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△-Convergence analysis of improved Kuhfittig iterative for asymptotically nonexpansive nonself-mappings in W-hyperbolic spaces

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Abstract

Throughout this paper, we introduce a class of asymptotically nonexpansive nonself-mapping and modify the classical Kuhfittig iteration algorithm in the general setup of hyperbolic space. Also, △-convergence results are obtained under a limit condition. The results presented in the paper extend various results in the existing literature.

Keywords: asymptotically nonexpansive nonself-mapping; improved Kuhfittig iterative; hyperbolic space; nonexpansive retraction

1 Introduction

As we know, fixed point theory proposed in the setting of normed linear spaces or Banach spaces mainly depends on the linear structure of the underlying space. However, a nonlinear framework for fixed point theory is a metric space embedded with a 'convex structure'. The class of hyperbolic spaces, nonlinear in nature, is a general abstract theoretic setting with rich geometrical structures for metric fixed point theory.

In fact, a few important results of the problems in various disciplines of science being nonlinear in nature were studied only in CAT(0) space. In 1976, the concept of △-convergence in general metric spaces was coined by Lim [1]. Since then, Kirk and Panyanak [2] specialized this concept to CAT(0) spaces and proved that it is very similar to the weak convergence in the Banach space setting. Dhompongsa and Panyanak [3] and Abbas *et al.* [4] obtained △-convergence theorems for the Mann and Ishikawa iterations in the CAT(0) space setting. Moreover, Yang and Zhao [5] studied the strong and △-convergence theorems for total asymptotically nonexpansive nonself-mappings in CAT(0) spaces. As for more details of this work, one can refer to the aforementioned papers and references therein.

In recent years, hyperbolic space has attracted much attention of many authors. The study of hyperbolic spaces has been largely motivated and dominated by questions about hyperbolic groups. It should be noted that one of the main object of study is in geometric group theory. For example, Wan [6] proved some △-convergence theorems in a hyperbolic

space, in which a mixed Agarwal-O'Regan-Sahu type iterative scheme for approximating a common fixed point of totally asymptotically nonexpansive mappings was constructed.

In this paper, following the work of Yang and Wan, by introducing a class of asymptotically nonexpansive nonself-mapping, we modify a classical Kuhfittig iteration algorithm in the general setup of hyperbolic space. Under a limit condition, we also establish some $\triangle$-convergence results, which extend various results in the existing literature.

2 Preliminaries

In this paper, we work in the setting of hyperbolic spaces introduced by Kohlenbach [7], which is more restrictive than the hyperbolic space introduced in Goebel and Kirk [8] and more general than the hyperbolic space in Reich and Shafrir [9]. Concretely, (E, d, W) is called a hyperbolic space if (E, d) is a metric space and $W : E \times E \times [0, 1] \to E$ is a function satisfying

$$(1) \quad d\big(z, W(x, y, \alpha)\big) \leq \alpha d(z, x) + (1 - \alpha)d(z, y); \tag{2.1}$$

$$(2) \quad d\big(W(x, y, \alpha), W(x, y, \beta)\big) = |\alpha - \beta| d(x, y); \tag{2.2}$$

$$(3) \quad W(x, y, \alpha) = W(y, x, 1 - \alpha); \tag{2.3}$$

$$(4) \quad d\big(W(x, z, \alpha), W(y, w, \alpha)\big) \leq (1 - \alpha)d(x, y) + \alpha d(z, w) \tag{2.4}$$

for all $x, y, z, w \in E$ and $\alpha, \beta \in [0, 1]$. A nonempty subset C of a hyperbolic space E is convex if $W(x, y, \alpha) \in E$ ($\forall x, y \in E$) and $\alpha \in [0, 1]$. The class of hyperbolic spaces contains normed spaces and convex subsets thereof, the Hilbert ball equipped with the hyperbolic metric [10], Hadamard manifolds as well as CAT(0) spaces in the sense of Gromov [11].

A hyperbolic space E is *uniformly convex* if for $u, x, y \in E$, $r > 0$ and $\epsilon \in (0, 2]$, there exists $\delta \in (0, 1]$ such that

$$d\left(W\left(x, y, \frac{1}{2}\right), u\right) \leq (1 - \delta)r,$$

provided that $d(x, u) \leq r$, $d(y, u) \leq r$ and $d(x, y) \geq \epsilon r$.

A map $\eta : (0, +\infty) \times (0, 2] \to (0, 1]$ is called *modulus of uniform convexity* if $\delta = \eta(r, \epsilon)$ for given $r > 0$. Besides, η is monotone if it decreases with r, that is,

$$\eta(r_2, \epsilon) \leq \eta(r_1, \epsilon), \quad \forall r_2 \geq r_1.$$

Let C be a nonempty subset of a metric space (E, d). Recall that a mapping $T : C \to E$ is said to be nonexpansive if

$$d(Tx, Ty) \leq d(x, y), \quad \forall x, y \in C. \tag{2.5}$$

Recall that C is said to be a retraction of E if there exists a continuous map $P : E \to C$ such that $Px = x$, for all $x \in C$. A map $P : E \to C$ is said to be a retraction if $P^2 = P$. Consequently, if P is a retraction, then $Py = y$ for all y in the range of P.

Definition 2.1 ([12]) Let C be a nonempty and closed subset of a metric space (E, d), a map $P : E \to C$ is a retraction, a mapping $T : C \to E$ is said to be

(1) asymptotically nonexpansive nonself-mapping if there exists a sequence $\{k_n\} \subset [0, +\infty)$ with $k_n \to 1$ such that

$$d\big(T(PT)^{n-1}x, T(PT)^{n-1}y\big) \le k_n d(x, y), \quad \forall x, y \in C, n \ge 1; \tag{2.6}$$

(2) totally asymptotically nonexpansive nonself-mapping if there exist nonnegative sequences $\{\mu_n\}$, $\{\nu_n\}$ with $\mu_n \to 0$, $\nu_n \to 0$ and a strictly continuous function $\zeta : [0, +\infty) \to [0, +\infty)$ with $\zeta(0) = 0$ such that

$$d\big(T(PT)^{n-1}x, T(PT)^{n-1}y\big) \le d(x, y) + \nu_n \zeta\big(d(x, y)\big) + \mu_n, \quad \forall x, y \in C, n \ge 1; \tag{2.7}$$

(3) uniformly L-Lipschitzian if there exists a constant $L > 0$ such that

$$d\big(T(PT)^{n-1}x, T(PT)^{n-1}y\big) \le L d(x, y), \quad \forall x, y \in C, n \ge 1. \tag{2.8}$$

Remark 2.1 From the definitions above, we know that each nonexpansive mapping is an asymptotically nonexpansive nonself-mapping, and each asymptotically nonexpansive nonself-mapping is a totally asymptotically nonexpansive nonself-mapping.

To study our results in the general setup of hyperbolic spaces, we first collect some basic concepts. Let $\{x_n\}$ be a bounded sequence in hyperbolic space E. For $p \in E$, define a continuous functional $r(\cdot, \{x_n\}) : E \to [0, +\infty)$ by

$$r\big(p, \{x_n\}\big) = \limsup_{n\to\infty} d(p, x_n).$$

The asymptotic radius $r(\{x_n\})$ of $\{x_n\}$ is given by

$$r\big(\{x_n\}\big) = \inf\big\{r\big(p, \{x_n\}\big) : p \in E\big\}.$$

The asymptotic radius $r_C(\{x_n\})$ of $\{x_n\}$ with respect to $C \subset E$ is given by

$$r_C\big(\{x_n\}\big) = \inf\big\{r\big(p, \{x_n\}\big) : p \in C\big\}.$$

The asymptotic center $A(\{x_n\})$ of $\{x_n\}$ is the set

$$A\big(\{x_n\}\big) = \big\{p \in E : r\big(p, \{x_n\}\big) = r\big(\{x_n\}\big)\big\}.$$

The asymptotic center $A_C(\{x_n\})$ of $\{x_n\}$ with respect to $C \subset E$ is the set

$$A_C\big(\{x_n\}\big) = \big\{p \in C : r\big(p, \{x_n\}\big) = r_C\big(\{x_n\}\big)\big\}.$$

A sequence $\{x_n\}$ in hyperbolic space E is said to $\triangle$-converge to $p \in E$, if p is the unique asymptotic center of $\{u_n\}$ for every subsequence $\{u_n\}$ of $\{x_n\}$. In this case, we call p the $\triangle$-limit of $\{x_n\}$.

The following lemmas are important in our paper.

Lemma 2.1 (see [13]) *Let (E, d, W) be a complete uniformly convex hyperbolic space with monotone modulus of uniform convexity, and let C be a nonempty, closed, convex subset of E. Then every bounded sequence $\{x_n\}$ in E has a unique asymptotic center with respect to C.*

Lemma 2.2 (see [13, 14]) *Let (E, d, W) be a uniformly convex hyperbolic space with monotone modulus of uniform convexity η. Let $q \in E$ and $\{\alpha_n\}$ be a sequence in $[a, b]$ for some $a, b \in (0, 1)$. If $\{x_n\}$ and $\{y_n\}$ are sequences in E such that $\limsup_{n\to\infty} d(x_n, q) \leq c$, $\limsup_{n\to\infty} d(y_n, q) \leq c$, and $\lim_{n\to\infty} d(W(x_n, y_n, \alpha_n), q) = c$ for some $c \geq 0$, then $\lim_{n\to\infty} d(x_n, y_n) = 0$.*

Lemma 2.3 (see [12]) *Let C be a nonempty, closed, convex subset of a uniformly convex hyperbolic space, and let $\{x_n\}$ be a bounded sequence in C such that $A(\{x_n\}) = \{p\}$ and $r(\{x_n\}) = \rho$. If $\{y_k\}$ is another sequence in C such that $\lim_{k\to\infty} r(y_k, \{x_n\}) = \rho$, then $\lim_{k\to\infty} y_k = p$.*

Lemma 2.4 *Let $\{\alpha_n\}$, $\{\beta_n\}$, and $\{\gamma_n\}$ be sequences of nonnegative numbers such that*

$$\alpha_{n+1} \leq (1 + \beta_n)\alpha_n + \gamma_n, \quad \forall n \geq 1.$$

If $\sum_{n=1}^{+\infty} \beta_n < +\infty$ and $\sum_{n=1}^{+\infty} \gamma_n < +\infty_n$, then $\lim_{n\to+\infty} \alpha_n$ exists. If there exists a subsequence $\{\alpha_{n_k}\} \subset \{\alpha_n\}$ such that $\lim_{k\to+\infty} \alpha_{n_k}$, then $\lim_{n\to+\infty} \alpha_n = 0$.

3 Main results

Now we modify a classical Kuhfittig iteration algorithm in the general setup of hyperbolic space, and prove a $\triangle$-convergence theorem for the following implicit iterative scheme:

$$x_n = PW\big(x_{n-1}, T(PT)^{n-1}x_n, \alpha_n\big), \quad x \in C, n \geq 1,$$

where C is a nonempty closed and convex subset of a complete uniformly hyperbolic space E, $T : C \to E$ is a uniformly L-Lipschitzian and $(\{k_n\})$-asymptotically nonexpansive nonself-mapping with $k_n \in [1, +\infty)$ and $\lim_{n\to+\infty} k_n = 1$, and $\alpha_n \in (0, 1)$ such that $\alpha_n k_n < 1$. P is nonexpansive retraction of E onto C.

Remark 3.1 For $\alpha_n \in (0, 1)$ and a fixed $u \in C$, define the mapping $K_n : C \to E$ by

$$K_n(x) = PW\big(u, T(PT)^{n-1}x, \alpha_n\big), \quad x \in C.$$

It can be seen obviously that K_n is contraction on C. Indeed, for $x, y \in C$, we have

$$
\begin{aligned}
d\big(K_n(x), K_n(y)\big) &= d\big(PW\big(u, T(PT)^{n-1}x, \alpha_n\big), PW\big(u, T(PT)^{n-1}y, \alpha_n\big)\big) \\
&\leq d\big(W(u, \alpha_n, \alpha_n), W\big(u, T(PT)^{n-1}y, \alpha_n\big)\big) \\
&\leq \alpha_n d\big(T(PT)^{n-1}x, T(PT)^{n-1}y\big) \\
&\leq \alpha_n k_n d(x, y).
\end{aligned}
$$

Therefore, K_n is a contraction mapping, that is,

$$x_n = PW\big(x_{n-1}, T(PT)^{n-1}x_n, \alpha_n\big), \quad n \geq 1,$$

is valid under the condition $\alpha_n k_n < 1$.

Theorem 3.1 *Let E be a complete hyperbolic space, C be a nonempty, bounded, closed, convex subset of E and $P : E \to C$ be the nonexpansive retraction. Let $T : C \to E$ be $\{k_n\}$-asymptotically nonexpansive nonself-mapping with sequence $\{k_n\} \subset [1, +\infty)$ and $\lim_{n \to +\infty} k_n = 1$ such that T is uniformly L-Lipschitz continuous, satisfying the following conditions*:

(i) $\sum_{n=1}^{+\infty}(k_n - 1) < +\infty$;

(ii) $\alpha_n k_n < 1$ *for all* $n \geq 1$;

(iii) *there exist constants* $a, b \in (0,1)$ *with* $0 < b(1-a) < \frac{1}{2}$ *such that* $\{\alpha_n\} \subset [a, b]$.

Define $\{x_n\}$ *as follows*: $x_0 \in C$,

$$x_n = PW\big(x_{n-1}, T(PT)^{n-1}x_n, \alpha_n\big), \quad n \geq 1, \tag{3.1}$$

and $F(T) \neq \emptyset$. *Then the sequence* $\{x_n\}$ $\triangle$-*converges to a point* $p^* \in F(T)$.

Proof (I) First, we prove that $\lim_{n \to \infty} d(x_n, p)$ $(\forall p \in F(T))$ and $\lim_{n \to \infty} d(x_n, F(T))$ exist, respectively.

Since $T : C \to E$ is $\{k_n\}$-asymptotically nonexpansive nonself-mapping with sequence $\{k_n\} \subset [1, +\infty)$ and $\lim_{n \to +\infty} k_n = 1$, from Definition 2.1(1), for any $x, y \in C$, we have

$$d\big(T(PT)^{n-1}x, T(PT)^{n-1}x\big) \leq k_n d(x, y), \quad \forall n \geq 1. \tag{3.2}$$

For each $p \in F(T)$, it follows from (3.1) and (2.1) that

$$\begin{aligned}
d(x_n, p) &= d\big(PW\big(x_{n-1}, T(PT)^{n-1}x_n, \alpha_n\big), p\big) \\
&\leq d\big(W\big(x_{n-1}, T(PT)^{n-1}x_n, \alpha_n\big), p\big) \\
&\leq (1 - \alpha_n)d(x_{n-1}, p) + \alpha_n d\big(T(PT)^{n-1}x_n, p\big) \\
&\leq (1 - \alpha_n)d(x_{n-1}, p) + \alpha_n k_n d(x_n, p), \tag{3.3}
\end{aligned}$$

which indicates

$$d(x_n, p) \leq \left(1 + \frac{\alpha_n(k_n - 1)}{1 - \alpha_n k_n}\right)d(x_{n-1}, p).$$

From condition (iii), we have

$$d(x_n, p) \leq \left(1 + \frac{b(k_n - 1)}{1 - bk_n}\right)d(x_{n-1}, p).$$

Since $\lim_{n \to \infty}(1 - bk_n) = 1 - b$, there exists an integer n_0 such that $1 - bk_n \geq \frac{1-b}{2}$ for all $n > n_0$. Hence we have

$$d(x_n, p) \leq (1 + \delta_n)d(x_{n-1}, p), \quad \forall n > n_0, \tag{3.4}$$

and so

$$d(x_n, F(T)) \leq (1 + \delta_n) d(x_{n-1}, F(T)), \quad \forall n > n_0, \tag{3.5}$$

where $\delta_n := \frac{2b(k_n - 1)}{1 - b}$. By condition (i), we get $\sum_{n=1}^{+\infty} \delta_n < +\infty$. Therefore, from Lemma 2.4, $\lim_{n \to \infty} d(x_n, p)$ ($\forall p \in F(T)$) and $\lim_{n \to \infty} d(x_n, F(T))$ exist.

(II) Next, we prove that $d(x_n, Tx_n) \to 0$ (as $n \to \infty$).

For $p \in F(T)$, according to the proof of (I), we know that $\lim_{n \to \infty} d(x_n, p)$ exists. Assume that

$$\lim_{n \to \infty} d(x_n, p) = r \geq 0. \tag{3.6}$$

From (3.4) and (3.6), we get

$$r = \lim_{n \to \infty} d(x_n, p)$$
$$\leq \lim_{n \to \infty} d\big(W(x_{n-1}, T(PT)^{n-1} x_n, \alpha_n), p\big)$$
$$\leq \lim_{n \to \infty} \big((1 + \delta_n) d(x_{n-1}, p)\big) = r,$$

which implies that

$$\lim_{n \to \infty} d\big(W(x_{n-1}, T(PT)^{n-1} x_n, \alpha_n), p\big) = r. \tag{3.7}$$

In addition, since

$$d\big(T(PT)^{n-1} x_n, p\big) \leq k_n d(x_n, p), \quad \forall n \geq 1,$$

from (3.6), we have

$$\limsup_{n \to \infty} d\big(T(PT)^{n-1} x_n, p\big) \leq r. \tag{3.8}$$

It follows from (3.6)-(3.8) and Lemma 2.2 that

$$\lim_{n \to \infty} d\big(x_{n-1}, T(PT)^{n-1} x_n\big) = 0. \tag{3.9}$$

We obtain

$$d(x_n, x_{n-1}) = d\big(PW(x_{n-1}, T(PT)^{n-1} x_n, \alpha_n), x_{n-1}\big)$$
$$\leq \alpha_n d\big(T(PT)^{n-1} x_n, x_{n-1}\big) \to 0 \quad (\text{as } n \to \infty). \tag{3.10}$$

Hence, from (3.9) and (3.10), we get

$$\lim_{n \to \infty} d\big(x_n, T(PT)^{n-1} x_n\big) = 0. \tag{3.11}$$

Since T is uniformly L-Lipschitzian, we have

$$
\begin{aligned}
d(x_n, Tx_n) &\leq d(x_n, x_{n+1}) + d\big(x_{n+1}, T(PT)^n x_{n+1}\big) \\
&\quad + d\big(T(PT)^n x_{n+1}, T(PT)^n x_n\big) + d\big(T(PT)^n x_n, Tx_n\big) \\
&\leq (1+L)d(x_n, x_{n+1}) + d\big(x_{n+1}, T(PT)^n x_{n+1}\big) + d\big(T(PT)^n x_n, Tx_n\big) \\
&\leq (1+L)d(x_n, x_{n+1}) + d\big(x_{n+1}, T(PT)^n x_{n+1}\big) + Ld\big((PT)^n x_n, x_n\big) \\
&\leq (1+L)d(x_n, x_{n+1}) + d\big(x_{n+1}, T(PT)^n x_{n+1}\big) + Ld\big(T(PT)^{n-1} x_n, x_n\big).
\end{aligned}
\tag{3.12}
$$

It follows from (3.11) and (3.12) that

$$
\lim_{n \to \infty} d(x_n, Tx_n) = 0.
$$

(III) Now we prove that $\{x_n\}$ $\triangle$-converges to a point $p^* F(T)$.

Since $\{x_n\}$ is bounded, by Lemma 2.1, it has a unique asymptotic center $A_C(\{x_n\}) = \{p^*\}$. If $\{u_n\}$ is any subsequence of $\{x_n\}$ with $A_C(\{u_n\}) = \{q\}$, then, from (3.12), we have

$$
\lim_{n \to \infty} d(u_n, Tu_n) = 0.
\tag{3.13}
$$

We claim that $q \in F(T)$. In fact, for any $m, n \geq 1$,

$$
\begin{aligned}
d\big(T(PT)^{m-1} q, u_n\big) &\leq d\big(T(PT)^{m-1} q, T(PT)^{m-1} u_n\big) \\
&\quad + d\big(T(PT)^{m-1} u_n, T(PT)^{m-2} u_n\big) + \cdots + d(Tu_n, u_n) \\
&\leq k_m d(q, u_n) + Ld(Tu_n, u_n) + \cdots + d(Tu_n, u_n).
\end{aligned}
$$

From (3.13), we get

$$
\limsup_{n \to \infty} d\big(T(PT)^{m-1} q, u_n\big) \leq \limsup_{n \to \infty} k_m d(q, u_n) = k_m r\big(q, \{u_n\}\big),
$$

and so

$$
\limsup_{m \to \infty} d\big(T(PT)^{m-1} q, u_n\big) \leq r\big(q, \{u_n\}\big).
$$

By the definition of the asymptotic center $A_c\{u_n\}$ of a bounded sequence $\{u_n\}$ with respect C, we have

$$
r\big(q, \{u_n\}\big) \leq r\big(x, \{u_n\}\big), \quad \forall x \in C.
$$

This implies that

$$
\liminf_{m \to \infty} r\big(T(PT)^{m-1} q, \{u_n\}\big) \leq r\big(q, \{u_n\}\big).
$$

Therefore, we have

$$
\lim_{m \to \infty} r\big(T(PT)^{m-1} q, \{u_n\}\big) = r\big(q, \{u_n\}\big).
$$

By Lemma 2.3, one shows that $\lim_{m\to\infty} T(PT)^{m-1}q = q$. Because T is uniformly continuous, we have

$$Tq = T \lim_{m\to\infty} T(PT)^{m-1}q = T \lim_{m\to\infty} PT(PT)^{m-1}q = \lim_{m\to\infty} T(PT)^{m}q = q.$$

Consequently, $q \in F(T)$. By the uniqueness of asymptotic centers, we get $p^* = q$. It implies that p^* is the unique asymptotic of $\{u_n\}$ for each subsequence $\{u_n\} \subset \{x_n\}$, that is, $\{x_n\}$ $\triangle$-converges to a point $p^* \in F(T)$. The proof of Theorem 3.1 is completed. □

From Remark 2.1, we have the following result.

Corollary 3.1 *Let E be a complete hyperbolic space, C be a nonempty, bounded, closed, convex subset of E, and $P : E \to D$ be the nonexpansive retraction. Let $T : C \to E$ be nonexpansive nonself-mapping such that T be uniformly L-Lipschitz continuous. Define $\{x_n\}$ as follows: $x_0 \in C$,*

$$x_n = PW\big(x_{n-1}, T(TP)^{n-1}x_n, \alpha_n\big), \quad n \geq 1. \tag{3.14}$$

If there exist constants $a, b \in (0,1)$ with $0 < b(1-a) < \frac{1}{2}$ such that $\{\alpha_n\} \subset [a, b]$, and $F(T) \neq \emptyset$, then the sequence $\{x_n\}$ $\triangle$-converges to a point $p^* \in F(T)$.

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