higher-order neutral differential equations with positive and negative coefficients

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Abstract

In this paper we consider the existence of nonoscillatory solutions of higher-order neutral differential equations with distributed coefficients and delays. We use the *Banach* contraction principle to obtain new sufficient condition for the existence of nonoscillatory solutions.

Keywords: nonlinear; neutral; distributed delays; nonoscillatory solutions; Banach contraction principle

1 Introduction

In this paper, we consider the higher-order nonlinear neutral differential equation with distributed coefficients

$$\left[\Phi \left[r(t)x(t) + \int_a^b p(t, \theta)x(t - \theta) d\theta \right] \right]^{(n)} + q_1(t)g_1(x(t - \tau)) - q_2(t)g_2(x(t - \sigma)) = h(t). \quad (1)$$

Throughout this paper, the following conditions are assumed to hold.

- (1) $n \geq 1$ is a positive integer, $r \in C([t_0, \infty), R^+)$, $0 < a < b$, $\tau > 0$, $\sigma > 0$;
- (2) $p \in C([t_0, \infty) \times [a, b], R)$, $q_1 \in C([t_0, \infty), R^+)$, $q_2 \in C([t_0, \infty), R^+)$, $h \in C([t_0, \infty), R)$;
- (3) $\Phi(u)$ is a continuously increasing real function with respect to u defined on R , and $\Phi^{-1}(u)$ satisfies the local Lipschitz condition;
- (4) $g_i \in C(R, R)$, $g_i(u)$ satisfy the local Lipschitz condition and $ug_i(u) > 0$, $i = 1, 2$, for $u \neq 0$.

Recently there have been a lot of activities concerning the existence of nonoscillatory solutions for neutral differential equations with positive and negative coefficients. In 2002, Zhou and Zhang [1] studied the higher-order linear neutral delay differential equation

$$\frac{d^n}{dt^n} [x(t) + cx(t - \tau)] + (-1)^{n+1} [P(t)x(t - \sigma) - Q(t)x(t - \delta)] = 0.$$

In 2005, the existence of nonoscillatory solutions of first-order linear neutral delay differential equations of the form

$$\frac{d}{dt} [x(t) + P(t)x(t - \tau)] + Q_1(t)x(t - \sigma_1) - Q_2(t)x(t - \sigma_2) = 0$$

was investigated by Zhang *et al.* [2] and, in the same year, Yu and Wang [3] studied nonoscillatory solutions of second-order nonlinear neutral differential equations of the form

$$[r(t)[x(t) + P(t)x(t - \tau)]'] + Q_1(t)f(x(t - \sigma_1)) - Q_2(t)g(x(t - \sigma_2)) = 0.$$

In 2010, Candan and Dahiya [4] studied nonoscillatory solutions of first-order and second-order nonlinear neutral differential equations with positive and negative coefficients,

$$\frac{d^k}{dt^k} [x(t) + P(t)x(t - \tau)] + \int_a^b q_1(t, \xi)x(t - \xi) d\xi - \int_c^d q_2(t, \mu)x(t - \mu) d\mu = 0, \quad k = 1, 2.$$

In 2012, Candan [5] studied the higher-order nonlinear differential equation

$$[r(t)[x(t) + P(t)x(t - \tau)]^{(n-1)'} + (-1)^n [Q_1(t)g_1(x(t - \sigma_1)) - Q_2(t)g_2(x(t - \mu)) - f(t)] = 0.$$

As can be seen from the development process of the above equations, the delays of the neutral part in the discussed differential equations were all constant delays, and the main thought in the employed verification method also was kept the same in which the coefficient neutral part in the neutral were all discussed in four cases, that is, $(-\infty, -1)$, $(-1, 0)$, $(0, 1)$, $(1, +\infty)$, and then they were verified by constructing the corresponding operator. However, studies of the case for distributed deviating arguments are rather rare. In 2013, Candan [6] studied a first-order neutral differential equation with distributed deviating arguments,

$$\left[\left[x(t) - \int_a^b p_2(t, \xi)x(t - \xi) d\xi \right]^\gamma \right]' + \int_c^d Q_2(t, \xi)G(x(t - \xi)) d\xi = 0, \tag{2}$$

where γ is a ratio of odd positive integers, however, the discussion only covered the condition for the coefficient being $0 < \int_a^b p_2(t, \xi) d\xi < 1$, while without the other three conditions, which might be caused by the difficulty in establishing feasible operator. In view of the above, here, in this paper, the difficulty of an operator establishment was settled and sufficiency conditions for the existence of nonoscillatory solutions of differential equation with coefficient of $\int_a^b p_2(t, \xi) d\xi$ in the four cases were obtained. Thus, this paper may have theoretical value as well as practical application value. For related work, we refer the reader to [7–17].

As usual, a solution of equation (1) is said to be oscillatory if it has arbitrarily large zeros. Otherwise the solution is said to be nonoscillatory.

A solution of equation (1) is a continuous function $x(t)$ defined on $[t_1 - \mu, \infty)$, for some $t_1 > t_0$, such that $\Phi[r(t)x(t) + \int_a^b p(t, \theta)x(t - \theta) d\theta]$ is n times continuously differentiable and equation (1) holds for all $n \geq 1$. Here, $\mu = \max\{b, \tau, \sigma\}$.

Let $L_i, i = 1, 2$, denote the Lipschitz constants of the functions $g_i, i = 1, 2$ on the set A , K denote the Lipschitz constants of functions $\Phi^{-1}(u)$, respectively, and $L = \max\{L_1, L_2\}$, $\beta_i = \max_{x \in A} g_i(x), i = 1, 2$.

Let Ω be the set of all continuous and bounded functions on $[t_0, \infty)$ and the norm be $\|x(t)\| = \sup_{t_0 \leq t < +\infty} x(t)$.

2 The main results

Theorem Assume that

$$\int_t^\infty s^{n-1}q_i(s) ds < \infty, \quad i = 1, 2, \tag{3}$$

$$\int_t^\infty s^{n-1}h(s) ds < \infty,$$

and $\int_a^b p(t, \theta) d\theta$ satisfies one of the following conditions:

- (a) $0 \leq \int_a^b p(t, \theta) d\theta < 1,$
- (b) $1 < \int_a^b p(t, \theta) d\theta < +\infty,$
- (c) $-1 < \int_a^b p(t, \theta) d\theta \leq 0,$
- (d) $-\infty < \int_a^b p(t, \theta) d\theta < -1.$

Then equation (1) has a bounded nonoscillatory solution.

Proof Case (a): $0 \leq \int_a^b p(t, \theta) d\theta \leq p_1 < 1$. Set $A = \{x \in \Lambda, M_1 \leq x(t) \leq M_2, t \geq t_0\}$, where M_1, M_2 are two positive constants such that $p_1M_2 + \frac{M_1}{p_1} < \alpha < M_2, 1 \leq r(t) \leq \frac{1}{p_1}$. From (3), one can choose a $t_1 \geq t_0 + b$, and a sufficiently large $t \geq t_1$ such that

$$\int_t^\infty \frac{(s-t)^{n-1}}{(n-1)!} [\beta_1q_1(s) + |h(s)|] ds \leq \Phi(M_2) - \Phi(\alpha), \tag{4}$$

$$\int_t^\infty \frac{(s-t)^{n-1}}{(n-1)!} [\beta_2q_2(s) + |h(s)|] ds \leq \Phi(\alpha) - \Phi\left(p_1M_2 + \frac{M_1}{p_1}\right), \tag{5}$$

$$\int_t^\infty \frac{(s-t)^{n-1}}{(n-1)!} [q_1(s) + q_2(s)] ds \leq \frac{1-p_1}{KL}, \tag{6}$$

and define an operator T on A as follows:

$$(Tx)(t) = \begin{cases} \frac{1}{r(t)} \left\{ -\int_a^b p(t, \theta)x(t-\theta) d\theta \right. \\ \quad \left. + \Phi^{-1} \left[\Phi(\alpha) + \int_t^\infty \frac{(s-t)^{n-1}}{(n-1)!} [q_1(s)g_1(x(s-\tau)) \right. \right. \\ \quad \left. \left. - q_2(s)g_2(x(s-\sigma)) - h(s)] ds \right] \right\}, & t \geq t_1, \\ (Tx)(t_1), & t_0 \leq t \leq t_1. \end{cases}$$

It is easy to see that T is continuous, for $t \geq t_1, x \in A$. By using (4), we have

$$\begin{aligned} (Tx)(t) &\leq \frac{1}{r(t)} \left\{ \Phi^{-1} \left[\Phi(\alpha) + \int_t^\infty \frac{(s-t)^{n-1}}{(n-1)!} [q_1(s)g_1(x(s-\tau)) - h(s)] ds \right] \right\} \\ &\leq \Phi^{-1} \left[\Phi(\alpha) + \int_t^\infty \frac{(s-t)^{n-1}}{(n-1)!} (\beta_1q_1(s) + |h(s)|) ds \right] \\ &\leq M_2, \end{aligned}$$

and taking (5) into account, we have

$$\begin{aligned} (Tx)(t) &\geq \frac{1}{r(t)} \left\{ - \int_a^b p(t, \theta)x(t - \theta) d\theta \right. \\ &\quad \left. - \Phi^{-1} \left[\Phi(\alpha) + \int_t^\infty \frac{(s-t)^{n-1}}{(n-1)!} [q_2(s)g_2(x(s-\sigma)) - h(s)] ds \right] \right\} \\ &\geq p_1 \left\{ -p_1M_2 - \Phi^{-1} \left[\Phi(\alpha) + \int_t^\infty \frac{(s-t)^{n-1}}{(n-1)!} (\beta_2q_2(s) + |h(s)|) ds \right] \right\} \\ &\geq M_1. \end{aligned}$$

These show that $TA \subset A$. Since A is a bounded, closed, convex subset of Λ , in order to apply the contraction principle we have to show that T is a contraction mapping on A . For $\forall x_1, x_2 \in A$, and $t \geq t_1$,

$$\begin{aligned} &|(Tx_1)(t) - (Tx_2)(t)| \\ &\leq \frac{1}{r(t)} \left\{ \int_a^b p(t, \theta) |x_1(t - \theta) - x_2(t - \theta)| d\theta \right. \\ &\quad + \Phi^{-1} \left[\Phi(\alpha) + \int_t^\infty \frac{(s-t)^{n-1}}{(n-1)!} |q_1(s)g_1(x_1(s-\tau)) - q_2(s)g_2(x_1(s-\sigma)) - h(s)| ds \right] \\ &\quad - \Phi^{-1} \left[\Phi(\alpha) + \int_t^\infty \frac{(s-t)^{n-1}}{(n-1)!} |q_1(s)g_1(x_2(s-\tau)) \right. \\ &\quad \left. - q_2(s)g_2(x_2(s-\sigma)) - h(s)| ds \right] \left. \right\} \\ &\leq \frac{1}{r(t)} \left\{ p_1|x_1 - x_2| + K \int_t^\infty \frac{(s-t)^{n-1}}{(n-1)!} [q_1(s)|g_1(x_1(s-\tau)) - g_1(x_2(s-\tau))| \right. \\ &\quad \left. + q_2(s)|g_2(x_1(s-\sigma)) - g_2(x_2(s-\sigma))|] ds \right\}. \end{aligned}$$

Using (6),

$$\begin{aligned} |(Tx_1)(t) - (Tx_2)(t)| &\leq |x_1 - x_2| \left(p_1 + KL \int_t^\infty \frac{(s-t)^{n-1}}{(n-1)!} [q_1(s) + q_2(s)] ds \right) \\ &< |x_1 - x_2|. \end{aligned}$$

This implies with the sup norm that

$$\|Tx_1 - Tx_2\| < \|x_1 - x_2\|,$$

which shows that T is a contraction mapping on A and therefore there exists a unique solution, obviously a bounded positive solution of (1) $x \in A$, such that $Tx = x$.

Case (b): $1 < p_3 \leq \int_a^b p(t, \theta) d\theta \leq p_2 < 2p_3 < +\infty$. Set $A = \{x \in \Lambda, M_3 \leq x(t) \leq M_4, t \geq t_0\}$, where M_3, M_4 are two positive constants such that $p_2M_4 + 2p_2M_3 < \alpha < 2p_3M_4, 2p_3 \leq r(t) \leq 2p_2$. From (3), one can choose a $t_1 \geq t_0 + b$, and a sufficiently large $t \geq t_1$ such that

$$\int_t^\infty \frac{(s-t)^{n-1}}{(n-1)!} [\beta_1q_1(s) + |h(s)|] ds \leq \Phi(2p_3M_4) - \Phi(\alpha), \tag{7}$$

$$\int_t^\infty \frac{(s-t)^{n-1}}{(n-1)!} [\beta_2 q_2(s) + |h(s)|] ds \leq \Phi(\alpha) - \Phi(p_2 M_4 - 2p_2 M_3), \tag{8}$$

$$\int_t^\infty \frac{(s-t)^{n-1}}{(n-1)!} [q_1(s) + q_2(s)] ds \leq \frac{2p_3 - p_2}{KL}, \tag{9}$$

and we define an operator T on A as follows:

$$(Tx)(t) = \begin{cases} \frac{1}{r(t)} \left\{ - \int_a^b p(t, \theta) x(t - \theta) d\theta \right. \\ \quad + \Phi^{-1} \left[\Phi(\alpha) + \int_t^\infty \frac{(s-t)^{n-1}}{(n-1)!} [q_1(s) g_1(x(s - \tau)) \right. \\ \quad \left. \left. - q_2(s) g_2(x(s - \sigma)) - h(s)] ds \right] \right\}, & t \geq t_1, \\ (Tx)(t_1), & t_0 \leq t \leq t_1. \end{cases}$$

It is easy to see that T is continuous, for $t \geq t_1, x \in A$. By using (7), we have

$$\begin{aligned} (Tx)(t) &\leq \frac{1}{r(t)} \left\{ \Phi^{-1} \left[\Phi(\alpha) + \int_t^\infty \frac{(s-t)^{n-1}}{(n-1)!} [q_1(s) g_1(x(s - \tau)) - h(s)] ds \right] \right\} \\ &\leq \frac{1}{2p_3} \left\{ \Phi^{-1} \left[\Phi(\alpha) + \int_t^\infty \frac{(s-t)^{n-1}}{(n-1)!} [\beta_1 q_1(s) + |h(s)|] ds \right] \right\} \\ &\leq M_4, \end{aligned}$$

and taking (8) into account, we have

$$\begin{aligned} (Tx)(t) &\geq \frac{1}{r(t)} \left\{ - \int_a^b p(t, \theta) x(t - \theta) d\theta \right. \\ &\quad \left. + \Phi^{-1} \left[\Phi(\alpha) + \int_t^\infty \frac{(s-t)^{n-1}}{(n-1)!} [q_2(s) g_2(x(s - \sigma)) - h(s)] ds \right] \right\} \\ &\geq \frac{1}{2p_2} \left\{ -p_2 M_4 - \Phi^{-1} \left[\Phi(\alpha) + \int_t^\infty \frac{(s-t)^{n-1}}{(n-1)!} (\beta_2 q_2(s) + |h(s)|) ds \right] \right\} \\ &\geq M_3. \end{aligned}$$

These show that $TA \subset A$. Since A is a bounded, closed, convex subset of Λ , in order to apply the contraction principle, we have to show that T is a contraction mapping on A . For $\forall x_1, x_2 \in A$, and $t \geq t_1$,

$$\begin{aligned} &|(Tx_1)(t) - (Tx_2)(t)| \\ &\leq \frac{1}{r(t)} \left\{ \int_a^b p(t, \theta) |x_1(t - \theta) - x_2(t - \theta)| d\theta \right. \\ &\quad + \Phi^{-1} \left[\Phi(\alpha) + \int_t^\infty \frac{(s-t)^{n-1}}{(n-1)!} |q_1(s) g_1(x_1(s - \tau)) - q_2(s) g_2(x_1(s - \sigma)) - h(s)| ds \right] \\ &\quad \left. - \Phi^{-1} \left[\Phi(\alpha) + \int_t^\infty \frac{(s-t)^{n-1}}{(n-1)!} |q_1(s) g_1(x_2(s - \tau)) - q_2(s) g_2(x_2(s - \sigma)) - h(s)| ds \right] \right\} \\ &\leq \frac{1}{r(t)} \left\{ p_2 \|x_1 - x_2\| + K \int_t^\infty \frac{(s-t)^{n-1}}{(n-1)!} [q_1(s) |g_1(x_1(s - \tau)) - g_1(x_2(s - \tau))| \right. \\ &\quad \left. + q_2(s) |g_2(x_1(s - \sigma)) - g_2(x_2(s - \sigma))|] ds \right\}, \end{aligned}$$

or using (9),

$$\begin{aligned} |(Tx_1)(t) - (Tx_2)(t)| &\leq |x_1 - x_2| \frac{1}{2p_3} \left\{ p_2 + KL \int_t^\infty \frac{(s-t)^{n-1}}{(n-1)!} [q_1(s) + q_2(s)] ds \right\} \\ &< |x_1 - x_2|. \end{aligned}$$

This implies with the sup norm that

$$\|Tx_1 - Tx_2\| < \|x_1 - x_2\|,$$

which shows that T is a contraction mapping on A and therefore there exists a unique solution, obviously a bounded positive solution of (1) $x \in A$, such that $Tx = x$.

Case (c): $-1 < p_4 \leq \int_a^b p(t, \theta) d\theta \leq 0$. Set $A = \{x \in \Lambda, M_5 \leq x(t) \leq M_6, t \geq t_0\}$, where M_5, M_6 are two positive constants such that $\frac{M_5}{-p_4} < \alpha < (1 + p_4)M_6, 1 \leq r(t) \leq \frac{1}{-p_4}$. From (3), one can choose a $t_1 \geq t_0 + b$, and a sufficiently large $t \geq t_1$ such that

$$\begin{aligned} \int_t^\infty \frac{(s-t)^{n-1}}{(n-1)!} [\beta_1 q_1(s) + |h(s)|] ds &\leq \Phi((1 + p_4)M_6) - \Phi(\alpha), \\ \int_t^\infty \frac{(s-t)^{n-1}}{(n-1)!} [\beta_2 q_2(s) + |h(s)|] ds &\leq \Phi(\alpha) - \Phi\left(-\frac{M_5}{p_4}\right), \\ \int_t^\infty \frac{(s-t)^{n-1}}{(n-1)!} [q_1(s) + q_2(s)] ds &\leq \frac{1 + p_4}{KL}. \end{aligned}$$

The remaining part of the proof of (c) is similar to the proof of part (a), therefore it is omitted.

Case (d): $-\infty < 2p_5 < p_6 \leq \int_a^b p(t, \theta) d\theta \leq p_5 < -1$. Set $A = \{x \in \Lambda, M_7 \leq x(t) \leq M_8, t \geq t_0\}$, where M_7, M_8 are two positive constants such that $-2p_6M_7 < \alpha < (-2p_5 + p_6)M_8, -2p_5 < r(t) < -2p_6$. From (3), one can choose a $t_1 \geq t_0 + b$, and a sufficiently large $t \geq t_1$ such that

$$\begin{aligned} \int_t^\infty \frac{(s-t)^{n-1}}{(n-1)!} [\beta_1 q_1(s) d\tau + |h(s)|] ds &\leq \Phi(-2p_5M_8 + p_6M_8) - \Phi(\alpha), \\ \int_t^\infty \frac{(s-t)^{n-1}}{(n-1)!} [\beta_2 q_2(s) + |h(s)|] ds &\leq \Phi(\alpha) - \Phi(-2p_6M_7), \\ \int_t^\infty \frac{(s-t)^{n-1}}{(n-1)!} [q_1(s) + q_2(s) d] ds &\leq \frac{p_6 - 2p_5}{KL}. \end{aligned}$$

The remaining part of the proof of (d) is similar to the proof of part (b), therefore it is omitted. The proof is complete. □

3 Example

Example Consider a high-order neutral differential equation with distributed deviating arguments

$$\begin{aligned} &\left[\left(\frac{3}{2 + \sin t} x(t) + \int_{\frac{\pi}{2}}^\pi e^{-t} x(t - \theta) d\theta \right)^3 \right]'' + 36e^{-t} x\left(t - \frac{\pi}{2}\right) - 36\pi e^{-2t} x(t - \pi) \\ &= e^{-3t} \left(15 \cos^3 t - 30 \sin^3 t + 63\pi \sin^2 t - 18\pi \cos^2 t \right) \end{aligned}$$

$$\begin{aligned} & + \frac{1}{2} \sin 2t(15 \cos t - 27 \sin^2 t + 33\pi) + 6\pi^2(\cos t - 7 \sin t) + 9\pi^3 \\ & + 18e^{-2t}(4\pi \cos t + \cos 2t - \sin 2t - 4\pi) - 18e^{-t}(2 \sin t - \pi - 4). \end{aligned} \quad (10)$$

Here, $n = 2$, $r(t) = \frac{3}{2+\sin t}$, $P(t, \theta) = e^{-t}$, $q_1(t) = 36e^{-t}$, $q_2(t) = 36\pi e^{-2t}$, $\Phi(u) = u^3$, $g_1(u) = g_2(u) = u$, $a = \frac{\pi}{2}$, $b = \pi$, $\tau = \frac{\pi}{2}$, $\sigma = \pi$. $1 < r(t) = \frac{3}{2+\sin t} < 3$, $\int_{\frac{\pi}{2}}^{\pi} e^{-t} d\theta = \frac{\pi}{2} e^{-t} < 1$, $\int_{t_0}^{\infty} se^{-s} ds < \infty$, $\int_{t_0}^{\infty} se^{-2s} ds < \infty$, $\int_{t_0}^{\infty} e^{-3s} |15 \cos^3 s - 30 \sin^3 s + 63\pi \sin^2 s - 18\pi \cos^2 s + \frac{1}{2} \sin 2s(15 \cos s - 27 \sin^2 s + 33\pi) + 6\pi^2(\cos s - 7 \sin s) + 9\pi^3| + 18e^{-2s} |4\pi \cos s + \cos 2s - \sin 2s - 4\pi| - 18e^{-s} |2 \sin s - \pi - 4| ds < \infty$.

Then it is easy to see that all the conditions of Theorem are satisfied. In fact, $x(t) = 2 + \sin t$ is a nonoscillatory solution of equation (10).

4 Results and discussion

We obtained a new sufficiency condition for the existence of nonoscillatory solutions of higher-order neutral differential equations with distributed coefficients and delays.

5 Conclusions

In this paper we use the Banach contraction principle to obtain a sufficiency condition for the existence of nonoscillatory solutions of a differential equation with coefficient of $\int_a^b p_2(t, \xi) d\xi$ in the four cases.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

YL conceived of the study and helped to draft the manuscript. HZ undertook the numerical calculation and a revision of the English. JY established the feasible operator and participated in its design. All authors read and approved the final manuscript.

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