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Common fixed point for generalized weakly G-contraction mappings satisfying common $(E.A)$ property in G-metric spaces

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Abstract

In this paper, using the concept of common $(E.A)$ property, we prove some common fixed point theorems for three pairs of weakly compatible self-maps satisfying a generalized weakly G-contraction condition in the framework of a generalized metric space. Our results do not rely on any commuting or continuity condition of mappings. An example is provided to support our result in nonsymmetric G-metric space.

Keywords: generalized metric space; common fixed point; generalized weakly G-contraction; weakly compatible mappings; common $(E.A)$ property

1 Introduction and preliminaries

The study of fixed points and common fixed points of mappings satisfying certain contractive conditions has been at the center of rigorous research activity. In 2006, Mustafa and Sims [1] introduced the concept of generalized metric spaces or simply G-metric spaces as a generalization of the notion of metric space. Based on the notion of generalized metric spaces, Mustafa *et al.* [2–5], Obiedat and Mustafa [6], Aydi [7], Gajié and Stojakovié [8], Shatanawi *et al.* [9], Zhou and Gu [10] obtained some fixed point results for mappings satisfying different contractive conditions. Shatanawi [11] obtained some fixed point results for Φ-maps in G-metric spaces. Chugh *et al.* [12] obtained some fixed point results for maps satisfying property P in G-metric spaces. Al-khaleel *et al.* [13] obtained several fixed point results for mappings that satisfy certain contractive conditions in generalized cone metric spaces. The study of common fixed point problems in G-metric spaces was initiated by Abbas and Rhoades [14]. Subsequently, many authors have obtained many common fixed point theorems for the mappings satisfying different contractive conditions; see [15–34] for more details. Recently, Abbas *et al.* [35] and Mustafa *et al.* [36] obtained some common fixed point results for a pair of mappings satisfying the $(E.A)$ property under certain generalized strict contractive conditions in G-metric spaces. Long *et al.* [37] obtained some common coincidence and common fixed points results of two pairs of mappings when only one pair satisfies the $(E.A)$ property in G-metric spaces. Very recently, Gu and Yin [38] obtained some common fixed point theorems of three pairs of mappings for which only two pairs need to satisfy the common $(E.A)$ property in the framework of G-metric spaces.

Now we give preliminaries and basic definitions which are used throughout the paper.

Definition 1.1 (see [1]) Let X be a nonempty set, and let $G : X \times X \times X \to R^+$ be a function satisfying the following axioms:

(G1) $G(x,y,z) = 0$ if $x = y = z$;

(G2) $0 < G(x,x,y)$ for all $x,y \in X$ with $x \neq y$;

(G3) $G(x,x,y) \leq G(x,y,z)$ for all $x,y,z \in X$ with $z \neq y$;

(G4) $G(x,y,z) = G(x,z,y) = G(y,z,x) = \cdots$ (symmetry in all three variables);

(G5) $G(x,y,z) \leq G(x,a,a) + G(a,y,z)$ for all $x,y,z,a \in X$ (rectangle inequality);

then the function G is called a generalized metric or, more specifically, a G-metric on X and the pair (X,G) is called a G-metric space.

It is known that the function $G(x,y,z)$ on a G-metric space X is jointly continuous in all three of its variables, and $G(x,y,z) = 0$ if and only if $x = y = z$; for more details, see [1] and the references therein.

Definition 1.2 (see [1]) Let (X,G) be a G-metric space, and let $\{x_n\}$ be a sequence of points in X. A point x in X is said to be the limit of the sequence $\{x_n\}$ if $\lim_{m,n\to\infty} G(x,x_n,x_m) = 0$, and one says that the sequence $\{x_n\}$ is G-convergent to x.

Thus, if $x_n \to x$ in a G-metric space (X,G), then for any $\epsilon > 0$, there exists $N \in \mathbb{N}$ (throughout this paper we mean by $\mathbb{N}$ the set of all natural numbers) such that $G(x,x_n,x_m) < \epsilon$ for all $n,m \geq N$.

Proposition 1.3 (see [1]) *Let (X,G) be a G-metric space, then the following are equivalent*:

(1) $\{x_n\}$ *is G-convergent to x*;

(2) $G(x_n,x_n,x) \to 0$ *as* $n \to \infty$;

(3) $G(x_n,x,x) \to 0$ *as* $n \to \infty$;

(4) $G(x_n,x_m,x) \to 0$ *as* $n,m \to \infty$.

Definition 1.4 (see [1]) Let (X,G) be a G-metric space. The sequence $\{x_n\}$ is called a G-Cauchy sequence if for each $\epsilon > 0$, there exists $N \in \mathbb{N}$ such that $G(x_n,x_m,x_l) < \epsilon$ for all $n,m,l \geq N$; *i.e.*, if $G(x_n,x_m,x_l) \to 0$ as $n,m,l \to \infty$.

Definition 1.5 (see [1]) A G-metric space (X,G) is said to be G-complete (or a complete G-metric space) if every G-Cauchy sequence in (X,G) is G-convergent in X.

Proposition 1.6 (see [1]) *Let (X,G) be a G-metric space. Then the following are equivalent*.

(1) *The sequence $\{x_n\}$ is G-Cauchy.*

(2) *For every $\epsilon > 0$, there exists $k \in \mathbb{N}$ such that $G(x_n,x_m,x_m) < \epsilon$ for all $n,m \geq k$.*

Proposition 1.7 (see [1]) *Let (X,G) be a G-metric space. Then the function $G(x,y,z)$ is jointly continuous in all three of its variables.*

Proposition 1.8 (see [1]) *Let (X,G) be a G-metric space. Then, for all x, y in X, it follows that $G(x,y,y) \leq 2G(y,x,x)$.*

Definition 1.9 (see [39]) Let f and g be self-maps of a set X. If $w = fx = gx$ for some x in X, then x is called a coincidence point of f and g, and w is called a point of coincidence of f and g.

Definition 1.10 (see [39]) Two self-mappings f and g on X are said to be weakly compatible if they commute at coincidence points.

Definition 1.11 (see [35]) Let X be a G-metric space. Self-maps f and g on X are said to satisfy the G-$(E.A)$ property if there exists a sequence $\{x_n\}$ in X such that $\{fx_n\}$ and $\{gx_n\}$ are G-convergent to some $t \in X$.

Definition 1.12 Let (X,d) be a G-metric space and A, B, S and T be four self-maps on X. The pairs (A,S) and (B,T) are said to satisfy the common $(E.A)$ property if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = t$ for some $t \in X$.

Definition 1.13 (see [19]) Self-mappings f and g of a G-metric space (X,G) are said to be compatible if $\lim_{n\to\infty} G(fgx_n,gfx_n,gfx_n) = 0$ and $\lim_{n\to\infty} G(gfx_n,fgx_n,fgx_n) = 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to} fx_n = \lim_{n\to\infty} gx_n = t$ for some $t \in X$.

Definition 1.14 (see [18]) A pair of self-mappings (f,g) of a G-metric space is said to be weakly commuting if

$$G(fgx,gfx,gfx) \leq G(fx,gx,gx), \quad \forall x \in X.$$

Definition 1.15 (see [18]) A pair of self-mappings (f,g) of a G-metric space is said to be R-weakly commuting if there exists some positive real number R such that

$$G(fgx,gfx,gfx) \leq RG(fx,gx,gx), \quad \forall x \in X.$$

Recently, Shatanawi *et al.* [9] introduced the following definitions.

Definition 1.16 (see [9]) Let (X,G) be a G-metric space. A mapping $f : X \to X$ is said to be weakly G-contractive if for all $x,y,z \in X$, the following inequality holds:

$$\begin{aligned} G(fx,fy,fz) \leq{}& \frac{1}{3}\bigl(G(x,fy,fy) + G(y,fz,fz) + G(z,fx,fx)\bigr) \\ &- \phi\bigl(G(x,fy,fy), G(y,fz,fz), G(z,fx,fx)\bigr), \end{aligned}$$

where $\phi : [0,+\infty)^3 \to [0,+\infty)$ is a continuous function with $\phi(t,s,u) = 0$ if and only if $t = s = u = 0$.

Definition 1.17 (see [9]) Let (X,G) be a G-metric space. A mapping $f : X \to X$ is said to be a weakly G-contractive-type mapping if for all $x,y,z \in X$, the following inequality holds:

$$\begin{aligned} G(fx,fy,fz) \leq{}& \frac{1}{3}\bigl(G(x,x,fy) + G(y,y,fz) + G(z,z,fx)\bigr) \\ &- \phi\bigl(G(x,x,fy), G(y,y,fz), G(z,z,fx)\bigr), \end{aligned}$$

where $\phi : [0,+\infty)^3 \to [0,+\infty)$ is a continuous function with $\phi(t,s,u) = 0$ if and only if $t = s = u = 0$.

Khan *et al.* [40] introduced the concept of altering distance function that is a control function employed to alter the metric distance between two points enabling one to deal with relatively new classes of fixed point problems. Here, we consider the following notion.

Definition 1.18 (see [15]) The function $\psi : [0,+\infty) \to [0,+\infty)$ is called an altering distance function if the following properties are satisfied:
(1) ψ is continuous and increasing;
(2) $\psi(t) = 0$ if and only if $t = 0$.

In 2011, Aydi *et al.* [15] introduced the concept of generalized weakly G-contraction mapping of A and B as follows.

Definition 1.19 (see [15]) Let (X,G) be a G-metric space and $f,g : X \to X$ be two mappings. We say that f is a generalized weakly G-contraction mapping of type A with respect to g if for all $x,y,z \in X$, the following inequality holds:

$$\begin{aligned}\psi\bigl(G(fx,fy,fz)\bigr) &\le \psi\left(\frac{1}{3}\bigl(G(gx,fy,fy) + G(gy,fz,fz) + G(gz,fx,fx)\bigr)\right) \\ &\quad - \phi\bigl(G(gx,fy,fy), G(gy,fz,fz), G(gz,fx,fx)\bigr),\end{aligned}$$

where
(1) ψ is an altering distance function;
(2) $\phi : [0,+\infty)^3 \to [0,+\infty)$ is a continuous function with $\phi(t,s,u) = 0$ if and only if $t = s = u = 0$.

Definition 1.20 (see [15]) Let (X,G) be a G-metric space and $f,g : X \to X$ be two mappings. We say that f is a generalized weakly G-contraction mapping of type B with respect to g if for all $x,y,z \in X$, the following inequality holds:

$$\begin{aligned}\psi\bigl(G(fx,fy,fz)\bigr) &\le \psi\left(\frac{1}{3}\bigl(G(gx,gx,fy) + G(gy,gy,fz) + G(gz,gz,fx)\bigr)\right) \\ &\quad - \phi\bigl(G(gx,gx,fy), G(gy,gy,fz), G(gz,gz,fx)\bigr),\end{aligned}$$

where
(1) ψ is an altering distance function;
(2) $\phi : [0,+\infty)^3 \to [0,+\infty)$ is a continuous function with $\phi(t,s,u) = 0$ if and only if $t = s = u = 0$.

In this paper, using the concept of common $(E.A)$ property, we prove some common fixed point results for six self-mappings f, g, h, R, S and T, where the triple (f,g,h) is a generalized weakly G-contraction mapping of types A and B with respect to the triple (R,S,T). These notions will be given by Definitions 2.1 and 2.5.

2 Main results

We start with the following definition.

Definition 2.1 Let (X,G) be a G-metric space and $f,g,h,R,S,T : X \to X$ be six mappings. We say that the triple (f,g,h) is a generalized weakly G-contraction mapping of type A

with respect to the triple (R,S,T) if for all $x,y,z \in X$, the following inequality holds:

$$\psi\big(G(fx,gy,hz)\big) \le \psi\left(\frac{1}{3}\big(G(Rx,gy,gy) + G(Sy,hz,hz) + G(Tz,fx,fx)\big)\right) - \phi\big(G(Rx,gy,gy), G(Sy,hz,hz), G(Tz,fx,fx)\big), \tag{2.1}$$

where

(1) ψ is an altering distance function;
(2) $\phi : [0,+\infty)^3 \to [0,+\infty)$ is a continuous function with $\phi(t,s,u) = 0$ if and only if $t = s = u = 0$.

Theorem 2.2 *Let (X,G) be a G-metric space and $f,g,h,R,S,T : X \to X$ be six mappings such that (f,g,h) is a generalized weakly G-contraction mapping of type A with respect to (R,S,T). If one of the following conditions is satisfied, then the pairs (f,R), (g,S) and (h,T) have a common point of coincidence in X.*

(i) *The subspace RX is closed in X, $fX \subseteq SX$, $gX \subseteq TX$, and two pairs of (f,R) and (g,S) satisfy the common $(E.A)$ property;*
(ii) *The subspace SX is closed in X, $gX \subseteq TX$, $hX \subseteq RX$, and two pairs of (g,S) and (h,T) satisfy the common $(E.A)$ property;*
(iii) *The subspace TX is closed in X, $fX \subseteq SX$, $hX \subseteq RX$, and two pairs of (f,R) and (h,T) satisfy the common $(E.A)$ property.*

Moreover, if the pairs (f,R), (g,S) and (h,T) are weakly compatible, then f, g, h, R, S and T have a unique common fixed point in X.

Proof First, we suppose that the subspace RX is closed in X, $fX \subseteq SX$, $gX \subseteq TX$, and two pairs of (f,R) and (g,S) satisfy the common $(E.A)$ property. Then by Definition 1.12 we know that there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} Rx_n = \lim_{n\to\infty} gy_n = \lim_{n\to\infty} Sy_n = t$$

for some $t \in X$.

Since $gX \subseteq TX$, there exists a sequence $\{z_n\}$ in X such that $gy_n = Tz_n$. Hence $\lim_{n\to\infty} Tz_n = t$. Next, we will show $\lim_{n\to\infty} hz_n = t$. In fact, from condition (2.1), we can get

$$\psi\big(G(fx_n,gy_n,hz_n)\big) \le \psi\left(\frac{1}{3}\big(G(Rx_n,gy_n,gy_n) + G(Sy_n,hz_n,hz_n) + G(Tz_n,fx_n,fx_n)\big)\right) - \phi\big(G(Rx_n,gy_n,gy_n), G(Sy_n,hz_n,hz_n), G(Tz_n,fx_n,fx_n)\big).$$

On letting $n \to \infty$ and using the continuities of ψ and ϕ, we can obtain

$$\psi\Big(G\Big(t,t,\lim_{n\to\infty} hz_n\Big)\Big) \le \psi\left(\frac{1}{3}\Big(0 + G\Big(t,\lim_{n\to\infty} hz_n,\lim_{n\to\infty} hz_n\Big) + 0\Big)\right) - \phi\Big(0, G\Big(t,\lim_{n\to\infty} hz_n,\lim_{n\to\infty} hz_n\Big), 0\Big). \tag{2.2}$$

By Proposition 1.8, we have

$$G\Big(t,\lim_{n\to\infty} hz_n,\lim_{n\to\infty} hz_n\Big) \le 2G\Big(t,\lim_{n\to\infty} hz_n,\lim_{n\to\infty} hz_n\Big),$$

and hence using the fact that ψ is increasing, (2.2) becomes

$$\begin{aligned}\psi\Big(G\Big(t,t,\lim_{n\to\infty} hz_n\Big)\Big) &\le \psi\left(\frac{2}{3}\Big(G\Big(t,t,\lim_{n\to\infty} hz_n\Big)\Big)\right) - \phi\Big(0, G\Big(t,\lim_{n\to\infty} hz_n,\lim_{n\to\infty} hz_n\Big),0\Big)\\ &\le \psi\Big(G\Big(t,t,\lim_{n\to\infty} hz_n\Big)\Big) - \phi\Big(0, G\Big(t,\lim_{n\to\infty} hz_n,\lim_{n\to\infty} hz_n\Big),0\Big),\end{aligned}$$

which implies that $\phi(0, G(t,\lim_{n\to\infty} hz_n,\lim_{n\to\infty} hz_n),0) = 0$, and so $\lim_{n\to\infty} hz_n = t$.

Since RX is a closed subspace of X and $\lim_{n\to\infty} Rx_n = t$, there exists p in X such that $t = Rp$. We claim that $fp = t$. In fact, by using (2.1), we obtain

$$\begin{aligned}\psi\big(G(fp,gy_n,hz_n)\big) \le{}& \psi\left(\frac{1}{3}\big(G(Rp,gy_n,gy_n) + G(Sy_n,hz_n,hz_n) + G(Tz_n,fp,fp)\big)\right)\\ &- \phi\big(G(Rp,gy_n,gy_n), G(Sy_n,hz_n,hz_n), G(Tz_n,fp,fp)\big).\end{aligned}$$

Taking $n \to \infty$ on the two sides of the above inequality, using the continuities of ψ and ϕ, Proposition 1.8 and the fact that ψ is increasing, we can get

$$\begin{aligned}\psi\big(G(fp,t,t)\big) &\le \psi\left(\frac{1}{3}\big(0+0+G(t,fp,fp)\big)\right) - \phi\big(0,0,G(t,fp,fp)\big)\\ &\le \psi\left(\frac{2}{3}\big(G(fp,t,t)\big)\right) - \phi\big(0,0,G(t,fp,fp)\big)\\ &\le \psi\big(G(fp,t,t)\big) - \phi\big(0,0,G(t,fp,fp)\big),\end{aligned}$$

which implies that $\phi(0,0,G(t,fp,fp)) = 0$, and hence $fp = t = Rp$. Therefore, p is the coincidence point of a pair (f,R).

By the condition $fX \subseteq SX$ and $fp = t$, there exist a point u in X such that $t = Su$. Now, we claim that $gu = t$. In fact, from (2.1) we have

$$\begin{aligned}\psi\big(G(fp,gu,hz_n)\big) \le{}& \psi\left(\frac{1}{3}\big(G(Rp,gu,gu) + G(Su,hz_n,hz_n) + G(Tz_n,fp,fp)\big)\right)\\ &- \phi\big(G(Rp,gu,gu), G(Su,hz_n,hz_n), G(Tz_n,fp,fp)\big).\end{aligned}$$

Letting $n \to \infty$ on the two sides of the above inequality, using the continuities of ψ and ϕ, Proposition 1.8 and the fact that ψ is increasing, we can obtain

$$\begin{aligned}\psi\big(G(t,gu,t)\big) &\le \psi\left(\frac{1}{3}\big(G(t,gu,gu)+0+0\big)\right) - \phi\big(G(t,gu,gu),0,0\big)\\ &\le \psi\left(\frac{2}{3}\big(G(t,gu,t)\big)\right) - \phi\big(G(t,gu,gu),0,0\big)\\ &\le \psi\big(G(t,gu,t)\big) - \phi\big(G(t,gu,gu),0,0\big),\end{aligned}$$

which implies that $\phi(G(t,gu,gu),0,0) = 0$, hence $gu = t = Su$, and so u is the coincidence point of a pair (g,S).

Since $gX \subseteq TX$ and $gu = t$, there exist a point v in X such that $t = Tv$. We claim that $hv = t$. In fact, from (2.1), using $fp = Rp = gu = Su = t$, Proposition 1.8 and the fact that ψ is increasing, we have

$$\begin{aligned}
\psi\big(G(t,t,hv)\big) &= \psi\big(G(fp,gu,hv)\big) \\
&\le \psi\left(\frac{1}{3}\big(G(Rp,gu,gu)+G(Su,hv,hv)+G(Tv,fp,fp)\big)\right) \\
&\quad -\phi\big(G(Rp,gu,gu),G(Su,hv,hv),G(Tv,fp,fp)\big) \\
&= \psi\left(\frac{1}{3}\big(G(t,t,t)+G(t,hv,hv)+G(t,t,t)\big)\right) \\
&\quad -\phi\big(G(t,t,t),G(t,hv,hv),G(t,t,t)\big) \\
&\le \psi\left(\frac{2}{3}\big(G(t,t,hv)\big)\right)-\phi\big(0,G(t,hv,hv),0\big) \\
&\le \psi\big(G(t,t,hv)\big)-\phi\big(0,G(t,hv,hv),0\big).
\end{aligned}$$

This implies that $\phi(0,G(t,hv,hv),0)=0$, and so $hv=t=Tv$, hence v is the coincidence point of a pair (h,T).

Therefore, in all the above cases, we obtain $fp=Rp=gu=Su=hv=Tv=t$. Now, weak compatibility of the pairs (f,R), (g,S) and (h,T) gives that $ft=Rt$, $gt=St$ and $ht=Tt$.

Next, we show that $ft=t$. In fact, using (2.1), Proposition 1.8 and the fact that ψ is increasing, we have

$$\begin{aligned}
\psi\big(G(ft,t,t)\big) &= \psi\big(G(ft,gu,hv)\big) \\
&\le \psi\left(\frac{1}{3}\big(G(Rt,gu,gu)+G(Su,hv,hv)+G(Tv,ft,ft)\big)\right) \\
&\quad -\phi\big(G(Rt,gu,gu),G(Su,hv,hv),G(Tv,ft,ft)\big) \\
&= \psi\left(\frac{1}{3}\big(G(ft,t,t)+G(t,t,t)+G(t,ft,ft)\big)\right) \\
&\quad -\phi\big(G(ft,t,t),G(t,t,t),G(t,ft,ft)\big) \\
&\le \psi\big(G(ft,t,t)\big)-\phi\big(G(ft,t,t),0,G(t,ft,ft)\big),
\end{aligned}$$

which implies that $\phi(G(ft,t,t),0,G(t,ft,ft))=0$, and so $G(ft,t,t)=G(t,ft,ft)=0$, that is, $ft=t$, and so $ft=Rt=t$. Similarly, it can be shown that $gt=St=t$ and $ht=Tt=t$, so we get $ft=gt=ht=Rt=St=Tt=t$, which means that t is a common fixed point of f, g, h, R, S and T.

Next, we will show that the common fixed point of f, g, h, R, S and T is unique. Actually, suppose that $w\in X$ is another common fixed point of f, g, h, R, S and T, then by condition (2.1), Proposition 1.8 and the fact that ψ is increasing, we have

$$\begin{aligned}
\psi\big(G(w,t,t)\big) &= \psi\big(G(fw,gt,ht)\big) \\
&\le \psi\left(\frac{1}{3}\big(G(Rw,gt,gt)+G(St,ht,ht)+G(Tt,fw,fw)\big)\right) \\
&\quad -\phi\big(G(Rw,gt,gt),G(St,ht,ht),G(Tt,fw,fw)\big) \\
&= \psi\left(\frac{1}{3}\big(G(w,t,t)+G(t,t,t)+G(t,w,w)\big)\right)
\end{aligned}$$

$$- \phi\big(G(w,t,t), G(t,t,t), G(t,w,w)\big)$$
$$\le \psi\big(G(w,t,t)\big) - \phi\big(G(w,t,t), 0, G(t,w,w)\big),$$

which implies that $\phi(G(w,t,t),0,G(t,w,w)) = 0$, and so $G(w,t,t) = G(t,w,w) = 0$, hence $w = t$, that is, mappings f, g, h, R, S and T have a unique common fixed point.

Finally, if condition (ii) or (iii) holds, then the argument is similar to that above, so we delete it.

This completes the proof of Theorem 2.2. □

Now we introduce an example to support Theorem 2.2.

Example 2.3 Let $X = \{0,1,2\}$ be a set with G-metric defined by Table 1.

Note that G is non-symmetric as $G(1,2,2) \neq G(1,1,2)$. Let $f,g,h,R,S,T : X \to X$ be defined by Table 2.

Clearly, the subspace RX is closed in X, $fX \subseteq SX$ and $gX \subseteq TX$ with the pairs (f,R), (g,S) and (h,T) being weakly compatible. Also, two pairs (f,R) and (g,S) satisfy the common $(E.A)$ property, indeed, $x_n = 0$ and $y_n = 1$ for each $n \in \mathbb{N}$ are the required sequences. The control functions $\psi : [0,\infty) \to [0,\infty)$ and $\phi : [0,\infty)^3 \to [0,\infty)$ are defined by

$$\psi(t) = 3t \quad \text{and} \quad \phi(t,s,u) = \frac{t+s+u}{4}.$$

It is easy to show that the triple (f,g,h) is a generalized weakly G-contraction mapping of type A with respect to the triple (R,S,T). In fact, contractive condition (2.1) and the following inequality are equivalent:

$$\psi\big(G(fx,gy,hz)\big) \le \frac{3}{4}\big(G(Rx,gy,gy) + G(Sy,hz,hz) + G(Tz,fx,fx)\big). \tag{2.3}$$

To check contractive condition (2.3) for all $x,y,z \in X$, we consider the following cases.

Note that for Cases (1) $x = y = z = 0$, (2) $x = y = 0$, $z = 2$, (3) $x = z = 0$, $y = 1$, (4) $x = 0$, $y = 1$, $z = 2$, (5) $x = 1$, $y = z = 0$, (6) $x = 1$, $y = 0$, $z = 2$, (7) $x = y = 1$, $z = 0$, (8) $x = y = 1$, $z = 2$,

Table 1 The definition of G-metric on X

(x,y,z)	$G(x,y,z)$
(0,0,0), (1,1,1), (2,2,2),	0
(0,0,1), (0,1,0), (1,0,0), (0,1,1), (1,0,1), (1,1,0),	1
(1,2,2), (2,1,2), (2,2,1),	2
(0,0,2), (0,2,0), (2,0,0), (0,2,2), (2,0,2), (2,2,0),	3
(1,1,2), (1,2,1), (2,1,1), (0,1,2), (0,2,1), (1,0,2), (1,2,0), (2,0,1), (2,1,0)	4

Table 2 The definition of maps f, g, h, R, S and T on X

x	$f(x)$	$g(x)$	$h(x)$	$R(x)$	$S(x)$	$T(x)$
0	0	0	0	0	0	0
1	0	0	1	2	0	2
2	0	1	0	2	2	1

(9) $x = 2$, $y = z = 0$, (10) $x = z = 2$, $y = 0$, (11) $x = 2$, $y = 1$, $z = 0$ and (12) $x = z = 2$, $y = 1$, we have $G(fx, gy, hz) = G(0, 0, 0) = 0$, and hence (2.3) is obviously satisfied.

Case (13) If $x = y = 0$, $z = 1$, then $fx = gy = 0$, $hz = 1$, $Rx = Sy = 0$, $Tz = 2$, hence we have

$$\begin{aligned}\psi\bigl(G(fx,gy,hz)\bigr) &= 3G(0,0,1) = 3 = \frac{3}{4}\cdot 4\\ &= \frac{3}{4}\bigl(G(0,0,0) + G(0,1,1) + G(2,0,0)\bigr)\\ &= \frac{3}{4}\bigl(G(R0,g0,g0) + G(S0,h1,h1) + G(T1,f0,f0)\bigr)\\ &= \frac{3}{4}\bigl(G(Rx,gy,gy) + G(Sy,hz,hz) + G(Tz,fx,fx)\bigr).\end{aligned}$$

Case (14) If $x = 0$, $y = z = 1$, then $fx = gy = 0$, $hz = 1$, $Rx = Sy = 0$, $Tz = 2$, hence we have

$$\begin{aligned}\psi\bigl(G(fx,gy,hz)\bigr) &= 3G(0,0,1) = 3 = \frac{3}{4}\cdot 4\\ &= \frac{3}{4}\bigl(G(0,0,0) + G(0,1,1) + G(2,0,0)\bigr)\\ &= \frac{3}{4}\bigl(G(R0,g1,g1) + G(S1,h1,h1) + G(T1,f0,f0)\bigr)\\ &= \frac{3}{4}\bigl(G(Rx,gy,gy) + G(Sy,hz,hz) + G(Tz,fx,fx)\bigr).\end{aligned}$$

Case (15) If $x = z = 0$, $y = 2$, then $fx = hz = 0$, $gy = 1$, $Rx = 0$, $Sy = 2$, $Tz = 0$, hence we have

$$\begin{aligned}\psi\bigl(G(fx,gy,hz)\bigr) &= 3G(0,1,0) = 3 = \frac{3}{4}\cdot 4\\ &= \frac{3}{4}\bigl(G(0,1,1) + G(2,0,0) + G(0,0,0)\bigr)\\ &= \frac{3}{4}\bigl(G(R0,g2,g2) + G(S2,h0,h0) + G(T0,f0,f0)\bigr)\\ &= \frac{3}{4}\bigl(G(Rx,gy,gy) + G(Sy,hz,hz) + G(Tz,fx,fx)\bigr).\end{aligned}$$

Case (16) If $x = 0$, $y = 2$, $z = 1$, then $fx = 0$, $gy = hz = 1$, $Rx = 0$, $Sy = Tz = 2$, hence we have

$$\begin{aligned}\psi\bigl(G(fx,gy,hz)\bigr) &= 3G(0,1,1) = 3 < \frac{3}{4}\cdot 8\\ &= \frac{3}{4}\bigl(G(0,1,1) + G(2,1,1) + G(2,0,0)\bigr)\\ &= \frac{3}{4}\bigl(G(R0,g2,g2) + G(S2,h1,h1) + G(T1,f0,f0)\bigr)\\ &= \frac{3}{4}\bigl(G(Rx,gy,gy) + G(Sy,hz,hz) + G(Tz,fx,fx)\bigr).\end{aligned}$$

Case (17) If $x = 0$, $y = z = 2$, then $fx = hz = 0$, $gy = 1$, $Rx = 0$, $Sy = 2$, $Tz = 1$, hence we have

$$\begin{aligned}\psi\bigl(G(fx,gy,hz)\bigr) &= 3G(0,1,0) = 3 < \frac{3}{4}\cdot 5\\ &= \frac{3}{4}\bigl(G(0,1,1) + G(2,0,0) + G(1,0,0)\bigr)\end{aligned}$$

$$\begin{aligned}&= \frac{3}{4}\big(G(R0,g2,g2) + G(S2,h2,h2) + G(T2,f0,f0)\big)\\&= \frac{3}{4}\big(G(Rx,gy,gy) + G(Sy,hz,hz) + G(Tz,fx,fx)\big).\end{aligned}$$

Case (18) If $x = z = 1$, $y = 0$, then $fx = gy = 0$, $hz = 1$, $Rx = 2$, $Sy = 0$, $Tz = 2$, hence we have

$$\begin{aligned}\psi\big(G(fx,gy,hz)\big) &= 3G(0,0,1) = 3 < \frac{3}{4}\cdot 7\\&= \frac{3}{4}\big(G(2,0,0) + G(0,1,1) + G(2,0,0)\big)\\&= \frac{3}{4}\big(G(R1,g0,g0) + G(S0,h1,h1) + G(T1,f1,f1)\big)\\&= \frac{3}{4}\big(G(Rx,gy,gy) + G(Sy,hz,hz) + G(Tz,fx,fx)\big).\end{aligned}$$

Case (19) $x = y = z = 1$, then $fx = gy = 0$, $hz = 1$, $Rx = 2$, $Sy = 0$, $Tz = 2$, hence we have

$$\begin{aligned}\psi\big(G(fx,gy,hz)\big) &= 3G(0,0,1) = 3 < \frac{3}{4}\cdot 7\\&= \frac{3}{4}\big(G(2,0,0) + G(0,1,1) + G(2,0,0)\big)\\&= \frac{3}{4}\big(G(R1,g1,g1) + G(S1,h1,h1) + G(T1,f1,f1)\big)\\&= \frac{3}{4}\big(G(Rx,gy,gy) + G(Sy,hz,hz) + G(Tz,fx,fx)\big).\end{aligned}$$

Case (20) If $x = 1$, $y = 2$, $z = 0$, then $fx = hz = 0$, $gy = 1$, $Rx = 2$, $Sy = 2$, $Tz = 0$, hence we have

$$\begin{aligned}\psi\big(G(fx,gy,hz)\big) &= 3G(0,1,0) = 3 < \frac{3}{4}\cdot 7\\&= \frac{3}{4}\big(G(2,1,1) + G(2,0,0) + G(0,0,0)\big)\\&= \frac{3}{4}\big(G(R1,g2,g2) + G(S2,h0,h0) + G(T0,f1,f1)\big)\\&= \frac{3}{4}\big(G(Rx,gy,gy) + G(Sy,hz,hz) + G(Tz,fx,fx)\big).\end{aligned}$$

Case (21) If $x = z = 1$, $y = 2$, then $fx = 0$, $gy = hz = 1$, $Rx = Sy = Tz = 2$, hence we have

$$\begin{aligned}\psi\big(G(fx,gy,hz)\big) &= 3G(0,1,1) = 3 < \frac{3}{4}\cdot 11\\&= \frac{3}{4}\big(G(2,1,1) + G(2,1,1) + G(2,0,0)\big)\\&= \frac{3}{4}\big(G(R1,g2,g2) + G(S2,h1,h1) + G(T1,f1,f1)\big)\\&= \frac{3}{4}\big(G(Rx,gy,gy) + G(Sy,hz,hz) + G(Tz,fx,fx)\big).\end{aligned}$$

Case (22) If $x = 1$, $y = z = 2$, then $fx = hz = 0$, $gy = 1$, $Rx = Sy = 2$, $Tz = 1$, hence we have

$$\begin{aligned}\psi\bigl(G(fx,gy,hz)\bigr) = 3G(0,1,0) = 3 &< \frac{3}{4}\cdot 8\\ &= \frac{3}{4}\bigl(G(2,1,1)+G(2,0,0)+G(1,0,0)\bigr)\\ &= \frac{3}{4}\bigl(G(R1,g2,g2)+G(S2,h2,h2)+G(T2,f1,f1)\bigr)\\ &= \frac{3}{4}\bigl(G(Rx,gy,gy)+G(Sy,hz,hz)+G(Tz,fx,fx)\bigr).\end{aligned}$$

Case (23) If $x = 2$, $y = 0$, $z = 1$, then $fx = gy = 0$, $hz = 1$, $Rx = 2$, $Sy = 0$, $Tz = 2$, hence we have

$$\begin{aligned}\psi\bigl(G(fx,gy,hz)\bigr) = 3G(0,0,1) = 3 &< \frac{3}{4}\cdot 7\\ &= \frac{3}{4}\bigl(G(2,0,0)+G(0,1,1)+G(2,0,0)\bigr)\\ &= \frac{3}{4}\bigl(G(R2,g0,g0)+G(S0,h1,h1)+G(T1,f2,f2)\bigr)\\ &= \frac{3}{4}\bigl(G(Rx,gy,gy)+G(Sy,hz,hz)+G(Tz,fx,fx)\bigr).\end{aligned}$$

Case (24) If $x = 2$, $y = z = 1$, then $fx = gy = 0$, $hz = 1$, $Rx = 2$, $Sy = 0$, $Tz = 2$, hence we have

$$\begin{aligned}\psi\bigl(G(fx,gy,hz)\bigr) = 3G(0,0,1) = 3 &< \frac{3}{4}\cdot 7\\ &= \frac{3}{4}\bigl(G(2,0,0)+G(0,1,1)+G(2,0,0)\bigr)\\ &= \frac{3}{4}\bigl(G(R2,g1,g1)+G(S1,h1,h1)+G(T1,f2,f2)\bigr)\\ &= \frac{3}{4}\bigl(G(Rx,gy,gy)+G(Sy,hz,hz)+G(Tz,fx,fx)\bigr).\end{aligned}$$

Case (25) If $x = y = 2$, $z = 0$, then $fx = hz = 0$, $gy = 1$, $Rx = 2$, $Sy = 2$, $Tz = 0$, hence we have

$$\begin{aligned}\psi\bigl(G(fx,gy,hz)\bigr) = 3G(0,1,0) = 3 &< \frac{3}{4}\cdot 7\\ &= \frac{3}{4}\bigl(G(2,1,1)+G(2,0,0)+G(0,0,0)\bigr)\\ &= \frac{3}{4}\bigl(G(R2,g2,g2)+G(S2,h0,h0)+G(T0,f2,f2)\bigr)\\ &= \frac{3}{4}\bigl(G(Rx,gy,gy)+G(Sy,hz,hz)+G(Tz,fx,fx)\bigr).\end{aligned}$$

Case (26) $x = y = 2$, $z = 1$, then $fx = 0$, $gy = hz = 1$, $Rx = Sy = Tz = 2$, hence we have

$$\begin{aligned}\psi\bigl(G(fx,gy,hz)\bigr) = 3G(0,1,1) = 3 &< \frac{3}{4}\cdot 11\\ &= \frac{3}{4}\bigl(G(2,1,1)+G(2,1,1)+G(2,0,0)\bigr)\end{aligned}$$

$$\begin{aligned}
&= \frac{3}{4}\big(G(R2,g2,g2) + G(S2,h1,h1) + G(T1,f2,f2)\big) \\
&= \frac{3}{4}\big(G(Rx,gy,gy) + G(Sy,hz,hz) + G(Tz,fx,fx)\big).
\end{aligned}$$

Case (27) If $x = y = z = 2$, then $fx = hz = 0$, $gy = 1$, $Rx = Sy = 2$, $Tz = 1$, hence we have

$$\begin{aligned}
\psi\big(G(fx,gy,hz)\big) &= 3G(0,1,0) = 3 < \frac{3}{4}\cdot 8 \\
&= \frac{3}{4}\big(G(2,1,1) + G(2,0,0) + G(1,0,0)\big) \\
&= \frac{3}{4}\big(G(R2,g2,g2) + G(S2,h2,h2) + G(T2,f2,f2)\big) \\
&= \frac{3}{4}\big(G(Rx,gy,gy) + G(Sy,hz,hz) + G(Tz,fx,fx)\big).
\end{aligned}$$

Hence, all of the conditions of Theorem 2.2 are satisfied. Moreover, 0 is the unique common fixed point of f, g, h, R, S and T.

Corollary 2.4 *Let (X,G) be a G-metric space. Suppose that mappings $f,g,h,R,S,T : X \to X$ satisfy the following conditions*:

$$G(fx,gy,hz) \le \alpha\big(G(Rx,gy,gy) + G(Sy,hz,hz) + G(Tz,fx,fx)\big) \tag{2.4}$$

for all $x,y,z \in X$, where $\alpha \in [0,\frac{1}{3})$. If one of the following conditions is satisfied, then the pairs (f,R), (g,S) and (h,T) have a common point of coincidence in X.

(i) *The subspace RX is closed in X, $fX \subseteq SX$, $gX \subseteq TX$, and two pairs of (f,R) and (g,S) satisfy the common (E.A) property*;

(ii) *The subspace SX is closed in X, $gX \subseteq TX$, $hX \subseteq RX$, and two pairs of (g,S) and (h,T) satisfy the common (E.A) property*;

(iii) *The subspace TX is closed in X, $fX \subseteq SX$, $hX \subseteq RX$, and two pairs of (f,R) and (h,T) satisfy the common (E.A) property.*

Moreover, if the pairs (f,R), (g,S) and (h,T) are weakly compatible, then f, g, h, R, S and T have a unique common fixed point in X.

Proof It suffices to take $\psi(t) = t$ and $\phi(t,s,u) = (\frac{1}{3} - \alpha)(t + s + u)$ in Theorem 2.2. □

Definition 2.5 Let (X,G) be a G-metric space and $f,g,h,R,S,T : X \to X$ be six mappings. We say that the triple (f,g,h) is a generalized weakly G-contraction mapping of type B with respect to the triple (R,S,T) if for all $x,y,z \in X$, the following inequality holds:

$$\begin{aligned}
\psi\big(G(fx,gy,hz)\big) \le{}& \psi\left(\frac{1}{3}\big(G(Rx,Rx,gy) + G(Sy,Sy,hz) + G(Tz,Tz,fx)\big)\right) \\
&- \phi\big(G(Rx,Rx,gy), G(Sy,Sy,hz), G(Tz,Tz,fx)\big),
\end{aligned} \tag{2.5}$$

where

(1) ψ is an altering distance function;

(2) $\phi : [0,+\infty)^3 \to [0,+\infty)$ is a continuous function with $\phi(t,s,u) = 0$ if and only if $t = s = u = 0$.

Using arguments similar to those in Theorem 2.2, we can prove the following theorem.

Theorem 2.6 *Let (X, G) be a G-metric space and $f, g, h, R, S, T : X \to X$ be six mappings such that (f, g, h) is a generalized weakly G-contraction mapping of type B with respect to (R, S, T). If one of the following conditions is satisfied, then the pairs (f, R), (g, S) and (h, T) have a common point of coincidence in X.*

(i) *The subspace RX is closed in X, $fX \subseteq SX$, $gX \subseteq TX$, and two pairs of (f, R) and (g, S) satisfy the common $(E.A)$ property*;

(ii) *The subspace SX is closed in X, $gX \subseteq TX$, $hX \subseteq RX$, and two pairs of (g, S) and (h, T) satisfy the common $(E.A)$ property*;

(iii) *The subspace TX is closed in X, $fX \subseteq SX$, $hX \subseteq RX$, and two pairs of (f, R) and (h, T) satisfy the common $(E.A)$ property.*

Moreover, if the pairs (f, R), (g, S) and (h, T) are weakly compatible, then f, g, h, R, S and T have a unique common fixed point in X.

As in the case of Theorem 2.2, we can deduce the following corollary from Theorem 2.6.

Corollary 2.7 *Let (X, G) be a G-metric space. Suppose that mappings $f, g, h, R, S, T : X \to X$ satisfy the following conditions*:

$$G(fx, gy, hz) \le \alpha\big(G(Rx, Ry, gy) + G(Sy, Sz, hz) + G(Tz, Tx, fx)\big) \tag{2.6}$$

for all $x, y, z \in X$, where $\alpha \in [0, \frac{1}{3})$. If one of the following conditions is satisfied, then the pairs (f, R), (g, S) and (h, T) have a common point of coincidence in X.

(i) *The subspace RX is closed in X, $fX \subseteq SX$, $gX \subseteq TX$, and two pairs of (f, R) and (g, S) satisfy the common $(E.A)$ property*;

(ii) *The subspace SX is closed in X, $gX \subseteq TX$, $hX \subseteq RX$, and two pairs of (g, S) and (h, T) satisfy the common $(E.A)$ property*;

(iii) *The subspace TX is closed in X, $fX \subseteq SX$, $hX \subseteq RX$, and two pairs of (f, R) and (h, T) satisfy the common $(E.A)$ property.*

Moreover, if the pairs (f, R), (g, S) and (h, T) are weakly compatible, then f, g, h, R, S and T have a unique common fixed point in X.

Remark 2.8 If we take: (1) $R = S = T$; (2) $f = g = h$; (3) $R = S = T = I$ (I is an identity mapping); (4) $S = T$ and $g = h$; (5) $S = T$, $g = h = I$ in Theorems 2.2 and 2.6, Corollaries 2.4 and 2.7, then several new results can be obtained.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

Both authors contributed equally to this work. Both authors read and approved the final manuscript.

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