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# Strong convergence theorems for system of equilibrium problems and asymptotically strict pseudocontractions in the intermediate sense

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**Abstract**

Let $\{S_i\}_{i=1}^N$ be $N$ uniformly continuous asymptotically $\lambda_i$-strict pseudocontractions in the intermediate sense defined on a nonempty closed convex subset $C$ of a real Hilbert space $H$. Consider the problem of finding a common element of the fixed point set of these mappings and the solution set of a system of equilibrium problems by using hybrid method. In this paper, we propose new iterative schemes for solving this problem and prove these schemes converge strongly.

**MSC:** 47H05; 47H09; 47H10.

**Keywords:** asymptotically strict pseudocontraction in the intermediate sense, system of equilibrium problem, hybrid method, fixed point

## 1. Introduction

Let $H$ be a real Hilbert space and let $C$ be a nonempty closed convex subset of $H$.

A nonlinear mapping $S : C \to C$ is a self mapping of $C$. We denote the set of fixed points of $S$ by $F(S)$ (i.e., $F(S) = \{x \in C : Sx = x\}$). Recall the following concepts.

(1) $S$ is uniformly Lipschitzian if there exists a constant $L > 0$ such that

$$||S^n x - -S^n y|| \leq L||x - y|| \text{ for all integers } n \geq 1 \text{ and } x, y \in C.$$

(2) $S$ is nonexpansive if

$$||Sx - Sy|| \leq ||x - y|| \text{ for all } x, y \in C.$$

(3) $S$ is asymptotically nonexpansive if there exists a sequence $k_n$ of positive numbers satisfying the property $\lim_{n\to\infty} k_n = 1$ and

$$||S^n x - S^n y|| \leq k_n ||x - y|| \text{ for all integers } n \geq 1 \text{ and } x, y \in C.$$

(4) $S$ is asymptotically nonexpansive in the intermediate sense [1] provided $S$ is continuous and the following inequality holds:

$$\limsup_{n\to\infty} \sup_{x,y\in C} (||S^n x - S^n y|| - ||x - y||) \leq 0.$$

(5) $S$ is asymptotically $\lambda$-strict pseudocontractive mapping [2] with sequence $\{\gamma_n\}$ if there exists a constant $\lambda \in [0, 1)$ and a sequence $\{\gamma_n\}$ in $[0, \infty)$ with $\lim_{n\to\infty} \gamma_n = 0$ such that

$$||S^n x - S^n y||^2 \leq (1 + \gamma_n)||x - y||^2 + \lambda||x - S^n x - (y - S^n y)||^2$$

for all $x, y \in C$ and $n \in \mathbb{N}$.

(6) $S$ is asymptotically $\lambda$-strict pseudocontractive mapping in the intermediate sense [3,4] with sequence $\{\gamma_n\}$ if there exists a constant $\lambda \in [0, 1)$ and a sequence $\{\gamma_n\}$ in $[0, \infty)$ with $\lim_{n\to\infty} \gamma_n = 0$ such that

$$\limsup_{n\to\infty} \sup_{x,y\in C} (||S^n x - S^n y||^2 - (1 + \gamma_n)||x - y||^2 - \lambda||x - S^n x - (y - S^n y)||^2) \leq 0 \quad (1.1)$$

for all $x, y \in C$ and $n \in \mathbb{N}$.

Throughout this paper, we assume that

$$c_n = \max\{0, \sup_{x,y\in C} (||S^n x - S^n y||^2 - (1 + \gamma_n)||x - y||^2 - \lambda||x - S^n x - (y - S^n y)||^2)\}.$$

Then, $c_n \geq 0$ for all $n \in N$, $c_n \to 0$ as $n \to \infty$ and (1.1) reduces to the relation

$$||S^n x - S^n y||^2 \leq (1 + \gamma_n)||x - y||^2 + \lambda||x - S^n x - (y - S^n y)||^2 + c_n \quad (1.2)$$

for all $x, y \in C$ and $n \in \mathbb{N}$.

When $c_n = 0$ for all $n \in N$ in (1.2), then $S$ is an asymptotically $\lambda$-strict pseudocontractive mapping with sequence $\{\gamma_n\}$. We note that $S$ is not necessarily uniformly $L$-Lipschitzian (see [4]), more examples can also be seen in [3].

Let $\{F_k\}$ be a countable family of bifunctions from $C \times C$ to $\mathbb{R}$, where $\mathbb{R}$ is the set of real numbers. Combettes and Hirstoaga [5] considered the following system of equilibrium problems:

$$\text{Finding } x \in C \text{ such that } F_k(x, y) \geq 0, \forall k \in \Gamma \text{ and } \forall y \in C, \quad (1.3)$$

where $\Gamma$ is an arbitrary index set. If $\Gamma$ is a singleton, then problem (1.3) becomes the following equilibrium problem:

$$\text{Finding } x \in C \text{ such that } F(x, y) \geq 0, \forall y \in C. \quad (1.4)$$

The solution set of (1.4) is denoted by $EP(F)$.

The problem (1.3) is very general in the sense that it includes, as special cases, optimization problems, variational inequalities, minimax problems, Nash equilibrium problem in noncooperative games and others; see, for instance, [6,7] and the references therein. Some methods have been proposed to solve the equilibrium problem (1.3), related work can also be found in [8-11].

For solving the equilibrium problem, let us assume that the bifunction $F$ satisfies the following conditions:

(A1) $F(x, x) = 0$ for all $x \in C$;
(A2) $F$ is monotone, i.e.$F(x, y) + F(y, x) \leq 0$ for any $x, y \in C$;
(A3) for each $x, y, z \in C$, $\limsup_{t \to 0} F(tz + (1 - t)x, y) \leq F(x, y)$;
(A4) $F(x,\cdot)$ is convex and lower semicontionuous for each $x \in C$.

Recall Mann's iteration algorithm was introduced by Mann [12]. Since then, the construction of fixed points for nonexpansive mappings and asymptotically strict pseudo-contractions via Mann' iteration algorithm has been extensively investigated by many authors (see, e.g., [2,6]).

Mann's iteration algorithm generates a sequence $\{x_n\}$ by the following manner:

$$\forall x_0 \in C, x_{n+1} = \alpha_n x_n + (1 - \alpha_n)Sx_n, n \geq 0,$$

where $\alpha_n$ is a real sequence in (0, 1) which satisfies certain control conditions.

On the other hand, Qin et al. [13] introduced the following algorithm for a finite family of asymptotically $\lambda_i$-strict pseudocontractions. Let $x_0 \in C$ and $\{\alpha_n\}_{n=0}^{\infty}$ be a sequence in (0, 1). The sequence $\{x_n\}$ by the following way:

$$\begin{array}{l} x_1 = \alpha_0 x_0 + (1 - \alpha_0)S_1 x_0, \\ x_2 = \alpha_1 x_1 + (1 - \alpha_1)S_2 x_1, \\ \cdots \\ x_N = \alpha_{N-1} x_{N-1} + (1 - \alpha_{N-1})S_N x_{N-1}, \\ x_{N+1} = \alpha_N x_N + (1 - \alpha_N)S_1^2 x_N, \\ \cdots \\ x_{2N} = \alpha_{2N-1} x_{2N-1} + (1 - \alpha_{2N-1})S_N^2 x_{2N-1}, \\ x_{2N+1} = \alpha_{2N} x_{2N} + (1 - \alpha_{2N})S_1^3 x_{2N}, \\ \cdots . \end{array}$$

It is called the explicit iterative sequence of a finite family of asymptotically $\lambda_i$-strict pseudocontractions $\{S_1, S_2, \ldots, S_N\}$. Since, for each $n \geq 1$, it can be written as $n = (h - 1)N + i$, where $i = i(n) \in \{1, 2, \ldots, N\}$, $h = h(n) \geq 1$ is a positive integer and $h(n) \to \infty$, as $n \to \infty$. We can rewrite the above table in the following compact form:

$$x_n = \alpha_{n-1} x_{n-1} + (1 - \alpha_{n-1})S_{i(n)}^{h(n)} x_{n-1}, \forall n \geq 1.$$

Recently, Sahu et al. [4] introduced new iterative schemes for asymptotically strict pseudocontractive mappings in the intermediate sense. To be more precise, they proved the following theorem.

**Theorem 1.1**. *Let $C$ be a nonempty closed convex subset of a real Hilbert space $H$ and $T: C \to C$ a uniformly continuous asymptotically $\kappa$-strict pseudocontractive mapping in the intermediate sense with sequence $\gamma_n$ such that $F(T)$ is nonempty and bounded. Let $\alpha_n$ be a sequence in* [0, 1] *such that $0 < \delta \leq \alpha_n \leq 1 - \kappa$ for all $n \in N$. Let $\{x_n\} \subset C$ be sequences generated by the following (CQ) algorithm:*

$$\begin{cases} u = x_1 \in C \text{ chosen arbitrary}, \\ y_n = (1 - \alpha_n)x_n + \alpha_n T^n x_n, \\ C_n = \{z \in C : ||y_n - z||^2 \leq ||x_n - z||^2 + \theta_n\}, \\ Q_n = \{z \in C : \langle x_n - z, u - x_n \rangle \geq 0\}, \\ x_{n+1} = P_{C_n \cap Q_n}(u), \text{ for all } n \in \mathbb{N}, \end{cases}$$

*where* $\theta_n = c_n + \gamma_n \Delta_n$ *and* $\Delta_n = \sup \{||x_n - z||: z \in F(T)\} < \infty$. *Then,* $\{x_n\}$ *converges strongly to* $P_{F(T)}(u)$.

Very recently, Hu and Cai [3] further considered the asymptotically strict pseudocontractive mappings in the intermediate sense concerning equilibrium problem. They obtained the following result in a real Hilbert space.

**Theorem 1.2**. *Let C be a nonempty closed convex subset of a real Hilbert space H and* $N \geq 1$ *be an integer,* $\varphi : C \to C$ *be a bifunction satisfying (A1)-(A4) and* $A : C \to H$ *be an* $\alpha$*-inverse-strongly monotone mapping. Let for each* $1 \leq i \leq N$, $T_i : C \to C$ *be a uniformly continuous* $k_i$*-strictly asymptotically pseudocontractive mapping in the intermediate sense for some* $0 \leq k_i < 1$ *with sequences* $\{\gamma_{n,i}\} \subset [0, \infty)$ *such that* $\lim_{n\to\infty} \gamma_{n,i} = 0$ *and* $\{c_{n,i}\} \subset [0, \infty)$ *such that* $\lim_{n\to\infty} c_{n,i} = 0$. *Let* $k = \max\{k_i : 1 \leq i \leq N\}$, $\gamma_n = \max\{\gamma_{n,i} : 1 \leq i \leq N\}$ *and* $c_n = \max\{c_{n,i} : 1 \leq i \leq N\}$. *Assume that* $F = \cap_{i=1}^{N} F(T_i) \cap EP$ *is nonempty and bounded. Let* $\{\alpha_n\}$ *and* $\{\beta_n\}$ *be sequences in* $[0, 1]$ *such that* $0 < a \leq \alpha_n \leq 1$, $0 < \delta \leq \beta_n \leq 1 - k$ *for all* $n \in N$ *and* $0 < b \leq r_n \leq c < 2\alpha$. *Let* $\{x_n\}$ *and* $\{u_n\}$ *be sequences generated by the following algorithm:*

$$\begin{cases} x_0 \in C \text{ chosen arbitrary}, \\ u_n \in C, \text{ such that } \phi(u_n, y) + \langle Ax_n, y - u_n \rangle + \frac{1}{s}\langle y - u_n, u_n - x_n \rangle \geq 0, \forall y \in C, \\ z_n = (1 - \beta_n)u_n + \beta_n T_{i(n)}^{h(n)} u_n, \\ y_n = (1 - \alpha_n)u_n + \alpha_n z_n, \\ C_n = \{v \in C : \ ||y_n - v||^2 \leq \ ||x_n - v||^2 + \theta_n\}, \\ Q_n = \{v \in C : \langle x_n - v, x_0 - x_n \rangle \geq 0\}, \\ x_{n+1} = P_{C_n \cap Q_n} x_0, \forall n \in \mathbb{N} \cup \{0\}, \end{cases}$$

*where* $\theta_n = c_{h(n)} + \gamma_{h(n)} \rho_n^2 \to 0$, *as* $n \to \infty$, *where* $\rho_n = \sup\{||x_n - v||: v \in F\} < \infty$. *Then,* $\{x_n\}$ *converges strongly to* $P_{F(T)} x_0$.

Motivated by Hu and Cai [3], Sahu et al. [4], and Duan [8], the main purpose of this paper is to introduce a new iterative process for finding a common element of the fixed point set of a finite family of asymptotically $\lambda_i$-strict pseudocontractions and the solution set of the problem (1.3). Using the hybrid method, we obtain strong convergence theorems that extend and improve the corresponding results [3,4,13,14].

We will adopt the following notations:

1. $\rightharpoonup$ for the weak convergence and $\to$ for the strong convergence.
2. $\omega_w(x_n) = \{x : \exists x_{n_j} \rightharpoonup x\}$ denotes the weak $\omega$-limit set of $\{x_n\}$.

## 2. Preliminaries

We need some facts and tools in a real Hilbert space $H$ which are listed below.

**Lemma 2.1**. *Let H be a real Hilbert space. Then, the following identities hold.*

*(i)* $||x - y||^2 = ||x||^2 - ||y||^2 - 2\langle x - y, y\rangle$, $\forall x, y \in H$.

*(ii)* $||tx + (1 - t)y||^2 = t||x||^2 + (1 - t)||y||^2 - t(1 - t)||x - y||^2$, $\forall t \in [0, 1]$, $\forall x, y \in H$.

**Lemma 2.2**. ([10]) *Let H be a real Hilbert space. Given a nonempty closed convex subset* $C \subset H$ *and points* $x, y, z \in H$ *and given also a real number* $a \in \mathbb{R}$, *the set*

$$\{v \in C : \ ||y - v||^2 \leq \ ||x - v||^2 + \langle z, v \rangle + a\}$$

*is convex (and closed).*

**Lemma 2.3**. ([15]) *Let C be a nonempty, closed and convex subset of H. Let* $\{x_n\}$ *be a sequence in H and* $u \in H$. *Let* $q = P_C u$. *Suppose that* $\{x_n\}$ *is such that* $\omega_w(x_n) \subset C$ *and satisfies the following condition*

$$||x_n - u|| \leq ||u - q|| \text{ for all } n.$$

*Then,* $x_n \to q$.

**Lemma 2.4**. ([4]) *Let C be a nonempty closed convex subset of a real Hilbert space H and* $T : C \to C$ *a continuous asymptotically* $\kappa$*-strict pseudocontractive mapping in the intermediate sense. Then I - T is demiclosed at zero in the sense that if* $\{x_n\}$ *is a sequence in C such that* $x_n \rightharpoonup x \in C$ *and* $\limsup_{m\to\infty} \limsup_{n\to\infty} ||x_n - T^m x_n|| = 0$, *then* $(I - T)x = 0$.

**Lemma 2.5**. ([4]) *Let C be a nonempty subset of a Hilbert space H and* $T : C \to C$ *an asymptotically* $\kappa$ *- strict pseudocontractive mapping in the intermediate sense with sequence* $\{\gamma_n\}$. *Then*

$$||T^n x - T^n y|| \leq \frac{1}{1-\kappa}\left(\kappa||x-y|| + \sqrt{(1+(1-\kappa)\gamma_n)||x-y||^2 + (1-\kappa)c_n}\right)$$

*for all* $x, y \in C$ *and* $n \in N$.

**Lemma 2.6**. ([6]) *Let C be a nonempty closed convex subset of H, let F be bifunction from* $C \times C$ *to* $\mathbb{R}$ *satisfying (A1)-(A4) and let* $r > 0$ *and* $x \in H$. *Then there exists* $z \in C$ *such that*

$$F(z, y) + \frac{1}{r}\langle y - z, z - x\rangle \geq 0, \quad \text{for all } y \in C.$$

**Lemma 2.7**. ([5]) *For* $r > 0$, $x \in H$, *define a mapping* $T_r : H \to C$ *as follows:*

$$T_r(x) = \{z \in C \,|\, F(z, y) + \frac{1}{r}\langle y - z, z - x\rangle \geq 0, \ \forall y \in C\}$$

*for all* $x \in H$. *Then, the following statements hold:*

(i) $T_r$ *is single-valued*;
(ii) $T_r$ *is firmly nonexpansive, i.e., for any* $x, y \in H$,

$$||T_r x - T_r y||^2 \leq \langle T_r x - T_r y, x - y\rangle;$$

(iii) $F(T_r) = EP(F)$;
(iv) $EP(F)$ *is closed and convex.*

## 3. Main result

**Theorem 3.1**. *Let C be a nonempty closed convex subset of a real Hilbert space H and* $N \geq 1$ *be an integer, let* $F_k$, $k \in \{1, 2, \dots M\}$, *be a bifunction from* $C \times C$ *to* $\mathbb{R}$ *which satisfies conditions (A1)-(A4). Let, for each* $1 \leq i \leq N$, $S_i : C \to C$ *be a uniformly continuous asymptotically* $\lambda_i$*-strict pseudocontractive mapping in the intermediate sense for some* $0 \leq \lambda_i < 1$ *with sequences* $\{\gamma_{n,i}\} \subset [0, \infty)$ *such that* $\lim_{n\to\infty} \gamma_{n,i} = 0$ *and* $\{c_{n,i}\} \subset [0, \infty)$ *such that* $\lim_{n\to\infty} c_{n,i} = 0$. *Let* $\lambda = \max\{\lambda_i : 1 \leq i \leq N\}$, $\gamma_n = \max\{\gamma_{n,i} : 1 \leq i \leq N\}$ *and* $c_n = \max\{c_{n,i} : 1 \leq i \leq N\}$. *Assume that* $\Omega = \cap_{i=1}^N F(S_i) \cap (\cap_{k=1}^M EP(F_k))$*is nonempty* *and bounded. Let* $\{\alpha_n\}$ *and* $\{\beta_n\}$ *be sequences in* $[0, 1]$ *such that* $0 < a \le \alpha_n \le 1$, $0 < \delta \le \beta_n \le 1 - \lambda$ *for all* $n \in \mathbb{N}$ *and* $\{r_{k,n}\} \subset (0, \infty)$ *satisfies* $\liminf_{n\to\infty} r_{k,n} > 0$ *for all* $k \in \{1, 2, \dots M\}$. *Let* $\{x_n\}$ *and* $\{u_n\}$ *be sequences generated by the following algorithm:*

$$\begin{cases} x_1 \in C \text{ chosen arbitrary}, \\ u_n = T^{F_M}_{r_{M,n}} T^{F_{M-1}}_{r_{M-1,n}} \cdots T^{F_2}_{r_{2,n}} T^{F_1}_{r_{1,n}} x_n, \\ z_n = (1-\beta_n)u_n + \beta_n S^{h(n)}_{i(n)} u_n, \\ y_n = (1-\alpha_n)u_n + \alpha_n z_n, \\ C_n = \{v \in C : \ ||y_n - v||^2 \le \ ||x_n - v||^2 + \theta_n\}, \\ Q_n = \{v \in C : \langle x_n - v, x_1 - x_n\rangle \ge 0\}, \\ x_{n+1} = P_{C_n \cap Q_n} x_1, \forall n \in \mathbb{N}, \end{cases} \tag{3.1}$$

*where* $\theta_n = c_{h(n)} + \gamma_{h(n)}\rho_n^2 \to 0$, *as* $n \to \infty$, *where* $\rho_n = \sup\{||x_n - v|| : v \in \Omega\} < \infty$. *Then* $\{x_n\}$ *converges strongly to* $P_\Omega x_1$.

*Proof.* Denote $\Theta^k_n = T^{F_k}_{r_{k,n}} \dots T^{F_2}_{r_{2,n}} T^{F_1}_{r_{1,n}}$ for every $k \in \{1, 2, \dots, M\}$ and $\Theta^0_n = I$ for all $n \in \mathbb{N}$. Therefore $u_n = \Theta^M_n x_n$. The proof is divided into six steps.

**Step 1**. The sequence $\{x_n\}$ is well defined.

It is obvious that $C_n$ is closed and $Q_n$ is closed and convex for every $n \in \mathbb{N}$. From Lemma 2.2, we also get that $C_n$ is convex.

Take $p \in \Omega$, since for each $k \in \{1, 2, \dots, M\}$, $T^{F_k}_{r_{k,n}}$ is nonexpansive, $p = T^{F_k}_{r_{k,n}} p$ and $u_n = \Theta^M_n x_n$, we have

$$||u_n - p|| \ = \ ||\Theta^M_n x_n - \Theta^M_n p|| \ \le \ ||x_n - p|| \text{ for all } n \in \mathbb{N}. \tag{3.2}$$

It follows from the definition of $S_i$ and Lemma 2.1(ii), we get

$$\begin{aligned} ||z_n - p||^2 &= ||(1-\beta_n)(u_n - p) + \beta_n(S^{h(n)}_{i(n)} u_n - p)||^2 \\ &= (1-\beta_n)||u_n - p||^2 + \beta_n||S^{h(n)}_{i(n)} u_n - p||^2 - \beta_n(1-\beta_n)||S^{h(n)}_{i(n)} u_n - u_n||^2 \\ &\le (1-\beta_n)||u_n - p||^2 + \beta_n\left[||(1+\gamma_{h(n)})||u_n - p||^2 + \lambda||S^{h(n)}_{i(n)} u_n - u_n||^2 + c_{h(n)}\right] \\ &\quad - \beta_n(1-\beta_n)||S^{h(n)}_{i(n)} u_n - u_n||^2 \\ &\le (1+\gamma_{h(n)})||u_n - p||^2 - \beta_n(1-\beta_n-\lambda)||S^{h(n)}_{i(n)} u_n - u_n||^2 + \beta_n c_{h(n)} \\ &\le (1+\gamma_{h(n)})||u_n - p||^2 + \beta_n c_{h(n)}. \end{aligned} \tag{3.3}$$

By virtue of the convexity of $||\cdot||^2$, one has

$$||y_n - p||^2 = \ ||(1-\alpha_n)(u_n - p) + \alpha_n(z_n - p)||^2 \le (1-\alpha_n)||u_n - p||^2 + \alpha_n||z_n - p||^2. \tag{3.4}$$

Substituting (3.2) and (3.3) into (3.4), we obtain

$$\begin{aligned} ||y_n - p||^2 &\le (1-\alpha_n)||u_n - p||^2 + \alpha_n\left[(1+\gamma_{h(n)})||u_n - p||^2 + \beta_n c_{h(n)}\right] \\ &\le ||u_n - p||^2 + \gamma_{h(n)}||u_n - p||^2 + \beta_n c_{h(n)} \\ &\le ||u_n - p||^2 + \gamma_{h(n)}||x_n - p||^2 + c_{h(n)} \\ &\le ||u_n - p||^2 + \theta_n \\ &\le ||x_n - p||^2 + \theta_n. \end{aligned} \tag{3.5}$$

It follows that $p \in C_n$ for all $n \in \mathbb{N}$. Thus, $\Omega \subset C_n$.

Next, we prove that $\Omega \subset Q_n$ for all $n \in \mathbb{N}$ by induction. For $n = 1$, we have $\Omega \subset C = Q_1$. Assume that $\Omega \subset Q_n$ for some $n \ge 1$. Since $x_{n+1} = P_{C_n \cap Q_n} x_1$, we obtain

$$\langle x_{n+1} - z, x_1 - x_{n+1} \rangle \geq 0, \quad \forall z \in C_n \cap Q_n.$$

As $\Omega \subset C_n \cap Q_n$ by induction assumption, the inequality holds, in particular, for all $z \in \Omega$. This together with the definition of $Q_{n+1}$ implies that $\Omega \subset Q_{n+1}$.

Hence $\Omega \subset Q_n$ holds for all $n \geq 1$. Thus $\Omega \subset C_n \cap Q_n$ and therefore the sequence $\{x_n\}$ is well defined.

**Step 2**. Set $q = P_\Omega x_1$, then

$$||x_{n+1} - x_1|| \leq ||q - x_1|| \text{ for all } n \in \mathbb{N}. \tag{3.6}$$

Since $\Omega$ is a nonempty closed convex subset of $H$, there exists a unique $q \in \Omega$ such that $q = P_\Omega x_1$.

From $x_{n+1} = P_{C_n \cap Q_n} x_1$, we have

$$||x_{n+1} - x_1|| \leq ||v - x_1|| \text{ for all } v \in C_n \cap Q_n, \text{ for all } n \in \mathbb{N}.$$

Since $q \in \Omega \subset C_n \cap Q_n$, we get (3.6).

Therefore, $\{x_n\}$ is bounded. So are $\{u_n\}$ and $\{y_n\}$.

**Step 3**. The following limits hold:

$$\lim_{n\to\infty} ||u_n - u_{n+i}|| = 0, \lim_{n\to\infty} ||x_n - x_{n+i}|| = 0; \forall i = 1, 2, ..., N.$$

From the definition of $Q_n$, we have $x_n = P_{Q_n} x_1$, which together with the fact that $x_{n+1} \in C_n \cap Q_n \subset Q_n$ implies that

$$||x_n - x_1|| \leq ||x_{n+1} - x_1||, \quad \langle x_n - x_{n+1}, x_1 - x_n \rangle \geq 0. \tag{3.7}$$

This shows that the sequence $\{||x_n - x_1||\}$ is nondecreasing. Since $\{x_n\}$ is bounded, the limit of $\{||x_n - x_1||\}$ exists.

It follows from Lemma 2.1(i) and (3.7) that

$$\begin{aligned} ||x_{n+1} - x_n||^2 &= ||x_{n+1} - x_1 - (x_n - x_1)||^2 \\ &= ||x_{n+1} - x_1||^2 - ||x_n - x_1||^2 - 2\langle x_n - x_{n+1}, x_1 - x_n \rangle \\ &\leq ||x_{n+1} - x_1||^2 - ||x_n - x_1||^2. \end{aligned}$$

Noting that $\lim_{n\to\infty} ||x_n - x_1||$ exists, this implies

$$\lim_{n\to\infty} ||x_n - x_{n+1}|| = 0. \tag{3.8}$$

It is easy to get

$$||x_{n+i} - x_n|| \to 0, \forall i = 1, 2, ..., N, \text{ as } n \to \infty. \tag{3.9}$$

Since $x_{n+1} \in C_n$, we have

$$||y_n - x_{n+1}||^2 \leq ||x_n - x_{n+1}||^2 + \theta_n.$$

So, we get $\lim_{n\to\infty} ||y_n - x_{n+1}|| = 0$. It follows that

$$||y_n - x_n|| \leq ||y_n - x_{n+1}|| + ||x_n - x_{n+1}|| \to 0, \text{ as } n \to \infty. \tag{3.10}$$

Next we will show that

$$\lim_{n\to\infty} ||\Theta_n^k x_n - \Theta_n^{k-1} x_n|| = 0, \ k = 1, 2, \ldots, M. \tag{3.11}$$

Indeed, for $p \in \Omega$, it follows from the firmly nonexpansivity of $T_{r_{k,n}}^{F_k}$ that for each $k \in \{1, 2,..., M\}$, we have

$$\begin{aligned} ||\Theta_n^k x_n - p||^2 &= ||T_{r_{k,n}}^{F_k} \Theta_n^{k-1} x_n - T_{r_{k,n}}^{F_k} p||^2 \\ &\le \langle \Theta_n^k x_n - p, \Theta_n^{k-1} x_n - p \rangle \\ &= \frac{1}{2}\left(||\Theta_n^k x_n - p||^2 + ||\Theta_n^{k-1} x_n - p||^2 - ||\Theta_n^k x_n - \Theta_n^{k-1} x_n||^2\right). \end{aligned}$$

Thus we get

$$||\Theta_n^k x_n - p||^2 \le ||\Theta_n^{k-1} x_n - p||^2 - ||\Theta_n^k x_n - \Theta_n^{k-1} x_n||^2, k = 1, 2, \dots, M,$$

which implies that for each $k \in \{1, 2,..., M\}$,

$$\begin{aligned} ||\Theta_n^k x_n - p||^2 &\le ||\Theta_n^0 x_n - p||^2 - ||\Theta_n^k x_n - \Theta_n^{k-1} x_n||^2 - ||\Theta_n^{k-1} x_n - \Theta_n^{k-2} x_n||^2 \\ &\quad - \cdots - ||\Theta_n^2 x_n - \Theta_n^1 x_n||^2 - ||\Theta_n^1 x_n - \Theta_n^0 x_n||^2 \\ &\le ||x_n - p||^2 - ||\Theta_n^k x_n - \Theta_n^{k-1} x_n||^2. \end{aligned} \tag{3.12}$$

Therefore, by the convexity of $||\cdot||^2$, (3.5) and the nonexpansivity of $T_{r_{k,n}}^{F_k}$, we get

$$\begin{aligned} ||y_n - p||^2 &\le ||u_n - p||^2 + \theta_n \\ &= ||\Theta_n^M x_n - \Theta_n^M p||^2 + \theta_n \\ &\le ||\Theta_n^k x_n - p||^2 + \theta_n \\ &\le ||x_n - p||^2 - ||\Theta_n^k x_n - \Theta_n^{k-1} x_n||^2 + \theta_n. \end{aligned}$$

It follows that

$$||\Theta_n^k x_n - \Theta_n^{k-1} x_n||^2 \le ||x_n - p||^2 - ||y_n - p||^2 + \theta_n \le ||x_n - y_n||(||x_n - p|| + ||y_n - p||) + \theta_n \tag{3.13}$$

From (3.10) and (3.13), we obtain (3.11). Then, we have

$$||u_n - x_n|| \le ||u_n - \Theta_n^{M-1} x_n|| + ||\Theta_n^{M-1} x_n - \Theta_n^{M-2} x_n|| + \cdots + ||\Theta_n^1 x_n - x_n|| \to 0 \tag{3.14}$$

Combining (3.8) and (3.14), we have

$$||u_{n+1} - u_n|| \le ||u_{n+1} - x_{n+1}|| + ||x_{n+1} - x_n|| + ||x_n - u_n|| \to 0, \text{ as } n \to \infty. \tag{3.15}$$

It follows that

$$||u_{n+i} - u_n|| \to 0, \forall i = 1, 2, ..., N, \text{ as } n \to \infty. \tag{3.16}$$

**Step 4**. Show that $||u_n - S_i u_n|| \to 0$, $||x_n - S_i x_n|| \to 0$, as $n \to \infty$; $\forall i \in \{1, 2,..., N\}$.

Since, for any positive integer $n \ge N$, it can be written as $n = (h(n) - 1) N + i(n)$, where $i(n) \in \{1, 2,..., N\}$. Observe that

$$\begin{aligned} ||u_n - S_n u_n|| &\le ||u_n - S_{i(n)}^{h(n)} u_n|| + ||S_{i(n)}^{h(n)} u_n - S_n u_n|| \\ &= ||u_n - S_{i(n)}^{h(n)} u_n|| + ||S_{i(n)}^{h(n)} u_n - S_{i(n)} u_n||. \end{aligned} \tag{3.17}$$

From (3.10), (3.14), the conditions $0 < a \le \alpha_n \le 1$ and $0 < \delta \le \beta_n \le 1 - \lambda$, we obtain

$$\begin{aligned} ||S_{i(n)}^{h(n)} u_n - u_n|| &= \frac{1}{\beta_n} ||z_n - u_n|| \\ &= \frac{1}{\alpha_n \beta_n} ||y_n - u_n|| \\ &\le \frac{1}{a\delta}(||y_n - x_n|| + ||u_n - x_n||) \to 0, \text{ as } n \to \infty. \end{aligned} \tag{3.18}$$

Next, we prove that

$$\lim_{n\to\infty} ||S_{i(n)}^{h(n)-1}u_n - u_n|| = 0. \tag{3.19}$$

It is obvious that the relations hold: $h(n) = h(n - N) + 1$, $i(n) = i(n - N)$.

Therefore,

$$\begin{aligned} ||S_{i(n)}^{h(n)-1}u_n - u_n|| &\le ||S_{i(n)}^{h(n)-1}u_n - S_{i(n-N)}^{h(n)-1}u_{n-N+1}|| + ||S_{i(n-N)}^{h(n)-1}u_{n-N+1} - S_{i(n-N)}^{h(n-N)}u_{n-N}|| \\ &\quad + ||S_{i(n-N)}^{h(n-N)}u_{n-N} - u_{n-N}|| + ||u_{n-N} - u_{n-N+1}|| + ||u_{n-N+1} - u_n|| \\ &= ||S_{i(n)}^{h(n)-1}u_n - S_{i(n)}^{h(n)-1}u_{n-N+1}|| + ||S_{i(n-N)}^{h(n-N)}u_{n-N+1} - S_{i(n-N)}^{h(n-N)}u_{n-N}|| \\ &\quad + ||S_{i(n-N)}^{h(n-N)}u_{n-N} - u_{n-N}|| + ||u_{n-N} - u_{n-N+1}|| + ||u_{n-N+1} - u_n||. \end{aligned} \tag{3.20}$$

Applying Lemma 2.5 and (3.16), we get (3.19). Using the uniformly continuity of $S_i$, we obtain

$$\lim_{n\to\infty} ||S_{i(n)}^{h(n)}u_n - S_{i(n)}u_n|| = 0, \tag{3.21}$$

this together with (3.17) yields

$$\lim_{n\to\infty} ||u_n - S_n u_n|| = 0.$$

We also have

$$||u_n - S_{n+i}u_n|| \le ||u_n - u_{n+i}|| + ||u_{n+i} - S_{n+i}u_{n+i}|| + ||S_{n+i}u_{n+i} - S_{n+i}u_n|| \to 0, \text{ as } n \to \infty,$$

for any $i = 1, 2, \dots N$, which gives that

$$\lim_{n\to\infty} ||u_n - S_i u_n|| = 0; \forall i = 1, 2, \dots N. \tag{3.22}$$

Moreover, for each $i \in \{1, 2, \dots N\}$, we obtain that

$$||x_n - S_i x_n|| \le ||x_n - u_n|| + ||u_n - S_i u_n|| + ||S_i u_n - S_i x_n|| \to 0, \text{ as } n \to \infty. \tag{3.23}$$

**Step 5**. The following implication holds:

$$\omega_w(x_n) \subset \Omega. \tag{3.24}$$

We first show that $\omega_w(x_n) \subset \cap_{i=1}^N F(S_i)$. To this end, we take $\omega \in \omega_w(x_n)$ and assume that $x_{n_j} \rightharpoonup \omega$ as $j \to \infty$ for some subsequence $\{x_{n_j}\}$ of $x_n$.

Note that $S_i$ is uniformly continuous and (3.23), we see that $||x_n - S_i^m x_n|| \to 0$, for all $m \in \mathbb{N}$. So by Lemma 2.4, it follows that $\omega \in \cap_{i=1}^N F(S_i)$ and hence $\omega_w(x_n) \subset \cap_{i=1}^N F(S_i)$.

Next we will show that $\omega \in \cap_{k=1}^M EP(F_k)$. Indeed, by Lemma 2.6, we have that for each $k = 1, 2, \dots, M$,

$$F_k(\Theta_n^k x_n, y) + \frac{1}{r_n}\langle y - \Theta_n^k x_n, \Theta_n^k x_n - \Theta_n^{k-1} x_n\rangle \ge 0, \forall y \in C.$$

From (A2), we get

$$\frac{1}{r_n}\langle y - \Theta_n^k x_n, \Theta_n^k x_n - \Theta_n^{k-1} x_n\rangle \ge F_k(y, \Theta_n^k x_n), \forall y \in C.$$

Hence,

$$\langle y - \Theta_{n_j}^k x_{n_j}, \frac{\Theta_{n_j}^k x_{n_j} - \Theta_{n_j}^{k-1} x_{n_j}}{r_{n_j}}\rangle \ge F_k(y, \Theta_{n_j}^k x_{n_j}), \forall y \in C.$$

From (3.11), we obtain that $\Theta_{n_j}^k x_{n_j} \rightharpoonup \omega$ as $j \to \infty$ for each $k = 1, 2, ..., M$ (especially, $u_{n_j} = \Theta_{n_j}^M x_{n_j}$). Together with (3.11) and (A4) we have, for each $k = 1, 2, ..., M$, that

$$0 \geq F_k(y, \omega), \forall y \in C.$$

For any, $0 < t \leq 1$ and $y \in C$, let $y_t = ty + (1 - t)\omega$. Since $y \in C$ and $\omega \in C$, we obtain that $y_t \in C$ and hence $F_k(y_t, \omega) \leq 0$. So, we have

$$0 = F_k(y_t, y_t) \leq tF_k(y_t, y) + (1 - t)F_k(y_t, \omega) \leq tF_k(y_t, y).$$

Dividing by $t$, we get, for each $k = 1, 2, ..., M$, that

$$F_k(y_t, y) \geq 0, \forall y \in C.$$

Letting $t \to 0$ and from (A3), we get

$$F_k(\omega, y) \geq 0$$

for all $y \in C$ and $\omega \in EP(F_k)$ for each $k = 1, 2, ..., M$, i.e., $\omega \in \cap_{k=1}^M EP(F_k)$.

Hence (3.24) holds.

**Step 6**. Show that $x_n \to q = P_\Omega x_1$.

From (3.6), (3.24) and Lemma 2.3, we conclude that $x_n \to q$, where $q = P_\Omega x_1$. □

**Corollary 3.2**. *Let $C$ be a nonempty closed convex subset of a real Hilbert space $H$ and $N \geq 1$ be an integer, let $F$ be a bifunction from $C \times C$ to $\mathbb{R}$ which satisfies conditions (A1)-(A4). Let, for each $1 \leq i \leq N$, $S_i : C \to C$ be a uniformly continuous $\lambda_i$-strict asymptotically pseudocontractive mapping in the intermediate sense for some $0 \leq \lambda_i < 1$ with sequences $\{\gamma_{n,i}\} \subset [0, \infty)$ such that $\lim_{n\to\infty} \gamma_{n,i} = 0$ and $\{c_{n,i}\} \subset [0, \infty)$ such that $\lim_{n\to\infty} c_{n,i} = 0$. Let $\lambda = max\{\lambda_i : 1 \leq i \leq N\}$, $\gamma_n = max\{\gamma_{n,i} : 1 \leq i \leq N\}$ and $c_n = max\{c_{n,i} : 1 \leq i \leq N\}$. Assume that $\Omega = \cap_{i=1}^N F(S_i) \cap EP(F)$ is nonempty and bounded. Let $\{\alpha_n\}$ and $\{\beta_n\}$ be sequences in $[0, 1]$ such that $0 < a \leq \alpha_n \leq 1, 0 < \delta \leq \beta_n \leq 1 - \lambda$ for all $n \in N$ and $\{r_n\} \subset (0,\infty)$ satisfies $\liminf_{n\to\infty} r_n > 0$ for all $k \in \{1, 2, ... M\}$.*

*Let $\{x_n\}$ and $\{u_n\}$ be sequences generated by the following algorithm*:

$$\begin{cases} x_1 \in C \text{ chosen arbitrary}, \\ u_n = T_{r_n}^F x_n, \\ z_n = (1 - \beta_n)u_n + \beta_n S_{i(n)}^{h(n)} u_n, \\ y_n = (1 - \alpha_n)u_n + \alpha_n z_n, \\ C_n = \{v \in C : \ ||y_n - v||^2 \leq ||x_n - v||^2 + \theta_n\}, \\ Q_n = \{v \in C : \langle x_n - v, x_1 - x_n \rangle \geq 0\}, \\ x_{n+1} = P_{C_n \cap Q_n} x_1, \forall n \in \mathbb{N}, \end{cases} \tag{3.25}$$

*where $\theta_n = c_{h(n)} + \gamma_{h(n)}\rho_n^2 \to 0$, as $n \to \infty$, where $\rho_n = sup\{||x_n - v|| : v \in \Omega\} < \infty$. Then $\{x_n\}$ converges strongly to $P_\Omega x_1$.*

*Proof*. Putting $M = 1$, we can draw the desired conclusion from Theorem 3.1.
□

*Remark* 3.3. Corollary 3.2 extends the theorem of Tada and Takahashi [14] from a nonexpansive mapping to a finite family of asymptotically $\lambda_i$-strict pseudocontractive mappings in the intermediate sense.

**Corollary 3.4**. *Let $C$ be a nonempty closed convex subset of a real Hilbert space $H$ and $N \geq 1$ be an integer, let, for each $1 \leq i \leq N$, $S_i : C \to C$ be a uniformly continuous $\lambda_i$-strict asymptotically pseudocontractive mapping in the intermediate sense for some $0 \leq \lambda_i < 1$ with sequences $\{\gamma_{n,i}\} \subset [0, \infty)$ such that $\lim_{n\to\infty} \gamma_{n,i} = 0$ and $\{c_{n,i}\} \subset [0, \infty)$ such that $\lim_{n\to\infty} c_{n,i} = 0$. Let $\lambda = max\{\lambda_i : 1 \leq i \leq N\}$, $\gamma_n = max\{\gamma_{n,i} : 1 \leq i \leq N\}$ and $c_n = \max\{c_{n,i} : 1$*

*≤ i ≤ N}. Assume that* $\Omega = \cap_{i=1}^{N} F(S_i)$*is nonempty and bounded. Let* $\{\alpha_n\}$ *and* $\{\beta_n\}$ *be sequences in* [0, 1] *such that* $0 < a \le \alpha_n \le 1$, $0 < \delta \le \beta_n \le 1 - \lambda$ *for all* $n \in \mathbb{N}$. *Let* $\{x_n\}$ *and* $\{u_n\}$ *be sequences generated by the following algorithm*:

$$\begin{cases} x_1 \in C \text{ chosen arbitrary}, \\ y_n = (1-\beta_n)x_n + \beta_n S_{i(n)}^{h(n)} x_n, \\ C_n = \{v \in C : \ ||y_n - v||^2 \le ||x_n - v||^2 + \theta_n\}, \\ Q_n = \{v \in C : \langle x_n - v, x_1 - x_n \rangle \ge 0\}, \\ x_{n+1} = P_{C_n \cap Q_n} x_1, \forall n \in \mathbb{N}, \end{cases} \tag{3.26}$$

*where* $\theta_n = c_{h(n)} + \gamma_{h(n)} \rho_n^2 \to 0$, *as* $n \to \infty$, *where* $\rho_n = sup\{||x_n - v|| : v \in \Omega\} < \infty$. *Then* $\{x_n\}$ *converges strongly to* $P_\Omega x_1$.

*Proof.* If $F_k(x, y) = 0$, $\alpha_n = 1$ in Theorem 3.1, we can draw the conclusion easily. □

*Remark* 3.5. Corollary 3.4 extends the Theorem 4.1 of [4] and Theorem 2.2 of [13], respectively.

## 4. Numerical result

In this section, in order to demonstrate the effectiveness, realization and convergence of the algorithm in Theorem 3.1, we consider the following simple example ever appeared in the reference [4]:

**Example 4.1**. *Let* $x = R$ *and* $C = [0, 1]$ *For each* $x \in C$, *we define*

$$Tx = \begin{cases} kx, & \text{if } x \in [0, \frac{1}{2}], \\ 0, & \text{if } x \in (\frac{1}{2}, 1], \end{cases}$$

*where* $0 < k < 1$.

Set $C_1 := [0, 1/2]$ and $C_2 := (1/2, 1]$. Hence,

$$|T^n x - T^n y| = k^n |x - y| \le |x - y| \text{ for all } x, y \in C_1 \text{ and } n \in \mathbb{N}$$

and

$$|T^n x - T^n y| = 0 \le |x - y| \text{ for all } x, y \in C_2 \text{ and } n \in \mathbb{N}.$$

For $x \in C_1$ and $y \in C_2$, we have

$$|T^n x - T^n y| = |k^n x - 0| \le k^n |x - y| + k^n |y| \le |x - y| + k^n \text{ for all } n \in \mathbb{N}.$$

Thus

$$|T^n x - T^n y|^2 \le (|x - y| + k^n)^2 \le |x - y|^2 + k|x - T^n x - (y - T^n y)|^2 + k^n K.$$

for all $x, y \in C$, $n \in \mathbb{N}$ and some $K > 0$. Therefore, $T$ is an asymptotically $k$-strict pseudocontractive mapping in the intermediate sense.

In the algorithm (3.1), set $F_k(x, y) = 0, N = 1, \beta_n = 1 - k, \alpha_n = \frac{n+1}{2n}$. We apply it to find the fixed point of $T$ of Example 4.1.

Under the above assumptions, (3.1) is simplified as follows:

$$\begin{cases} x_1 \in C \text{ chosen arbitrary }, \\ z_n = kx_n + (1-k)T^n x_n, \\ y_n = \frac{n-1}{2n} x_n + \frac{n+1}{2n} z_n, \\ C_n = \{v \in C : |y_n - v|^2 \le |x_n - v|^2 + \theta_n\}, \\ Q_n = \{v \in C : (x_n - v)(x_1 - x_n) \ge 0\}, \\ x_{n+1} = P_{C_n \cap Q_n} x_1, \forall n \in \mathbb{N}, \end{cases}$$

In fact, in one dimensional case, the $C_n \cap Q_n$ is an closed interval. If we set $[a_n, b_n] := C_n \cap Q_n$, then the projection point $x_{n+1}$ of $x_1 \in C$ onto $C_n \cap Q_n$ can be expressed as:

$$x_{n+1} = P_{C_n \cap Q_n} x_1 = \begin{cases} x_1, & \text{if } x_1 \in [a_n, b_n], \\ b_n, & \text{if } x_1 > b_n, \\ a_n, & \text{if } x_1 < a_n. \end{cases}$$

Since the conditions of Theorem 3.1 are satisfied in Example 4.1, the conclusion holds, i.e., $x_n \to 0 \in F(T)$.

Now we turn to realizing (3.1) for approximating a fixed point of $T$. Take the initial guess $x_1 = 1/2$, $1/5$ and $5/8$, respectively. All the numerical results are given in Tables 1, 2 and 3. The corresponding graph appears in Figure 1a,b,c.

**Table 1** $x_1 = 0.5$

| *n* (iterative number) | $x_1$ (initial guess) | Errors (*n*) |
|---|---|---|
| 5 | 0.2471 | $2.471 \times 10^{-1}$ |
| 20 | 0.0527 | $5.27 \times 10^{-2}$ |
| 50 | 0.0028 | $2.8 \times 10^{-3}$ |
| 93 | 0.0000 | 0 |

**Table 2** $x_1 = 0.2$

| *n* (iterative number) | $x_1$ (initial guess) | Errors (*n*) |
|---|---|---|
| 5 | 0.0998 | $9.98 \times 10^{-2}$ |
| 20 | 0.0211 | $2.11 \times 10^{-2}$ |
| 50 | 0.0011 | $1.1 \times 10^{-3}$ |
| 83 | 0.0000 | 0 |

**Table 3** $x_1 = \frac{5}{8}$

| *n* (iterative number) | $x_1$ (initial guess) | Errors (*n*) |
|---|---|---|
| 5 | 0.2636 | $2.636 \times 10^{-1}$ |
| 20 | 0.0562 | $5.62 \times 10^{-2}$ |
| 50 | 0.0030 | $3.0 \times 10^{-3}$ |
| 93 | 0.0000 | 0 |

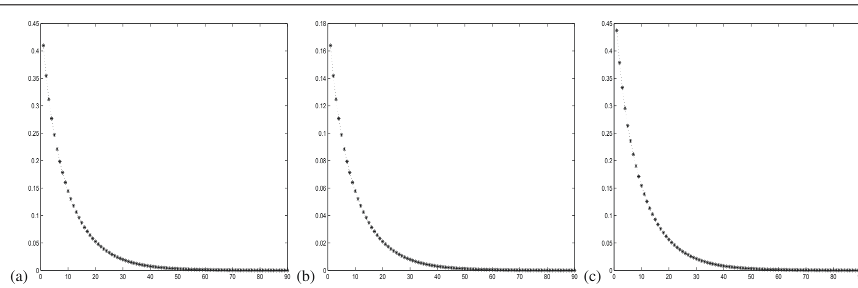

**Figure 1 The iteration comparison chart of different initial values**. **(a)** $x_1 = 0.5$; **(b)** $x_1 = 0.2$; **(c)** $x_1 = \frac{5}{8}$.

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**Authors' contributions**
PD carried out the proof of convergence of the theorems and realization of numerical examples. JZ carried out the check of the manuscript. All authors read and approved the final manuscript.

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The authors declare that they have no competing interests.

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