View metadata, citation and similar papers at core.ac.uk

brought to you by CORE

provided by Crossref

Attiya and Nasr *Journal of Inequalities and Applications* 2013, **2013**:191
http://www.journalofinequalitiesandapplications.com/content/2013/1/191

Journal of Inequalities and Applications
a SpringerOpen Journal

RESEARCH **Open Access**

On sufficient conditions for Carathéodory functions with applications

Adel A Attiya[1,2*] and Mohamed AM Nasr[3]

*Correspondence:
aattiy@mans.edu.eg
[1]Department of Mathematics, Faculty of Science, University of Mansoura, Mansoura, 35516, Egypt
[2]Current address: Department of Mathematics, College of Science, University of Hail, Hail, Saudi Arabia
Full list of author information is available at the end of the article

Abstract

In the present paper, we derive some interesting relations associated with the Carathéodory functions which yield sufficient conditions for the Carathéodory functions in the open unit disk $\mathbb{U} = \{z : |z| < 1\}$. Some interesting applications of the main results are also obtained.

MSC: Primary 30C45; 30C80

Keywords: analytic functions; starlike functions; convex functions; spirallike functions; Carathéodory functions

1 Introduction

Let P denote the class of functions of the form

$$p(z) = \sum_{n=0}^{\infty} p_n z^n,$$

which are analytic in the unit disc $\mathbb{U} = \{z : |z| < 1\}$. The function $p(z)$ is called a Carathéodory function if it satisfies the condition

$$\operatorname{Re}\bigl(p(z)\bigr) > 0.$$

Moreover, let A denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{1.1}$$

which are analytic in the unit disc $\mathbb{U}$.

A function $f(z) \in A$ is in K, the class of convex functions, if it satisfies

$$\operatorname{Re}\left(1 + \frac{zf''(z)}{f(z)}\right) > 0 \quad (z \in \mathbb{U}). \tag{1.2}$$

Also, a function $f(z) \in A$ is in S^λ ($|\lambda| < \frac{\pi}{2}$), the class of λ-spirallike functions, if it satisfies

$$\operatorname{Re}\left(e^{i\lambda}\frac{zf'(z)}{f(z)}\right) > 0 \quad (z \in \mathbb{U}). \tag{1.3}$$

Moreover, we denote by $S^* = S^0$ the class of starlike functions in $\mathbb{U}$.

© 2013 Attiya and Nasr; licensee Springer. This is an Open Access article distributed under the terms of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/2.0), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Definition 1.1 Let $f(z)$ and $F(z)$ be analytic functions. The function $f(z)$ is said to be *subordinate* to $F(z)$, written $f(z) \prec F(z)$, if there exists a function $w(z)$ analytic in $\mathbb{U}$, with $w(0) = 0$ and $|w(z)| \leq 1$, and such that $f(z) = F(w(z))$. If $F(z)$ is univalent, then $f(z) \prec F(z)$ *if and only if* $f(0) = F(0)$ and $f(\mathbb{U}) \subset F(\mathbb{U})$.

Definition 1.2 Let $\mathbb{D}$ be the set of analytic functions $q(z)$ and injective on $\bar{\mathbb{U}} \backslash E(q)$, where

$$E(q) = \left\{\zeta \in \partial\mathbb{U} : \lim_{z\to\zeta} q(z) = \infty\right\}$$

and $q'(\zeta) \neq 0$ for $\zeta \in \partial\mathbb{U}\backslash E(q)$. Further, let $\mathbb{D}_a = \{q(z) \in \mathbb{D} : q(0) = a\}$.

Many authors have obtained several relations of Carathéodory functions, *e.g.*, see ([1–13]).

In the present paper, we derive some relations associated with the Carathéodory functions which yield the sufficient conditions for Carathéodory functions in $\mathbb{U}$. Some applications of the main results are also obtained.

2 Main results

To prove our results, we need the following lemma due to Miller and Mocanu [14, p.24]

Lemma 2.1 *Let* $q(z) \in \mathbb{D}_a$ *and let*

$$p(z) = b + b_n z^n + \cdots \tag{2.1}$$

be analytic in $\mathbb{U}$ *with* $p(z) \neq b$. *If* $p(z) \not\prec q(z)$, *then there exist points* $z_0 \in \mathbb{U}$ *and* $\zeta_0 \in \partial\mathbb{U}\backslash E(q)$ *and on* $m \geq n \geq 1$ *for which*

(i) $p(z_0) = q(\zeta_0)$,

(ii) $z_0 p'(z_0) = m\zeta_0 q'(\zeta_0)$.

Theorem 2.1 *Let*

$$P : \mathbb{U} \to \mathbb{C}$$

with

$$\operatorname{Re}\big(\bar{a}P(z)\big) > 0 \quad (a \in \mathbb{C}).$$

If $p(z)$ *is an analytic function in* $\mathbb{U}$ *with* $p(0) = 1$ *and*

$$\operatorname{Re}\big(p(z) + P(z)zp'(z)\big) > \frac{E}{2|a|^2 \operatorname{Re}(\bar{a}P(z))}, \tag{2.2}$$

then

$$\operatorname{Re}\big(ap(z)\big) > 0,$$

where

$$E = -\big(\operatorname{Re}(a)\big)\big(\operatorname{Re}\big(\bar{a}P(z)\big)\big)^2 + 2\operatorname{Re}\big(\bar{a}P(z)\big)\big(\operatorname{Im}(a)\big)^2 + \big(\operatorname{Re}(a)\big)\big(\operatorname{Im}(a)\big)^2 \tag{2.3}$$

with $\operatorname{Re}(a) > 0$.

Proof Let us define both $q(z)$ and $h(z)$ as follows:

$$q(z) = ap(z)$$

and

$$h(z) = \frac{a + \bar{a}z}{1 - z} \quad \bigl(\operatorname{Re}(a) > 0\bigr),$$

where $p(z)$ is defined by (2.1) since $q(z)$ and $h(z)$ are analytic functions in $\mathbb{U}$ with $q(0) = h(0) = a \in \mathbb{C}$ with

$$h(\mathbb{U}) = \bigl\{w : \operatorname{Re}(w) > 0\bigr\}.$$

Now, we suppose that $q(z) \not\prec h(z)$. Therefore, by using Lemma 2.1, there exist points

$$z_0 \in \mathbb{U} \quad \text{and} \quad \zeta_0 \in \partial\mathbb{U}\setminus\{1\}$$

such that $q(z_0) = h(\zeta_0)$ and $z_0 q'(z_0) = m\zeta_0 h'(\zeta_0)$, $m \geq n \geq 1$.

We note that

$$\zeta_0 = h^{-1}\bigl(q(z_0)\bigr) = \frac{q(z_0) - a}{q(z_0) + \bar{a}} \tag{2.4}$$

and

$$\varsigma_0 h'(\varsigma_0) = -\frac{|q(z_0) - a|^2}{2\operatorname{Re}(a - q(z_0))}. \tag{2.5}$$

We have $h(\zeta_0) = \rho i$ ($\rho \in \mathbb{R}$); therefore,

$$\begin{aligned}
&\operatorname{Re}\bigl(p(z_0) + P(z_0)zp'(z_0)\bigr) \\
&\quad = \operatorname{Re}\left(\frac{1}{a}h(\zeta_0) + \frac{1}{a}P(z_0)m\zeta_0 h'(\zeta_0)\right) \\
&\quad = \operatorname{Re}\left(\frac{\rho i}{a}\right) - m\frac{|\rho i - a|^2}{2\operatorname{Re}(a)}\operatorname{Re}\left(\frac{P(z_0)}{a}\right) \\
&\quad \leq \operatorname{Re}\left(\frac{\rho i}{a}\right) - \frac{|\rho i - a|^2}{2\operatorname{Re}(a)}\operatorname{Re}\left(\frac{P(z_0)}{a}\right) \\
&\quad = A\rho^2 + B\rho + C \\
&\quad = g(\rho),
\end{aligned} \tag{2.6}$$

where

$$\begin{aligned}
A &= -\frac{\operatorname{Re}(\bar{a}p(z_0))}{2|a|^2\operatorname{Re}(a)}, \\
B &= \frac{\operatorname{Im}(a)}{|a|^2}\left(1 + \frac{\operatorname{Re}(\bar{a}p(z_0))}{\operatorname{Re}(a)}\right)
\end{aligned}$$

and

$$C = -\frac{\mathrm{Re}(\bar{a}p(z_0))}{2\,\mathrm{Re}(a)}.$$

We can see that the function $g(\rho)$ in (2.6) takes the maximum value at ρ_1 given by

$$\rho_1 = \mathrm{Im}(a)\left(1 + \frac{\mathrm{Re}(a)}{\mathrm{Re}(\bar{a}p(z_0))}\right).$$

Hence, we have

$$\begin{aligned}\mathrm{Re}\bigl(p(z_0) + P(z_0)zp'(z_0)\bigr) &\le g(\rho_1)\\ &= \frac{E}{2|a|^2\,\mathrm{Re}(\bar{a}P(z))},\end{aligned}$$

where E is defined by (2.3). This is a contradiction to (2.2). Then we obtain $\mathrm{Re}(ap(z)) > 0$. □

Theorem 2.2 *Let $p(z)$ be a nonzero analytic function in $\mathbb{U}$ and $p(0) = 1$. If*

$$\gamma_1 < \mathrm{Im}\left(p(z) + \frac{zp'(z)}{p(z)}\right) < \gamma_2, \tag{2.7}$$

where

$$\gamma_1 = -\frac{\sqrt{|a|^2 + 2(\mathrm{Re}(a))^2} - \mathrm{Im}(a)}{\mathrm{Re}\,a}$$

and

$$\gamma_2 = \frac{\sqrt{|a|^2 + 2(\mathrm{Re}(a))^2} + \mathrm{Im}(a)}{\mathrm{Re}(a)},$$

then

$$\mathrm{Re}\bigl(ap(z)\bigr) > 0,$$

where $\mathrm{Re}(a) > 0$.

Proof Let us define both $q(z)$ and $h(z)$ as follows:

$$q(z) = ap(z)$$

and

$$h(z) = \frac{a + \bar{a}z}{1 - z} \quad \bigl(\mathrm{Re}(a) > 0\bigr),$$

where $p(z)$ is defined by (2.1) since $q(z)$ and $h(z)$ are analytic functions in $\mathbb{U}$ with $q(0) = h(0) = a \in \mathbb{C}$ with

$$h(\mathbb{U}) = \bigl\{w : \mathrm{Re}(w) > 0\bigr\}.$$

Now, we suppose that $q(z) \nprec h(z)$. Therefore, by using Lemma 2.1, there exist points

$$z_0 \in \mathbb{U} \quad \text{and} \quad \zeta_0 \in \partial\mathbb{U}\backslash\{1\}$$

such that $q(z_0) = h(\zeta_0)$ and $z_0 q'(z_0) = m\zeta_0 h'(\zeta_0)$, $m \geq n \geq 1$.

We note that

$$\zeta_0 h'(\zeta_0) = -\frac{|q(z_0) - a|^2}{2\operatorname{Re}(a - q(z_0))}. \tag{2.8}$$

We have $h(\zeta_0) = \rho i$ ($\rho \in \mathbb{R}$); therefore,

$$\begin{aligned}
\operatorname{Im}\left(p(z_0) + \frac{z_0 p'(z_0)}{p(z_0)}\right) &= \operatorname{Im}\left(q(z_0) + \frac{z_0 q'(z_0)}{q(z_0)}\right) \\
&= \operatorname{Im}\left(\frac{h(\zeta_0)}{a} + \frac{m\zeta_0 h'(\zeta_0)}{h(\zeta_0)}\right) \\
&= \operatorname{Im}\left(\frac{\rho i}{a} - \frac{m|\rho i - a|^2}{2\operatorname{Re}(a)\rho i}\right) \\
&= \frac{\rho}{|a|^2}\operatorname{Re}(a) + \frac{m|\rho i - a|^2}{2\rho\operatorname{Re}(a)}.
\end{aligned}$$

For the case $\rho > 0$, we obtain

$$\begin{aligned}
\operatorname{Im}\left(p(z_0) + \frac{z_0 p'(z_0)}{p(z_0)}\right) &\geq \frac{\rho}{|a|^2}\operatorname{Re}(a) + \frac{|\rho i - a|^2}{2\rho\operatorname{Re}(a)} \\
&= \frac{1}{2\rho\operatorname{Re}(a)}\left[\left(1 + 2\left(\frac{\operatorname{Re}(a)}{|a|}\right)^2\right)\rho^2 + 2\operatorname{Im}(a)\rho + |a|\right] \\
&= g(\rho).
\end{aligned} \tag{2.9}$$

We can see that the function $g(\rho)$ in (2.9) takes the minimum value at ρ_1 given by

$$\rho_1 = \frac{|a|^2}{\sqrt{|a|^2 + 2(\operatorname{Re}(a))^2}}.$$

Hence, we have

$$\begin{aligned}
\operatorname{Im}\left(p(z_0) + \frac{z_0 p'(z_0)}{p(z_0)}\right) &\geq g(\rho_1) \\
&= \gamma_2.
\end{aligned}$$

This is a contradiction to (2.7). Then we obtain $\operatorname{Re}(ap(z)) > 0$.

For the case $\rho < 0$, we obtain

$$\begin{aligned}
\operatorname{Im}\left(p(z_0) + \frac{z_0 p'(z_0)}{p(z_0)}\right) &\leq \frac{\rho}{|a|^2}\operatorname{Re}(a) + \frac{|\rho i - a|^2}{2\rho\operatorname{Re}(a)} \\
&= \frac{1}{2\rho\operatorname{Re}(a)}\left[\left(1 + 2\left(\frac{\operatorname{Re}(a)}{|a|}\right)^2\right)\rho^2 + 2\operatorname{Im}(a)\rho + |a|^2\right] \\
&= g(\rho).
\end{aligned} \tag{2.10}$$

We can see that the function $g(\rho)$ in (2.10) takes the maximum value at ρ_2 given by

$$\rho_2 = -\frac{|a|^2}{\sqrt{|a|^2 + 2(\operatorname{Re}(a))^2}}.$$

Hence, we have

$$\begin{aligned}\operatorname{Im}\left(p(z_0) + \frac{z_0 p'(z_0)}{p(z_0)}\right) &\le g(\rho_2) \\ &= \gamma_1.\end{aligned}$$

This is a contradiction to (2.7). Then we obtain $\operatorname{Re}(ap(z)) > 0$. □

Theorem 2.3 *Let $p(z)$ be a nonzero analytic function in $\mathbb{U}$ with $p(0) = 1$. If*

$$\left|p(z) + \frac{zp'(z)}{p(z)} - 1\right| < \frac{3\operatorname{Re}(a)}{2|a|},$$

then

$$\operatorname{Re}\left(\frac{a}{p(z)}\right) > 0,$$

where $\operatorname{Re}(a) > 0$.

Proof Let us define both $q(z)$ and $h(z)$ as follows:

$$q(z) = ap(z)$$

and

$$h(z) = \frac{a + \bar{a}z}{1 - z} \quad \bigl(\operatorname{Re}(a) > 0\bigr),$$

where $p(z)$ is defined by (2.1) since $q(z)$ and $h(z)$ are analytic functions in $\mathbb{U}$ with $q(0) = h(0) = a \in \mathbb{C}$ with

$$h(\mathbb{U}) = \{w : \operatorname{Re} w > 0\}.$$

Now, we suppose that $q(z) \nprec h(z)$. Therefore, by using Lemma 2.1, there exist points

$$z_0 \in \mathbb{U} \quad \text{and} \quad \zeta_0 \in \partial\mathbb{U}\backslash\{1\}$$

such that $q(z_0) = h(\zeta_0)$ and $z_0 q'(z_0) = m\zeta_0 h'(\zeta_0)$, $m \ge n \ge 1$.

We note that

$$\zeta_0 h'(\zeta_0) = -\frac{|q(z_0) - a|^2}{2\operatorname{Re}(a - q(z_0))}. \tag{2.11}$$

We have $h(\zeta_0) = \rho i$ $(\rho \in \mathbb{R})$.

Therefore,

$$\begin{aligned}\frac{|p(z_0)+\frac{zp'(z_0)}{p(z_0)}-1|}{|p(z_0)|} &= \left|\frac{\rho i}{a}-\frac{m}{a}\frac{|a-i\rho|^2}{2\operatorname{Re}(a)}-1\right| \\ &\geq \frac{1}{|a|}\left|\frac{m|a-i\rho|^2}{2\operatorname{Re}(a)}+\operatorname{Re}(a)\right| \\ &\geq \frac{1}{|a|}\left(\frac{|a-i\rho|^2}{2\operatorname{Re}(a)}+\operatorname{Re}(a)\right) \\ &\geq \frac{1}{2|a|\operatorname{Re}(a)}\left(3\big(\operatorname{Re}(a)\big)^2+\big(\operatorname{Im}(a)-\rho\big)^2\right) \\ &\geq \frac{3\operatorname{Re}(a)}{2|a|}.\end{aligned}$$

This is a contradiction to (2.7). Then we obtain $\operatorname{Re}(\frac{a}{p(z)}) > 0$. □

3 Applications and examples

Putting $P(z) = \beta$ ($\beta > 0$; real) in Theorem 2.1, we have the following corollary.

Corollary 3.1 *If* $p(z)$ *is an analytic function in* $\mathbb{U}$ *with* $p(0) = 1$ *and*

$$\operatorname{Re}\big(p(z)+\beta zp'(z)\big) > \frac{E}{2\beta|a|^2\operatorname{Re}(a)},$$

then

$$\operatorname{Re}\big(ap(z)\big) > 0,$$

where

$$E = -\big(\operatorname{Re}(a)\big)\big[\beta^2\big(\operatorname{Re}(a)\big)^2+(1+2\beta)\big(\operatorname{Im}(a)\big)^2\big]$$

with $\operatorname{Re}(a) > 0$.

Putting $\beta = 1$ in Corollary 3.1, we obtain the following corollary.

Corollary 3.2 *If* $p(z)$ *is an analytic function in* $\mathbb{U}$ *with* $p(0) = 1$ *and*

$$\operatorname{Re}\big(p(z)+zp'(z)\big) > \frac{3}{2}-2\left(\frac{\operatorname{Re}(a)}{|a|}\right)^2,$$

then

$$\operatorname{Re}\big(ap(z)\big) > 0,$$

where $\operatorname{Re}(a) > 0$.

Putting $p(z) = \frac{f(z)}{g(z)}$ and $P(z) = \frac{g(z)}{zg'(z)}$ in Theorem 2.1, we have the following corollary.

Corollary 3.3 *Let* $f(z) \in A$, $g(z) \in S^*$ *and*

$$\operatorname{Re}\left(\frac{f'(z)}{g'(z)}\right) > \frac{3}{2} - 2\left(\frac{\operatorname{Re}(a)}{|a|}\right)^2.$$

Then

$$\operatorname{Re}\left(a\frac{f(z)}{g(z)}\right) > 0,$$

where $\operatorname{Re}(a) > 0$.

Example 3.1 Let $f(z) \in A$ satisfy the following relation:

$$\operatorname{Re}\bigl(f'(z)\bigr) > \frac{3}{2} - 2\left(\frac{\operatorname{Re}(a)}{|a|}\right)^2.$$

Then

$$\operatorname{Re}\left(a\frac{f(z)}{z}\right) > 0,$$

where $\operatorname{Re}(a) > 0$.

Example 3.2 Let $f(z) \in A$ satisfy the following relation:

$$\operatorname{Re}\left(\left(2 + \frac{zf''(z)}{f'(z)} - \frac{zf'(z)}{f(z)}\right)\frac{zf'(z)}{f(z)}\right) > \frac{3}{2} - 2\left(\frac{\operatorname{Re}(a)}{|a|}\right)^2.$$

Then

$$\operatorname{Re}\left(a\frac{zf'(z)}{f(z)}\right) > 0,$$

where $\operatorname{Re}(a) > 0$.

Remark 3.1

(i) Putting $a = e^{i\lambda}$ ($|\lambda| < \frac{\pi}{2}$) in Theorem 2.1, we have Theorem 1 due to Kim and Cho [3].

(ii) Putting $a = e^{i\lambda}$ ($|\lambda| < \frac{\pi}{2}$), $P(z) = \beta$ ($\beta > 0$; real) in Theorem 2.1, we have Corollary 1 due to Kim and Cho [3].

(iii) Putting $a = 0$ and $P(z) = 1$ in Theorem 2.1, we have the result due to Nunokawa *et al.* [15].

(iv) Putting $a = e^{i\lambda}$ ($|\lambda| < \frac{\pi}{2}$), $P(z) = 1$ in Theorem 2.1, we have Corollary 2 due to Kim and Cho [3].

Putting $p(z) = \frac{zf'(z)}{f(z)}$ in Theorem 2.2, we have the following corollary.

Corollary 3.4 *Let* $f(z) \in A$. *If*

$$\gamma_1 < \operatorname{Im}\left(1 + \frac{zf''(z)}{f'(z)}\right) < \gamma_2,$$

where

$$\gamma_1 = -\frac{\sqrt{|a|^2 + 2(\mathrm{Re}(a))^2} - \mathrm{Im}(a)}{\mathrm{Re}(a)}$$

and

$$\gamma_2 = \frac{\sqrt{|a|^2 + 2(\mathrm{Re}(a))^2} + \mathrm{Im}(a)}{\mathrm{Re}(a)},$$

then

$$\mathrm{Re}\left(a\frac{zf'(z)}{f(z)}\right) > 0,$$

where $\mathrm{Re}(a) > 0$.

Putting $p(z) = \frac{zf'(z)}{f(z)}$ in Theorem 2.3, we have the following corollary.

Corollary 3.5 *Let* $p(z)$ *be a nonzero analytic function in* $\mathbb{U}$ *with* $p(0) = 1$. *If*

$$\left|\frac{zf''(z)}{f'(z)}\right| < \frac{3\,\mathrm{Re}(a)}{2|a|},$$

then

$$\mathrm{Re}\left(\frac{1}{a}\frac{zf'(z)}{f(z)}\right) > 0,$$

where $\mathrm{Re}(a) > 0$.

Remark 3.2 Putting $a = e^{i\lambda}$ ($|\lambda| < \frac{\pi}{2}$) in Corollary 3.5, we have the result due to Kim and Cho [3].

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the paper. Also, all authors have read and approved the final version of the paper.

Author details
[1]Department of Mathematics, Faculty of Science, University of Mansoura, Mansoura, 35516, Egypt. [2]Current address: Department of Mathematics, College of Science, University of Hail, Hail, Saudi Arabia. [3]Department of Mathematics, College of Science, King Khaled University, Abha, Saudi Arabia.

Acknowledgements
Dedicated to Professor Hari M Srivastava.
The authors would like to express their gratitude to the referees for the valuable advices to improve this paper.

Received: 25 November 2012 Accepted: 16 January 2013 Published: 19 April 2013

References
1. Cho, NE, Hwa, KI: Conditions for Carathéodory functions. J. Inequal. Appl. **2009**, Article ID 601597 (2009)
2. Delsarte, P, Genin, Y: A simple proof of Livingston's inequality for Carathéodory functions. Proc. Am. Math. Soc. **107**, 1017-1020 (1989)
3. Kim, IH, Cho, NE: Sufficient conditions for Carathéodory functions. Comput. Math. Appl. **59**(6), 2067-2073 (2010)
4. Li, JL, Owa, S: Sufficient conditions for starlikeness. Indian J. Pure Appl. Math. **33**, 313-318 (2002)

5. Miller, S: Differential inequalities and Carathéodory functions. Bull. Am. Math. Soc. **81**, 79-81 (1975)
6. Nunokawa, M: Differential inequalities and Carathéodory functions. Proc. Jpn. Acad., Ser. A, Math. Sci. **65**, 326-328 (1989)
7. Nunokawa, M: On properties of non-Carathéodory functions. Proc. Jpn. Acad., Ser. A, Math. Sci. **68**, 152-153 (1992)
8. Nunokawa, M, Owa, S, Takahashi, N, Saitoh, H: Sufficient conditions for Carathéodory functions. Indian J. Pure Appl. Math. **33**, 1385-1390 (2002)
9. Obradović, M, Owa, S: On certain properties for some classes of starlike functions. J. Math. Anal. Appl. **145**, 357-364 (1990)
10. Padmanabhan, KS: On sufficient conditions for starlikeness. Indian J. Pure Appl. Math. **32**, 543-550 (1990)
11. Shiraishi, H, Owa, S, Srivastava, HM: Sufficient conditions for strongly Carathéodory functions. Comput. Math. Appl. **62**(8), 2978-2987 (2011)
12. Tuneski, N: On certain sufficient conditions for starlikeness. Int. J. Math. Math. Sci. **23**, 521-527 (2000)
13. Yang, D, Owa, S, Ochiai, K: Sufficient conditions for Carathéodory functions. Comput. Math. Appl. **51**, 467-474 (2006)
14. Miller, SS, Mocanu, PT: Differential Subordination, Theory and Application. Dekker, New York (2000)
15. Nunokawa, M, Owa, S, Nishiwaki, J, Saitoh, H: Sufficient conditions for starlikeness and convexity of analytic functions with real coefficients. Southeast Asian Bull. Math. **33**(6), 1149-1155 (2009)

doi:10.1186/1029-242X-2013-191
Cite this article as: Attiya and Nasr: On sufficient conditions for Carathéodory functions with applications. *Journal of Inequalities and Applications* 2013 **2013**:191.

Submit your manuscript to a SpringerOpen journal and benefit from:

- ► Convenient online submission
- ► Rigorous peer review
- ► Immediate publication on acceptance
- ► Open access: articles freely available online
- ► High visibility within the field
- ► Retaining the copyright to your article

Submit your next manuscript at ► springeropen.com