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On a product operator from weighted Bergman-Nevanlinna spaces to weighted Zygmund spaces

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Abstract

Let $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ be the open unit disk, φ an analytic self-map of $\mathbb{D}$ and ψ an analytic function in $\mathbb{D}$. Let $\mathcal{D}$ be the differentiation operator and $W_{\varphi,\psi}$ the weighted composition operator. The boundedness and compactness of the product operator $\mathcal{D}W_{\varphi,\psi}$ from weighted Bergman-Nevanlinna spaces to weighted Zygmund spaces on $\mathbb{D}$ are characterized.

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1 Introduction

Let $\mathbb{C}$ be the complex plane, $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ the open unit disk, $H(\mathbb{D})$ the space of all analytic functions on $\mathbb{D}$ and $dA(z) = \frac{1}{\pi} \, dx \, dy$ the normalized Lebesgue measure on $\mathbb{D}$. For $\alpha > -1$, let $dA_\alpha(z) = (\alpha + 1)(1 - |z|^2)^\alpha dA(z)$ be the weighted Lebesgue measure on $\mathbb{D}$. The *weighted Bergman-Nevanlinna space* $\mathcal{A}^\alpha_{\log}$ consists of all $f \in H(\mathbb{D})$ such that

$$\|f\|_{\mathcal{A}^\alpha_{\log}} = \int_{\mathbb{D}} \log\big(1 + \big|f(z)\big|\big) \, dA_\alpha(z) < \infty.$$

It is a Fréchet space with the translation invariant metric

$$d(f, g) = \|f - g\|_{\mathcal{A}^\alpha_{\log}}.$$

For some details of this space, see, *e.g.*, [1–3] and [4].

For $\beta > 0$, the *weighted-type* $\mathcal{A}_\beta$ consists of all $f \in H(\mathbb{D})$ such that

$$\sup_{z \in \mathbb{D}}\big(1 - |z|^2\big)^\beta \big|f(z)\big| < \infty.$$

This space is a non-separable Banach space with the norm defined by

$$\|f\|_{\mathcal{A}_\beta} = \sup_{z \in \mathbb{D}}\big(1 - |z|^2\big)^\beta \big|f(z)\big|.$$

The closure of the set of polynomials in $\mathcal{A}_\beta$ is denoted by $\mathcal{A}_{\beta,0}$, which is a separable Banach space and consists exactly of those functions f in $\mathcal{A}_\beta$ satisfying the condition

$$\lim_{|z|\to 1^-}\left(1-|z|^2\right)^\beta\left|f(z)\right| = 0.$$

For $\beta > 0$, the *weighted Bloch space* is defined by

$$\mathcal{B}_\beta = \left\{f \in H(\mathbb{D}) : \sup_{z\in\mathbb{D}}\left(1-|z|^2\right)^\beta\left|f'(z)\right| < \infty\right\},$$

and it is well known (see, *e.g.*, [5]) that, if $\beta > 1$, then $f \in \mathcal{B}_\beta$ if and only if $f \in \mathcal{A}_{\beta-1}$. Under the norm

$$\|f\|_{\mathcal{B}_\beta} = \left|f(0)\right| + \sup_{z\in\mathbb{D}}\left(1-|z|^2\right)^\beta\left|f'(z)\right|,$$

it is a Banach space. The closure of the set of polynomials in $\mathcal{B}_\beta$ is called the *little weighted Bloch space* and denoted by $\mathcal{B}_{\beta,0}$. For a good source for such spaces, we refer to [5].

For $\beta > 0$, the *weighted Zygmund space* $\mathcal{Z}_\beta$ consists of all $f \in H(\mathbb{D})$ such that

$$\sup_{z\in\mathbb{D}}\left(1-|z|^2\right)^\beta\left|f''(z)\right| < \infty.$$

It is a Banach space with the norm

$$\|f\|_{\mathcal{Z}_\beta} = \left|f(0)\right| + \left|f'(0)\right| + \sup_{z\in\mathbb{D}}\left(1-|z|^2\right)^\beta\left|f''(z)\right|.$$

The *little weighted Zygmund space* $\mathcal{Z}_{\beta,0}$ consists those functions f in $\mathcal{Z}_\beta$ satisfying

$$\lim_{|z|\to 1^-}\left(1-|z|^2\right)^\beta\left|f''(z)\right| = 0,$$

and it is a closed subspace of the weighted Zygmund space.

Recently, many authors have studied the properties of some concrete operators between various spaces of analytic functions in the unit disk, the upper half plane, the unit ball and the unit polydisk. For some operators on weighted-type spaces, weighted Bloch spaces, and weighted Zygmund spaces on these domains, see, *e.g.*, [4, 6–38] and the references therein.

Let φ be an analytic self-map of $\mathbb{D}$ and let ψ be an analytic function in $\mathbb{D}$. It is well known that the *weighted composition operator* $W_{\varphi,\psi}$ on $H(\mathbb{D})$ is defined by

$$W_{\varphi,\psi}f(z) = \psi(z)\cdot f\big(\varphi(z)\big), \quad z \in \mathbb{D}.$$

If $\psi \equiv 1$, $W_{\varphi,\psi} := C_\varphi$ is called the *composition operator*. If $\varphi(z) = z$, $W_{\varphi,\psi} := M_\psi$ it is called the *multiplication operator*. It is of interest to provide function theoretic characterizations when φ and ψ induce a bounded or compact weighted composition operator. Sharma and Abbas have studied the boundedness and compactness of weighted composition operators from weighted Bergman-Nevanlinna spaces to Bloch spaces in [39]. Kumar and Sharma

have characterized the boundedness and compactness of weighted composition operators from weighted Bergman-Nevanlinna spaces to Zygmund spaces in [40].

Now we list several operators, which will be considered in this paper. Let $\mathcal{D}$ be the differentiation operator on $H(\mathbb{D})$ defined by

$$\mathcal{D}f(z) = f'(z), \quad z \in \mathbb{D}.$$

Operator $\mathcal{D}C_\varphi$ has been studied, for example, in [10, 15, 19, 20, 31, 35, 41–45]. Here we want to mention that Sharma has studied the following two operators from Bergman-Nevanlinna spaces to Bloch-type spaces in [46]. They are defined as follows:

$$\mathcal{D}M_\psi C_\varphi f(z) = \psi'(z)f\big(\varphi(z)\big) + \psi(z)\varphi'(z)f'\big(\varphi(z)\big)$$

and

$$\mathcal{D}C_\varphi M_\psi f(z) = \varphi'(z)\psi'\big(\varphi(z)\big)f\big(\varphi(z)\big) + \psi\big(\varphi(z)\big)\varphi'(z)f'\big(\varphi(z)\big),$$

for $z \in \mathbb{D}$ and $f \in H(\mathbb{D})$. These operators on weighted Bergman spaces have been also studied by Stević *et al.* in [47] and [48]. If we consider the *product operator* $\mathcal{D}W_{\varphi,\psi}$, it is clear that $\mathcal{D}M_\psi C_\varphi = \mathcal{D}W_{\varphi,\psi}$ and $\mathcal{D}C_\varphi M_\psi = \mathcal{D}W_{\varphi,\psi\circ\varphi}$. Quite recently, the operator $\mathcal{D}W_{\varphi,\psi}$ from weighted Bergman spaces to weighted Zygmund spaces has been considered in [49]. This paper is devoted to characterizing the boundedness and compactness of the operator $\mathcal{D}W_{\varphi,\psi}$ from weighted Bergman-Nevanlinna spaces to weighted Zygmund spaces. This paper can be regarded as an continuousness of our work.

Since the weighted Bergman-Nevanlinna space is a Fréchet space and not a Banach space, it is necessary to introduce several definitions needed in this paper. Let X and Y be topological vector spaces whose topologies are given by translation invariant metrics d_X and d_Y, respectively, and let $L: X \to Y$ be a linear operator. It is said that L is *metrically bounded* if there exists a positive constant K such that

$$d_Y(Lf, 0) \leq K d_X(f, 0)$$

for all $f \in X$. When X and Y are Banach spaces, the metrical boundedness coincides with the usual definition of bounded operators between Banach spaces. Recall that $L: X \to Y$ is *metrically compact* if it maps bounded sets into relatively compact sets. When X and Y are Banach spaces, the metrical compactness coincides with the usual definition of compact operators between Banach spaces. When $X = \mathcal{A}_{\log}^\alpha$ and Y is a Banach space, we define

$$\|L\|_{\mathcal{A}_{\log}^\alpha \to Y} = \sup_{\|f\|_{\mathcal{A}_{\log}^\alpha} \leq 1} \|Lf\|_Y,$$

and we often write $\|L\|_{\mathcal{A}_{\log}^\alpha \to Y}$ by $\|L\|$.

Throughout this paper, an operator is bounded (respectively, compact), if it is metrically bounded (respectively, metrically compact). Constants are denoted by C, they are positive and may differ from one occurrence to the next. The notation $a \asymp b$ means that there exists a positive constant C such that $a/C \leq b \leq Ca$.

2 The operator $\mathcal{D}W_{\varphi,\psi} : \mathcal{A}^\alpha_{\log} \to \mathcal{Z}_\beta(\mathcal{Z}_{\beta,0})$

Our first lemma characterizes the compactness in terms of sequential convergence. Since the proof is standard, it is omitted (see Proposition 3.11 in [50]).

Lemma 2.1 *Let* $\alpha > -1$, $\beta > 0$ *and* $\mathcal{Y} \in \{\mathcal{Z}_\beta, \mathcal{Z}_{\beta,0}\}$. *Then the bounded operator* $\mathcal{D}W_{\varphi,\psi} : \mathcal{A}^\alpha_{\log} \to \mathcal{Y}$ *is compact if and only if for every bounded sequence* $(f_n)_{n\in\mathbb{N}}$ *in* $\mathcal{A}^\alpha_{\log}$ *such that* $f_n \to 0$ *uniformly on every compact subset of* $\mathbb{D}$ *as* $n \to \infty$, *it follows that*

$$\lim_{n\to\infty} \|\mathcal{D}W_{\varphi,\psi} f_n\|_{\mathcal{Y}} = 0.$$

The next result can be found, for example, in [4].

Lemma 2.2 *Let* $\alpha > -1$ *and* $n \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$. *Then, for all* $f \in \mathcal{A}^\alpha_{\log}$ *and* $z \in \mathbb{D}$, *there exists a positive constant* C *independent of* f *such that*

$$(1 - |z|^2)^n |f^{(n)}(z)| \leq \exp\frac{C\|f\|_{\mathcal{A}^\alpha_{\log}}}{(1 - |z|^2)^{\alpha+2}}.$$

Now we consider the boundedness of the operator $\mathcal{D}W_{\varphi,\psi} : \mathcal{A}^\alpha_{\log} \to \mathcal{Z}_\beta$.

Theorem 2.3 *Let* $\alpha > -1$, $\beta > 0$, φ *an analytic self-map of* $\mathbb{D}$ *and* $\psi \in H(\mathbb{D})$. *Then, for all* $c > 0$, *the following statements are equivalent*:

  (i) *The operator* $\mathcal{D}W_{\varphi,\psi} : \mathcal{A}^\alpha_{\log} \to \mathcal{Z}_\beta$ *is bounded.*
 (ii) *The operator* $\mathcal{D}W_{\varphi,\psi} : \mathcal{A}^\alpha_{\log} \to \mathcal{Z}_\beta$ *is compact.*
(iii) $\psi' \in \mathcal{Z}_\beta$,

$$M_0 := \sup_{z\in\mathbb{D}}(1 - |z|^2)^\beta |\psi(z)\varphi'''(z) + 3\psi''(z)\varphi'(z) + 3\psi'(z)\varphi''(z)| < \infty,$$

$$M_1 := \sup_{z\in\mathbb{D}}(1 - |z|^2)^\beta |\psi(z)||\varphi'(z)|^3 < \infty,$$

$$M_2 := \sup_{z\in\mathbb{D}}(1 - |z|^2)^\beta |\psi'(z)\varphi'(z)^2 + \psi(z)\varphi'(z)\varphi''(z)| < \infty,$$

$$\lim_{\varphi(z)\to\partial\mathbb{D}} (1 - |z|^2)^\beta |\psi'''(z)| \exp\frac{c}{(1 - |\varphi(z)|^2)^{\alpha+2}} = 0,$$

$$\lim_{\varphi(z)\to\partial\mathbb{D}} \frac{(1 - |z|^2)^\beta}{(1 - |\varphi(z)|^2)^3} |\psi(z)||\varphi'(z)|^3 \exp\frac{c}{(1 - |\varphi(z)|^2)^{\alpha+2}} = 0,$$

$$\lim_{\varphi(z)\to\partial\mathbb{D}} \frac{(1 - |z|^2)^\beta}{(1 - |\varphi(z)|^2)^2} |\psi'(z)\varphi'(z)^2 + \psi(z)\varphi'(z)\varphi''(z)| \exp\frac{c}{(1 - |\varphi(z)|^2)^{\alpha+2}} = 0,$$

and

$$\lim_{\varphi(z)\to\partial\mathbb{D}} \frac{(1 - |z|^2)^\beta}{1 - |\varphi(z)|^2} |\psi(z)\varphi'''(z) + 3\psi''(z)\varphi'(z) + 3\psi'(z)\varphi''(z)| \exp\frac{c}{(1 - |\varphi(z)|^2)^{\alpha+2}} = 0.$$

Proof (i) $\Rightarrow$ (iii). Suppose that (i) holds. Take the functions $f(z) = z$ and $f(z) \equiv 1$, respectively. Since the operator $\mathcal{D}W_{\varphi,\psi} : \mathcal{A}^\alpha_{\log} \to \mathcal{Z}_\beta$ is bounded, we have

$$\sup_{z\in\mathbb{D}}(1 - |z|^2)^\beta |\psi'''(z)\varphi(z) + 3\psi''(z)\varphi'(z) + 3\psi'(z)\varphi''(z) + \psi(z)\varphi'''(z)|$$

$$\leq \|\mathcal{D}W_{\varphi,\psi} z\|_{\mathcal{Z}_\beta} \leq C\|\mathcal{D}W_{\varphi,\psi}\|, \tag{1}$$

and

$$\sup_{z\in\mathbb{D}}\left(1-|z|^2\right)^{\beta}\left|\psi'''(z)\right| \leq \|\mathcal{D}W_{\varphi,\psi}1\|_{\mathcal{Z}_{\beta}} \leq C\|\mathcal{D}W_{\varphi,\psi}\|. \tag{2}$$

Inequality (2) shows $\psi' \in \mathcal{Z}_{\beta}$. From (1), (2), and the boundedness of φ, it follows that

$$M_0 = \sup_{z\in\mathbb{D}}\left(1-|z|^2\right)^{\beta}\left|\psi(z)\varphi'''(z) + 3\psi''(z)\varphi'(z) + 3\psi'(z)\varphi''(z)\right| \leq C\|\mathcal{D}W_{\varphi,\psi}\|. \tag{3}$$

Taking the functions $f(z) = z^2$ and $f(z) = z^3$, respectively, we have

$$\begin{aligned}
&\sup_{z\in\mathbb{D}}\left(1-|z|^2\right)^{\beta}\big|\psi(z)'''(z)\varphi(z)^2 + 6\psi''(z)\varphi'(z)\varphi(z) + 6\psi'(z)\varphi''(z)\varphi(z)\\
&\quad + 6\psi'(z)\varphi'(z)^2 + 6\psi(z)\varphi'(z)\varphi''(z) + 2\psi(z)\varphi'''(z)\varphi(z)\big|\\
&\leq \left\|\mathcal{D}W_{\varphi,\psi}z^2\right\|_{\mathcal{Z}_{\beta}} \leq C\|\mathcal{D}W_{\varphi,\psi}\|,
\end{aligned} \tag{4}$$

and

$$\begin{aligned}
&\sup_{z\in\mathbb{D}}\left(1-|z|^2\right)^{\beta}\big|\psi'''(z)\varphi(z)^3 + 9\psi''(z)\varphi'(z)\varphi(z)^2 + 18\psi'(z)\varphi(z)\varphi'(z)^2\\
&\quad + 6\psi(z)\varphi'(z)^3 + 18\psi(z)\varphi(z)\varphi'(z)\varphi''(z) + 9\psi'(z)\varphi''(z)\varphi(z)^2 + 3\psi(z)\varphi(z)^2\varphi'''(z)\big|\\
&\leq \left\|\mathcal{D}W_{\varphi,\psi}z^3\right\|_{\mathcal{Z}_{\beta}} \leq C\|\mathcal{D}W_{\varphi,\psi}\|.
\end{aligned} \tag{5}$$

By (2) and the boundedness of φ, we have

$$\sup_{z\in\mathbb{D}}\left(1-|z|^2\right)^{\beta}\left|\psi'''(z)\right|\left|\varphi(z)\right|^2 \leq C\|\mathcal{D}W_{\varphi,\psi}\|, \tag{6}$$

and

$$\sup_{z\in\mathbb{D}}\left(1-|z|^2\right)^{\beta}\left|\psi'''(z)\right|\left|\varphi(z)\right|^3 \leq C\|\mathcal{D}W_{\varphi,\psi}\|. \tag{7}$$

From (3), (4), (6), and the boundedness of φ, it follows that

$$M_2 = \sup_{z\in\mathbb{D}}\left(1-|z|^2\right)^{\beta}\left|\psi'(z)\varphi'(z)^2 + \psi(z)\varphi'(z)\varphi''(z)\right| \leq C\|\mathcal{D}W_{\varphi,\psi}\|. \tag{8}$$

Inequalities (3), (5), (7), (8), and the boundedness of φ give

$$M_1 = \sup_{z\in\mathbb{D}}\left(1-|z|^2\right)\left|\psi(z)\right|\left|\varphi'(z)\right|^3 \leq C\|\mathcal{D}W_{\varphi,\psi}\|. \tag{9}$$

For $w \in \mathbb{D}$, we choose the functions

$$\begin{aligned}
f_1(z) = {}&\frac{\alpha+4}{3\alpha+6}\frac{(1-|\varphi(w)|^2)^{\alpha+2}}{(1-\overline{\varphi(w)}z)^{2(\alpha+2)}} - \frac{2\alpha+8}{\alpha+3}\frac{(1-|\varphi(w)|^2)^{\alpha+4}}{(1-\overline{\varphi(w)}z)^{2(\alpha+2)+2}}\\
&+ \frac{16\alpha+64}{6\alpha+21}\frac{(1-|\varphi(w)|^2)^{\alpha+5}}{(1-\overline{\varphi(w)}z)^{2(\alpha+2)+3}} - \frac{(1-|\varphi(w)|^2)^{\alpha+6}}{(1-\overline{\varphi(w)}z)^{2(\alpha+2)+4}}
\end{aligned}$$

and

$$g_1(z) = \frac{2\alpha + 7}{4\alpha + 8} \frac{(1 - |\varphi(w)|^2)^{\alpha+2}}{(1 - \overline{\varphi(w)}z)^{2(\alpha+2)}} - \frac{6\alpha + 21}{4\alpha + 12} \frac{(1 - |\varphi(w)|^2)^{\alpha+4}}{(1 - \overline{\varphi(w)}z)^{2(\alpha+2)+2}} + \frac{(1 - |\varphi(w)|^2)^{\alpha+5}}{(1 - \overline{\varphi(w)}z)^{2(\alpha+2)+3}}.$$

By a direct calculation, it follows that

$$f_1'\big(\varphi(w)\big) = f_1''\big(\varphi(w)\big) = f_1'''\big(\varphi(w)\big) = 0 \tag{10}$$

and

$$g_1'\big(\varphi(w)\big) = g_1''\big(\varphi(w)\big) = 0. \tag{11}$$

Using f_1 and g_1, we define the function $f(z) = f_1(z) \exp cg_1(z)$. Applying (10) and (11) to f', f'' and f''', we find

$$f'\big(\varphi(w)\big) = f''\big(\varphi(w)\big) = f'''\big(\varphi(w)\big) = 0.$$

It is obvious that

$$f\big(\varphi(w)\big) = \frac{C}{(1 - |\varphi(w)|^2)^{\alpha+2}} \exp \frac{c}{(1 - |\varphi(w)|^2)^{\alpha+2}},$$

where

$$C = \frac{\alpha + 4}{3\alpha + 6} - \frac{2\alpha + 8}{\alpha + 3} + \frac{16\alpha + 64}{6\alpha + 21} - 1.$$

From the proof in [46], we see that $f \in \mathcal{A}_{\log}^{\alpha}$ and $\|f\|_{\mathcal{A}_{\log}^{\alpha}} \le C$. Since the operator $\mathcal{D}W_{\varphi,\psi} : \mathcal{A}_{\log}^{\alpha} \to \mathcal{Z}_{\beta}$ is bounded, it follows that

$$\|\mathcal{D}W_{\varphi,\psi}f\|_{\mathcal{Z}_{\beta}} \le C\|\mathcal{D}W_{\varphi,\psi}\|,$$

which means that, for all $z \in \mathbb{D}$,

$$\big(1 - |z|^2\big)^{\beta} \big|(\mathcal{D}W_{\varphi,\psi}f)''(z)\big| \le C. \tag{12}$$

Replacing z by w in (12), we obtain

$$\frac{(1 - |w|^2)^{\beta}}{(1 - |\varphi(w)|^2)^{\alpha+2}} \big|\psi'''(w)\big| \exp \frac{c}{(1 - |\varphi(w)|^2)^{\alpha+2}} \le C$$

and then

$$\big(1 - |w|^2\big)^{\beta} \big|\psi'''(w)\big| \exp \frac{c}{(1 - |\varphi(w)|^2)^{\alpha+2}} \le C\big(1 - \big|\varphi(w)\big|^2\big)^{\alpha+2}. \tag{13}$$

Taking the limit as $\varphi(w) \to \partial\mathbb{D}$ in (13), it gives

$$\lim_{\varphi(w) \to \partial\mathbb{D}} \big(1 - |w|^2\big)^{\beta} \big|\psi'''(w)\big| \exp \frac{c}{(1 - |\varphi(w)|^2)^{\alpha+2}} = 0.$$

For $w \in \mathbb{D}$, we choose the functions

$$f_2(z) = \frac{1}{3} \frac{(1 - |\varphi(w)|^2)^{\alpha+2}}{(1 - \overline{\varphi(w)}z)^{2(\alpha+2)}} - 2 \frac{(1 - |\varphi(w)|^2)^{\alpha+4}}{(1 - \overline{\varphi(w)}z)^{2(\alpha+2)+2}}$$
$$+ \frac{8}{3} \frac{(1 - |\varphi(w)|^2)^{\alpha+5}}{(1 - \overline{\varphi(w)}z)^{2(\alpha+2)+3}} - \frac{(1 - |\varphi(w)|^2)^{\alpha+6}}{(1 - \overline{\varphi(w)}z)^{2(\alpha+2)+4}}$$

and

$$g_2(z) = \frac{(1 - |\varphi(w)|^2)^{\alpha+2}}{(1 - \overline{\varphi(w)}z)^{2(\alpha+2)}}.$$

From a calculation, we obtain

$$f_2(\varphi(w)) = f_2'(\varphi(w)) = f_2''(\varphi(w)) = 0. \tag{14}$$

Define the function $g(z) = f_2(z) \exp cg_2(z)$. Then by (14),

$$g(\varphi(w)) = g'(\varphi(w)) = g''(\varphi(w)) = 0,$$

and by a direct calculation,

$$g'''(\varphi(w)) = C \frac{\overline{\varphi(w)}^3}{(1 - |\varphi(w)|^2)^{\alpha+5}} \exp \frac{c}{(1 - |\varphi(w)|^2)^{\alpha+2}},$$

where $C = -30(\alpha + 2)^2 - 8$. Since $\mathcal{D}W_{\varphi,\psi} : \mathcal{A}_{\log}^\alpha \to \mathcal{Z}_\beta$ is bounded, we have

$$\|\mathcal{D}W_{\varphi,\psi}g\|_{\mathcal{Z}_\beta} \leq C\|\mathcal{D}W_{\varphi,\psi}\|,$$

and so

$$(1 - |z|^2)^\beta |(\mathcal{D}W_{\varphi,\psi}g)''(z)| \leq C\|\mathcal{D}W_{\varphi,\psi}\|, \tag{15}$$

for all $z \in \mathbb{D}$. Letting $z = w$ in (15) yields

$$\frac{(1 - |w|^2)^\beta |\varphi'(w)|^3 |\varphi(w)|^3 |\psi(w)|}{(1 - |\varphi(w)|^2)^{\alpha+5}} \exp \frac{c}{(1 - |\varphi(w)|^2)^{\alpha+2}} \leq C\|W_{\varphi,\psi}\|.$$

Thus

$$\frac{(1 - |w|^2)^\beta}{(1 - |\varphi(w)|^2)^3} |\varphi'(w)|^3 |\psi(w)| \exp \frac{c}{(1 - |\varphi(w)|^2)^{\alpha+2}} \leq \frac{C(1 - |\varphi(w)|^2)^{\alpha+2}}{|\varphi(w)|^3}. \tag{16}$$

Taking the limit as $\varphi(w) \to \partial\mathbb{D}$ in (16), we have

$$\lim_{\varphi(w) \to \partial\mathbb{D}} \frac{(1 - |w|^2)^\beta}{(1 - |\varphi(w)|^2)^3} |\varphi'(w)|^3 |\psi(w)| \exp \frac{c}{(1 - |\varphi(w)|^2)^{\alpha+2}} = 0.$$

For $w \in \mathbb{D}$, we choose the function

$$f_3(z) = c_1 \frac{(1 - |\varphi(w)|^2)^{\alpha+2}}{(1 - \overline{\varphi(w)}z)^{2(\alpha+2)}} + c_2 \frac{(1 - |\varphi(w)|^2)^{\alpha+4}}{(1 - \overline{\varphi(w)}z)^{2(\alpha+2)+2}}$$
$$+ c_3 \frac{(1 - |\varphi(w)|^2)^{\alpha+5}}{(1 - \overline{\varphi(w)}z)^{2(\alpha+2)+3}} - \frac{(1 - |\varphi(w)|^2)^{\alpha+6}}{(1 - \overline{\varphi(w)}z)^{2(\alpha+2)+4}},$$

where

$$c_2 = -\frac{48\alpha^3 + 460\alpha^2 + 1398\alpha + 1340}{24\alpha^3 + 214\alpha^2 + 655\alpha + 682}, \qquad c_3 = \frac{16\alpha^2 + 104\alpha + 164}{6\alpha^2 + 37\alpha + 62},$$

and

$$c_1 = 1 - c_2 - c_3.$$

We also choose the function

$$g_3(z) = \frac{2\alpha + 7}{4\alpha + 8} \frac{(1 - |\varphi(w)|^2)^{\alpha+2}}{(1 - \overline{\varphi(w)}z)^{2(\alpha+2)}} - \frac{6\alpha + 21}{4\alpha + 12} \frac{(1 - |\varphi(w)|^2)^{\alpha+4}}{(1 - \overline{\varphi(w)}z)^{2(\alpha+2)+2}} + \frac{(1 - |\varphi(w)|^2)^{\alpha+5}}{(1 - \overline{\varphi(w)}z)^{2(\alpha+2)+3}}.$$

For the functions f_3 and g_3, we have

$$f_3\big(\varphi(w)\big) = f_3''\big(\varphi(w)\big) = f_3'''\big(\varphi(w)\big) = 0 \tag{17}$$

and

$$g_3'\big(\varphi(w)\big) = g_3''\big(\varphi(w)\big) = 0. \tag{18}$$

Consequently, (17) and (18) make the function $h(z) = f_3(z) \exp c g_3(z)$ to satisfy

$$h\big(\varphi(w)\big) = h''\big(\varphi(w)\big) = h'''\big(\varphi(w)\big) = 0$$

and

$$h'\big(\varphi(w)\big) = C \frac{\overline{\varphi(w)}}{(1 - |\varphi(w)|^2)^{\alpha+3}} \exp \frac{c}{(1 - |\varphi(w)|^2)^{\alpha+2}},$$

where

$$C = 2c_2 + 3c_3 - 4.$$

By the boundedness of the operator $\mathcal{D}W_{\varphi,\psi} : \mathcal{A}_{\log}^{\alpha} \to \mathcal{Z}_{\beta}$, we find

$$\frac{|\varphi(w)|(1 - |w|^2)^{\beta}}{(1 - |\varphi(w)|^2)^{\alpha+3}} \big|3\psi''(w)\varphi'(w) + 3\psi'(w)\varphi''(w) + \psi(w)\varphi'''(w)\big|$$
$$\times \exp \frac{c}{(1 - |\varphi(w)|^2)^{\alpha+2}} \leq C.$$

Thus

$$\lim_{\varphi(w)\to\partial\mathbb{D}} \frac{(1-|w|^2)^\beta}{1-|\varphi(w)|^2} \left| 3\psi''(w)\varphi'(w) + 3\psi'(w)\varphi''(w) + \psi(w)\varphi'''(w) \right|$$
$$\times \exp\frac{c}{(1-|\varphi(w)|^2)^{\alpha+2}} = 0.$$

For $w \in \mathbb{D}$, we choose the functions

$$f_4(z) = \frac{3\alpha+8}{3\alpha+10} \frac{(1-|\varphi(w)|^2)^{\alpha+2}}{(1-\overline{\varphi(w)}z)^{2(\alpha+2)}} - \frac{6\alpha+22}{3\alpha+10} \frac{(1-|\varphi(w)|^2)^{\alpha+4}}{(1-\overline{\varphi(w)}z)^{2(\alpha+2)+2}}$$
$$+ \frac{6\alpha+24}{3\alpha+10} \frac{(1-|\varphi(w)|^2)^{\alpha+5}}{(1-\overline{\varphi(w)}z)^{2(\alpha+2)+3}} - \frac{(1-|\varphi(w)|^2)^{\alpha+6}}{(1-\overline{\varphi(w)}z)^{2(\alpha+2)+4}}$$

and

$$g_4(z) = \frac{\alpha+3}{\alpha+2} \frac{(1-|\varphi(w)|^2)^{\alpha+2}}{(1-\overline{\varphi(w)}z)^{2(\alpha+2)}} - \frac{(1-|\varphi(w)|^2)^{\alpha+4}}{(1-\overline{\varphi(w)}z)^{2(\alpha+2)+2}}.$$

Then

$$f_4\big(\varphi(w)\big) = f_4'\big(\varphi(w)\big) = f_4'''\big(\varphi(w)\big) = 0 \tag{19}$$

and $g_4'(\varphi(w)) = 0$. From this and (19), for the function $u(z) = f_4(z) \exp c g_4(z)$ we have

$$u\big(\varphi(w)\big) = u'\big(\varphi(w)\big) = u'''\big(\varphi(w)\big) = 0$$

and

$$u''\big(\varphi(w)\big) = C \frac{\overline{\varphi(w)}^2}{(1-|\varphi(w)|^2)^{\alpha+4}} \exp\frac{c}{(1-|\varphi(w)|^2)^{\alpha+2}},$$

where

$$C = -\frac{24\alpha+120\alpha+141}{3\alpha+10}.$$

By the boundedness of $\mathcal{D}W_{\varphi,\psi} : \mathcal{A}_{\log}^\alpha \to \mathcal{Z}_\beta$,

$$\|\mathcal{D}W_{\varphi,\psi} u_{\varphi(w)}\|_{\mathcal{Z}_\beta} \le C\|\mathcal{D}W_{\varphi,\psi}\|,$$

and from which we get

$$\frac{|\varphi(w)|^2(1-|w|^2)^\beta}{(1-|\varphi(w)|^2)^{\alpha+4}} \left| \psi'(w)\varphi'(w)^2 + \psi(w)\varphi'(w)\varphi''(w) \right| \exp\frac{c}{(1-|\varphi(w)|^2)^{\alpha+2}} \le C.$$

This shows that

$$\lim_{\varphi(w)\to\partial\mathbb{D}} \frac{(1-|w|^2)^\beta}{(1-|\varphi(w)|^2)^2} \left| \psi'(w)\varphi'(w)^2 + \psi(w)\varphi'(w)\varphi''(w) \right| \exp\frac{c}{(1-|\varphi(w)|^2)^{\alpha+2}} = 0.$$

The proof of the implication is finished.

(iii) $\Rightarrow$ (ii). Let $(f_n)_{n\in\mathbb{N}}$ be a sequence in $\mathcal{A}^\alpha_{\log}$ with $\sup_{n\in\mathbb{N}}\|f_n\|_{\mathcal{A}^\alpha_{\log}} \leq M$ and $f_n \to 0$ uniformly on every compact subset of $\mathbb{D}$ as $n \to \infty$. We have, for all $c > 0$, and especially for constant C in Lemma 2.2, for any $\varepsilon > 0$, the result that there exists a constant $\delta \in (0,1)$ such that whenever $\delta < |\varphi(z)| < 1$, it follows that

$$(1 - |z|^2)^\beta |\psi'''(z)| \exp \frac{C}{(1 - |\varphi(z)|^2)^{\alpha+2}} < \varepsilon,$$

$$\frac{(1 - |z|^2)^\beta}{(1 - |\varphi(z)|^2)^3} |\varphi'(z)|^3 |\psi(z)| \exp \frac{C}{(1 - |\varphi(z)|^2)^{\alpha+2}} < \varepsilon,$$

$$\frac{(1 - |z|^2)^\beta}{(1 - |\varphi(z)|^2)^2} |\psi'(z)\varphi'(z)^2 + \psi(z)\varphi'(z)\varphi''(z)| \exp \frac{C}{(1 - |\varphi(z)|^2)^{\alpha+2}} < \varepsilon,$$

and

$$\frac{(1 - |z|^2)^\beta}{1 - |\varphi(z)|^2} |\psi(z)\varphi'''(z) + 3\psi''(z)\varphi'(z) + 3\psi'(z)\varphi''(z)| \exp \frac{C}{(1 - |\varphi(z)|^2)^{\alpha+2}} < \varepsilon.$$

Then by Lemma 2.2, we have

$$\|\mathcal{D}W_{\varphi,\psi}f_n\|_{\mathcal{Z}_\beta}$$

$$= \left|(\psi \cdot f_n \circ \varphi)'(0)\right| + \left|(\psi \cdot f_n \circ \varphi)''(0)\right| + \sup_{z\in\mathbb{D}}(1 - |z|^2)^\beta \left|(\psi \cdot f_n \circ \varphi)'''(z)\right|$$

$$\leq \left|(\psi \cdot f_n \circ \varphi)'(0)\right| + \left|(\psi \cdot f_n \circ \varphi)''(0)\right| + \sup_{z\in\mathbb{D}}(1 - |z|^2)^\beta \left|\psi'''(z)f_n(\varphi(z))\right|$$

$$+ \sup_{z\in\mathbb{D}}(1 - |z|^2)^\beta \left|(3\psi''(z)\varphi'(z) + 3\psi'(z)\varphi''(z) + \psi(z)\varphi'''(z))f_n'(\varphi(z))\right|$$

$$+ 3\sup_{z\in\mathbb{D}}(1 - |z|^2)^\beta \left|(\psi'(z)\varphi'(z)^2 + \psi(z)\varphi'(z)\varphi''(z))f_n''(\varphi(z))\right|$$

$$+ \sup_{z\in\mathbb{D}}(1 - |z|^2)^\beta \left|\psi(z)\varphi'(z)^3 f_n'''(\varphi(z))\right|$$

$$\leq \left|(\psi \cdot f_n \circ \varphi)'(0)\right| + \left|(\psi \cdot f_n \circ \varphi)''(0)\right| + \sup_{|\varphi(z)|\leq\delta}(1 - |z|^2)^\beta \left|\psi'''(z)f_n(\varphi(z))\right|$$

$$+ \sup_{\delta<|\varphi(z)|<1}(1 - |z|^2)^\beta \left|\psi'''(z)f_n(\varphi(z))\right|$$

$$+ \sup_{|\varphi(z)|\leq\delta}(1 - |z|^2)^\beta \left|(3\psi''(z)\varphi'(z) + 3\psi'(z)\varphi''(z) + \psi(z)\varphi'''(z))f_n'(\varphi(z))\right|$$

$$+ \sup_{\delta<|\varphi(z)|<1}(1 - |z|^2)^\beta \left|(3\psi''(z)\varphi'(z) + 3\psi'(z)\varphi''(z) + \psi(z)\varphi'''(z))f_n'(\varphi(z))\right|$$

$$+ 3\sup_{|\varphi(z)|\leq\delta}(1 - |z|^2)^\beta \left|(\psi'(z)\varphi'(z)^2 + \psi(z)\varphi'(z)\varphi''(z))f_n''(\varphi(z))\right|$$

$$+ 3\sup_{\delta<|\varphi(z)|<1}(1 - |z|^2)^\beta \left|(\psi'(z)\varphi'(z)^2 + \psi(z)\varphi'(z)\varphi''(z))f_n''(\varphi(z))\right|$$

$$+ \sup_{|\varphi(z)|\leq\delta}(1 - |z|^2)^\beta \left|\psi(z)\varphi'(z)^3 f_n'''(\varphi(z))\right| + \sup_{\delta<|\varphi(z)|<1}(1 - |z|^2)^\beta \left|\psi(z)\varphi'(z)^3 f_n'''(\varphi(z))\right|$$

$$\leq \left|(\psi \cdot f_n \circ \varphi)'(0)\right| + \left|(\psi \cdot f_n \circ \varphi)''(0)\right| + \|\psi'\|_{\mathcal{Z}_\beta} \sup_{|\varphi(z)|\leq\delta}\left|f_n(\varphi(z))\right|$$

$$+ \sup_{\delta<|\varphi(z)|<1}(1 - |z|^2)^\beta \left|\psi'''(z)\right| \exp \frac{C}{(1 - |\varphi(z)|^2)^{\alpha+2}} + M_0 \sup_{|\varphi(z)|\leq\delta}\left|f_n'(\varphi(z))\right|$$

$$+ \sup_{\delta < |\varphi(z)| < 1} \frac{(1-|z|^2)^\beta}{1-|\varphi(z)|^2} \left| 3\psi''(z)\varphi'(z) + 3\psi'(z)\varphi''(z) + \psi(z)\varphi'''(z) \right| \exp \frac{C}{(1-|\varphi(z)|^2)^{\alpha+2}}$$

$$+ 3 \sup_{\delta < |\varphi(z)| < 1} \frac{(1-|z|^2)^\beta}{(1-|\varphi(z)|^2)^2} \left| \psi'(z)\varphi'(z)^2 + \psi(z)\varphi'(z)\varphi''(z) \right| \exp \frac{C}{(1-|\varphi(z)|^2)^{\alpha+2}}$$

$$+ 3M_2 \sup_{|\varphi(z)| \leq \delta} \left| f_n''(\varphi(z)) \right| + M_1 \sup_{|\varphi(z)| \leq \delta} \left| f_n'''(\varphi(z)) \right|$$

$$+ \sup_{\delta < |\varphi(z)| < 1} \frac{(1-|z|^2)^\beta}{(1-|\varphi(z)|^2)^3} \left| \psi(z) \right| \left| \varphi'(z) \right|^3 \exp \frac{C}{(1-|\varphi(z)|^2)^{\alpha+2}}.$$

By Cauchy's estimation, if $(f_n)_{n \in \mathbb{N}}$ converges to zero on each compact subset of $\mathbb{D}$, then $(f_n')_{n \in \mathbb{N}}$, $(f_n'')_{n \in \mathbb{N}}$, and $(f_n''')_{n \in \mathbb{N}}$ also do as $n \to \infty$. From this, and since both $\{z \in \mathbb{D} : |z| \leq \delta\}$ and $\{0\}$ are compact subset of $\mathbb{D}$, we can choose a natural number N such that, whenever $n > N$, it follows that

$$\left| (\psi \cdot f_n \circ \varphi)'(0) \right| + \left| (\psi \cdot f_n \circ \varphi)''(0) \right| < \varepsilon$$

and

$$\sup_{|\varphi(z)| \leq \delta} \left| f_n^{(i)}(\varphi(z)) \right| < \varepsilon,$$

where $i = 0, 1, 2, 3$. Consequently, for all $n > N$ it follows that

$$\| \mathcal{D}W_{\varphi,\psi} f_n \|_{\mathcal{Z}_\beta} \leq \left(7 + \| \psi' \|_{\mathcal{Z}_\beta} + M_0 + M_1 + 3M_2 \right) \varepsilon,$$

which shows that the operator $\mathcal{D}W_{\varphi,\psi} : \mathcal{A}_{\log}^\alpha \to \mathcal{Z}_\beta$ is compact.

(ii) $\Rightarrow$ (i). This implication is obvious. The proof is finished. $\qquad \square$

Now, we consider the boundedness of operator $\mathcal{D}W_{\varphi,\psi} : \mathcal{A}_{\log}^\alpha \to \mathcal{Z}_{\beta,0}$. We first have the following result.

Lemma 2.4 *Let $\alpha > -1$ and $\beta > 0$. Let φ be an analytic self-map of $\mathbb{D}$ and ψ an analytic function on $\mathbb{D}$. Then, for all $c > 0$, the following statements are equivalent:*

(i)

$$\lim_{z \to \partial \mathbb{D}} \frac{(1-|z|^2)^\beta}{1-|\varphi(z)|^2} \left| \psi(z)\varphi'''(z) + 3\psi''(z)\varphi'(z) + 3\psi'(z)\varphi''(z) \right|$$

$$\times \exp \frac{c}{(1-|\varphi(z)|^2)^{\alpha+2}} = 0.$$

(ii)

$$\lim_{\varphi(z) \to \partial \mathbb{D}} \frac{(1-|z|^2)^\beta}{1-|\varphi(z)|^2} \left| \psi(z)\varphi'''(z) + 3\psi''(z)\varphi'(z) + 3\psi'(z)\varphi''(z) \right|$$

$$\times \exp \frac{c}{(1-|\varphi(z)|^2)^{\alpha+2}} = 0,$$

and $\psi\varphi''' + 3\psi''\varphi' + 3\psi'\varphi'' \in \mathcal{A}_{\beta,0}$.

Proof Suppose that (i) holds. Since

$$\frac{1}{1 - |\varphi(z)|^2} \exp \frac{c}{(1 - |\varphi(z)|^2)^{\alpha+2}} \geq 1$$

for all $z \in \mathbb{D}$, we have

$$(1 - |z|^2)^{\beta} |\psi(z)\varphi'''(z) + 3\psi''(z)\varphi'(z) + 3\psi'(z)\varphi''(z)|$$

$$\leq \frac{(1 - |z|^2)^{\beta}}{1 - |\varphi(z)|^2} |\psi(z)\varphi'''(z) + 3\psi''(z)\varphi'(z) + 3\psi'(z)\varphi''(z)| \exp \frac{c}{(1 - |\varphi(z)|^2)^{\alpha+2}}$$

$$\to 0,$$

as $z \to \partial\mathbb{D}$. Hence $\psi\varphi''' + 3\psi''\varphi' + 3\psi'\varphi'' \in \mathcal{A}_{\beta,0}$. Since $\varphi(z) \to \partial\mathbb{D}$ implies $z \to \partial\mathbb{D}$, it follows that the first assertion in (ii) holds.

Now suppose that (ii) holds, but (i) is not true. Then there exist constants $c_0 > 0$, $\varepsilon_0 > 0$ and a sequence $\{z_n\}$ tending to $\partial\mathbb{D}$ as $n \to \infty$ such that

$$\frac{(1 - |z_n|^2)^{\beta}}{1 - |\varphi(z_n)|^2} |\psi(z)\varphi'''(z) + 3\psi''(z)\varphi'(z) + 3\psi'(z)\varphi''(z)| \exp \frac{c}{(1 - |\varphi(z_n)|^2)^{\alpha+2}} \geq \varepsilon_0. \quad (20)$$

Since $\psi\varphi''' + 3\psi''\varphi' + 3\psi'\varphi'' \in \mathcal{A}_{\beta,0}$, it follows from (20) that the sequence $(z_n)_{n\in\mathbb{N}}$ has a subsequence $(z_{n_k})_{k\in\mathbb{N}}$ with $\varphi(z_{n_k}) \to \partial\mathbb{D}$. Therefore, applying $(z_{n_k})_{k\in\mathbb{N}}$ to the first assertion in (ii), we arrive at a contradiction to (20), finishing the proof. □

By Lemma 2.4, the following result follows similar to the proof of Theorem 2.3. Hence, we omit the details.

Theorem 2.5 *Let $\alpha > -1$ and $\beta > 0$. Let φ be an analytic self-map of $\mathbb{D}$ and ψ an analytic function on $\mathbb{D}$. Then, for all $c > 0$, the following statements are equivalent*:

(i) *The operator $\mathcal{D}W_{\varphi,\psi} : \mathcal{A}^{\alpha}_{\log} \to \mathcal{Z}_{\beta,0}$ is bounded.*

(ii) *The operator $\mathcal{D}W_{\varphi,\psi} : \mathcal{A}^{\alpha}_{\log} \to \mathcal{Z}_{\beta,0}$ is compact.*

(iii)

$$\psi\varphi''' + 3\psi''\varphi' + 3\psi'\varphi'', \varphi'^3\psi, \psi'\varphi'^2 + \psi\varphi'\varphi'', \psi''' \in \mathcal{A}_{\beta,0},$$

$$\lim_{\varphi(z)\to\partial\mathbb{D}} (1 - |z|^2)^{\beta} |\psi'''(z)| \exp \frac{c}{(1 - |\varphi(z)|^2)^{\alpha+2}} = 0,$$

$$\lim_{\varphi(z)\to\partial\mathbb{D}} \frac{(1 - |z|^2)^{\beta}}{1 - |\varphi(z)|^2} |\psi(z)\varphi'''(z) + 3\psi''(z)\varphi'(z) + 3\psi'(z)\varphi''(z)|$$

$$\times \exp \frac{c}{(1 - |\varphi(z)|^2)^{\alpha+2}} = 0,$$

$$\lim_{\varphi(z)\to\partial\mathbb{D}} \frac{(1 - |z|^2)^{\beta}}{(1 - |\varphi(z)|^2)^2} |\psi'(z)\varphi'(z)^2 + \psi(z)\varphi'(z)\varphi''(z)| \exp \frac{c}{(1 - |\varphi(z)|^2)^{\alpha+2}} = 0,$$

and

$$\lim_{\varphi(z)\to\partial\mathbb{D}} \frac{(1 - |z|^2)^{\beta}}{(1 - |\varphi(z)|^2)^3} |\psi(z)||\varphi'(z)|^3 \exp \frac{c}{(1 - |\varphi(z)|^2)^{\alpha+2}} = 0.$$

(iv)

$$\lim_{z \to \partial \mathbb{D}} \left(1 - |z|^2\right)^\beta \left|\psi'''(z)\right| \exp \frac{c}{(1 - |\varphi(z)|^2)^{\alpha+2}} = 0,$$

$$\lim_{z \to \partial \mathbb{D}} \frac{(1 - |z|^2)^\beta}{1 - |\varphi(z)|^2} \left|\psi(z)\varphi'''(z) + 3\psi''(z)\varphi'(z) + 3\psi'(z)\varphi''(z)\right|$$

$$\times \exp \frac{c}{(1 - |\varphi(z)|^2)^{\alpha+2}} = 0,$$

$$\lim_{z \to \partial \mathbb{D}} \frac{(1 - |z|^2)^\beta}{(1 - |\varphi(z)|^2)^2} \left|\psi'(z)\varphi'(z)^2 + \psi(z)\varphi'(z)\varphi''(z)\right| \exp \frac{c}{(1 - |\varphi(z)|^2)^{\alpha+2}} = 0,$$

and

$$\lim_{z \to \partial \mathbb{D}} \frac{(1 - |z|^2)^\beta}{(1 - |\varphi(z)|^2)^3} \left|\psi(z)\right| \left|\varphi'(z)\right|^3 \exp \frac{c}{(1 - |\varphi(z)|^2)^{\alpha+2}} = 0.$$

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