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Multiple positive doubly periodic solutions for a singular semipositone telegraph equation with a parameter

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Abstract

In this paper, we study the multiplicity of positive doubly periodic solutions for a singular semipositone telegraph equation. The proof is based on a well-known fixed point theorem in a cone.

MSC: 34B15; 34B18

Keywords: semipositone telegraph equation; doubly periodic solution; singular; cone; fixed point theorem

1 Introduction

Recently, the existence and multiplicity of positive periodic solutions for a scalar singular equation or singular systems have been studied by using some fixed point theorems; see [1–9]. In [10], the authors show that the method of lower and upper solutions is also one of common techniques to study the singular problem. In addition, the authors [11] use the continuation type existence principle to investigate the following singular periodic problem:

$$\left(\left|u'\right|^{p-2}u'\right)' + h(u)u' = g(u) + c(t).$$

More recently, using a weak force condition, Wang [12] has built some existence results for the following periodic boundary value problem:

$$\begin{cases} u_{tt} - u_{xx} + c_1 u_t + a_{11}(t,x)u + a_{12}(t,x)v = f_1(t,x,u,v) + \chi_1(t,x), \\ v_{tt} - v_{xx} + c_2 v_t + a_{21}(t,x)u + a_{22}(t,x)v = f_2(t,x,u,v) + \chi_2(t,x). \end{cases}$$

The proof is based on Schauder's fixed point theorem. For other results concerning the existence and multiplicity of positive doubly periodic solutions for a single regular telegraph equation or regular telegraph system, see, for example, the papers [13–17] and the references therein. In these references, the nonlinearities are nonnegative.

On the other hand, the authors [18] study the semipositone telegraph system

$$\begin{cases} u_{tt} - u_{xx} + c_1 u_t + a_1(t,x)u = b_1(t,x)f(t,x,u,v), \\ v_{tt} - v_{xx} + c_2 v_t + a_2(t,x)v = b_2(t,x)g(t,x,u,v), \end{cases}$$

where the nonlinearities f, g may change sign. In addition, there are many authors who have studied the semipositone equations; see [19, 20].

Inspired by the above references, we are concerned with the multiplicity of positive doubly periodic solutions for a general singular semipositone telegraph equation

$$\begin{cases} u_{tt} - u_{xx} + cu_t + a(t,x)u = \lambda f(t,x,u), \\ u(t+2\pi,x) = u(t,x+2\pi) = u(t,x), \end{cases} \tag{1}$$

where $c > 0$ is a constant, $\lambda > 0$ is a positive parameter, $a(t,x) \in C(R \times R, R)$, $f(t,x,u)$ may change sign and is singular at $u = 0$, namely,

$$\lim_{u \to 0^+} f(t,x,u) = +\infty.$$

The main method used here is the following fixed-point theorem of a cone mapping.

Lemma 1.1 [21] *Let E be a Banach space, and $K \subset E$ be a cone in E. Assume Ω_1, Ω_2 are open subsets of E with $0 \in \Omega_1$, $\overline{\Omega}_1 \subset \Omega_2$, and let $T : K \cap (\overline{\Omega}_2 \setminus \Omega_1) \to K$ be a completely continuous operator such that either*

(i) $\|Tu\| \leq \|u\|$, $u \in K \cap \partial\Omega_1$ *and* $\|Tu\| \geq \|u\|$, $u \in K \cap \partial\Omega_2$; *or*
(ii) $\|Tu\| \geq \|u\|$, $u \in K \cap \partial\Omega_1$ *and* $\|Tu\| \leq \|u\|$, $u \in K \cap \partial\Omega_2$.

Then T has a fixed point in $K \cap (\overline{\Omega}_2 \setminus \Omega_1)$.

The paper is organized as follows. In Section 2, some preliminaries are given. In Section 3, we give the main result.

2 Preliminaries

Let $\top^2$ be the torus defined as

$$\top^2 = (R/2\pi Z) \times (R/2\pi Z).$$

Doubly 2π-periodic functions will be identified to be functions defined on $\top^2$. We use the notations

$$L^p(\top^2), \qquad C(\top^2), \qquad C^\alpha(\top^2), \qquad D(\top^2) = C^\infty(\top^2), \ldots$$

to denote the spaces of doubly periodic functions with the indicated degree of regularity. The space $D'(\top^2)$ denotes the space of distributions on $\top^2$.

By a doubly periodic solution of Eq. (1) we mean that a $u \in L^1(\top^2)$ satisfies Eq. (1) in the distribution sense, *i.e.*,

$$\int_{\top_2} u(\varphi_{tt} - \varphi_{xx} - c\varphi_t + a(t,x)\varphi)\,dt\,dx = \lambda \int_{\top^2} f(t,x,u)\varphi\,dt\,dx.$$

First, we consider the linear equation

$$u_{tt} - u_{xx} + cu_t - \xi u = h(t,x), \quad \text{in } D'(\top^2), \tag{2}$$

where $c > 0$, $\mu \in R$, and $h(t,x) \in L^1(\top^2)$.

Let $\pounds_\xi$ be the differential operator

$$\pounds_\xi u = u_{tt} - u_{xx} + cu_t - \xi u,$$

acting on functions on $\top^2$. Following the discussion in [14], we know that if $\xi < 0$, $\pounds_\xi$ has the resolvent R_ξ,

$$R_\xi : L^1(\top^2) \to C(\top^2), \qquad h_i(t,x) \mapsto u_i(t,x),$$

where $u(t,x)$ is the unique solution of Eq. (2), and the restriction of R_ξ on $L^p(\top^2)$ $(1 < p < \infty)$ or $C(\top^2)$ is compact. In particular, $R_\xi : C(\top^2) \to C(\top^2)$ is a completely continuous operator.

For $\xi = -c^2/4$, the Green function $G(t,x)$ of the differential operator $\pounds_\xi$ is explicitly expressed; see Lemma 5.2 in [14]. From the definition of $G(t,x)$, we have

$$\underline{G} := \operatorname{ess\,inf} G(t,x) = e^{-3c\pi/2}/\left(1 - e^{-c\pi}\right)^2,$$
$$\overline{G} := \operatorname{ess\,sup} G(t,x) = \left(1 + e^{-c\pi}\right)/2\left(1 - e^{-c\pi}\right)^2.$$

For convenience, we assume the following condition holds throughout this paper:

(H1) $a(t,x) \in C(\top^2, R)$, $0 \le a(t,x) \le \frac{c^2}{4}$ on $\top^2$, and $\int_{\top^2} a(t,x)\,dt\,dx > 0$.

Finally, if $-\xi$ is replaced by $a(t,x)$ in Eq. (2), the author [13] has proved the following unique existence and positive estimate result.

Lemma 2.1 *Let $h(t,x) \in L^1(\top^2)$. Then Eq.* (2) *has a unique solution $u(t,x) = P[h(t,x)]$, $P : L^1(\top^2) \to C(\top^2)$ is a linear bounded operator with the following properties*:

(i) *$P : C(\top^2) \to C(\top^2)$ is a completely continuous operator*;

(ii) *If $h(t,x) > 0$, a.e $(t,x) \in \top^2$, $P[h(t,x)]$ has the positive estimate*

$$\underline{G}\|h\|_{L^1} \le P\left[h(t,x)\right] \le \frac{\overline{G}}{\underline{G}\|a\|_{L^1}}\|h\|_{L^1}. \tag{3}$$

3 Main result

Theorem 3.1 *Assume* (H1) *holds. In addition, if $f(t,x,u)$ satisfies*

(H2) $\lim_{u\to 0^+} f(t,x,u) = +\infty$, *uniformly* $(t,x) \in \top^2$,

(H3) $f : \top^2 \times (0,+\infty) \to (-\infty,+\infty)$ *is continuous*,

(H4) *there exists a nonnegative function $h(t,x) \in C(\top^2)$ such that*

$$f(t,x,u) + h(t,x) \ge 0, \quad (t,x) \in \top^2, u > 0,$$

(H5) $\int_{\top^2} F_\infty(t,x)\,dt\,dx = +\infty$, *where the limit function* $F_\infty(t,x) = \liminf_{u\to+\infty} \frac{f(t,x,u)}{u}$, *then Eq.* (1) *has at least two positive doubly periodic solutions for sufficiently small* λ.

$C(\top^2)$ is a Banach space with the norm $\|u\| = \max_{(t,x)\in\top^2} |u(t,x)|$. Define a cone $K \subset C(\top^2)$ by

$$K = \left\{u \in C\left(\top^2\right) : u \ge 0, u(t,x) \ge \delta\|u\|\right\},$$

where $\delta = \frac{\underline{G}^2\|a\|_{L^1}}{\overline{G}} \in (0,1)$. Let $\partial K_r = \{u \in K : \|u\| = r\}$, $[u]^+ = \max\{u, 0\}$. By Lemma 2.1, it is easy to obtain the following lemmas.

Lemma 3.2 *If $h(t,x) \in C(\top^2)$ is a nonnegative function, the linear boundary value problem*

$$\begin{cases} u_{tt} - u_{xx} + cu_t + a(t,x)u = \lambda h(t,x), \\ u(t+2\pi, x) = u(t, x+2\pi) = u(t,x) \end{cases}$$

has a unique solution $\omega(t,x)$. The function $\omega(t,x)$ satisfies the estimates

$$\lambda \underline{G}\|h\|_{L^1} \le \omega(t,x) = \lambda P\big(h(t,x)\big) \le \lambda \frac{\overline{G}}{\underline{G}\|a\|_{L^1}}\|h\|_{L^1}.$$

Lemma 3.3 *If the boundary value problem*

$$\begin{cases} u_{tt} - u_{xx} + cu_t + a(t,x)u = \lambda[f(t,x,[u(t,x) - \omega(t,x)]^+) + h(t,x)], \\ u(t+2\pi, x) = u(t, x+2\pi) = u(t,x) \end{cases}$$

has a solution $\widetilde{u}(t,x)$ with $\|\widetilde{u}\| > \lambda \frac{\overline{G}^2}{\underline{G}^3\|a\|_{L^1}^2}\|h\|_{L^1}$, then $u^*(t,x) = \widetilde{u}(t,x) - \omega(t,x)$ is a positive doubly periodic solution of Eq. (1).

Proof of Theorem 3.1 Step 1. Define the operator T as follows:

$$(Tu)(t,x) = \lambda P\big[f\big(t,x,\big[u(t,x) - \omega(t,x)\big]^+\big) + h(t,x)\big].$$

We obtain the conclusion that $T(K\backslash\{u \in K : [u(t,x) - \omega(t,x)]^+ = 0\}) \subseteq K$, and $T : K\backslash\{u \in K : [u(t,x) - \omega(t,x)]^+ = 0\} \to K$ is completely continuous.

For any $u \in K\backslash\{u \in K : [u(t,x) - \omega(t,x)]^+ = 0\}$, then $[u(t,x) - \omega(t,x)]^+ > 0$, and T is defined. On the other hand, for $u \in K\backslash\{u \in K : [u(t,x) - \omega(t,x)]^+ = 0\}$, the complete continuity is obvious by Lemma 2.1. And we can have

$$\begin{aligned} (Tu)(t,x) &= \lambda P\big[f\big(t,x,\big[u(t,x) - \omega(t,x)\big]^+\big) + h(t,x)\big] \\ &\ge \lambda \underline{G}\big\|f\big(t,x,\big[u(t,x) - \omega(t,x)\big]^+\big) + h(t,x)\big\|_{L^1} \\ &\ge \underline{G}\frac{\underline{G}\|a\|_{L^1}}{\overline{G}}\big\|T(u)\big\| \\ &\ge \delta\|Tu\|. \end{aligned}$$

Thus, $T(K\backslash\{u \in K : u(t,x) \le \omega(t,x)\}) \subseteq K$.

Now we prove that the operator T has one fixed point $\widetilde{u} \in K$ and $\|\widetilde{u}\| > \lambda \frac{\overline{G}^2}{\underline{G}^3\|a\|_{L^1}^2}\|h\|_{L^1}$ for all sufficiently small λ.

Since $\int_{\top^2} F_\infty(t,x)\,dt\,dx = +\infty$, there exists $r_1 \ge 2$ such that

$$\int_{\top^2} \frac{f(t,x,u)}{u}\,dt\,dx \ge \frac{1}{\delta}, \quad u \ge \delta r_1.$$

Furthermore, we have $\int_{\top^2} f(t,x,\delta r_1)\,dt\,dx \geq r_1 \geq 2$. It follows that

$$\begin{aligned}&\int_{\top^2}\left[\max\left\{f(t,x,u):\frac{\delta}{2}r_1 \leq u \leq r_1\right\} + h(t,x)\right]dt\,dx\\&\quad\geq \int_{\top^2} f(t,x,\delta r_1)\,dt\,dx \geq r_1 \geq 2.\end{aligned}$$

Let $\Phi(t,x) = \max\{f(t,x,u) : \frac{\delta}{2}r_1 \leq u \leq r_1\} + h(t,x)$. Then $\Phi \in L^1(\top^2)$ and $\int_{\top^2}\Phi(t,x)\,dt\,dx > 0$. Set

$$\lambda^* = \min\left\{\frac{\delta^2}{2\underline{G}\|h\|_{L^1}}, \frac{2\underline{G}\|a\|_{L^1}}{\overline{G}\|\Phi\|_{L^1}}\right\}.$$

For any $u \in \partial K_{r_1}$ and $0 < \lambda < \lambda^*$, we can verify that

$$\begin{aligned}u(t,x) - \omega(t,x) &\geq \delta\|u\| - \omega(t,x)\\&= \delta r_1 - \omega(t,x)\\&\geq \delta r_1 - \lambda\frac{\overline{G}}{\underline{G}\|a\|_{L^1}}\|h\|_{L^1}\\&\geq \delta r_1 - \frac{\delta r_1}{2}\\&= \frac{\delta r_1}{2}.\end{aligned}$$

Then we have

$$\begin{aligned}\|Tu\| &= \lambda\left\|P\left[f\left(t,x,\left[u(t,x)-\omega(t,x)\right]^+\right) + h(t,x)\right]\right\|\\&\leq \lambda\frac{\overline{G}}{\underline{G}\|a\|_{L^1}}\left\|f\left(t,x,\left[u(t,x)-\omega(t,x)\right]^+\right) + h(t,x)\right\|_{L^1}\\&\leq \lambda\frac{\overline{G}}{\underline{G}\|a\|_{L^1}}\left\|\Phi(t,x)\right\|_{L^1}\\&< 2 \leq r_1 = \|u\|.\end{aligned}$$

On the other hand,

$$\liminf_{u\to+\infty}\frac{f(t,x,u-\omega(t,x))}{u} = \liminf_{u\to+\infty}\frac{f(t,x,u)}{u} = F_\infty(t,x).$$

By the Fatou lemma, one has

$$\begin{aligned}&\liminf_{u\to+\infty}\int_{\top^2}\frac{f(t,x,u-\omega(t,x)) + h(t,x)}{u}\,dt\,dx\\&\quad\geq \int_{\top^2}\liminf_{u\to+\infty}\frac{f(t,x,u)+h(t,x)}{u}\,dt\,dx\\&\quad= \int_{\top^2}F_\infty(t,x)\,dt\,dx = +\infty.\end{aligned}$$

Hence, there exists a positive number $r_2 > \delta r_2 > r_1$ such that

$$\int_{\top^2} \frac{f(t,x,u-\omega(t,x)) + h(t,x)}{u}\,dt\,dx \geq \lambda^{-1}\delta^{-1}\underline{G}^{-1}\bigl(4\pi^2\bigr)^{-1}, \quad u \geq \delta r_2.$$

Hence, we have

$$\int_{\top^2} f\bigl(t,x,u-\omega(t,x)\bigr) + h(t,x)\,dt\,dx \geq \lambda^{-1}\underline{G}^{-1}\bigl(4\pi^2\bigr)^{-1} r_2, \quad u \geq \delta r_2.$$

For any $u \in \partial K_{r_2}$, we have $\delta r_2 = \delta\|u\| \leq u(t,x) \leq \|u\| = r_2$. On the other hand, since $0 < \lambda < \lambda^*$, we can get

$$\begin{aligned} u(t,x) - \omega(t,x) &\geq \delta r_2 - \omega(t,x) \\ &\geq \delta \frac{r_2}{\delta} - \lambda \frac{\overline{G}}{\underline{G}\|a\|_{L^1}} \\ &\geq \delta r_2 - \delta \\ &> 0. \end{aligned}$$

From above, we can have

$$\begin{aligned} \|Tu\| &\geq \lambda P\bigl[f\bigl(t,x,\bigl[u(t,x)-\omega(t,x)\bigr]^+\bigr) + h(t,x)\bigr] \\ &\geq \lambda \underline{G}\bigl\|f\bigl(t,x,\bigl[u(t,x)-\omega(t,x)\bigr]^+\bigr) + h(t,x)\bigr\|_{L^1} \\ &\geq \lambda \underline{G} 4\pi^2 \lambda^{-1}\underline{G}^{-1}\bigl(4\pi^2\bigr)^{-1} r_2 \\ &= r_2. \end{aligned}$$

Therefore, by Lemma 1.1, the operator T has a fixed point $\widetilde{u}(t,x) \in K$ and

$$\begin{aligned} &r_2 \geq \|\widetilde{u}\| \geq r_1, \\ &\widetilde{u}(t,x) - \omega(t,x) \geq \delta r_1 - \lambda \frac{\overline{G}}{\underline{G}\|a\|_{L^1}}\|h\|_{L^1} \geq \delta r_1 - \frac{\overline{G}}{\underline{G}\|a\|_{L^1}}\|h\|_{L^1}\frac{\delta^2}{\underline{G}\|h\|_{L^1}} \geq \delta. \end{aligned}$$

So, Eq. (1) has a positive solution $\widehat{u}(t,x) = \widetilde{u}(t,x) - \omega(t,x) \geq \delta$.

Step 2. By conditions (H2) and (H3), it is clear to obtain that

$$u_0 = \inf\bigl\{u \in K : f(t,x,u) \leq 0, (t,x) \in \top^2\bigr\} > 0.$$

Let $r_4 = \min\{\frac{\delta}{2}, \frac{\delta\|u_0\|}{2}\}$. For any $u \in (0, r_4]$, we have $f(t,x,u) > 0$. Then define the operator A as follows:

$$(Au)(t,x) = \lambda\widehat{P}\bigl[f\bigl(t,x,u(t,x)\bigr)\bigr].$$

It is easy to prove that $A(K \cap \{u \in C(\top^2) : 0 < \|u\| < r_4\}) \subseteq K$, and $A : K \cap \{u \in C(\top^2) : 0 < \|u\| < r_4\} \to K$ is completely continuous.

And for any $\rho > 0$, define

$$M(\rho) = \max\bigl\{f(t,x,u) : u \in R^+, \delta\rho \leq u \leq \rho, (t,x) \in \top^2\bigr\} > 0.$$

Furthermore, for any $u \in \partial K_{r_4}$, we have

$$\begin{aligned}\|Au\| &= \lambda \big\| \widehat{P}\big[f\big(t,x,u(t,x)\big)\big]\big\| \\ &\le \lambda \frac{\overline{G}}{\underline{G}\|a\|_{L^1}} \big\|f\big(t,x,u(t,x)\big)\big\|_{L^1} \\ &\le \lambda \frac{\overline{G}}{\underline{G}\|a\|_{L^1}} M(r_4) 4\pi^2.\end{aligned}$$

Thus, from the above inequality, there exists $\overline{\lambda}$ such that

$$\|Au\| < \|u\|, \quad \text{for } u \in \partial K_{r_4}, 0 < \lambda < \overline{\lambda}.$$

Since $\lim_{u\to 0^+} f(t,x,u) = \infty$, then there is $0 < r_3 < \frac{r_4}{2}$ such that

$$f(t,x,u) \ge \mu u, \quad \text{for } u \in R^+ \text{ with } 0 < u \le r_3,$$

where μ satisfies $\lambda \underline{G} \mu \delta > 1$. For any $u \in \partial K_{r_3}$, then we have

$$f(t,x,u) \ge \mu u(t,x), \quad \text{for } (t,x) \in \top^2.$$

By Lemma 2.1, it is clear to obtain that

$$\begin{aligned}\|Au\| &= \lambda \big\| \widehat{P}\big[f\big(t,x,u(t,x)\big)\big]\big\| \\ &\ge \lambda \underline{G} \big\|f\big(t,x,u(t,x)\big)\big\|_{L^1} \\ &\ge \lambda \underline{G} \mu \delta r_3 \\ &> r_3 = \|u\|.\end{aligned}$$

Therefore, by Lemma 1.1, A has a fixed point in $\overline{u}(t,x) \in K$ and $\|\overline{u}\| \le r_4 \le \frac{\delta}{2}$, which is another positive periodic solution of Eq. (1).

Finally, from Step 1 and Step 2, Eq. (1) has two positive doubly periodic solutions $\widehat{u}(t,x)$ and $\overline{u}(t,x)$ for sufficiently small λ. □

Example Consider the following problem:

$$\begin{cases} u_{tt} - u_{xx} + 2u_t + \sin^2(t+x)u = \lambda[\frac{1}{u} + \min\{u^2, \frac{u}{|1-\frac{t}{\pi}||1-\frac{x}{\pi}|}\} - 10], \\ u(t+2\pi,x) = u(t,x+2\pi) = u(t,x). \end{cases}$$

It is clear that $f(t,x,u)$ satisfies the conditions (H1)-(H5).

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
This paper is concerned with a singular semipositone telegraph equation with a parameter and represents a somewhat interesting contribution in the investigation of the existence and multiplicity of doubly periodic solutions of the telegraph equation. All authors typed, read and approved the final manuscript.

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