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Mazur-Ulam theorem under weaker conditions in the framework of 2-fuzzy 2-normed linear spaces

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Abstract

The purpose of this paper is to prove that every 2-isometry without any other conditions from a fuzzy 2-normed linear space to another fuzzy 2-normed linear space is affine, and to give a new result of the Mazur-Ulam theorem for 2-isometry in the framework of 2-fuzzy 2-normed linear spaces.

MSC: 03E72; 46B20; 51M25; 46B04; 46S40

Keywords: α-2-norm; 2-fuzzy 2-normed linear spaces; 2-isometry; Mazur-Ulam theorem

1 Introduction

A satisfactory theory of 2-norm and n-norm on a linear space has been introduced and developed by Gähler in [1, 2]. Freese and Cho [3] gave some isometry conditions in linear 2-normed spaces. Raja and Vaezpour [4] introduced the notion of 2-normed hyperset in a hypervector and also constructed some special 2-normed hypersets of strong homomorphisms over hypervector spaces. Different authors introduced the definitions of fuzzy norms on a linear space. For reference, one may see [5]. Following Cheng and Mordeson [6], Bag and Samanta [7] introduced the concept of fuzzy norm on a linear space.

Somasundaram and Beaula [8] introduced the concept of 2-fuzzy 2-normed linear space or fuzzy 2-normed linear space of the set of all fuzzy sets of a set. They gave the notion of α-2-norm on a linear space corresponding to a 2-fuzzy 2-norm with the help of [7] and also gave some fundamental properties of this space.

Let X and Y be metric spaces. A mapping $f : X \to Y$ is called an isometry if f satisfies $d_Y(f(x), f(y)) = d_X(x, y)$ for all $x, y \in X$, where $d_X(\cdot,\cdot)$ and $d_Y(\cdot,\cdot)$ denote the metrics in the spaces X and Y, respectively. Two metric spaces X and Y are defined to be isometric if there exists an isometry of X onto Y. In 1932, Mazur and Ulam [9] proved the following theorem.

Mazur-Ulam theorem *Every isometry of a real normed linear space onto a real normed linear space is a linear mapping up to translation.*

Baker [10] showed that an isometry from a real normed linear space into a strictly convex real normed linear space is affine. Also, Jian [11] investigated the generalizations of the Mazur-Ulam theorem in F^*-spaces. Th.M. Rassias and Wagner [12] described all volume preserving mappings from a real finite dimensional vector space into itself and Väisälä [13] gave a short and simple proof of the Mazur-Ulam theorem. Chu [14] proved that the Mazur-Ulam theorem holds when X is a linear 2-normed space. Chu *et al.* [15] generalized the Mazur-Ulam theorem when X is a linear n-normed space, that is, the Mazur-Ulam theorem holds, when the n-isometry mapped to a linear n-normed space is affine. They also obtained extensions of Th.M. Rassias and Šemrl's theorem [16]. The Mazur-Ulam theorem has been extensively studied by many authors in different aspects (see [12, 17–20]).

Recently, Cho *et al.* [21] investigated the Mazur-Ulam theorem on probabilistic 2-normed spaces. Moslehian and Sadeghi [22] investigated the Mazur-Ulam theorem in non-Archimedean spaces. Choy and Ku [23] proved that the barycenter of a triangle carries the barycenter of a corresponding triangle. They showed the Mazur-Ulam problem on non-Archimedean 2-normed spaces using the above statement. Chen and Song [24] introduced the concept of weak n-isometry, and then they got that under some conditions a weak n-isometry is also an n-isometry. Alaca [25] gave the concepts of 2-isometry, collinearity, 2-Lipschitz mapping in 2-fuzzy 2-normed linear spaces. Also, he gave a new generalization of the Mazur-Ulam theorem when X is a 2-fuzzy 2-normed linear space or $\Im(X)$ is a fuzzy 2-normed linear space. Park and Alaca [26] introduced the concept of 2-fuzzy n-normed linear space or fuzzy n-normed linear space of the set of all fuzzy sets of a non-empty set. They defined the concepts of n-isometry, n-collinearity, n-Lipschitz mapping in this space. Also, they generalized the Mazur-Ulam theorem, that is, when X is a 2-fuzzy n-normed linear space or $\Im(X)$ is a fuzzy n-normed linear space, the Mazur-Ulam theorem holds. Moreover, it is shown that each n-isometry in 2-fuzzy n-normed linear spaces is affine. Ren [27] showed that every generalized area n preserving mapping between real 2-normed linear spaces X and Y which is strictly convex is affine under some conditions.

In the present paper, we give a new version of Mazur-Ulam theorem with a new method when X is a 2-fuzzy 2-normed linear space or $\Im(X)$ is a fuzzy 2-normed linear space.

2 On 2-fuzzy 2-normed linear spaces

In this section, at first we give the concept of linear 2-normed space and later the concept of 2-fuzzy 2-normed linear space and its fundamental properties with help of [8]. For more details, we refer the readers to [7, 8, 28, 29].

Definition 2.1 [28] Let X be a real vector space of dimension greater than 1 and let $\|\bullet,\bullet\|$ be a real-valued function on $X \times X$ satisfying the following four properties:

(1) $\|x,y\| = 0$ if and only if x and y are linearly dependent,

(2) $\|x,y\| = \|y,x\|$,

(3) $\|x,\alpha y\| = |\alpha|\|x,y\|$ for any $\alpha \in \mathbb{R}$,

(4) $\|x,y+z\| \le \|x,y\| + \|x,z\|$,

$\|\bullet,\bullet\|$ is called a 2-norm on X and the pair $(X,\|\bullet,\bullet\|)$ is called a linear 2-normed space.

Definition 2.2 [7] Let X be a linear space over S (a field of real or complex numbers). A fuzzy subset N of $X \times \mathbb{R}$ ($\mathbb{R}$, the set of real numbers) is called a fuzzy norm on X if and only if:

(N1) For all $t \in \mathbb{R}$ with $t \le 0$, $N(x,t) = 0$,

(N2) For all $t \in \mathbb{R}$ with $t > 0$, $N(x,t) = 1$ if and only if $x = 0$,
(N3) For all $t \in \mathbb{R}$ with $t > 0$, $N(\lambda x, t) = N(x, \frac{t}{|\lambda|})$, if $\lambda \neq 0$, $\lambda \in S$,
(N4) For all $s, t \in \mathbb{R}$, $x, y \in X$, $N(x + y, s + t) \geq \min\{N(x,s), N(y,t)\}$,
(N5) $N(x,\cdot)$ is a non-decreasing function of $t \in \mathbb{R}$ and $\lim_{t\to\infty} N(x,t) = 1$.
Then (X,N) is called a fuzzy normed linear space or, in short, f-NLS.

Theorem 2.1 [7] *Let (X,N) be an f-NLS. Assume the condition that*
(N6) $N(x,t) > 0$ *for all* $t > 0$ *implies* $x = 0$.
Define $\|x\|_\alpha = \inf\{t : N(x,t) \geq \alpha\}$, $\alpha \in (0,1)$. *Then* $\{\| \bullet \|_\alpha : \alpha \in (0,1)\}$ *is an ascending family of norms on* X. *We call these norms* α*-norms on* X *corresponding to the fuzzy norm on* X.

Definition 2.3 Let X be any non-empty set and $\Im(X)$ be the set of all fuzzy sets on X. For $U, V \in \Im(X)$ and $\lambda \in S$ the field of real numbers, define

$$U + V = \{(x + y, \nu \wedge \mu) : (x,\nu) \in U, (y,\mu) \in V\}$$

and $\lambda U = \{(\lambda x, \nu) : (x,\nu) \in U\}$.

Definition 2.4 A fuzzy linear space $\widehat{X} = X \times (0,1]$ over the number field S, where the addition and scalar multiplication operation on X are defined by $(x,\nu) + (y,\mu) = (x + y, \nu \wedge \mu)$, $\lambda(x,\nu) = (\lambda x, \nu)$ is a fuzzy normed space if to every $(x,\nu) \in \widehat{X}$, there is associated a non-negative real number, $\|(x,\nu)\|$, called the fuzzy norm of (x,ν), in such a way that
(i) $\|(x,\nu)\| = 0$ iff $x = 0$ the zero element of X, $\nu \in (0,1]$,
(ii) $\|\lambda(x,\nu)\| = |\lambda|\|(x,\nu)\|$ for all $(x,\nu) \in \widehat{X}$ and all $\lambda \in S$,
(iii) $\|(x,\nu) + (y,\mu)\| \leq \|(x,\nu \wedge \mu)\| + \|(y,\nu \wedge \mu)\|$ for all $(x,\nu), (y,\mu) \in \widehat{X}$,
(iv) $\|(x, \bigvee_t \nu_t)\| = \bigwedge_t \|(x,\nu_t)\|$ for all $\nu_t \in (0,1]$.

Definition 2.5 [8] Let X be a non-empty and $\Im(X)$ be the set of all fuzzy sets in X. If $f \in \Im(X)$, then $f = \{(x,\mu) : x \in X$ and $\mu \in (0,1]\}$. Clearly, f is a bounded function for $|f(x)| \leq 1$. Let S be the space of real numbers, then $\Im(X)$ is a linear space over the field S where the addition and multiplication are defined by

$$f + g = \{(x,\mu) + (y,\eta)\} = \{(x + y, \mu \wedge \eta) : (x,\mu) \in f \text{ and } (y,\eta) \in g\}$$

and

$$\lambda f = \{(\lambda x, \mu) : (x,\mu) \in f\},$$

where $\lambda \in S$.

The linear space $\Im(X)$ is said to be a normed space if for every $f \in \Im(X)$, there is associated a non-negative real number $\|f\|$ called the norm of f in such a way that
(i) $\|f\| = 0$ if and only if $f = 0$. For

$$\begin{aligned} &\|f\| = 0 \\ &\iff \quad \{\|(x,\mu)\| : (x,\mu) \in f\} = 0 \\ &\iff \quad x = 0, \qquad \mu \in (0,1] \quad \iff \quad f = 0. \end{aligned}$$

(ii) $\|\lambda f\| = |\lambda|\|f\|$, $\lambda \in S$. For

$$\begin{aligned}\|\lambda f\| &= \{\|\lambda(x,\mu)\| : (x,\mu) \in f, \lambda \in S\} \\ &= \{|\lambda|\|(x,\mu)\| : (x,\mu) \in f\} = |\lambda|\|f\|.\end{aligned}$$

(iii) $\|f+g\| \le \|f\| + \|g\|$ for every $f, g \in \Im(X)$. For

$$\begin{aligned}\|f+g\| &= \{\|(x,\mu) + (y,\eta)\| : x, y \in X, \mu, \eta \in (0,1]\} \\ &= \{\|(x+y), (\mu \wedge \eta)\| : x, y \in X, \mu, \eta \in (0,1]\} \\ &= \{\|(x, \mu \wedge \eta)\| + \|(y, \mu \wedge \eta)\| : (x,\mu) \in f, (y,\eta) \in g\} \\ &= \|f\| + \|g\|.\end{aligned}$$

Then $(\Im(X), \|\bullet\|)$ is a normed linear space.

Definition 2.6 [8] A 2-fuzzy set on X is a fuzzy set on $\Im(X)$.

Definition 2.7 [8] Let $\Im(X)$ be a linear space over the real field S. A fuzzy subset N of $\Im(X) \times \Im(X) \times \mathbb{R}$ ($\mathbb{R}$, a set of real numbers) is called a 2-fuzzy 2-norm on X (or a fuzzy 2-norm on $\Im(X)$) if and only if

(2-N1) for all $t \in \mathbb{R}$ with $t \le 0$, $N(f_1, f_2, t) = 0$,
(2-N2) for all $t \in \mathbb{R}$ with $t > 0$, $N(f_1, f_2, t) = 1$ if and only if f_1 and f_2 are linearly dependent,
(2-N3) $N(f_1, f_2, t)$ is invariant under any permutation of f_1, f_2,
(2-N4) for all $t \in \mathbb{R}$ with $t > 0$, $N(f_1, \lambda f_2, t) = N(f_1, f_2, \frac{t}{|\lambda|})$, if $\lambda \neq 0$, $\lambda \in S$,
(2-N5) for all $s, t \in \mathbb{R}$,

$$N(f_1, f_2 + f_3, s + t) \ge \min\{N(f_1, f_2, s), N(f_1, f_3, t)\},$$

(2-N6) $N(f_1, f_2, \cdot) : (0, \infty) \to [0,1]$ is continuous,
(2-N7) $\lim_{t\to\infty} N(f_1, f_2, t) = 1$.

Then $(\Im(X), N)$ is a fuzzy 2-normed linear space or (X, N) is a 2-fuzzy 2-normed linear space.

Remark 2.1 In a 2-fuzzy 2-normed linear space (X, N), $N(f_1, f_2, \cdot)$ is a non-decreasing function of $\mathbb{R}$ for all $f_1, f_2 \in \Im(X)$.

Theorem 2.2 [8] *Let* $(\Im(X), N)$ *be a fuzzy* 2*-normed linear space. Assume that*

(2-N8) $N(f_1, f_2, t) > 0$ *for all* $t > 0$ *implies* f_1 *and* f_2 *are linearly dependent.*

Define $\|f_1, f_2\|_\alpha = \inf\{t : N(f_1, f_2 t) \ge \alpha, \alpha \in (0,1)\}$.

Then $\{\|\bullet, \bullet\|_\alpha : \alpha \in (0,1)\}$ *is an ascending family of* 2*-norms on* $\Im(X)$. *These* 2*-norms are called* α*-*2*-norms on* $\Im(X)$ *corresponding to the* 2*-fuzzy* 2*-norm on* X.

3 On the Mazur-Ulam theorem

Recently, Alaca [25] introduced the concept of 2-isometry which is suitable to represent the notion of area-preserving mappings in fuzzy 2-normed linear spaces as follows.

For $f, g, h \in \Im(X)$ and $\alpha, \beta \in (0,1)$, $\|f-h, g-h\|_\alpha$ is called an area of f, g and h. We call Ψ a 2-isometry if $\|f-h, g-h\|_\alpha = \|\Psi(f)-\Psi(h), \Psi(g)-\Psi(h)\|_\beta$ for all $f, g, h \in \Im(X)$ and $\alpha, \beta \in (0,1)$.

A version of the Mazur-Ulam theorem has been obtained in [25] as follows.

Theorem 3.1 [25] *Assume that $\Im(X)$ and $\Im(Y)$ are fuzzy 2-normed linear spaces. If $\Psi : \Im(X) \to \Im(Y)$ is a 2-isometry and satisfies $\Psi(f)$, $\Psi(g)$ and $\Psi(h)$ are collinear when f, g and h are collinear, then Ψ is affine.*

A natural question is whether the 2-isometry in the fuzzy 2-normed linear spaces is also affine without the condition of preserving collinearity. In this section, we find a reply to this question when X is a 2-fuzzy 2-normed linear space or $\Im(X)$ is a fuzzy 2-normed linear space.

Lemma 3.1 [25] *For all $f, g \in \Im(X)$, $\alpha \in (0,1)$ and $\lambda \in \mathbb{R}$. Then*

$$\|f, g\|_\alpha = \|f, g+\lambda f\|_\alpha.$$

Lemma 3.2 *Let $f, g, h \in \Im(X)$ and $\alpha \in (0,1)$. Then $v = \frac{f+g}{2}$ is the unique element of $\Im(X)$ satisfying*

$$\|f-h, f-v\|_\alpha = \|g-v, g-h\|_\alpha = \frac{1}{2}\|f-h, g-h\|_\alpha$$

with $\|f-h, g-h\|_\alpha \neq 0$ and $v \in \{kf+(1-k)g : k \in \mathbb{R}\}$.

Proof From Lemma 3.1, it is obvious that $v = \frac{f+g}{2}$ satisfies

$$\|f-h, f-v\|_\alpha = \|g-v, g-h\|_\alpha = \frac{1}{2}\|f-h, g-h\|_\alpha$$

with $\|f-h, g-h\|_\alpha \neq 0$ and $v \in \{kf+(1-k)g : k \in \mathbb{R}\}$.

For the uniqueness of v, assume that $u \in \Im(X)$ also satisfies

$$\|f-h, f-u\|_\alpha = \|g-u, g-h\|_\alpha = \frac{1}{2}\|f-h, g-h\|_\alpha$$

with $\|f-h, g-h\|_\alpha \neq 0$ and $u \in \{kf+(1-k)g : k \in \mathbb{R}\}$. Let $u = kf+(1-k)g$ for some $k \in \mathbb{R}$. From Lemma 3.1, we have

$$\begin{aligned}\|f-h, g-h\|_\alpha &= 2\|f-h, f-u\|_\alpha \\ &= 2\big\|f-h, f-\big(kf+(1-k)g\big)\big\|_\alpha \\ &= 2|1-k|\,\|f-h, f-g\|_\alpha \\ &= 2|1-k|\,\|f-h, g-h\|_\alpha\end{aligned}$$

and

$$\begin{aligned}\|f-h, g-h\|_\alpha &= 2\|g-h, g-u\|_\alpha \\ &= 2\big\|g-h, g-\big(kf+(1-k)g\big)\big\|_\alpha\end{aligned}$$

$$= 2|k|\|g-h, g-f\|_\alpha$$
$$= 2|k|\|f-h, g-h\|_\alpha.$$

Since $\|f-h, g-h\|_\alpha \neq 0$, we have $1 = 2|1-k| = 2|k|$. So, $k = \frac{1}{2}$ and $u = v = \frac{f+g}{2}$. □

Theorem 3.2 *Let $\Im(X)$ and $\Im(Y)$ be fuzzy 2-normed linear spaces. If $\Psi : \Im(X) \to \Im(Y)$ is a 2-isometry, then Ψ is affine.*

Proof Let $\Phi(f) = \Psi(f) - \Psi(0)$. Obviously, $\Phi(0) = 0$ and Φ is a 2-isometry. Now, we prove that Φ is linear.

Firstly, we show that Φ is additive. For $f, g, h \in \Im(X)$, $\alpha, \beta \in (0,1)$ with $\|f-h, g-h\|_\alpha \neq 0$, $\|\Phi(f)-\Phi(h), \Phi(g)-\Phi(h)\|_\beta \neq 0$ and from Lemma 3.1, we have

$$\begin{aligned}\left\|\Phi(f)-\Phi(h), \Phi(f)-\Phi\left(\frac{f+g}{2}\right)\right\|_\beta &= \left\|f-h, f-\frac{f+g}{2}\right\|_\alpha \\ &= \left\|f-h, \frac{f-g}{2}\right\|_\alpha \\ &= \frac{1}{2}\|f-h, f-g\|_\alpha \\ &= \frac{1}{2}\|f-h, g-h\|_\alpha \\ &= \frac{1}{2}\big\|\Phi(f)-\Phi(h), \Phi(g)-\Phi(h)\big\|_\beta.\end{aligned}$$

Similarly,

$$\left\|\Phi(g)-\Phi(h), \Phi(g)-\Phi\left(\frac{f+g}{2}\right)\right\|_\beta = \frac{1}{2}\big\|\Phi(f)-\Phi(h), \Phi(g)-\Phi(h)\big\|_\beta.$$

And

$$\begin{aligned}\left\|\Phi\left(\frac{f+g}{2}\right)-\Phi(g), \Phi(f)-\Phi(g)\right\|_\beta &= \left\|\frac{f+g}{2}-g, f-g\right\|_\alpha \\ &= \frac{1}{2}\|f-g, f-g\|_\alpha = 0.\end{aligned}$$

So, we get

$$\Phi\left(\frac{f+g}{2}\right) - \Phi(g) = k\big(\Phi(f)-\Phi(g)\big)$$

for some $k \in \mathbb{R}$ by Definition 2.7. That is,

$$\Phi\left(\frac{f+g}{2}\right) = k\Phi(f) + (1-k)\Phi(g).$$

Thus, from Lemma 3.2,

$$\Phi\left(\frac{f+g}{2}\right) = \frac{\Phi(f)+\Phi(g)}{2}$$

for all $f, g \in \Im(X)$.

Since $\Phi(0) = 0$, we have

$$\Phi\left(\frac{f}{2}\right) = \Phi\left(\frac{f+0}{2}\right) = \frac{\Phi(f) + \Phi(0)}{2} = \frac{\Phi(f)}{2}$$

and

$$\begin{aligned}\Phi(f+g) &= \Phi\left(\frac{2f+2g}{2}\right) = \frac{\Phi(2f)+\Phi(2g)}{2} = \frac{\Phi(2f)}{2} + \frac{\Phi(2g)}{2} \\ &= \Phi(f) + \Phi(g).\end{aligned}$$

It follows that Φ is additive.

Secondly, we show that $\Phi(rf) = r\Phi(f)$ for every $r \in \mathbb{R}$, $f \in \Im(X)$ and $\alpha, \beta \in (0,1)$. Let $r \in \mathbb{R}^+$ and $f \in \Im(X)$ and $\alpha, \beta \in (0,1)$. Since $\Phi(0) = 0$ and Φ is a 2-isometry, we have

$$\begin{aligned}\left\|\Phi(rf), \Phi(f)\right\|_\beta &= \left\|\Phi(rf) - \Phi(0), \Phi(f) - \Phi(0)\right\|_\beta \\ &= \|rf - 0, f - 0\|_\alpha \\ &= \|rf, f\|_\alpha \\ &= 0.\end{aligned}$$

So, $\Phi(rf) = s\Phi(f)$ for some $s \in \mathbb{R}$ from Definition 2.7. As $\dim \Im(X) > 1$, there exists a $g \in \Im(X)$ such that $\|f,g\|_\alpha \neq 0$. It is easy to see that

$$\begin{aligned}r\|f,g\|_\alpha = \|rf,g\|_\alpha = \left\|\Phi(rf), \Phi(g)\right\|_\beta &= \left\|s\Phi(f), \Phi(g)\right\|_\beta \\ = |s|\left\|\Phi(f), \Phi(g)\right\|_\beta &= |s|\|f,g\|_\alpha.\end{aligned}$$

So, $s = r$ or $s = -r$. If $s = -r$, then

$$\begin{aligned}|r-1|\|f,g\|_\alpha &= \left\|(r-1)f, g\right\|_\alpha = \|rf - f, g - 0\|_\alpha \\ &= \left\|\Phi(rf) - \Phi(f), \Phi(g) - \Phi(0)\right\|_\beta \\ &= \left\|-r\Phi(f) - \Phi(f), \Phi(g)\right\|_\beta \\ &= (r+1)\left\|\Phi(f), \Phi(g)\right\|_\beta \\ &= (r+1)\|f,g\|_\alpha.\end{aligned}$$

So, $|r-1| = r+1$. This is a contradiction since $r \in \mathbb{R}^+$. Thus, $\Phi(rf) = r\Phi(f)$ for every $r \in \mathbb{R}^+$, $f \in \Im(X)$ and $\alpha, \beta \in (0,1)$.

Similarly, we can prove $\Phi(rf) = r\Phi(f)$ for every $r \in \mathbb{R}^-$, $f \in \Im(X)$ and $\alpha, \beta \in (0,1)$.

Hence, we prove that Φ is linear and Ψ is affine. □

Remark 3.1 Theorem 3.1 has been substantially improved by Theorem 3.2.

Remark 3.2 It is clear that the Mazur-Ulam theorem has been proved under much weaker conditions than the main result of Alaca [25] in the framework of 2-fuzzy 2-normed linear spaces.

Open problem How can obtain some results for the Aleksandrov problem in fuzzy 2-normed linear spaces with the help of this technique?

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors conceived of the study, participated in its design and coordination, drafted the manuscript, participated in the sequence alignment, and read and approved the final manuscript.

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