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Strong convergence theorems of general split equality problems for quasi-nonexpansive mappings

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Abstract

The purpose of this paper is to introduce and study the general split equality problem and general split equality fixed point problem in the setting of infinite-dimensional Hilbert spaces. Under suitable conditions, we prove that the sequences generated by the proposed new algorithm converges strongly to a solution of the general split equality fixed point problem and the general split equality problem for quasi-nonexpansive mappings in Hilbert spaces. As an application, we shall utilize our results to study the null point problem of maximal monotone operators, the split feasibility problem, and the equality equilibrium problem. The results presented in the paper extend and improve the corresponding results announced by Moudafi *et al.* (Nonlinear Anal. 79:117-121, 2013; Trans. Math. Program. Appl. 1:1-11, 2013), Eslamian and Latif (Abstr. Appl. Anal. 2013:805104, 2013) and Chen *et al.* (Fixed Point Theory Appl. 2014:35, 2014), Censor and Elfving (Numer. Algorithms 8:221-239, 1994), Censor and Segal (J. Convex Anal. 16:587-600, 2009) and some others.

Keywords: general split equality problem; general split equality fixed point problem; quasi-nonexpansive mapping; split feasibility problem

1 Introduction

Let C and Q be nonempty closed convex subsets of real Hilbert spaces H_1 and H_2, respectively. The *split feasibility problem* (*SFP*) is formulated as

$$\text{to finding } x^* \in C \text{ and } Ax^* \in Q, \tag{1.1}$$

where $A : H_1 \to H_2$ is a bounded linear operator. In 1994, Censor and Elfving [1] first introduced the *SFP* in finite-dimensional Hilbert spaces for modeling inverse problems which arise from phase retrievals and in medical image reconstruction [2]. It has been found that the *SFP* can also be used in various disciplines such as image restoration, and computer tomograph and radiation therapy treatment planning [3–5]. The *SFP* in an infinite-dimensional real Hilbert space can be found in [2, 4, 6–10].

Assuming that the *SFP* is consistent, it is not hard to see that $x^* \in C$ solves *SFP* if and only if it solves the fixed point equation

$$x^* = P_C\big(I - \gamma A^*(I - P_Q)A\big)x^*,$$

where P_C and P_Q are the metric projection from H_1 onto C and from H_2 onto Q, respectively, $\gamma > 0$ is a positive constant and A^* is the adjoint of A.

A popular algorithm to be used to solve *SFP* (1.1) is due to Byrne's *CQ-algorithm* [2]:

$$x_{k+1} = P_C\bigl(I - \gamma A^*(I - P_Q)A\bigr)x_k, \quad k \geq 1,$$

where $\gamma \in (0, 2/\lambda)$ with λ being the spectral radius of the operator A^*A.

Recently, Moudafi [11] introduced the following *split equality problem* (*SEP*):

$$\text{to find } x \in C, y \in Q \text{ such that } Ax = By, \tag{1.2}$$

where $A : H_1 \to H_3$ and $B : H_2 \to H_3$ are two bounded linear operators. Obviously, if $B = I$ (identity mapping on H_2) and $H_3 = H_2$, then (1.2) reduces to (1.1). This kind of split equality problem (1.2) allows asymmetric and partial relations between the variables x and y. The interest is to cover many situations, such as decomposition methods for PDEs, applications in game theory, and intensity-modulated radiation therapy.

In order to solve the split equality problem (1.2), Moudafi [11] introduced the following relaxed alternating *CQ-algorithm*:

$$\begin{cases} x_{k+1} = P_{C_k}(x_k - \gamma A^*(Ax_k - By_k)), \\ y_{k+1} = P_{Q_k}(y_k + \beta B^*(Ax_k - By_k)), \end{cases} \tag{1.3}$$

where

$$\begin{aligned} C_k &= \bigl\{x \in H_1; c(x_k) + \langle \xi_k, x - x_k\rangle \leq 0\bigr\}, \quad \xi_k \in \partial c(x_k), \\ Q_k &= \bigl\{y \in H_2; q(y_k) + \langle \eta_k, y - y_k\rangle \leq 0\bigr\}, \quad \eta_k \in \partial q(y_k), \end{aligned} \tag{1.4}$$

and $c : H_1 \to R$ (respectively $q : H_2 \to R$) is a convex and subdifferentiable function. Under suitable conditions, he proved that the sequence $\{x_n\}$ defined by (1.4) converges weakly to a solution of the split equality problem (1.2).

Each nonempty closed convex subset of a Hilbert space can be regarded as a set of fixed points of a projection. In [12], Moudafi and Al-Shemas introduced the following *split equality fixed point problem*:

$$\text{find } x \in C := F(S), y \in Q := F(T) \text{ such that } Ax = By, \tag{1.5}$$

where $S : H_1 \to H_1$ and $T : H_2 \to H_2$ are two firmly quasi-nonexpansive mappings, $F(S)$ and $F(T)$ denote the fixed point sets of S and T, respectively.

To solve the split equality fixed point problem (1.5) for firmly quasi-nonexpansive mappings, Moudafi *et al.* [11–13] proposed the following iteration algorithm:

$$\begin{cases} x_{k+1} = S(x_k - \gamma_k A^*(Ax_k - By_k)), \\ y_{k+1} = T(y_k + \gamma_k B^*(Ax_k - By_k)). \end{cases} \tag{1.6}$$

Very recently, Eslamian and Latif [14] and Chen *et al.* [15] introduced and studied some kinds of *general split feasibility problem* and *split equality problem* in real Hilbert spaces, and under suitable conditions some strong convergence theorems are proved.

Motivated by the above works, the purpose of this paper is to introduce the following *general split equality fixed point problem*:

$$\text{(GSEFP)} \quad \text{to find } x \in C := \bigcap_{i=1}^{\infty} F(S_i), y \in Q := \bigcap_{i=1}^{\infty} F(T_i) \text{ such that } Ax = By, \tag{1.7}$$

and the *general split equality problem*:

$$\text{(GSEP)} \quad \text{to find } x \in Cy \in Q \text{ such that } Ax = By. \tag{1.8}$$

For solving the GSEFP (1.7) and GSEP (1.8), in Sections 3 and 4, we propose an algorithm for finding the solutions of the general split equality fixed point problem and general split equality problem in a Hilbert space. We establish the strong convergence of the proposed algorithms to a solution of GSEFP and GSEP. As applications, in Section 5 we utilize our results to study the split feasibility problem, the null point problem of maximal monotone operators, and the equality equilibrium problem.

2 Preliminaries

Let H be a real Hilbert space and C be a nonempty closed convex subset of H. In the sequel, denote by $F(T)$ the set of fixed points of a mapping T and by $x_n \to x^*$ and $x_n \rightharpoonup x^*$, the strong convergence and weak convergence of a sequence $\{x_n\}$ to a point x^*, respectively.

Recall that a mapping $T : H \to H$ is said to be nonexpansive, if $\|Tx - Ty\| \leq \|x - y\|$, $\forall x, y \in H$. A typical example of a nonexpansive mapping is the metric projection P_C from H onto $C \subseteq H$ defined by $\|x - P_C x\| = \inf_{y\in C} \|x - y\|$. The metric projection P_C is *firmly nonexpansive, i.e.*,

$$\|P_C x - P_C y\|^2 \leq \langle x - y, P_C x - P_C y\rangle, \quad \forall x, y \in H, \tag{2.1}$$

and it can be characterized by the fact that

$$P_C(x) \in C \quad \text{and} \quad \bigl\langle y - P_C(x), x - P_C(x)\bigr\rangle \leq 0, \quad \forall x \in H, y \in C. \tag{2.2}$$

Definition 2.1 A mapping $T : H \to H$ is said to be *quasi-nonexpansive*, if $F(T) \neq \emptyset$, and

$$\|Tx - p\| \leq \|x - p\| \quad \text{for each } x \in H \text{ and } p \in F(T).$$

Lemma 2.2 [16] *Let H be a real Hilbert space, and $\{x_n\}$ be a sequence in H. Then, for any given sequence $\{\lambda_n\}$ of positive numbers with $\sum_{i=1}^{\infty} \lambda_n = 1$ such that for any positive integers i, j with $i < j$, the following holds*:

$$\Bigl\|\sum_{i=1}^{\infty} \lambda_n x_n\Bigr\|^2 \leq \sum_{i=1}^{\infty} \lambda_n \|x_n\|^2 - \lambda_i\lambda_j\|x_i - x_j\|^2.$$

Lemma 2.3 [17] *Let H be a real Hilbert space. For any $x, y \in H$, the following inequality holds*:

$$\|x + y\|^2 \leq \|x\|^2 + 2\langle y, x + y\rangle.$$

Lemma 2.4 [18] *Let $\{t_n\}$ be a sequence of real numbers. If there exists a subsequence $\{n_i\}$ of $\{n\}$ such that $t_{n_i} < t_{n_i+1}$ for all $i \ge 1$, then there exists a nondecreasing sequence $\{\tau(n)\}$ with $\tau(n) \to \infty$ such that for all (sufficiently large) positive integer numbers n, the following holds*:

$$t_{\tau(n)} \le t_{\tau(n)+1}, \qquad t_n \le t_{\tau(n)+1}.$$

In fact,

$$\tau(n) = \max\{k \le n : t_k \le t_{k+1}\}.$$

Definition 2.5 (Demiclosedness principle) Let C be a nonempty closed convex subset of a real Hilbert space H, and $T : C \to C$ be a mapping with $F(T) \neq \emptyset$. Then $I - T$ is said to be *demi-closed at zero*, if for any sequence $\{x_n\} \subset C$ with $x_n \rightharpoonup x$ and $\|x_n - Tx_n\| \to 0$, then $x = Tx$.

Remark 2.6 It is well known that if $T : C \to C$ is a nonexpansive mapping, then $I - T$ is demi-closed at zero.

Lemma 2.7 *Let $\{a_n\}$, $\{b_n\}$, and $\{c_n\}$ be sequences of positive real numbers satisfying $a_{n+1} \le (1 - b_n)a_n + c_n$ for all $n \ge 1$. If the following conditions are satisfied*:

(1) $b_n \in (0,1)$ *and* $\sum_{n=1}^{\infty} b_n = \infty$,
(2) $\sum_{n=1}^{\infty} c_n < \infty$, *or* $\limsup_{n\to\infty} \frac{c_n}{b_n} \le 0$,

then $\lim_{n\to\infty} a_n = 0$.

3 Strong convergence theorem for general split equality fixed point problem

Throughout this section we always assume that

(1) H_1, H_2, H_3 are real Hilbert spaces;
(2) $\{S_i\}_{i=1}^{\infty} : H_1 \to H_1$ and $\{T_i\}_{i=1}^{\infty} : H_2 \to H_2$ are two families of one-to-one and quasi-nonexpansive mappings;
(3) $A : H_1 \to H_3$ and $B : H_2 \to H_3$ are two bounded linear operators;
(4) $f = \left[\begin{smallmatrix} f_1 \\ f_2 \end{smallmatrix}\right]$, where f_i, $i = 1, 2$ is a k-contractive mapping on H_i with $k \in (0,1)$;
(5) $C := \bigcap_{i=1}^{\infty} F(S_i)$, $Q := \bigcap_{i=1}^{\infty} F(T_i)$, Γ is the set of solutions of GSEFP (1.7),

$$P = \begin{bmatrix} P_C \\ P_Q \end{bmatrix}, \qquad K_i = \begin{bmatrix} S_i \\ T_i \end{bmatrix}, \qquad G = [A \quad -B], \qquad G^*G = \begin{bmatrix} A^*A & -A^*B \\ -B^*A & B^*B \end{bmatrix};$$

(6) for any given $w_0 \in H_1 \times H_2$, the iterative sequence $\{w_n\} \subset H_1 \times H_2$ is generated by

$$w_{n+1} = P\left[\alpha_n w_n + \beta_n f(w_n) + \sum_{i=1}^{\infty} \gamma_{n,i}\big(K_i\big(I - \lambda_{n,i} G^*G\big) w_n\big)\right], \quad n \ge 0, \tag{3.1}$$

where $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_{n,i}\}$ are the sequences of nonnegative numbers with

$$\alpha_n + \beta_n + \sum_{i=1}^{\infty} \gamma_{n,i} = 1 \quad \text{for each } n \ge 0.$$

We are now in a position to give the following main result.

Lemma 3.1 *Let $w^* = (x^*, y^*)$ be a point in $C \times Q$, i.e., $x^* \in C = \bigcap_{i=1}^{\infty} F(S_i)$ and $y^* \in Q = \bigcap_{i=1}^{\infty} F(T_i)$. Then the following statements are equivalent*:

(i) *w^* is a solution to GSEFP* (1.7);

(ii) *$w^* = K_i(w^*)$ for each $i \geq 1$ and $G(w^*) = 0$*;

(iii) *for each $i \geq 1$ and for each $\lambda > 0$, w^* solves the fixed point equations*:

$$w^* = K_i w^* \quad \textit{and} \quad w^* = K_i\big(I - \lambda G^* G\big) w^*. \tag{3.2}$$

Proof (i) $\Rightarrow$ (ii). If $w^* \in C \times Q$ is a solution to GSEFP (1.7), then for each $i \geq 1$, $w^* = K_i w^*$, and $Ax^* = By^*$. This implies that for each $i \geq 1$, $w^* = K_i w^*$, and

$$G\big(w^*\big) = [A \quad -B] \begin{bmatrix} x^* \\ y^* \end{bmatrix} = Ax^* - By^* = 0.$$

(ii) $\Rightarrow$ (iii). If $w^* = K_i(w^*)$, $\forall i \geq 1$ and $G(w^*) = 0$, it is easy to see that (3.2) holds.

(iii) $\Rightarrow$ (i). From (3.2), for each $i \geq 1$ we have $K_i w^* = K_i(I - \lambda G^* G) w^*$. Since S_i and T_i both are one-to-one, so is K_i. Hence we have $\|w^* - (I - \lambda G^* G) w^*\| = 0$, for any $\lambda > 0$. This implies that $G^* G(w^*) = 0$, and so

$$0 = \big\langle G^* G w^*, w^* \big\rangle = \big\langle G w^*, G w^* \big\rangle = \big\| G w^* \big\|^2,$$

i.e., $G(w^*) = Ax^* - By^* = 0$.

This completes the proof of Lemma 3.1. □

Lemma 3.2 *If $\lambda \in (0, \frac{2}{L})$, where $L = \|G\|^2$, then $(I - \lambda G^* G) : H_1 \times H_2 \to H_1 \times H_2$ is a nonexpansive mapping.*

Proof In fact for any $w, u \in H_1 \times H_2$, we have

$$\begin{aligned}
&\big\| \big(I - \lambda G^* G\big) u - \big(I - \lambda G^* G\big) w \big\|^2 \\
&\quad = \big\| (u - w) - \lambda G^* G(u - w) \big\|^2 \\
&\quad = \|u - w\|^2 + \lambda^2 \big\| G^* G(u - w) \big\|^2 - 2\lambda \big\langle u - w, G^* G(u - w) \big\rangle \\
&\quad \leq \|u - w\|^2 + \lambda^2 L \big\| G(u - w) \big\|^2 - 2\lambda \big\langle G(u - w), G(u - w) \big\rangle \\
&\quad = \|u - w\|^2 + \lambda^2 L \big\| G(u - w) \big\|^2 - 2\lambda \big\| G(u - w) \big\|^2 \\
&\quad = \|u - w\|^2 - \lambda(2 - \lambda L) \big\| G(u - w) \big\|^2 \\
&\quad \leq \|u - w\|^2.
\end{aligned}$$

This completes the proof. □

Theorem 3.3 *Let H_1, H_2, H_3, $\{S_i\}$, $\{T_i\}$, A, B, f, C, Q, Γ, P, G, K_i, $G^* G$ satisfy the above conditions* (1)-(5). *Let $\{w_n\}$ be the sequence defined by* (3.1). *If the solution set Γ of GSEFP* (1.7) *is nonempty and the following conditions are satisfied*:

(i) $\alpha_n + \beta_n + \sum_{i=1}^{\infty} \gamma_{n,i} = 1$, *for each* $n \geq 0$;
(ii) $\lim_{n\to\infty} \beta_n = 0$, *and* $\sum_{n=0}^{\infty} \beta_n = \infty$;
(iii) $\liminf_{n\to\infty} \alpha_n \gamma_{n,i} > 0$ *for each* $i \geq 1$;
(iv) $\{\lambda_{n,i}\} \subset (0, \frac{2}{L})$ *for each* $i \geq 1$, *where* $L = \|G\|^2$;
(v) *for each* $i \geq 1$, *the mapping* $I - K_i(I - \lambda_{n,i}G^*G)$ *is demi-closed at zero*,

then the sequence $\{w_n\}$ *converges strongly to* $w^* = P_\Gamma f(w^*)$ *which is a solution of GSEFP* (1.7).

Proof (I) *First we prove that the sequence* $\{w_n\}$ *is bounded.*

In fact, for any given $z \in \Gamma$, it follows from Lemma 3.1 that

$$G(z) = 0, \qquad K_i z = z \quad \text{and} \quad z = K_i\bigl(I - \lambda_{n,i}G^*G\bigr)z \quad \text{for each } i \geq 1.$$

By the assumptions and Lemma 3.2, for each $\lambda \in (0, \frac{2}{L})$, $(I - \lambda G^*G) : H_1 \times H_2 \to H_1 \times H_2$ is nonexpansive, and for each $i \geq 1$, $K_i = \left[\begin{smallmatrix} S_i \\ T_i \end{smallmatrix}\right]$ is quasi-nonexpansive, hence we have

$$\begin{aligned}
\|w_{n+1} - z\| &= \left\| P\left[\alpha_n w_n + \beta_n f(w_n) + \sum_{i=1}^{\infty} \gamma_{n,i} K_i\bigl(I - \lambda_{n,i}G^*G\bigr)w_n\right] - P(z)\right\| \\
&\leq \alpha_n\|w_n - z\| + \beta_n\bigl\|f(w_n) - z\bigr\| + \sum_{i=1}^{\infty} \gamma_{n,i}\bigl\|K_i\bigl(I - \lambda_{n,i}G^*G\bigr)w_n - z\bigr\| \\
&\leq \alpha_n\|w_n - z\| + \beta_n\bigl\|f(w_n) - z\bigr\| + \sum_{i=1}^{\infty} \gamma_{n,i}\bigl\|\bigl(I - \lambda_{n,i}G^*G\bigr)w_n - z\bigr\| \\
&\leq \alpha_n\|w_n - z\| + \beta_n\bigl\|f(w_n) - z\bigr\| + \sum_{i=1}^{\infty} \gamma_{n,i}\bigl\|\bigl(I - \lambda_{n,i}G^*G\bigr)w_n - \bigl(I - \lambda_{n,i}G^*G\bigr)z\bigr\| \\
&\leq \alpha_n\|w_n - z\| + \beta_n\bigl\|f(w_n) - z\bigr\| + \sum_{i=1}^{\infty} \gamma_{n,i}\|w_n - z\| \\
&= (1 - \beta_n)\|w_n - z\| + \beta_n\bigl\|f(w_n) - z\bigr\| \\
&\leq (1 - \beta_n)\|w_n - z\| + \beta_n\bigl\|f(w_n) - f(z)\bigr\| + \beta_n\bigl\|f(z) - z\bigr\| \\
&\leq (1 - \beta_n)\|w_n - z\| + k\beta_n\|w_n - z\| + \beta_n\bigl\|f(z) - z\bigr\| \\
&= \bigl(1 - (1 - k)\beta_n\bigr)\|w_n - z\| + (1 - k)\beta_n\frac{1}{1 - k}\bigl\|f(z) - z\bigr\| \\
&\leq \max\left\{\|w_n - z\|, \frac{1}{1 - k}\bigl\|f(z) - z\bigr\|\right\}.
\end{aligned}$$

By induction, we can prove that

$$\|w_n - z\| \leq \max\left\{\|w_0 - z\|, \frac{1}{1 - k}\bigl\|f(z) - z\bigr\|\right\}.$$

This shows that $\{w_n\}$ is bounded, and so is $\{f(w_n)\}$.

(II) *Now we prove that the following inequality holds*:

$$\begin{aligned}
&\alpha_n\gamma_{n,i}\bigl\|w_n - K_i\bigl(I - \lambda_{n,i}G^*G\bigr)w_n\bigr\|^2 \\
&\quad\leq \|w_n - z\|^2 - \|w_{n+1} - z\|^2 + \beta_n\bigl\|f(w_n) - z\bigr\|^2 \quad \text{for each } i \geq 1.
\end{aligned} \tag{3.3}$$

Indeed, it follows from (3.1) and Lemma 2.2 that for each $i \geq 1$

$$\begin{aligned}
\|w_{n+1} - z\|^2 &= \left\| P\left[\alpha_n(w_n - z) + \beta_n\big(f(w_n) - z\big) + \sum_{i=1}^{\infty} \gamma_{n,i}\big(K_i\big(I - \lambda_{n,i}G^*G\big)w_n\big)\right] - z\right\|^2 \\
&\leq \alpha_n\|w_n - z\|^2 + \beta_n\big\|f(w_n) - z\big\|^2 + \sum_{i=1}^{\infty}\gamma_{n,i}\big\|K_i\big(I - \lambda_{n,i}G^*G\big)w_n - z\big\|^2 \\
&\quad - \alpha_n\gamma_{n,i}\big\|w_n - K_i\big(I - \lambda_{n,i}G^*G\big)w_n\big\|^2 \\
&\leq \alpha_n\|w_n - z\|^2 + \beta_n\big\|f(w_n) - z\big\|^2 + \sum_{i=1}^{\infty}\gamma_{n,i}\|w_n - z\|^2 \\
&\quad - \alpha_n\gamma_{n,i}\big\|w_n - K_i\big(I - \lambda_{n,i}G^*G\big)w_n\big\|^2 \\
&= (1 - \beta_n)\|w_n - z\|^2 + \beta_n\big\|f(w_n) - z\big\|^2 \\
&\quad - \alpha_n\gamma_{n,i}\big\|w_n - K_i\big(I - \lambda_{n,i}G^*G\big)w_n\big\|^2.
\end{aligned}$$

This implies that for each $i \geq 1$

$$\alpha_n\gamma_{n,i}\big\|w_n - K_i\big(I - \lambda_{n,i}G^*G\big)w_n\big\|^2 \leq \|w_n - z\|^2 - \|w_{n+1} - z\|^2 + \beta_n\big\|f(w_n) - z\big\|^2.$$

Inequality (3.3) is proved.

It is easy to see that the solution set Γ of GSEFP (1.7) is a nonempty closed and convex subset in $C \times Q$, hence the metric projection P_Γ is well defined. In addition, since $P_\Gamma f : H_1 \times H_2 \to H_1 \times H_2$ is a contractive mapping, there exists a $w^* \in \Gamma$ such that

$$w^* = P_\Gamma f\big(w^*\big). \tag{3.4}$$

(III) *Now we prove that* $w_n \to w^*$.

For this purpose, we consider two cases.

Case I. Suppose that the sequence $\{\|w_n - w^*\|\}$ is monotone. Since $\{\|w_n - w^*\|\}$ is bounded, $\{\|w_n - w^*\|\}$ is convergent. Since $w^* \in \Gamma$, in (3.3) taking $z = w^*$ and letting $n \to \infty$, in view of conditions (ii) and (iii), we have

$$\lim_{n\to\infty}\big\|w_n - K_i\big(I - \lambda_{n,i}G^*G\big)w_n\big\| = 0 \quad \text{for each } i \geq 1. \tag{3.5}$$

On the other hand, by Lemma 2.3 and (3.1), we have

$$\begin{aligned}
\big\|w_{n+1} - w^*\big\|^2 &= \left\|P\left[\alpha_n w_n + \beta_n f(w_n) + \sum_{i=1}^{\infty}\gamma_{n,i}K_i\big(I - \lambda_{n,i}G^*G\big)w_n\right] - w^*\right\|^2 \\
&\leq \Bigg\|\alpha_n\big(w_n - w^*\big) + \beta_n\big(f(w_n) - w^*\big) \\
&\quad + \sum_{i=1}^{\infty}\gamma_{n,i}\big(K_i\big(I - \lambda_{n,i}G^*G\big)w_n - w^*\big)\Bigg\|^2 \\
&\leq \left\|\alpha_n\big(w_n - w^*\big) + \sum_{i=1}^{\infty}\gamma_{n,i}\big(K_i\big(I - \lambda_{n,i}G^*G\big)w_n - w^*\big)\right\|^2
\end{aligned}$$

$$\begin{aligned}
&\quad + 2\beta_n\langle f(w_n) - w^*, w_{n+1} - w^*\rangle \quad \text{(by Lemma 2.3)} \\
&\leq \left\{\alpha_n\|w_n - w^*\| + \sum_{i=1}^{\infty}\gamma_{n,i}\|w_n - w^*\|\right\}^2 \\
&\quad + 2\beta_n\langle f(w_n) - f(w^*), w_{n+1} - w^*\rangle + 2\beta_n\langle f(w^*) - w^*, w_{n+1} - w^*\rangle \\
&= (1-\beta_n)^2\|w_n - w^*\|^2 + 2\beta_n k\|w_n - w^*\|\|w_{n+1} - w^*\| \\
&\quad + 2\beta_n\langle f(w^*) - w^*, w_{n+1} - w^*\rangle \\
&\leq (1-\beta_n)^2\|w_n - w^*\|^2 + \beta_n k\{\|w_n - w^*\|^2 + \|w_{n+1} - w^*\|^2\} \\
&\quad + 2\beta_n\langle f(w^*) - w^*, w_{n+1} - w^*\rangle.
\end{aligned}$$

Simplifying we have

$$\begin{aligned}
\|w_{n+1} - w^*\|^2 &\leq \frac{(1-\beta_n)^2 + \beta_n k}{1 - \beta_n k}\|w_n - w^*\|^2 + \frac{2\beta_n}{1-\beta_n k}\langle f(w^*) - w^*, w_{n+1} - w^*\rangle \\
&= \frac{1 - 2\beta_n + \beta_n k}{1-\beta_n k}\|w_n - w^*\|^2 + \frac{\beta_n^2}{1-\beta_n k}\|w_n - w^*\|^2 \\
&\quad + \frac{2\beta_n}{1-\beta_n k}\langle f(w^*) - w^*, w_{n+1} - w^*\rangle \\
&= \left(1 - \frac{2(1-k)\beta_n}{1-\beta_n k}\right)\|w_n - w^*\|^2 \\
&\quad + \frac{2(1-k)\beta_n}{1-\beta_n k}\left\{\frac{\beta_n M}{2(1-k)} + \frac{1}{1-k}\langle f(w^*) - w^*, w_{n+1} - w^*\rangle\right\} \\
&= (1-\eta_n)\|w_n - w^*\|^2 + \eta_n\delta_n,
\end{aligned} \tag{3.6}$$

where $\eta_n = \frac{2(1-k)\beta_n}{1-\beta_n k}$, $\delta_n = \frac{\beta_n M}{2(1-k)} + \frac{1}{1-k}\langle f(w^*) - w^*, w_{n+1} - w^*\rangle$, $M := \sup_{n\geq 0}\|w_n - w^*\|$.

By condition (ii), $\lim_{n\to\infty}\beta_n = 0$ and $\sum_{n=1}^{\infty}\beta_n = \infty$, and so $\sum_{n=1}^{\infty}\eta_n = \infty$.

Next we prove that

$$\limsup_{n\to\infty}\delta_n \leq 0. \tag{3.7}$$

In fact, since $\{w_n\}$ is bounded in $C \times Q$, there exists a subsequence $\{w_{n_k}\} \subset \{w_n\}$ with $w_{n_k} \rightharpoonup v^*$ (some point in $C \times Q$), and $\lambda_{n_k,i} \to \lambda_i \in (0, \frac{2}{L})$ such that

$$\lim_{n\to\infty}\langle f(w^*) - w^*, w_{n_k} - w^*\rangle = \limsup_{n\to\infty}\langle f(w^*) - w^*, w_n - w^*\rangle.$$

In view of (3.5)

$$\|w_{n_k} - K_i(I - \lambda_{n_k,i}G^*G)w_{n_k}\| \to 0 \quad \text{for each } i \geq 1.$$

Again by the assumption that for each $i \geq 1$, the mapping $I - K_i(I - \lambda_{n,i}G^*G)$ is demi-closed at zero, hence we have

$$v^* = K_i v^* \quad \text{and} \quad v^* = K_i(I - \lambda_{n,i}G^*G)v^*, \quad \forall i \geq 1. \tag{3.8}$$

By Lemma 3.1, this implies that $v^* \in \Gamma$. In addition, since $w^* = P_\Gamma f(w^*)$, we have

$$\begin{aligned}\limsup_{n\to\infty}\langle f(w^*) - w^*, w_n - w^*\rangle &= \lim_{n\to\infty}\langle f(w^*) - w^*, w_{n_k} - w^*\rangle \\ &= \langle f(w^*) - w^*, v^* - w^*\rangle \le 0.\end{aligned}$$

This shows that (3.7) is true. Taking $a_n = \|w_n - w^*\|^2$, $b_n = \eta_n$, and $c_n = \delta_n\eta_n$ in Lemma 2.7, all conditions in Lemma 2.7 are satisfied. We have $w_n \to w^*$.

Case II. If the sequence $\{\|w_n - w^*\|\}$ is not monotone, by Lemma 2.4, there exists a sequence of positive integers: $\{\tau(n)\}$, $n \ge n_0$ (where n_0 is large enough) such that

$$\tau(n) = \max\{k \le n : \|w_k - w^*\| \le \|w_{k+1} - w^*\|\}. \tag{3.9}$$

Clearly $\{\tau(n)\}$ is nondecreasing, $\tau(n) \to \infty$ as $n \to \infty$, and for all $n \ge n_0$

$$\|w_{\tau(n)} - w^*\| \le \|w_{\tau(n)+1} - w^*\|; \qquad \|w_n - w^*\| \le \|w_{\tau(n)+1} - w^*\|. \tag{3.10}$$

Therefore $\{\|w_{\tau(n)} - w^*\|\}$ is a nondecreasing sequence. According to Case I, $\lim_{n\to\infty}\|w_{\tau(n)} - w^*\| = 0$ and $\lim_{n\to\infty}\|w_{\tau(n)+1} - w^*\| = 0$. Hence we have

$$0 \le \|w_n - w^*\| \le \max\{\|w_n - w^*\|, \|w_{\tau(n)} - w^*\|\} \le \|w_{\tau(n)+1} - w^*\| \to 0, \quad \text{as } n \to \infty.$$

This implies that $w_n \to w^*$ and $w^* = P_\Gamma f(w^*)$ is a solution of GSEFP (1.7).

This completes the proof of Theorem 3.3. □

Remark 3.4 Theorem 3.3 extends and improves the main results in Moudafi *et al.* [11–13] in the following aspects:

(a) For the mappings, we extend the mappings from firmly quasi-nonexpansive mappings to an infinite family of one-to-one quasi-nonexpansive mappings.
(b) For the algorithms, we propose new iterative algorithms which are different from ones given in [11–13].
(c) For the convergence, the iterative sequence proposed by our algorithm converges strongly to a solution of GSEFP (1.7). But the iterative sequences proposed in [11–13] are only of weak convergence to a solution of the split equality problem.

4 Strong convergence theorem for general split equality problem

Throughout this section we always assume that

(1) H_1, H_2, H_3 are real Hilbert spaces; $\{C_i\}_{i=1}^\infty \subset H_1$ and $\{Q_i\}_{i=1}^\infty \subset H_2$ are two families of nonempty closed and convex subsets with $C = \bigcap_{i=1}^\infty C_i \ne \emptyset$ and $Q = \bigcap_{i=1}^\infty Q_i \ne \emptyset$;
(2) P_{C_i} (resp. P_{Q_i}) is the metric projection from H_1 onto C_i (resp. H_2 onto Q_i), and $P_i := \begin{bmatrix} P_{C_i} \\ P_{Q_i}\end{bmatrix}$, $i = 1, 2, \ldots$, and $P := \begin{bmatrix} P_C \\ P_Q\end{bmatrix}$;
(3) $A : H_1 \to H_3$ and $B : H_2 \to H_3$ are two bounded linear operators;
(4) f, G, G^*G are the same as in Theorem 3.3.

The so-called *general split equality problem* (GSEP) is

$$\text{to find } x \in C y \in Q \text{ such that } Ax = By. \tag{4.1}$$

Lemma 4.1 *Let H_1, H_2, H_3, P, $\{P_i\}$, A, B, f, C, Q, G, G^*G be the same as above. Then a point $w^* = (x^*, y^*)$ is a solution to GSEP* (4.1), *if and only if for each $i \geq 1$ and for each $\lambda > 0$, w^* solves the following fixed point equations*:

$$w^* = P_i w^* \quad \text{and} \quad w^* = P_i\big(I - \lambda G^*G\big)w^*. \tag{4.2}$$

Proof In fact, a point $w^* = (x^*, y^*)$ is a solution of GSEP (4.1)

$$\begin{aligned}
&\Leftrightarrow \quad w^* = \big(x^*, y^*\big) \in C \times Q \quad \text{and} \quad Ax^* = By^* \\
&\Leftrightarrow \quad \text{for each } i \geq 1, \quad x^* = P_{C_i}\big(x^*\big), \qquad y^* = P_{Q_i}\big(y^*\big) \quad \text{and} \quad Ax^* = By^* \\
&\Leftrightarrow \quad w^* = P_i\big(w^*\big) \quad \text{and} \quad Ax^* = By^* \\
&\Leftrightarrow \quad \begin{cases} Ax^* = P_{A(C_i)\cap B(Q_i)}By^*, \\ By^* = P_{B(Q_i)\cap A(C_i)}Ax^* \end{cases} \\
&\Leftrightarrow \quad \begin{cases} \langle Ax^* - P_{B(Q_i)}By^*, Au - Ax^*\rangle \geq 0, \quad \forall u \in C_i, \\ \langle By^* - P_{A(C_i)}Ax^*, Bv - By^*\rangle \geq 0, \quad \forall v \in Q_i \end{cases} \\
&\Leftrightarrow \quad \begin{cases} \langle Ax^* - By^*, Au - Ax^*\rangle \geq 0, \quad \forall u \in C_i, \\ \langle By^* - Ax^*, Bv - By^*\rangle \geq 0, \quad \forall v \in Q_i \end{cases} \\
&\Leftrightarrow \quad \begin{cases} \langle \gamma A^*(Ax^* - By^*), u - x^*\rangle \geq 0, \quad \forall u \in C_i, \gamma > 0, \\ \langle \gamma B^*(By^* - Ax^*), v - y^*\rangle \geq 0, \quad \forall v \in Q_i, \gamma > 0 \end{cases} \\
&\Leftrightarrow \quad \begin{cases} \langle x^* - (x^* - \gamma A^*(Ax^* - By^*)), u - x^*\rangle \geq 0, \quad \forall u \in C_i, \gamma > 0, \\ \langle y^* - (y^* - \gamma B^*(By^* - Ax^*)), v - y^*\rangle \geq 0, \quad \forall v \in Q_i, \gamma > 0 \end{cases} \\
&\Leftrightarrow \quad \begin{cases} x^* = P_{C_i}(x^* - \gamma A^*(Ax^* - By^*)), \\ y^* = P_{Q_i}(y^* - \gamma B^*(By^* - Ax^*)) \end{cases} \\
&\Leftrightarrow \quad w^* = P_i\big(I - \gamma G^*G\big)w^* \quad \text{and} \quad w^* = P_i w^*.
\end{aligned}$$

This completes the proof of Lemma 4.1. □

The metric projections P_{C_i} and P_{Q_i} are nonexpansive with $F(P_{C_i}) = C_i$ and $F(P_{Q_i}) = Q_i$, $i \geq 1$. This implies that the metric projections P_{C_i} and P_{Q_i} all are quasi-nonexpansive. In addition, by Lemma 3.2, for each $i \geq 1$ and each $\lambda \in (0, \frac{2}{L})$, the mapping $P_i(I - \lambda G^*G) : H_1 \times H_2 \to C_i \times Q_i$ is nonexpansive. By Remark 2.6, for each $i \geq 1$ and each $\lambda \in (0, \frac{2}{L})$, the mapping $(I - P_i(I - \lambda G^*G))$ is demi-closed at zero.

Consequently, we have the following.

Theorem 4.2 *Let H_1, H_2, H_3, P, $\{P_i\}$, A, B, f, C, Q, G, G^*G be the same as above. Let $\{w_n\}$ be the sequence generated by $w_0 \in H_1 \times H_2$*

$$w_{n+1} = P\left[\alpha_n w_n + \beta_n f(w_n) + \sum_{i=1}^{\infty} \gamma_{n,i} P_i\big(I - \lambda_{n,i} G^*G\big)w_n\right], \quad n \geq 0. \tag{4.3}$$

If the solution set Γ_1 of GSEP (4.1) *is nonempty and the following conditions are satisfied*:

(i) $\alpha_n + \beta_n + \sum_{i=1}^{\infty} \gamma_{n,i} = 1$, *for each $n \geq 0$*;

(ii) $\lim_{n\to\infty} \beta_n = 0$, *and* $\sum_{n=0}^{\infty} \beta_n = \infty$;
(iii) $\liminf_{n\to\infty} \alpha_n \gamma_{n,i} > 0$ *for each* $i \geq 1$;
(iv) $\{\lambda_{n,i}\} \subset (0, \frac{2}{L})$ *for each* $i \geq 1$, *where* $L = \|G\|^2$,

then the sequence $\{w_n\}$ *defined by* (4.3) *converges strongly to a solution* w^* *of GSEP* (4.1) *and* $w^* = P_{\Gamma_1} f(w^*)$.

Proof Taking $S_i = P_{C_i}$, $T_i = P_{Q_i}$, and $K_i = P_i$, $i = 1, 2, \ldots$ in Theorem 3.3, we know that S_i and T_i both are nonexpansive with $F(S_i) = C_i$ and $F(T_i) = Q_i$ and so they are quasi-nonexpansive mappings, and $C = \bigcap_{i=1}^{\infty} F(S_i)$ and $Q = \bigcap_{i=1}^{\infty} F(T_i)$. Therefore all conditions in Theorem 3.3 are satisfied. The conclusion of Theorem 4.2 can be obtained from Lemma 4.1 and Theorem 3.3 immediately. □

Remark 4.3 Theorem 4.2 extends and improves the corresponding results in Censor and Elfving [1], Moudafi *et al.* [11, 12], Eslamian and Latif [14], Chen *et al.* [15], Censor and Segal [19].

5 Applications

In this section we shall utilize the results presented in the paper to give some applications.

5.1 Application to split feasibility problem

Let $C \subset H_1$ and $Q \subset H_2$ be two nonempty closed convex subsets and $A : H_1 \to H_2$ be a bounded linear operator. The so-called *split feasibility problem* (SFP) [1] is to find

$$x \in C, y \in Q \text{ such that } Ax = y. \tag{5.1}$$

Let P_C and P_Q be the metric projection from H_1 onto C and H_2 onto Q, respectively. Thus $F(P_C) = C$ and $F(P_Q) = Q$. From Theorem 4.2 we have the following.

Theorem 5.1 *Let* H_1, H_2 *be two real Hilbert spaces,* $A : H_1 \to H_2$ *be a bounded linear operator and* I *be the identity mapping on* H_2. *Let* $C \subset H_1$ *and* $Q \subset H_2$ *be nonempty closed convex subsets and* P_C *and* P_Q *are the metric projections from* H_1 *onto* C *and* H_2 *onto* Q, *respectively. Let* $\{w_n\}$ *be the sequence generated by* $w_0 \in H_1 \times H_2$:

$$w_{n+1} = P\big[\alpha_n w_n + \beta_n f(w_n) + \gamma_n P\big(I - \lambda_n U^* U\big) w_n\big], \quad n \geq 0, \tag{5.2}$$

where f *is the mapping as given in Theorem* 4.2 *and*

$$U = [A \quad -I], \qquad P = \begin{bmatrix} P_C \\ P_Q \end{bmatrix}, \qquad U^* U = \begin{bmatrix} A^* A & -A^* \\ -A & I \end{bmatrix}. \tag{5.3}$$

If the solution set Γ_2 *of SFP* (5.1) *is nonempty and the following conditions are satisfied*:
(i) $\alpha_n + \beta_n + \gamma_n = 1$, *for each* $n \geq 0$;
(ii) $\lim_{n\to\infty} \beta_n = 0$, *and* $\sum_{n=0}^{\infty} \beta_n = \infty$;
(iii) $\liminf_{n\to\infty} \alpha_n \gamma_n > 0$;
(iv) $\{\lambda_n\} \subset (0, \frac{2}{L})$, *where* $L = \|U\|^2$,

then the sequence $\{w_n\}$ *defined by* (5.2) *converges strongly to a solution* w^* *of SFP* (5.1) *and* $w^* = P_{\Gamma_2} f(w^*)$.

Proof In Theorem 4.2 taking $H_2 = H_3$, $B = I$, $G = U$, $\{C_i\} = \{C\}$, and $\{Q_i\} = \{Q\}$, the conclusions of Theorem 5.1 can be obtained from Theorem 4.2 immediately. □

Remark Theorem 5.1 generalizes and extends the main results of Censor and Elfving [1] and Censor and Segal [19] from weak convergence to strong convergence.

5.2 Application to null point problem of maximal monotone operators

Let H_1, H_2, H_3, A, B, be the same as in Theorem 3.3. Let $M : H_1 \to H_1$, and $N : H_2 \to H_2$ be two strictly maximal monotone operators. It is well known that the associated resolvent mappings $J_\mu^M(x) := (I + \mu M)^{-1}$ and $J_\mu^N(x) := (I + \mu N)^{-1}$ of M and N, respectively, are one-to-one nonexpansive mappings, and

$$x \in M^{-1}(0) \quad \Leftrightarrow \quad x \in F\bigl(J_\mu^M\bigr); \qquad y \in N^{-1}(0) \quad \Leftrightarrow \quad y \in F\bigl(J_\mu^N\bigr). \tag{5.4}$$

Denote $S = J_\mu^M$, $T = J_\mu^N$, $C = M^{-1}(0) = F(J_\mu^M)$, and $Q = N^{-1}(0) = F(J_\mu^N)$, then the general split equality fixed point problem (1.7) is reduced to the following *null point problem related to strictly maximal monotone operators M and N* (NPP(M,N)):

$$\text{to find } x^* \in M^{-1}(0), y^* \in N^{-1}(0) \text{ such that } Ax^* = By^*. \tag{5.5}$$

From Theorem 3.3 we can obtain the following.

Theorem 5.2 *Let H_1, H_2, H_3, A, B, f, G, be the same as in Theorem* 3.3. *Let C, Q, S, and T be the same as above. Let $\{w_n\}$ be the sequence generated by $w_0 \in H_1 \times H_2$*

$$w_{n+1} = P\bigl[\alpha_n w_n + \beta_n f(w_n) + \gamma_n K\bigl(I - \lambda_n G^* G\bigr) w_n\bigr], \quad n \ge 0, \tag{5.6}$$

where $P = \left[\begin{smallmatrix} P_C \\ P_Q \end{smallmatrix}\right]$, $K = \left[\begin{smallmatrix} S \\ T \end{smallmatrix}\right]$. If the solution set Γ_3 of NPP(M,N) (5.5) *is nonempty and the following conditions are satisfied*:

(i) $\alpha_n + \beta_n + \gamma_n = 1$, *for each* $n \ge 0$;
(ii) $\lim_{n\to\infty} \beta_n = 0$, *and* $\sum_{n=0}^\infty \beta_n = \infty$;
(iii) $\liminf_{n\to\infty} \alpha_n \gamma_n > 0$;
(iv) $\{\lambda_n\} \subset (0, \frac{2}{L})$, *where* $L = \|G\|^2$,

then the sequence $\{w_n\}$ defined by (5.6) *converges strongly to $w^* = P_{\Gamma_3} f(w^*)$, which is a solution of* NPP(M,N) (5.5).

Proof Since $S = J_\mu^M$ and $T = J_\mu^N$ both are one-to-one nonexpansive with $F(S) \neq \emptyset$ and $F(T) \neq \emptyset$. Hence they are one-to-one quasi-nonexpansive mappings and $I - K(I - \lambda_n G^* G)$ is demi-closed at zero. Therefore all conditions in Theorem 3.3 are satisfied. The conclusions of Theorem 5.2 can be obtained from Theorem 3.3 immediately. □

5.3 Application to equality equilibrium problem

Let D be a nonempty closed and convex subset of a real Hilbert H. A bifunction $g : D \times D \to (-\infty, +\infty)$ is said to be a *equilibrium function*, if it satisfies the following conditions:

(A1) $g(x,x) = 0$, for all $x \in D$;
(A2) g is monotone, *i.e.*, $g(x,y) + g(y,x) \le 0$ for all $x, y \in D$;
(A3) $\limsup_{t\downarrow 0} g(tz + (1-t)x, y) \le g(x,y)$ for all $x, y, z \in D$;

(A4) for each $x \in D$, $y \mapsto g(x,y)$ is convex and lower semi-continuous.

The so-called *equilibrium problem with respective to the equilibrium functions g and* D is

$$\text{to find } x^* \in D \text{ such that } g(x^*, y) \geq 0, \quad \forall y \in D. \tag{5.7}$$

Its solution set is denoted by $\mathrm{EP}(g,D)$.

For given $\lambda > 0$ and $x \in H$, the *resolvent of the equilibrium function g* is the operator $R_{\lambda,g} : H \to D$ defined by

$$R_{\lambda,g}(x) := \left\{ z \in D : g(z,y) + \frac{1}{\lambda}\langle y - z, z - x\rangle \geq 0, \forall y \in D \right\}. \tag{5.8}$$

It is well known that the resolvent $R_{\lambda,g}$ of the equilibrium function g has the following properties [20]:

(1) $R_{\lambda,g}$ is single-valued;

(2) $F(R_{\lambda,g}) = \mathrm{EP}(g,D)$ and $F(R_{\lambda,g})$ is a nonempty closed and convex subset of D;

(3) $R_{\lambda,g}$ is a nonexpansive mapping, and so it is quasi-nonexpansive.

Definition 5.3 Let $h, j : D \times D \to (-\infty, +\infty)$ be two equilibrium functions and, for given $\lambda > 0$, let $R_{\lambda,h}$ and $R_{\lambda,j}$ be the resolvents of h and j (defined by (5.8)), respectively. Denote by $S = R_{\lambda,h}$, $T = R_{\lambda,j}$, $C := F(R_{\lambda,h})$, and $Q := F(R_{\lambda,j})$. Then the *equality equilibrium problem with respective to the equilibrium functions h, j, and D* is

$$\begin{aligned} (\mathrm{EEP}(h,j,D)) \quad & \text{to find } x^* \in F(R_{\lambda,h}), y^* \in F(R_{\lambda,j}) \text{ such that } h(x^*, u) \geq 0, \\ & \forall u \in D, j(y^*, v) \geq 0, \forall v \in D \text{ and } Ax^* = By^*, \end{aligned} \tag{5.9}$$

where $A, B : H \to H$ are two linear and bounded operators.

The following theorem can be obtained from Theorem 3.3 immediately.

Theorem 5.4 *Let H be a real Hilbert space, D be a nonempty and closed convex subset of H. Let G, f be the same as in Theorem* 3.3. *For given $\lambda > 0$, let h, j, $R_{\lambda,h}$, $R_{\lambda,j}$, S, T, C, Q be the same as above. Let $\{w_n\}$ be the sequence generated by $w_0 \in H \times H$*:

$$w_{n+1} = P\left[\alpha_n w_n + \beta_n f(w_n) + \gamma_n K\left(I - \lambda_n G^* G\right) w_n\right], \quad n \geq 0, \tag{5.10}$$

where $P = \left[\begin{smallmatrix} P_C \\ P_Q \end{smallmatrix}\right]$, $K = \left[\begin{smallmatrix} S \\ T \end{smallmatrix}\right]$. If the solution set Γ_4 of $\mathrm{EEP}(h,j,D)$ (5.9) *is nonempty and the following conditions are satisfied*:

(i) $\alpha_n + \beta_n + \gamma_n = 1$, *for each* $n \geq 0$;

(ii) $\lim_{n\to\infty} \beta_n = 0$, *and* $\sum_{n=0}^{\infty} \beta_n = \infty$;

(iii) $\liminf_{n\to\infty} \alpha_n \gamma_n > 0$;

(iv) $\{\lambda_n\} \subset (0, \frac{2}{L})$, *where* $L = \|G\|^2$,

then the sequence $\{w_n\}$ converges strongly to $w^* = P_{\Gamma_4} f(w^*)$, which is a solution of $\mathrm{EEP}(h, j, D)$ (5.9).

Competing interests

The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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