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Sensitivity analysis for generalized set-valued parametric ordered variational inclusion with (α, λ) -NODSM mappings in ordered Banach spaces

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Abstract

In this paper, a new class of general set-valued parametric ordered variational inclusions, $\theta \in M(x, g(x, \rho), \rho)$, with (α, λ) -NODSM mappings is studied in ordered Banach spaces. Then, by using fixed point theory and the resolvent operator associated with (α, λ) -NODSM set-valued mappings, an existence theorem and a sensitivity analysis of the solution set for this kind of parametric variational inclusion is proved and discussed in ordered Banach spaces. The obtained results seem to be general in nature.

MSC: 49J40; 47H06

Keywords: sensitivity analysis; general set-valued parametric ordered variational inclusion; (α, λ) -NODSM set-valued mapping; ordered Banach spaces; comparison solution

1 Introduction

Generalized nonlinear ordered variational inequalities and inclusions (ordered equation) have wide applications in many fields including, for example, mathematics, physics, optimization and control, nonlinear programming, economics, and engineering sciences etc. In recent years, nonlinear mapping fixed point theory and applications have been extensively studied in ordered Banach spaces [1–3]. In 2008 the author introduced and studied the approximation algorithm and the approximation solution for a class of generalized nonlinear ordered variational inequalities and ordered equations, to find $x \in X$ such that $A(g(x)) \geq \theta(A(x))$ and $g(x)$ are single-valued mappings, in ordered Banach spaces [4]. By using the B -restricted-accretive method of the mapping A with constants α_1, α_2 , the author introduced and studied a new class of general nonlinear ordered variational inequalities and equations in ordered Banach spaces [5]. By using the resolvent operator associated with an RME set-valued mapping, the author introduced and studied a class of nonlinear inclusion problems for ordered MR set-valued mappings and the existence theorem of solutions and an approximation algorithm for this kind of nonlinear inclusion problems for ordered extended set-valued mappings in ordered Hilbert spaces [6]. In 2012, the author introduced and studied a class of nonlinear inclusion problems, to find $x \in X$ such that $0 \in M(x)$ ($M(x)$ is a set-valued mapping) for ordered (α, λ) -NODM

set-valued mappings, and he then, applying the resolvent operator associated with (α, λ) -*NODM* set-valued mappings, established the existence theorem on the solvability and a general algorithm applied to the approximation solvability of this class of nonlinear inclusion problems, based on the existence theorem and the new (α, λ) -*NODM* model in ordered Hilbert space [7]. For Banach spaces, the author made a sensitivity analysis of the solution for a new class of general nonlinear ordered parametric variational inequalities, to find $x = x(\lambda) : \Omega \rightarrow X$ such that $A(g(x, \lambda), \lambda) + f(x, \lambda) \geq \theta (A(x), g(x))$ and $F(\cdot, \cdot)$ are single-valued mappings) in 2012 [8]. In this field, the obtained results seem to be general in nature. In 2013, the author introduced and studied characterizations of ordered (α_A, λ) -weak-*ANODD* set-valued mappings, which was applied to finding an approximate solution for a new class of general nonlinear mixed-order quasi-variational inclusions involving the \oplus operator in ordered Banach spaces [9], and, applying the matrix analysis and the vector-valued mapping fixed point analysis method, he introduced and studied a new class of generalized nonlinear mixed-order variational inequalities systems with ordered B -restricted-accretive mappings for ordered Lipschitz continuous mappings in ordered Banach spaces [10].

On the other hand, as everyone knows, the sensitivity analysis for a class of general nonlinear variational inequalities (inclusions) has wide applications to many fields. In 1999, Noor and Noor have studied a sensitivity analysis for strongly nonlinear quasi-variational inclusions [11]. From 2000, Agarwal *et al.* have discussed a sensitivity analysis for strongly nonlinear quasi-variations in Hilbert spaces by using the resolvent operator technique [12]; furthermore, Bi *et al.* [13], Lan *et al.* [14, 15], Dong *et al.* [16], Jin [17], Verma [18], Li *et al.* [9], and Li [19] have shown the existence of solutions and made a sensitivity analysis for a class of nonlinear variational inclusions involving generalized nonlinear mappings in Banach spaces, respectively. Recently, it has become of the highest interest that we are studying a new class of nonlinear ordered inclusion problems for ordered (α, λ) -*NODSM* set-valued mappings and a sensitivity analysis of the solution set for this kind of parametric variational inclusions in ordered Banach spaces by using the resolvent operator technique [20] associated with ordered (α, λ) -*NODM* set-valued mappings. For details, we refer the reader to [1–35] and the references therein.

Let X be a real ordered Banach space with a norm $\|\cdot\|$, zero θ , and a partial ordering relation \leq defined by the normal cone \mathbf{P} , and a normal constant N of \mathbf{P} [4]. Let Ω be a nonempty open subset of X and we have the parametric $\rho \in \Omega$. Let $x = x(\rho) \in X$ ($\rho \in \Omega$), $g(x, \rho) : X \times \Omega \rightarrow X$ be a single-valued mapping and $M(x, g(x, \rho), \rho) : X \times X \times \Omega \rightarrow 2^X$ be a set-valued mapping. We consider the following problem:

Find $x = x(\rho) \in X$ ($\rho \in \Omega$) such that

$$0 \in M(x, g(x, \rho), \rho), \tag{1.1}$$

and the solution $x(\rho)$ of the inclusion problem (1.1) is continuous from Ω and X .

Problem (1.1) is called a nonlinear generalized set-valued parametric ordered variational inclusions for ordered (α, λ) -*NODSM* set-valued mappings in ordered Banach spaces.

Remark 1.1 When mapping M is single-valued and $M(x, y) = A(g(x))$, then the problem (1.1) reduces to problem (2.1) in [4].

When the mapping $M(x, y) = M(x)$ is set-valued, then the problem (1.1) reduces to problem (1.1) in [7].

Inspired and motivated by recent research work in this field, in this paper, a new class of nonlinear generalized parametric ordered variational inclusions with (α, λ) -NODSM mappings is studied in ordered Banach spaces. Then, by using the resolvent operator associated with (α, λ) -NODSM set-valued mappings, an existence theorem of this class of nonlinear inclusions is established, and a sensitivity analysis of the solution set for this kind of parametric variational inclusions is proved and discussed in ordered Banach spaces. The obtained results seem to be general in nature.

2 Preliminaries

Let X be a real ordered Banach space with a norm $\|\cdot\|$, a zero θ , a normal cone \mathbf{P} , a normal constant N and a partial ordering relation \leq defined by the cone \mathbf{P} . For arbitrary $x, y \in X$, $\text{lub}\{x, y\}$ and $\text{glb}\{x, y\}$ express the least upper bound of the set $\{x, y\}$ and the greatest lower bound of the set $\{x, y\}$ on the partial ordering relation \leq , respectively. Suppose that $\text{lub}\{x, y\}$ and $\text{glb}\{x, y\}$ exist. Let us recall some concepts and results.

Definition 2.1 [4, 24] Let X be a real Banach space with a norm $\|\cdot\|$, θ be a zero element in X .

- (i) A nonempty closed convex subset \mathbf{P} of X is said to be a cone if (1) for any $x \in \mathbf{P}$ and any $\lambda > 0$, $\lambda x \in \mathbf{P}$ holds, (2) if $x \in \mathbf{P}$ and $-x \in \mathbf{P}$, then $x = \theta$;
- (ii) \mathbf{P} is said to be a normal cone if and only if there exists a constant $N > 0$, a normal constant of \mathbf{P} such that for $\theta \leq x \leq y$, $\|x\| \leq N\|y\|$ holds;
- (iii) for arbitrary $x, y \in X$, $x \leq y$ if and only if $x - y \in \mathbf{P}$;
- (iv) for $x, y \in X$, x and y are said to be comparative to each other, if and only if $x \leq y$ (or $y \leq x$) holds (denoted by $x \propto y$ for $x \leq y$ and $y \leq x$).

Lemma 2.2 If $x \propto y$, then $\text{lub}\{x, y\}$ and $\text{glb}\{x, y\}$ exist, $x - y \propto y - x$, and $\theta \leq (x - y) \vee (y - x)$.

Proof If $x \propto y$, then $x \leq y$ or $y \leq x$. Let $x \leq y$, then $\text{lub}\{x, y\} = y$ and $\text{glb}\{x, y\} = x$, and $x - y \leq \theta \leq y - x$. It follows that $\text{lub}\{x, y\}$ and $\text{glb}\{x, y\}$ exist, and $x - y \propto y - x$. $(x - y) \vee (y - x) = (y - x)$, then $\theta \leq (x - y) \vee (y - x)$. \square

Lemma 2.3 If for any natural number n , $x \propto y_n$, and $y_n \rightarrow y^*$ ($n \rightarrow \infty$), then $x \propto y^*$.

Proof If for any natural number n , $x \propto y_n$ and $y_n \rightarrow y^*$ ($n \rightarrow \infty$), then $x - y_n \in \mathbf{P}$ or $y_n - x \in \mathbf{P}$ for any natural number n . Since \mathbf{P} is a nonempty closed convex subsets of X , we have $x - y^* = \lim_{n \rightarrow \infty} (x - y_n) \in \mathbf{P}$ or $y^* - x = \lim_{n \rightarrow \infty} (y_n - x) \in \mathbf{P}$. Therefore, $x \propto y^*$. \square

Lemma 2.4 [4–6] Let X be an ordered Banach space, let \mathbf{P} be a cone of X , let \leq be a relation defined by the cone \mathbf{P} in Definition 2.1(iii). For $x, y, v, u \in X$, the following relations hold:

- (1) the relation \leq in X is a partial ordering relation in X ;
- (2) $x \oplus y = y \oplus x$;
- (3) $x \oplus x = \theta$;
- (4) $\theta \leq x \oplus \theta$;
- (5) let λ be a real, then $(\lambda x) \oplus (\lambda y) = |\lambda|(x \oplus y)$;
- (6) if x, y , and w can be comparative to each other, then $(x \oplus y) \leq x \oplus w + w \oplus y$;

- (7) let $(x + y) \vee (u + v)$ exist, and if $x \propto u, v$ and $y \propto u, v$, then $(x + y) \oplus (u + v) \leq (x \oplus u + y \oplus v) \wedge (x \oplus v + y \oplus u)$;
- (8) if x, y, z, w can be compared with each other, then $(x \wedge y) \oplus (z \wedge w) \leq ((x \oplus z) \vee (y \oplus w)) \wedge ((x \oplus w) \vee (y \oplus z))$;
- (9) if $x \leq y$ and $u \leq v$, then $x + u \leq y + v$;
- (10) if $x \propto \theta$, then $-x \oplus \theta \leq x \leq x \oplus \theta$;
- (11) if $x \propto y$, then $(x \oplus \theta) \oplus (y \oplus \theta) \leq (x \oplus y) \oplus \theta = x \oplus y$;
- (12) $(x \oplus \theta) - (y \oplus \theta) \leq (x - y) \oplus \theta$;
- (13) if $\theta \leq x$ and $x \neq \theta$, and $\alpha > 0$, then $\theta \leq \alpha x$ and $\alpha x \neq \theta$.

Proof (1)-(8) come from Lemma 2.5 in [4] and Lemma 2.3 in [5], and (8)-(13) directly follow from (1)-(8). □

Definition 2.5 Let X be a real ordered Banach space, let Ω be a nonempty open subset of X in which the parametric ρ takes values, let $x = x(\rho) \in X$ ($\rho \in \Omega$), $g(x, \rho) : X \times \Omega \rightarrow X$ be a single-valued mapping and $M(x, g(x, \rho), \rho) : X \times X \times \Omega \rightarrow 2^X$ be a set-valued mapping and $M(x, \cdot, \rho)$ be a nonempty closed subset in X .

- (1) A set-valued mapping M is said to be a comparison mapping, if for any $v_x \in M(x, \cdot, \cdot)$, $x \propto v_x$, and if $x \propto y$, then for any $v_x \in M(x, \cdot, \cdot)$ and any $v_y \in M(y, \cdot, \cdot)$, $v_x \propto v_y$ ($\forall x, y \in X$).
- (2) A set-valued mapping M is said to be a comparison mapping with respect to g , if for any $v_x \in M(\cdot, g(x), \cdot)$, $x \propto v_x$, and if $x \propto y$, then for any $v_x \in M(\cdot, g(x), \cdot)$ and any $v_y \in M(\cdot, g(y), \cdot)$, $v_x \propto v_y$ ($\forall x, y \in X$).
- (3) A comparison mapping M is said to be an α -non-ordinary difference mapping, if there exists a constant $\alpha > 0$, for each $x, y \in X$, $v_x \in M(x, \cdot, \cdot)$, and $v_y \in M(y, \cdot, \cdot)$ such that

$$(v_x \oplus v_y) \oplus \alpha(x \oplus y) = \theta.$$

- (4) A comparison mapping M is said to be λ -ordered strongly monotonic increase mapping, if for $x \geq y$ there exists a constant $\lambda > 0$ such that

$$\lambda(v_x - v_y) \geq x - y \quad \forall x, y \in X, v_x \in M(x), v_y \in M(y, \cdot, \cdot).$$

- (5) A comparison mapping M is said to be a (α, λ) -NODSM mapping, if M is a α -non-ordinary difference and λ -ordered strongly monotone increasing mapping, and $(I + \lambda M(x, \cdot, \cdot))(X) = X$ for $\alpha, \lambda > 0$.

Obviously, if M is a comparison mapping, then $M(x, \cdot, \cdot) \propto I$ ($\forall x \in X$).

Definition 2.6 [4] Let X be a real ordered Banach space, \mathbf{P} be a normal cone with a normal constant N in X ; a mapping $A : X \times X \rightarrow X$ is said to be β -ordered compression, if A is comparison, and there exists a constant $0 < \beta < 1$ such that

$$(A(x, \cdot) \oplus A(y, \cdot)) \leq \beta(x \oplus y).$$

Definition 2.7 [4] Let X be a real ordered Banach space. A mapping $A : X \times X \rightarrow X$ is said to be a restricted-accretive mapping with constants (α_1, α_2) , if A is a comparison, and

there exist two constants $0 < \beta_1, \beta_2 \leq 1$ such that for arbitrary $x, y \in X$,

$$(A(x, \cdot) + I(x)) \oplus (A(y, \cdot) + I(y)) \leq \beta_1(A(x, \cdot) \oplus A(y, \cdot)) + \beta_2(x \oplus y)$$

holds, where I is the identity mapping on X .

Definition 2.8 Let X be a real ordered Banach space, let Ω be a nonempty open subset of X in which the parametric ρ takes values, let $x = x(\rho) \in X$ ($\rho \in \Omega$). $x = x(\rho)$ is said to be a comparison element when, if $\rho_1 \neq \rho_2$ then $x(\rho_1) \neq x(\rho_2)$ for any $\rho_1, \rho_2 \in \Omega$.

Lemma 2.9 Let $M = M(x, \cdot, \cdot) : X \times X \times X \rightarrow 2^X$. If M is a α -non-ordinary difference mapping, then an inverse mapping $J_{M,\lambda} = (I + \lambda M)^{-1} : X \times X \times X \rightarrow 2^X$ of $(I + \lambda M)$ is a single-valued mapping ($\alpha, \lambda > 0$), where I is the identity mapping on X .

Proof Let $u \in X$, and x and y be two elements in $(I + \lambda M)^{-1}(u)$. It follows that $u - x \in \lambda M(x, \cdot, \cdot)$ and $u - y \in \lambda M(y, \cdot, \cdot)$, and

$$\frac{1}{\lambda}(u - x) \oplus \frac{1}{\lambda}(u - y) = \left| \frac{1}{\lambda} \right| (x \oplus y).$$

Since M is a α -non-ordinary difference mapping, we have

$$\begin{aligned} 0 &= \left(\frac{1}{\lambda}(u - x) \oplus \frac{1}{\lambda}(u - y) \right) \oplus \alpha(x \oplus y) = \left| \frac{1}{\lambda} \right| (x \oplus y) \oplus \alpha(x \oplus y) \\ &= \left| \frac{1}{\lambda} \right| + \alpha \left| (x \oplus y) \right| \end{aligned}$$

and $x \oplus y = 0$ from Lemma 2.4. Also, $x = y$ holds. Thus $(I + \lambda M)^{-1}(u)$ is a single-valued mapping. The proof is completed. \square

Definition 2.10 Let X be a real ordered Banach space, let \mathbf{P} be a normal cone with normal constant N in the X , let $M = M(x, \cdot, \cdot) : X \times X \times X \rightarrow 2^X$ be a α -non-ordinary difference mapping. The resolvent operator $J_{M,\lambda} : X \times X \times X \rightarrow X$ of the $M(x, \cdot, \cdot)$ is defined by

$$J_{M,\lambda}(x) = (I + \lambda M)^{-1}(x) \quad \text{for all } x \in X,$$

where $\lambda > 0$ is a constant.

3 Existence theorem of the solution

In this section, we will show an existence theorem on the solvability of this class of non-linear inclusion problems (1.1).

Theorem 3.1 Let X be an ordered Banach space, let \mathbf{P} be a normal cone with the normal constant N in X , let \leq be an ordering relation defined by the cone \mathbf{P} . If $M = M(x, \cdot, \cdot) : X \times X \times \Omega \rightarrow 2^X$ is an α -non-ordinary difference mapping, then the inclusion problem (1.1) has a solution x if and only if $g(x, \cdot) = J_{M(x, \cdot, \cdot), \lambda} g(x, \cdot)$ in X .

Proof This directly follows from the definition of the resolvent operator $J_{M,\lambda}$ of $M(x, \cdot, \cdot)$. \square

Theorem 3.2 *Let X be an ordered Banach space, let \mathbf{P} be a normal cone with the normal constant N in the X , let \leq be an ordering relation defined by the cone \mathbf{P} , the operator \oplus be a XOR operator. If $M = M(x, \cdot, \cdot) : X \times X \times \Omega \rightarrow 2^X$ is an (α, λ) -NODSM mapping with respect to $J_{M,\lambda}$, then the resolvent operator $J_{M,\lambda} : X \rightarrow X$ is a comparison mapping.*

Proof Since $M = M(x, \cdot, \cdot) : X \times X \times \Omega \rightarrow 2^X$ is an α -non-ordinary difference mapping and a comparison mapping with respect to $J_{M,\lambda}$ so that $x \propto J_{M,\lambda}(x)$. For any $x, y \in X$, let $x \propto y$, and $v_x = \frac{1}{\lambda}(x - J_{M,\lambda}(x)) \in M(J_{M(x,\cdot),\lambda}(y))$ and $v_y = \frac{1}{\lambda}(y - J_{M(x,\cdot),\lambda}(y)) \in M(J_{M(x,\cdot),\lambda}(y))$. Setting

$$v_x - v_y = \frac{1}{\lambda}(x - y + J_{M(x,\cdot),\lambda}(y) - J_{M(x,\cdot),\lambda}(x)),$$

by using the λ -order strongly monotonicity of M , we have

$$\theta \leq \lambda(v_x - v_y) - (x - y) = J_{M,\lambda}(y) - J_{M,\lambda}(x), \tag{3.1}$$

and if $y \leq x$ then $\lambda(v_x - v_y) - (x - y) \in \mathbf{P}$, and if $x \leq y$ then $(x - y) - \lambda(v_x - v_y) \in \mathbf{P}$. Therefore $J_{M,\lambda}(y) \propto J_{M,\lambda}(x)$ for Lemma 2.4. \square

Theorem 3.3 *Let X be an ordered Banach space, let \mathbf{P} be a normal cone with the normal constant N in X , let \leq be an ordering relation defined by the cone \mathbf{P} . Let $M = M(\cdot, x, \cdot) : X \times X \times \Omega \rightarrow 2^X$ be a NODSM set-valued mapping with respect to $J_{M,\lambda}$. If $\alpha > \frac{1}{\lambda} > 0$, then for the resolvent operator $J_{M,\lambda} : X \rightarrow X$, the following relation holds:*

$$J_{M,\lambda}(y) \oplus J_{M,\lambda}(z) \leq \frac{1}{(\alpha\lambda - 1)}(y \oplus z). \tag{3.2}$$

Proof Let $M = M(\cdot, x, \cdot) : X \times X \times \Omega \rightarrow 2^X$ be a NODSM set-valued mapping with respect to $J_{M,\lambda}$. For $y, z \in X$, let $u_y = J_{M,\lambda}(y) \propto u_z = J_{M,\lambda}(z)$, $v_y = \frac{1}{\lambda}(y - u_y) \in M(\cdot, u_y, \cdot)$ and $v_z = \frac{1}{\lambda}(z - u_z) \in M(\cdot, u_z, \cdot)$, then $v_y \propto v_z$ for $y \propto z$. Since $M(\cdot, x, \cdot) : X \times X \times X \rightarrow 2^X$ is an (α, λ) -NODSM mapping with respect to the $J_{M,\lambda}$, the following relation holds by Lemma 2.4 and the condition $(v_y \oplus v_z) \oplus \alpha(u_y \oplus u_z) = \theta$:

$$\frac{1}{\lambda}(y \oplus z) + (u_y \oplus u_z) \geq v_y \oplus v_z = \alpha(u_y \oplus u_z).$$

It follows that $(\lambda\alpha - 1)(u_y \oplus u_z) \leq (y \oplus z)$ and $J_{M,\lambda}(y) \oplus J_{M,\lambda}(z) \leq \frac{1}{(\alpha\lambda - 1)}(y \oplus z)$ from the condition $\alpha > \frac{1}{\lambda} > 0$. The proof is completed. \square

Theorem 3.4 *Let X be an ordered Banach space, let \mathbf{P} be a normal cone with the normal constant N in the X , let \leq be an ordering relation defined by the cone \mathbf{P} . Let $M = M(x, \cdot, \cdot) : X \times X \times \Omega \rightarrow 2^X$ be an (α, λ) -NODSM set-valued mapping with respect to the first argument and $g : X \times \Omega \rightarrow X$ be a γ -ordered compression and an 1-ordered strongly monotonic increase with respect to the first argument and $\text{range}(g) \cap \text{dom} M(\cdot, x, \cdot) \neq \emptyset$, and $J_{M,\lambda}$ for M with respect to the first argument and $(J_{M,\lambda} - I)$ for M with respect to the second argument be two restricted-accretive mappings with constants (ξ_1, ξ_2) and (β_1, β_2) , respectively, and $g \propto J_{M,\lambda}$. Suppose that for any $x, y, z \in X$*

$$J_{M(x,\cdot),\lambda}(z) \oplus J_{M(y,\cdot),\lambda}(z) \leq \delta(x \oplus y) \tag{3.3}$$

and

$$\gamma \left(\frac{\xi_1}{\alpha\lambda - 1} \oplus \xi_2 \right) \oplus \delta < \frac{1 - N\beta_2}{N\beta_1} \tag{3.4}$$

hold. For any parametric $\rho \in \Omega$, for the nonlinear parametric inclusion problem (1.1) there exists a solution x^* .

Proof Let X be a real ordered Banach space, let \mathbf{P} be a normal cone with normal constant N in the X , let \leq be an ordering relation defined by the cone \mathbf{P} , let Ω be a nonempty open subset of X in which the parametric ρ takes values, let $M = M(x, \cdot, \cdot) : X \times X \times \Omega \rightarrow 2^X$, and for any given $\rho \in \Omega$ and $x_1 = x_1(\rho), x_2 = x_2(\rho) \in X$ for $\lambda > 0$. If $x_1(\rho) \propto x_2(\rho)$, and setting

$$F(x_i(\rho), \rho) = x_i(\rho) - g(x_i, \rho) + J_{M,\lambda}(g(x_i, \rho)), \tag{3.5}$$

where $i = 1, 2$, by (3.1) and the λ -ordered strongly monotonicity of M ,

$$\begin{aligned} & F(x_1(\rho), \rho) - F(x_2(\rho), \rho) \\ &= (x_1(\rho) - x_2(\rho)) + (g(x_2, \rho) - g(x_1, \rho)) + (J_{M,\lambda}(g(x_1, \rho)) - J_{M,\lambda}(g(x_2, \rho))) \\ &= (x_1(\rho) - x_2(\rho)) + \lambda(v_{g(x_2)} - v_{g(x_1)}) \\ &\leq (x_1(\rho) - x_2(\rho)) - (g(x_1, \rho) - g(x_2, \rho)) \\ &\leq \theta; \end{aligned}$$

by $(I + \lambda M)(X) = X$, the comparability of $J_{M,\lambda}$, and the 1-ordered monotonic increase of $g(x, \cdot)$, it follows from $x_1(\rho) \propto x_2(\rho)$ that $F(x_1(\rho), \rho) \propto F(x_2(\rho), \rho)$. Using (3.3), (3.5), Lemma 2.4, Theorem 3.3, and $\alpha > \frac{2}{\lambda} > 0$, from the conditions that $J_{M,\lambda}$ for M with respect to the first argument and $(J_{M,\lambda} - I)$ for M with respect to the second argument are two restricted-accretive mappings with constants (ξ_1, ξ_2) and (β_1, β_2) , respectively, it follows that

$$\begin{aligned} \theta &\leq F(x_1(\rho), \rho) \oplus F(x_2(\rho), \rho) \\ &\leq (x_1(\rho) - g(x_1, \rho) + J_{M,\lambda}(g(x_1, \rho))) \oplus (x_2(\rho) - g(x_2, \rho) + J_{M,\lambda}(g(x_2, \rho))) \\ &\leq \beta_2(x_1(\rho) \oplus x_2(\rho)) + \beta_1 [(J_{M(x_1(\rho), \cdot, \rho), \lambda}(g(x_1(\rho), \rho)) - g(x_1(\rho), \rho)) \\ &\quad \oplus (J_{M(x_2(\rho), \cdot, \rho), \lambda}(g(x_2(\rho), \rho)) - g(x_2(\rho), \rho)))] \\ &\leq \beta_2(x_1(\rho) \oplus x_2(\rho)) + \beta_1 \{ [(J_{M(x_1(\rho), \cdot, \rho), \lambda}(g(x_1(\rho), \rho)) - g(x_1(\rho), \rho)) \\ &\quad \oplus (J_{M(x_2(\rho), \cdot, \rho), \lambda}(g(x_1(\rho), \rho)) - g(x_1(\rho), \rho)))] \\ &\quad \oplus [(J_{M(x_2(\rho), \cdot, \rho), \lambda}(g(x_1(\rho), \rho)) - g(x_1(\rho), \rho)) \\ &\quad \oplus (J_{M(x_2(\rho), \cdot, \rho), \lambda}(g(x_2(\rho), \rho)) - g(x_2(\rho), \rho)))] \} \\ &\leq \beta_2(x_1(\rho) \oplus x_2(\rho)) + \beta_1 \{ \delta(x_1(\rho) \oplus x_2(\rho)) \oplus [\xi_2(g(x_1(\rho), \rho) \oplus g(x_2(\rho), \rho)) \\ &\quad + \xi_1(J_{M(x_2(\rho), \cdot, \rho), \lambda}(g(x_1(\rho), \rho)) \oplus J_{M(x_2(\rho), \cdot, \rho), \lambda}(g(x_2(\rho), \rho)))] \} \\ &\leq \beta_2(x_1(\rho) \oplus x_2(\rho)) + \beta_1 \left\{ \delta(x_1(\rho) \oplus x_2(\rho)) \right. \end{aligned}$$

$$\begin{aligned}
 & \oplus \left[\xi_2 \gamma(x_1(\rho) \oplus x_2(\rho)) + \frac{\xi_1}{\alpha\lambda - 1} (g(x_1(\rho), \rho) \oplus g(x_2(\rho), \rho)) \right] \Big\} \\
 & \leq \beta_2(x_1(\rho) \oplus x_2(\rho)) \\
 & \quad + \beta_1 \left\{ \delta(x_1(\rho) \oplus x_2(\rho)) \oplus \left[\xi_2 \gamma(x_1(\rho) \oplus x_2(\rho)) + \frac{\xi_1}{\alpha\lambda - 1} (\gamma(x_n \oplus x_{n-1})) \right] \right\} \\
 & \leq \beta_2(x_n \oplus x_{n-1}) + \beta_1 \left\{ \delta(x_1(\rho) \oplus x_2(\rho)) \oplus \left[\left(\xi_2 \oplus \frac{\xi_1}{\alpha\lambda - 1} \right) \gamma(x_1(\rho) \oplus x_2(\rho)) \right] \right\} \\
 & \leq \beta_2(x_1(\rho) \oplus x_2(\rho)) + \beta_1 \left(\left| \delta - \left(\xi_2 \oplus \frac{\xi_1}{\alpha\lambda - 1} \right) \gamma \right| (x_1(\rho) \oplus x_2(\rho)) \right) \\
 & \leq \beta_2(x_1(\rho) \oplus x_2(\rho)) + \beta_1 \left[\left(\xi_2 \oplus \xi_1 \frac{1}{\alpha\lambda - 1} \right) \gamma \oplus \delta \right] (x_1(\rho) \oplus x_2(\rho)) \\
 & \leq \beta_2(x_1(\rho) \oplus x_2(\rho)) + \beta_1 \left[\left(\frac{\xi_1}{\alpha\lambda - 1} \oplus \xi_2 \right) \gamma \oplus \delta \right] (x_1(\rho) \oplus x_2(\rho)) \\
 & \leq \left[\beta_2 + \beta_1 \left(\gamma \left(\frac{\xi_1}{\alpha\lambda - 1} \oplus \xi_2 \right) \oplus \delta \right) \right] (x_1(\rho) \oplus x_2(\rho)), \tag{3.6}
 \end{aligned}$$

and, by Definition 2.1(2), we obtain

$$\|F(x_1(\rho), \rho) - F(x_2(\rho), \rho)\| \leq hN \|x_1(\rho) - x_2(\rho)\|, \tag{3.7}$$

where $h = \beta_2 + \beta_1(\gamma(\frac{\xi_1}{\alpha\lambda-1} \oplus \xi_2) \oplus \delta)$. It follows from the condition (3.4) that $0 < hN < 1$, and $F(x(\rho), \rho)$ has a fixed point $x^* \in X$ and the x^* is a solution of the generalized nonlinear ordered parametric equation

$$x^*(\rho) = x^*(\rho) - g(x^*(\rho), \rho) + J_{M,\lambda}(g(x^*(\rho), \rho)).$$

Further, x^* satisfies the generalized nonlinear ordered parametric equation

$$g(x^*(\rho), \rho) = J_{M,\lambda}(g(x^*(\rho), \rho)).$$

Then for the nonlinear parametric inclusion problems (1.1) there exists a solution $x^* \in X$ for any parametric $\rho \in \Omega$. This completes the proof. \square

Remark 3.5 Though the method of solving problem by the resolvent operator is the same as in [20, 25–28] and [34] for the nonlinear inclusion problem, the character of the ordered (α, λ) -ANODM set-valued mapping is different from the one of the (A, η) -accretive mapping [25], the (H, η) -monotone mapping [26], the (G, η) -monotone mapping [27] and the monotone mapping [34].

4 Sensitivity analysis of the solution

Theorem 4.1 *Let X be an ordered Banach space, let \mathbf{P} be a normal cone with the normal constant N in the X , let \leq be an ordering relation defined by the cone \mathbf{P} . Let $M = M(x, \cdot, \cdot) : X \times X \times \Omega \rightarrow 2^X$ be a (α, λ) -NODSM set-valued mapping and $g : X \times \Omega \rightarrow X$ be a γ -ordered compression, continuous and 1-ordered monotonic increase of $g(x, \cdot)$ with respect to first argument $\rho \in \Omega$, and $\text{range}(g) \cap \text{dom } M(\cdot, x, \rho) \neq \emptyset$, and $J_{M,\lambda}$ for M with respect to first argument and $(J_{M,\lambda} - I)$ for M with respect to second argument be two restricted-accretive*

mappings with constants (ξ_1, ξ_2) and (β_1, β_2) , respectively, and $g \in J_{M,\lambda}$. Suppose that for any $x, y, z \in X$

$$J_{M(x,\cdot),\lambda}(z) \oplus J_{M(y,\cdot),\lambda}(z) \leq \delta(x \oplus y) \tag{4.1}$$

and

$$\gamma \left(\frac{\xi_1}{\alpha\lambda - 1} \oplus \xi_2 \right) \oplus \delta < \frac{1 - N\beta_2}{N\beta_1} \tag{4.2}$$

hold; if the solution $x(\rho)$ of the nonlinear parametric inclusion problem (1.1) is a comparison element, which is said to be a comparison solution of the nonlinear parametric inclusion problem (1.1), then $x(\rho)$, a comparison solution, is continuous on Ω .

Proof For any given $\rho, \bar{\rho} \in \Omega$, by Theorem 3.4, let $x(\rho)$ be a comparison solution, and $x(\rho)$ and $x(\bar{\rho})$ satisfy parametric problem (1.1), then for any $\lambda > 0$, we have

$$\begin{aligned} x(\rho) &= F(x(\rho), \rho) = x(\rho) - g(x(\rho), \rho) + J_{M,\lambda}(g(x(\rho), \rho)), \\ x(\bar{\rho}) &= F(x(\bar{\rho}), \bar{\rho}) = x(\bar{\rho}) - g(x(\bar{\rho}), \bar{\rho}) + J_{M,\lambda}(g(x(\bar{\rho}), \bar{\rho})). \end{aligned} \tag{4.3}$$

By the condition that M , g , $J_{M,\lambda}$, and $J_{M,\lambda} - I$ are comparisons for each other and by Lemma 2.4, we have

$$\begin{aligned} \theta &\leq x(\rho) \oplus x(\bar{\rho}) \leq F(x(\rho), \rho) \oplus F(x(\bar{\rho}), \bar{\rho}) \\ &\leq F(x(\rho), \rho) \oplus \theta \oplus F(x(\bar{\rho}), \bar{\rho}) \\ &\leq [F(x(\rho), \rho) \oplus F(x(\bar{\rho}), \rho)] \oplus [F(x(\bar{\rho}), \rho) \oplus F(x(\bar{\rho}), \bar{\rho})]. \end{aligned} \tag{4.4}$$

Further, $J_{M,\lambda}$ and $(J_{M,\lambda} - I)$ are two restricted-accretive mappings with constants (ξ_1, ξ_2) and (β_1, β_2) , respectively, so that from Lemma 2.4 and Theorem 3.3, $\alpha > \frac{2}{\lambda} > 0$, and from (3.6), it follows that

$$\begin{aligned} &F(x(\rho), \rho) \oplus F(x(\bar{\rho}), \rho) \\ &\leq (x(\rho) - g(x(\rho), \rho) + J_{M,\lambda}(g(x(\rho), \rho))) \oplus (x(\bar{\rho}) - g(x(\bar{\rho}), \rho) + J_{M,\lambda}(g(x(\bar{\rho}), \rho))) \\ &\leq h(x(\rho) \oplus x(\bar{\rho})), \end{aligned} \tag{4.5}$$

where $h = \beta_2 + \beta_1(\gamma_1(\frac{\xi_1}{\alpha\lambda - 1} \oplus \xi_2)) \oplus \delta < \frac{1}{N}$ for the condition (4.1), and

$$\begin{aligned} &F(x(\bar{\rho}), \rho) \oplus F(x(\bar{\rho}), \bar{\rho}) \\ &\leq (x(\bar{\rho}) - g(x(\bar{\rho}), \rho) + J_{M,\lambda}(g(x(\bar{\rho}), \rho))) \oplus (x(\bar{\rho}) - g(x(\bar{\rho}), \bar{\rho}) + J_{M,\lambda}(g(x(\bar{\rho}), \bar{\rho}))) \\ &\leq \beta_2\theta + \beta_1 [(J_{M(x(\bar{\rho}),\cdot),\lambda}(g(x(\bar{\rho}), \rho)) - g(x(\bar{\rho}), \rho)) \\ &\quad \oplus (J_{M(x(\bar{\rho}),\cdot),\lambda}(g(x(\bar{\rho}), \bar{\rho})) - g(x(\bar{\rho}), \bar{\rho}))] \\ &\leq \beta_2\theta + \beta_1 [\xi_2(g(x(\bar{\rho}), \rho) \oplus g(x(\bar{\rho}), \bar{\rho})) \\ &\quad + \xi_1(J_{M(x(\bar{\rho}),\cdot),\lambda}(g(x(\bar{\rho}), \rho)) \oplus J_{M(x(\bar{\rho}),\cdot),\lambda}(g(x(\bar{\rho}), \bar{\rho})))] \end{aligned}$$

$$\begin{aligned}
 &\leq \beta_2\theta + \beta_1[\xi_2(g(x(\bar{\rho}), \rho) \oplus g(x(\bar{\rho}), \bar{\rho})) \\
 &\quad + \xi_1((\rho \oplus \bar{\rho}) \oplus (J_{M(x(\bar{\rho}), \cdot, \bar{\rho}), \lambda}(g(x(\bar{\rho}), \rho)) \oplus J_{M(x(\bar{\rho}), \cdot, \bar{\rho}), \lambda}(g(x(\bar{\rho}), \bar{\rho})))))] \\
 &\leq \beta_2\theta + \beta_1[\xi_2(g(x(\bar{\rho}), \rho) \oplus g(x(\bar{\rho}), \bar{\rho})) \\
 &\quad + \xi_1((\rho \oplus \bar{\rho}) \oplus (\delta(g(x(\bar{\rho}), \rho) \oplus g(x(\bar{\rho}), \bar{\rho})))))] \\
 &\leq \beta_2\theta + \beta_1\xi_2(g(x(\bar{\rho}), \rho) \oplus g(x(\bar{\rho}), \bar{\rho})) \\
 &\quad + \beta_1\xi_1((\rho \oplus \bar{\rho}) \oplus (\delta(g(x(\bar{\rho}), \rho) \oplus g(x(\bar{\rho}), \bar{\rho}))))). \tag{4.6}
 \end{aligned}$$

Combining (4.4), (4.5), and (4.6), and by using Lemma 2.4, we get

$$\begin{aligned}
 (x(\rho) \oplus x(\bar{\rho})) &\leq h(x(\rho) \oplus x(\bar{\rho})) \oplus [\beta_2\theta + \beta_1\xi_2(g(x(\bar{\rho}), \rho) \oplus g(x(\bar{\rho}), \bar{\rho})) \\
 &\quad + \beta_1\xi_1((\rho \oplus \bar{\rho}) \oplus (\delta(g(x(\bar{\rho}), \rho) \oplus g(x(\bar{\rho}), \bar{\rho})))))].
 \end{aligned}$$

Therefore,

$$\begin{aligned}
 &(x(\rho) \oplus x(\bar{\rho})) \oplus h(x(\rho) \oplus x(\bar{\rho})) \\
 &\leq \beta_2\theta + \beta_1\xi_2(g(x(\bar{\rho}), \rho) \oplus g(x(\bar{\rho}), \bar{\rho})) \\
 &\quad + \beta_1\xi_1((\rho \oplus \bar{\rho}) \oplus (\delta(g(x(\bar{\rho}), \rho) \oplus g(x(\bar{\rho}), \bar{\rho}))))).
 \end{aligned}$$

It follows that

$$\begin{aligned}
 (x(\rho) \oplus x(\bar{\rho})) &\leq \frac{1}{1 \oplus h} [\beta_2\theta + \beta_1\xi_2(g(x(\bar{\rho}), \rho) \oplus g(x(\bar{\rho}), \bar{\rho})) \\
 &\quad + \beta_1\xi_1((\rho \oplus \bar{\rho}) \oplus (\delta(g(x(\bar{\rho}), \rho) \oplus g(x(\bar{\rho}), \bar{\rho})))))]. \tag{4.7}
 \end{aligned}$$

By Lemma 2.4, $\beta_2\theta = \theta$, and continuity of g with respect to the first argument $\rho \in \Omega$, we have

$$\lim_{\rho \rightarrow \bar{\rho}} x(\rho) \oplus x(\bar{\rho}) = \theta$$

and

$$\lim_{\rho \rightarrow \bar{\rho}} \|x(\rho) - x(\bar{\rho})\| = 0, \tag{4.8}$$

which implies that the solution $x(\rho)$ of problem (1.1) is continuous at $\rho = \bar{\rho}$. This completes the proof. □

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

The main idea of this paper was proposed by HGL, and HGL and LPL etc. prepared the manuscript initially and performed all the steps of the proofs in this research. All authors read and approved the final manuscript.

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Received: 24 October 2013 Accepted: 21 April 2014 Published: 17 May 2014

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10.1186/1687-1812-2014-122

Cite this article as: Li et al.: Sensitivity analysis for generalized set-valued parametric ordered variational inclusion with (α, λ) -NODSM mappings in ordered Banach spaces. *Fixed Point Theory and Applications* **2014**, **2014**:122

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