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Research Article

Reliability Optimization and Importance Analysis of Circular-Consecutive k-out-of-n System

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The circular-consecutive k-out-of-n:$F(G)$ system (Cir/Con/k/n:$F(G)$ system) usually consists of n components arranged in a circle where the system fails (works) if consecutive k components fail (work). The optimization of the Cir/Con/k/n system is a typical case in the component assignment problem. In this paper, the Birnbaum importance-based genetic algorithm (BIGA), which takes the advantages of genetic algorithm and Birnbaum importance, is introduced to deal with the reliability optimization for Cir/Con/k/n system. The detailed process and property of BIGA are put forward at first. Then, some numerical experiments are implemented, whose results are compared with two classic Birnbaum importance-based search algorithms, to evaluate the effectiveness and efficiency of BIGA in Cir/Con/k/n system. Finally, three typical cases of Cir/Con/k/n systems are introduced to demonstrate the relationships among the component reliability, optimal permutation, and component importance.

1. Introduction

The component assignment problem (CAP) [1] is a kind of classic problem in the optimization of system reliability. The system is composed of n positions and n components with different reliabilities, and then the n components should be assigned into n different positions to find the optimal assignment with the maximum system reliability. So, the CAP is usually a kind of combinatorial optimization and generally NP-hard problem [2]. The optimization of consecutive k-out-of-n system (Con/k/n system) is a typical case of CAP, and its purpose is to ensure that the system reliability remains largest. Con/k/n system contains linear-consecutive k-out-of-n:$F(G)$ system and circular-consecutive k-out-of-n:$F(G)$ system.

A linear-consecutive k-out-of-n:$F(G)$ system (Lin/Con/k/n:$F(G)$ system) is an ordered sequence of n components arranged in a line such that the system fails (works) if and only if at least k consecutive components fail (work). Lin/Con/k/n systems are used in most system designs, such as telecommunication or pipeline networks, the allocation of microwave towers and street lamps, and arrangement of spacecraft relay stations [3]. For a pipeline network, the petroleum or gas is sent to other places from the origin through the pipeline, which assumes that the pumps are arranged in the equidistant spacing. Each pump has the ability to send the gas to the following k pumps. If a pump is failed, the pipeline system also can work normally unless the k consecutive pumps all failed. Actually, the pipeline system is a Lin/Con/k/n:F system which is widely used in the practical engineering.

A circular-consecutive k-out-of-n:$F(G)$ system (Cir/Con/k/n:$F(G)$ system) is an ordered sequence of n components arranged in a circle such that the system fails (works) if and only if at least k consecutive components fail (work). Cir/Con/k/n systems are used in the camera system of nuclear accelerator and computer ring network [3]. For a nuclear accelerator, k high-speed cameras are arranged around the accelerator to record the motion state of various particles. If and only if at least k cameras work properly, the complete motion of particles can be recorded successfully. Actually, the camera system of nuclear accelerator is a Cir/Con/k/n:G system.

In recent years, many researchers have tried to optimize the Con/k/n system by combining the heuristic algorithm with importance measure.

The concept of importance measure was first proposed by Birnbaum for binary systems in 1969 [4]. Then, it made substantial progress and was applied to broad engineering practice. Vesely [5] and Fussell [6] put forward the Fussell-Vesely importance based on the fault tree analysis. Nakashima and Yamato [7] raised the uncertainty importance for the component to discriminate the component which affected the system reliability significantly. Hong and Lie [8] proposed the joint importance to evaluate the effect on system reliability which is caused by the interaction of the components. Borgonovo and Apostolakis [9] designed the differential importance measure for the probabilistic safety assessment. Si et al. [10–12] introduced the integrated importance measure and do some research of integrated importance measure in various situations. Based on the integrated importance measure, Dui et al. [13] extended the integrated importance measure from unit time to system lifetime and to different life stages; then Dui et al. [14] also studied how the transition of component states affects system performance under the semi-Markov process. Si et al. [15] studied the component reliability importance with the changes of the optimal component sequence for Lin/Con/k/n systems. Liu et al. [16] proposed the generalized Griffith importance measure to evaluate the accurate contribution of the components for continuous state systems.

For the optimal assignment, Kontoleon [17] gave an iterative algorithm to calculate the optimal permutation. This algorithm could assign the component with the lowest reliability into all the positions in the system and order the component based on the Birnbaum importance and finally place the component into the system through iterative allocation. Zuo and Kuo [18] proposed two similar heuristic algorithms, called ZKA and ZKB, which could be applied into the Lin/Con/k/n systems. Then, Zhu et al. [19] put forward two new algorithms, referred to as ZKC and ZKD, to improve the ZK algorithms. Lin and Kuo [20] established a greedy algorithm (which is called LKA) based on the Birnbaum importance by assigning components one by one into the system. On the basis of LKA, Yao et al. [21] built the LK type algorithms, which included LKA, LKB, LKC, and LKD, and designed a Birnbaum importance-based two-stage approach (BITA) according to the numerical experimentation of ZK and LK algorithms. BI is one of the most widely investigated importance measures and has been applied to the CAP especially in [17–21]; therefore, this article focuses on the BI applied to Cir/Con/k/n systems for the CAP.

Evolutionary algorithm is a kind of advanced heuristic algorithm, which combines the random algorithm with local search. Problem-independent technique is used to solve various complex problems, such as genetic algorithm [22], simulated annealing algorithm [23], particle swarm optimization [24], tabu search algorithm [25], and neural net algorithm [26]. The research of combining metaheuristics with importance measure is also developed to solve CAP. Cai et al. [27] proposed an improved genetic algorithm based on heuristic method (BGA) to deal with the CAP, and the research illustrates that BGA is more effective for the systems with arbitrary reliable components. Yao et al. [28] constructed a Birnbaum importance-based genetic local search algorithm (BIGLS), which is a comprehensive genetic algorithm to reduce the solution space of the optimal solution based on the local search. When the components of CAP are less, local search could improve the accuracy and convergence speed of the algorithm, but it will take longer time. Cai et al. [29] proposed a Birnbaum importance-based genetic algorithm (BIGA) to analyze the performance of the algorithm in the Lin/Con/k/n systems, which is stable and feasible to solve the general CAP with stronger robustness.

The rest of this paper is organized as follows. In Section 2, BIGA is introduced to optimize the reliability of Cir/Con/k/n systems which can break through the limitation of local optimal solution. By comparing with BITA and BIGLS, the numerical examples are implemented to discuss the optimization results of BIGA in the small and large systems. In Section 3, three typical cases with different n, k, and p in Cir/Con/k/n system are implemented, and the relationships among the Birnbaum importance and the optimal assignment are discussed. Finally, conclusions of the research are summarized in Section 4.

2. Reliability Optimization Method for Cir/Con/k/n System

2.1. BIGA for Cir/Con/k/n System. In order to optimize the reliability of Cir/Con/k/n systems efficiently, the BIGA [29] is introduced in this section. The detailed process of BIGA is as follows, and the flowchart is shown as in Figure 1.

(1) For the optimization problem, the objective function is system reliability of Cir/Con/k/n system, and the solution is the permutation of the components with maximum reliability. The system reliability can be calculated based on the literature [30].

(2) Choose the real-number encoding method and determine the encoding space for individuals.

(3) Generate an initial parent population $p(t)$ which contains M individuals. Perform Birnbaum importance-based local search on all the M individuals, and update the initial parent population $p(t)$.

(4) For the population $p(t)$, calculate the fitness of each individual.

(5) When the generation meets the termination condition 1, which is the limit of the generation scale, the algorithm will be terminated and the optimal solution will be output. If not, go to Step (6).

(6) Selection is performed on the current population $p(t)$, and the best chromosome will be selected and saved.

(7) Measure the fitness scaling of each individual.

(8) For the current population, when the termination condition 2 is satisfied, the process will be terminated and the optimal solution will be output. The termination condition refers to the convergence degree of

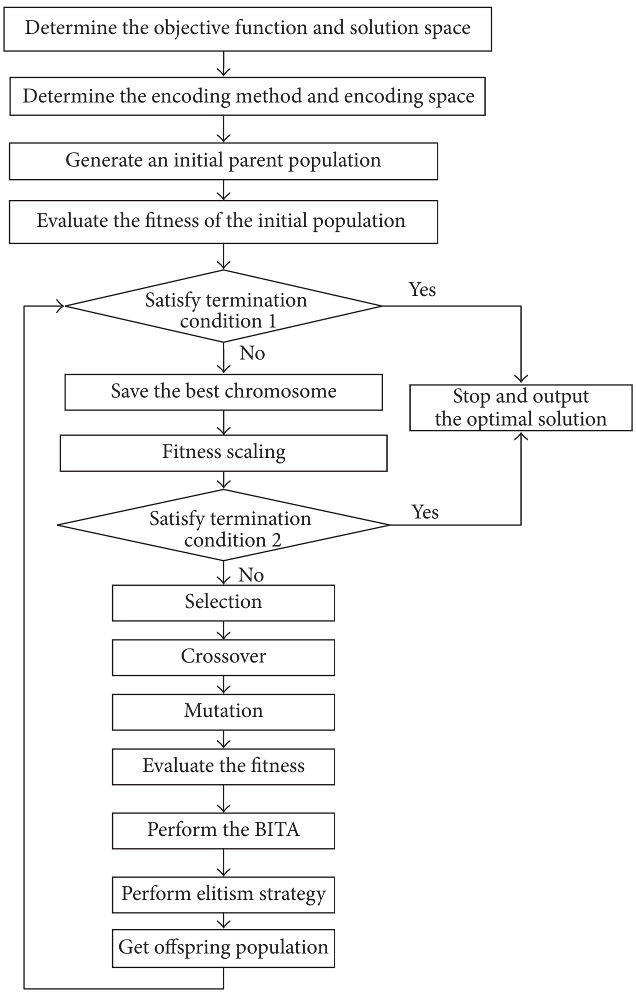


Figure 1: The flow chart of BIGA.

populations; that is, $(f_{avg} - f_{min}) \cdot S_f + (f_{max} - f_{avg}) < \alpha$, where f_{min}, f_{avg}, and f_{max} represent the minimum, average, and maximum fitness of chromosomes, respectively. $S_f > 1$ is conversion factor, and α is a very small positive number. If not, go to Step (9).

(9) For the current population $p(t)$, select M new individuals according to the fitness of each individual.

(10) Perform the crossover on the previous selected M individuals to generate offspring population which contains M individuals.

(11) Perform mutation on the previous selected offspring population to generate the new offspring population $p(t+1)$.

(12) Measure the fitness of every individual in the offspring population.

(13) Perform the BITA on the offspring chromosomes. If the fitness of individual is larger than that of the optimal chromosome, the individual will not perform the BITA.

(14) Perform the elitist strategy, replace the lowest fitness chromosome of offspring population $p(t+1)$ by the optimal chromosome, and then update the offspring population $p(t+1)$.

(15) Replace the initial population $p(t)$ by the new offspring population $p(t+1)$, and turn back to Step (5).

Table 1: The symbolic representation of small systems.

$g1$: Cir/con/2/5:G	$g2$: Cir/con/2/6:G	$g3$: Cir/con/3/5:G
$g4$: Cir/con/3/6:G	$f1$: Cir/con/3/5:F	$f1$: Cir/con/3/6:F

2.2. Numerical Experiments. System reliability depends not only on its inherent structure, but also on the reliabilities of components and the assigned positions of components in the system. The low, high, and arbitrary reliable components are introduced into the experiments to test the performance of BIGA. Their reliabilities are randomly distributed on $[0.01, 0.2]$, $[0.8, 0.99]$, and $[0.01, 0.99]$, respectively.

This paper defines that the number of components for the small system is below 8 (including 8), and the number is more than 15 for the large system. Cir/Con/k/n systems include F systems and G systems. In the experiments, the small system cases include 6 systems ($g1, g2, g3, g4, f1, f2$), and the large system cases include 12 F systems and 16 G systems. The symbols of small systems and large systems are shown in Tables 1 and 2, respectively.

Considering the fact that a single instance of algorithm may have error, thus 100 instances of one case as a test have been regarded. For each test, the reliabilities of components are randomly generated based on the type of components, which is the same in a test. Two typical experiments are introduced to compare the performance of the three algorithms, BIGA, BIGLS, and BITA, and all the three types of components should be considered in the experiments. Experiment 1 is used to compare the performance of three algorithms in small systems, and Experiment 2 is used for large systems. In all experiments, the BIGLS and BIGA are implemented with initial population size $M = 20$, the conversion factor $S_f = 3$, maximum generations $T = 200$, crossover probability $p_c = 0.8$, mutation probability $p_m = 0.05$, and $\alpha = 0.0001$.

Experiment 1. In order to verify the performance of BIGA for the CAP in small systems, the experiment is implemented to compare with three algorithms, respectively, for six small systems. In this experiment, the times of achieving the optimal solution are recorded, and the optimal solution is received by enumeration method. When the solution of algorithm is better than that of enumeration method, the value of "achieve" will be increased by 1.

Experiment 2. In order to verify the performance of BIGA for the CAP in large systems, all analysis methods are similar to Experiment 1. In the large system, the optimal solution cannot get in a reasonable time by enumeration method. In order to compare the performance of BIGA and BIGLS, the optimal solution is gained by BITA. When the solution of BIGA and BIGLS is better than that of BITA, the improved number (IN) will be increased by 1.

Table 2: The symbolic representation of large systems.

System	Mark
Cir/Con/3/20:*F*	*F*1
Cir/Con/4/20:*F*	*F*2
Cir/Con/5/20:*F*	*F*3
Cir/Con/3/30:*F*	*F*4
Cir/Con/4/30:*F*	*F*5
Cir/Con/5/30:*F*	*F*6
Cir/Con/3/50:*F*	*F*7
Cir/Con/4/50:*F*	*F*8
Cir/Con/5/50:*F*	*F*9
Cir/Con/3/100:*F*	*F*10
Cir/Con/4/100:*F*	*F*11
Cir/Con/5/100:*F*	*F*12
Cir/Con/2/20:*G*	*G*1
Cir/Con/3/20:*G*	*G*2
Cir/Con/4/20:*G*	*G*3
Cir/Con/5/20:*G*	*G*4
Cir/Con/2/30:*G*	*G*5
Cir/Con/3/30:*G*	*G*6
Cir/Con/4/30:*G*	*G*7
Cir/Con/5/30:*G*	*G*8
Cir/Con/2/50:*G*	*G*9
Cir/Con/3/50:*G*	*G*10
Cir/Con/4/50:*G*	*G*11
Cir/Con/5/50:*G*	*G*12
Cir/Con/2/100:*G*	*G*13
Cir/Con/3/100:*G*	*G*14
Cir/Con/4/100:*G*	*G*15
Cir/Con/5/100:*G*	*G*16

Table 3: Experimental results of three algorithms for small systems.

System	Algorithm	Achieve		
		Low	High	Arbitrary
*g*1	BITA	73	79	67
	BIGLS	100	95	99
	BIGA	100	96	100
*g*2	BITA	57	76	52
	BIGLS	99	87	99
	BIGA	98	89	100
*g*3	BITA	72	95	66
	BIGLS	100	98	100
	BIGA	100	100	100
*g*4	BITA	81	79	60
	BIGLS	100	100	99
	BIGA	99	100	99
*f*1	BITA	86	85	89
	BIGLS	100	98	100
	BIGA	100	100	100
*f*2	BITA	68	66	64
	BIGLS	99	100	98
	BIGA	100	100	100

2.3. Experimental Results Analysis. In this paper, all the results are implemented on MATLAB 7.11, and the computer configuration is the Intel (R) Core (TM) 2.3 GHZ processor and 2 GB of memory.

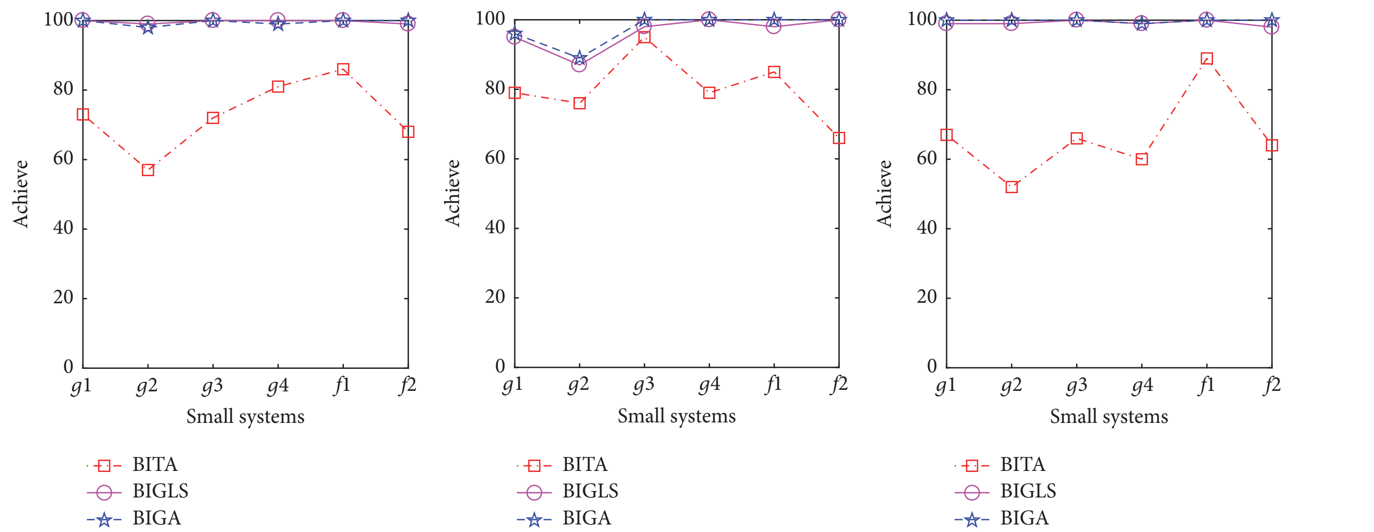
Experiment 1. The results of small systems are shown as Table 3. According to the result, we can construct Figure 2 to tell which algorithm is better. From Figure 2, it can be inferred that three algorithms can obtain the optimal solution, and the "achieve" of BIGA and BIGLS is better than that of BITA. For further analysis of Figure 2, the curve volatility of BIGLS and BIGA is stable in addition to the high reliability systems. In order to illustrate the performance of BIGA and BIGLS for different types of components, the mean achievement values of the two algorithms are calculated, and the results are shown in Figure 3. It can be clearly seen that BIGLS is slightly more efficient than BIGA in the low reliable systems, but less effective and inferior to BIGA in the high and arbitrary systems.

Experiment 2. The experimental results of the large systems are shown in Table 4. According to the IN values in Table 4, Figure 4 is obtained to illustrate the IN values of BIGLS and BIGA in the large systems. In each subgraph, the abscissa expresses the large systems, and the ordinate shows the IN values of BIGLS and BIGA. From the subgraphs (a) and (d), the IN values of two algorithms are close, and the IN value of BIGLS is worse than that of BIGA except 4 low reliable systems (*F*3, *F*7, *F*8, and *F*11) and 3 high reliable systems (*G*11, *G*12, and *G*15). Observing other subgraphs in Figure 4, the IN value of BIGA is better than that of BIGLS, which shows that BIGA can improve the performance of reliability optimization. In subgraph (d), it is not hard to find that the IN values of the two algorithms are close and small when n is more than 50. In these systems, the solution generated by the BITA is the optimal solution or approximate optimal solution, and the optimization space of BIGA and BIGLS is relatively small. Therefore, the three algorithms are all good for optimization of Cir/Con/k/n systems.

Through the typical experiments, it is clear that the BIGA can get better performance of reliability optimization than BIGLS and BITA. In the low reliable systems, sometimes BIGLS is better than BIGA, but the results of two algorithms are nearly identical. In the high or arbitrary reliable systems, the BIGA is better than BIGLS or BITA. The BIGA can improve the effectiveness and efficiency of solving the CAP.

3. Discussion of Optimal Permutation and Importance Measure

3.1. Optimal Assignment for Cir/Con/k/n System. The structure optimization problem of the system is through reassigning the positions of the n components to get the optimal system reliability. The optimal assignment also needs to reassign the n components with different reliabilities into the n different positions.



(a) Low reliable components (b) High reliable components (c) Arbitrary reliable components

FIGURE 2: The achievement of three algorithms for small systems.

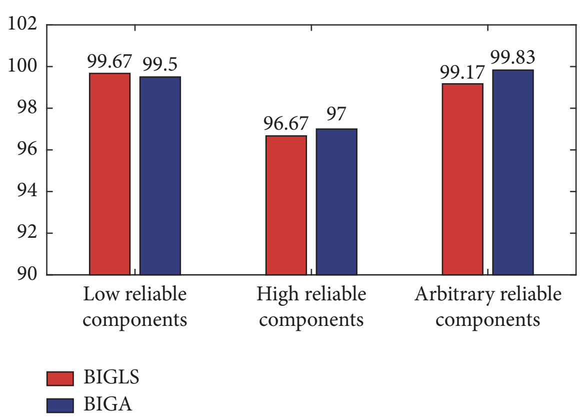


FIGURE 3: Average of achievement for BIGLS and BIGA.

Definition 1. If the sequence $\tau^* \in S$ is the optimal system structure, τ^* should satisfy the following conditions:

$$\varphi(\tau^*, p) = \max_{\tau\in S} \varphi(\tau, p). \quad (1)$$

The reliability of component $\tau(i)$ changes from $p_{\tau(i)}$ to $p^*_{\tau(i)}$, and the system reliability after changing is presented as follows:

$$\begin{aligned}\varphi(\tau_i^*, p) = {} & p^*_{\tau(i)} p_{\tau(j)} R(1_i, 1_j, p) \\ & + p^*_{\tau(i)}(1 - p_{\tau(j)}) R(1_i, 0_j, p) \\ & + (1 - p^*_{\tau(i)})(1 - p_{\tau(j)}) R(0_i, 0_j, p) \\ & + (1 - p^*_{\tau(i)}) p_{\tau(j)} R(0_i, 1_j, p).\end{aligned} \quad (2)$$

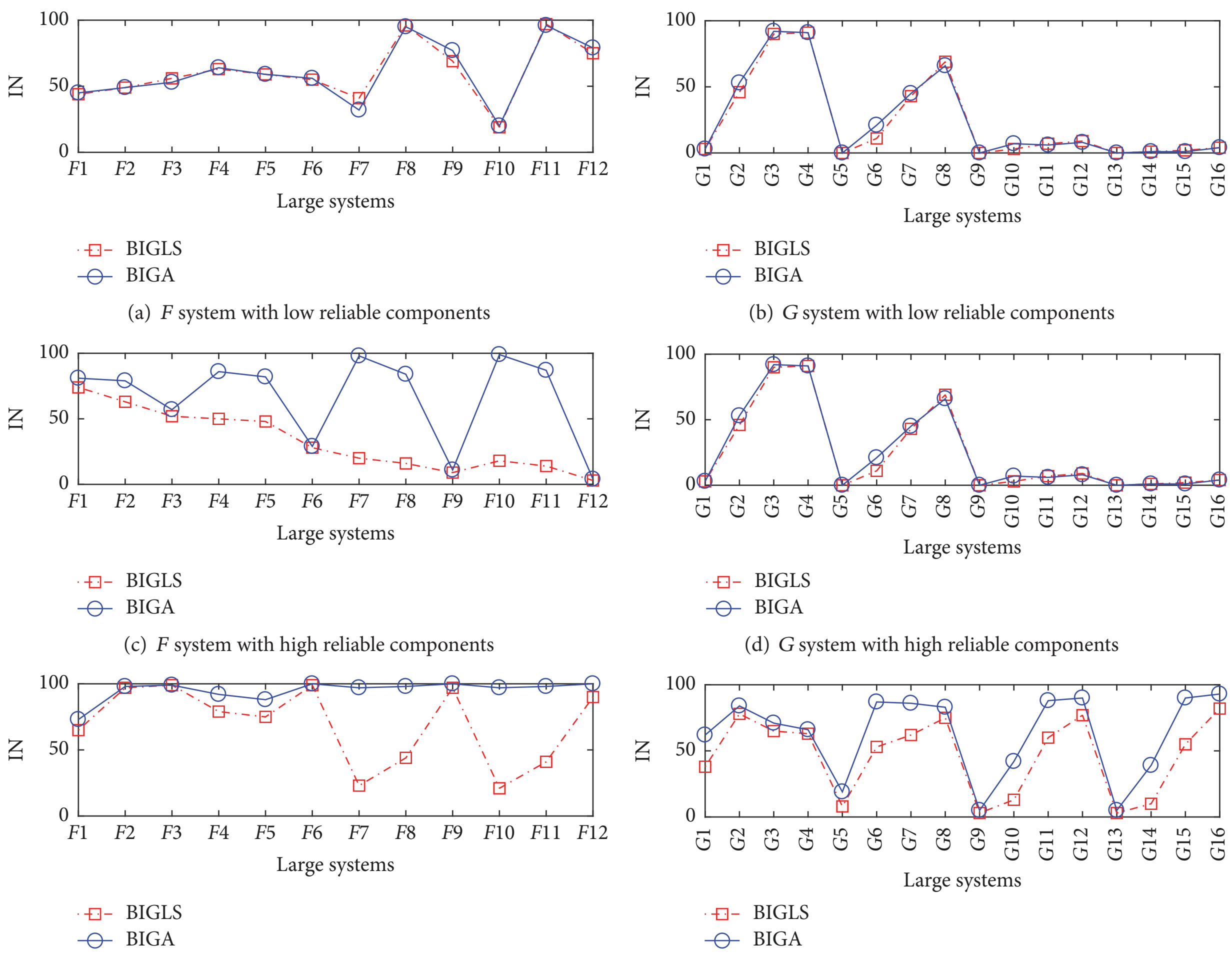
However, when the reliability of $p_{\tau(i)}$ is changed, sequence $\tau^* \in S$ may not be the optimal structure. The system reliability, after exchanging the positions i and j in the sequence τ, is as follows:

$$\begin{aligned}\varphi(\tau_{ij}^*, p) = {} & p^*_{\tau(i)} p_{\tau(j)} R(1_i, 1_j, p) \\ & + p_{\tau(j)}(1 - p^*_{\tau(i)}) R(1_i, 0_j, p) \\ & + (1 - p^*_{\tau(i)})(1 - p_{\tau(j)}) R(0_i, 0_j, p) \\ & + (1 - p_{\tau(j)}) p^*_{\tau(i)} R(0_i, 1_j, p).\end{aligned} \quad (3)$$

In order to compare the improvement of system reliability in these two methods, the difference of system can be calculated by (4), which is as follows:

$$\begin{aligned}& \varphi(\tau_i^*, p) - \varphi(\tau_{ij}^*, p) \\ & = p^*_{\tau(i)}(1 - p_{\tau(j)}) R(1_i, 0_j, p) \\ & \quad + (1 - p^*_{\tau(i)}) p_{\tau(j)} R(0_i, 1_j, p) \\ & \quad - p_{\tau(j)}(1 - p^*_{\tau(i)}) R(1_i, 0_j, p) \\ & \quad - (1 - p_{\tau(j)}) p^*_{\tau(i)} R(0_i, 1_j, p) \\ & = (p^*_{\tau(i)} - p_{\tau(j)}) R(1_i, 0_j, p) \\ & \quad - (p^*_{\tau(i)} - p_{\tau(j)}) R(0_i, 1_j, p) \\ & = (p^*_{\tau(i)} - p_{\tau(j)})(R(1_i, 0_j, p) - R(0_i, 1_j, p)).\end{aligned} \quad (4)$$

$\varphi(\tau_i^*, p) - \varphi(\tau_{ij}^*, p) < 0$ means that the method exchanging the positions of components is better than that of improving the component reliability. Thus, the components in the positions i and j in the sequence τ should be exchanged on this occasion.



(a) F system with low reliable components
(b) G system with low reliable components
(c) F system with high reliable components
(d) G system with high reliable components
(e) F system with arbitrary reliable components
(f) G system with arbitrary reliable components

Figure 4: Experimental results of BIGA and BIGLS.

The importance of component is used to measure the degree of impact that the change of component reliability has on the system reliability. Birnbaum [4] first proposed the calculation methods of the component importance

$$I_i = \frac{\partial R(p)}{\partial p_i} = R(1_i, p) - R(0_i, p). \quad (5)$$

When the reliability of component i changes from p_i to $p_i + \Delta$, the system reliability after changing the component reliability is shown as follows:

$$\begin{aligned} R((p_i + \Delta)_i, p) &= (p_i + \Delta) R(1_i, p) \\ &\quad + (1 - p_i - \Delta) R(0_i, p) \\ &= p_i R(1_i, p) + (1 - p_i) R(0_i, p) + \Delta \\ &\quad \cdot (R(1_i, p) - \Delta R(0_i, p)) \\ &= R(p) + \Delta \cdot I_i. \end{aligned} \quad (6)$$

From (6), when the difference of component reliability is Δ, the difference of system reliability will become $\Delta \cdot I_i$. When the reliability of specific component changes, the initial optimal assignment also may change.

Kuo et al. [30] presented the calculation method of the BI of component i in the Cir/Con/k/n systems. It was shown as follows:

$$I_i^G = \frac{R(n;k) - R(i-1;k) - R'(n-i;k) + R(i-1;k) R'(n-i;k)}{p_i}. \quad (7)$$

Papastavridis [31] presented the calculation of the BI of component i in the Cir/Con/k/n systems. It was shown as follows:

$$I_i^F = \frac{R(i-1;k) R'(n-i;k) - R(n;k)}{q_i}. \quad (8)$$

Zuo and Kuo [18] gave the completed invariant permutations of Cir/Con/k/n system, which is shown in Table 5. If the system reliability only depends on the order of the component reliability, and when the reliability meets the condition $p_1 < p_2 < \cdots < p_n$, the optimal assignment will be invariant. Whatever the value of component reliability is,

Table 4: Experimental results of the large systems by BIGA and BIGLS.

System	Algorithm	IN		
		Low	High	Arbitrary
$F1$	BIGLS	44	74	65
	BIGA	45	81	73
$F2$	BIGLS	49	63	97
	BIGA	49	79	98
$F3$	BIGLS	56	52	99
	BIGA	53	57	99
$F4$	BIGLS	63	50	79
	BIGA	64	86	92
$F5$	BIGLS	59	48	75
	BIGA	59	82	88
$F6$	BIGLS	55	28	99
	BIGA	56	29	100
$F7$	BIGLS	41	20	23
	BIGA	32	98	97
$F8$	BIGLS	96	16	44
	BIGA	95	84	98
$F9$	BIGLS	69	9	97
	BIGA	77	11	100
$F10$	BIGLS	19	18	21
	BIGA	20	99	97
$F11$	BIGLS	97	14	41
	BIGA	96	87	98
$F12$	BIGLS	75	3	90
	BIGA	79	4	100
G1	BIGLS	32	3	38
	BIGA	53	3	62
G2	BIGLS	23	46	78
	BIGA	32	53	84
G3	BIGLS	23	90	65
	BIGA	30	92	71
G4	BIGLS	24	91	63
	BIGA	25	91	66
G5	BIGLS	15	0	8
	BIGA	46	0	19
G6	BIGLS	7	11	53
	BIGA	22	21	87
G7	BIGLS	4	43	62
	BIGA	21	45	86
G8	BIGLS	4	69	75
	BIGA	8	66	83
G9	BIGLS	12	0	3
	BIGA	41	0	5
G10	BIGLS	1	3	13
	BIGA	9	7	42
G11	BIGLS	1	7	60
	BIGA	2	6	88
G12	BIGLS	1	9	77
	BIGA	1	8	90
G13	BIGLS	0	0	3
	BIGA	31	0	5
G14	BIGLS	0	1	10
	BIGA	7	1	39

Table 4: Continued.

System	Algorithm	IN		
		Low	High	Arbitrary
G15	BIGLS	0	2	55
	BIGA	1	1	90
G16	BIGLS	1	4	82
	BIGA	1	4	93

the permutation of component reliability is invariant when the order of component reliability is determined, and the design of the system according to Table 5 will be the optimal assignment.

If the invariant permutation exits, we can find the optimal assignment. However, if it does not exist, we can use the BIGA to find the approximate optimal assignment.

3.2. Analysis of Component Importance with Varied Reliability. In practical engineering, the components reliabilities usually degrade, respectively. So, the change of the optimal assignment and the component importance will be discussed in this condition. According to the values of component reliability, the order of the n components in the system is $p_1 < p_2 < \cdots < p_n$. The reliability of component n is 0.99, and the reliability of component i is $p_i = 0.99 - 0.025(n - i)$ when $n \leq 40$. In order to verify the changes of importance measure in the optimal assignment by the instances of Cir/Con/k/n systems, we construct three typical cases and set $n = 5$ (small system), $n = 9$ (medium system), and $n = 15$ (large system). According to Table 5, we set $k = 2, 2 < k < n/2, n/2 \leq k \leq n-2, k = n-2$ and $k = n - 1$ to analyze the changes of importance measure and the reasons of the changes.

Case 1 (Cir/Con/k/5 system). Considering all kinds of Cir/Con/k/5 systems, the value of k is 2, 3, and 4. Figure 5 represents the relationship between the importance measure of component 5 and its reliability in the optimal permutation for Cir/Con/k/5 systems, and $p_1 = 0.89$, $p_2 = 0.915$, $p_3 = 0.94$, and $p_4 = 0.965$. With decreasing of p_5, I_5^G will drop sharply at the node where the permutation of components is reassigned. Such as, for Cir/Con/3/5 G system, when $p_5 = 0.91575$, $I_5^G = p_2 p_4 p_3 + q_2 p_4 p_3 p_1 = 0.8528$, and the optimal permutation is 5-4-3-2-1-5; but when $p_5 = 0.9141$, $I_5^G = q_1 p_4 p_3 p_2 = 0.0865$, and the optimal permutation is 5-3-4-2-1-5.

For Cir/Con/k/5 systems, Figure 6 demonstrates the changes of all components' importance measure with different p_5 in the optimal permutations. When the reliability of component 5 is highest, the BI of component 5 is the largest. With decreasing of p_5, the BI of component 5 will decrease. In order to get the maximum system reliability, the position with highest BI should be assigned the component with largest reliability. For example, in the Cir/Con/2/5 F system, when $p_5 \in [0.962, 0.99)$, component 5 should be given greater priority to be assigned to position 5; when $p_5 \in [0.913, 0.962)$, component 4 should be given greater priority to be assigned to position 5; when $p_5 \in [0.889, 0.913)$, component 3 should be given greater priority to be assigned to position 5. Similarly, we can find the phenomenon in other

Table 5: The invariant permutation of Cir/Con/k/n systems.

k	F system	G system
$k = 1$	Any permutation	Any permutation
$k = 2$	$(1, n-1, 3, n-3, \ldots, n-4, 4, n-2, 2, n, 1)$	Inexistence
$2 < k < (n/2)$	Inexistence	Inexistence
$(n/2) \le k < n-2$	Inexistence	Inexistence
$k = n-2$	$(1, n-1, 3, n-3, \ldots, n-4, 4, n-2, 2, n, 1)$	$(1, 3, 5, \ldots, n, \ldots, 6, 4, 2, 1)$
$k = n-1$	Any permutation	Any permutation
$k = n$	Any permutation	Any permutation

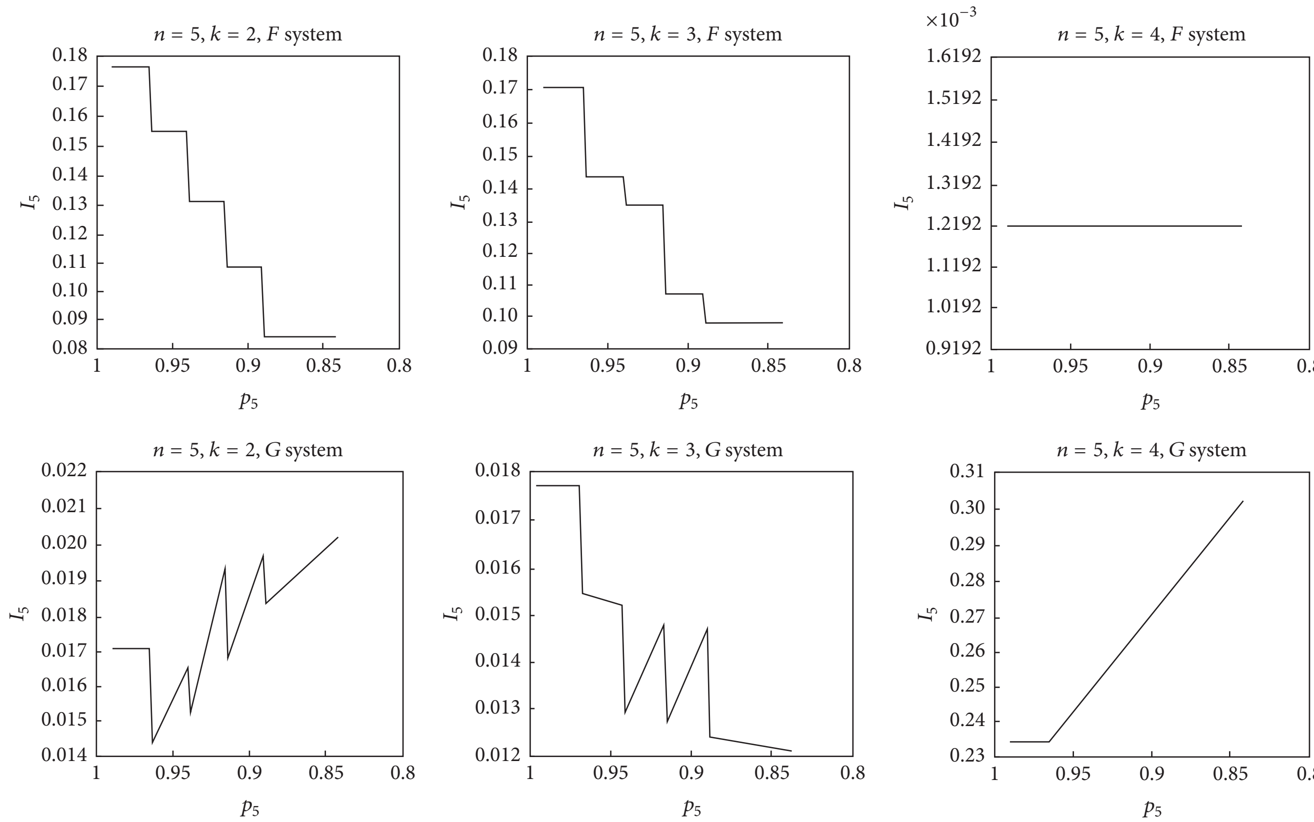


Figure 5: Changes of I_5 with different p_5 in the Cir/Con/k/5 systems.

Cir/Con/k/5 systems. Therefore, in the Cir/Con/k/5 systems, the BI of components will drop sharply at the node where the components are reassigned, which demonstrates that the changes of optimal permutation will have an effect on the BI of components.

Case 2 (Cir/Con/k/9 system). Considering all kinds of Cir/Con/k/9 systems, the value of k is 2, 3, 4, 5, 6, 7, and 8. From Figure 7, the BI of component 9 has volatility with decreasing of p_9 in the optimal permutation. In the Cir/Con/2/9 system, the volatility of I_9 in G system is more obvious than that of F system. We can learn from Table 5 that the Cir/Con/2/9:G system does not have the invariant permutation because of $2 \le k < (n/2)$; we can only find the approximate optimal assignment based on the BIGA. In order to analyze the volatility of I_9, we only study the fact that p_9 is in the interval [0.9302, 0.9500] for the Cir/Con/2/9 G system. The I_9 and the optimal permutation with different p_9 are shown in Table 6, and the changes of I_9 are shown in Figure 8. As shown in the table, when $p_9 = 0.9401$, the optimal permutation is 2-8-7-5-9-3-4-6-1-2, and $R(2875934612) - R(2874395612) = -1.8652 \times 10^{-5}$, which illustrates that 2-8-7-5-9-3-4-6-1-2 is not the optimal permutation; when $p_9 = 0.9385$, the system optimal permutation is 2-8-7-4-3-9-5-6-1-2, $R(2875934612) - R(2874395612) = 1.8964 \times 10^{-5}$, which illustrates that permutation 2-8-7-5-9-3-4-6-1-2 is superior to permutation 2-8-7-4-3-9-5-6-1-2.

In the Cir/Con/2/9 $F(G)$ system, the change of system reliability with the decrease of p_9 is shown in Figures 9 and 10, respectively. With the p_9 decreasing, the system reliability of F system and G system both decrease, but the system reliability of G system is more fluctuant. In the

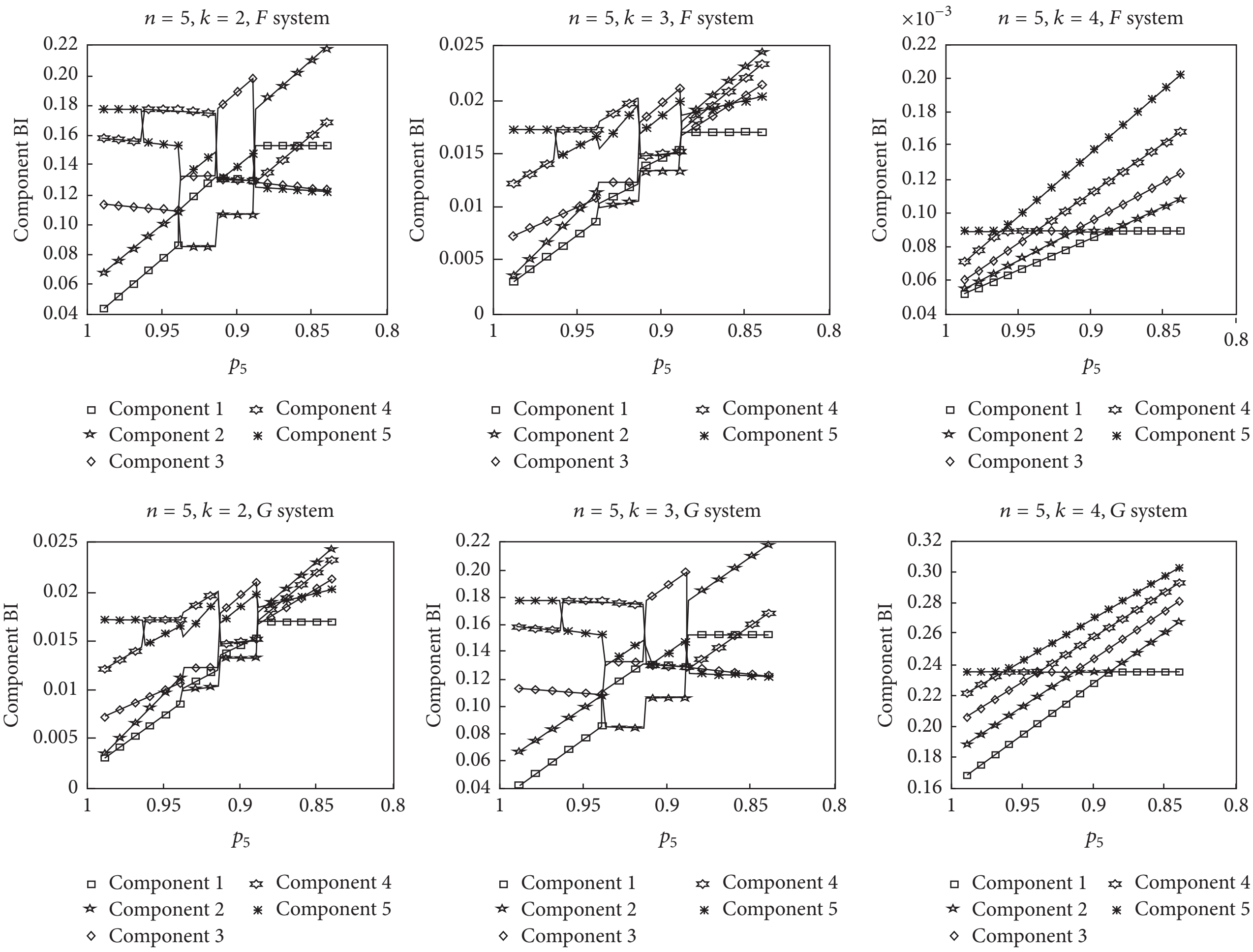


Figure 6: Changes of BI for all components with different p_5 in the Cir/Con/k/5 systems.

Table 6: Optimal permutation and I_9 with different p_9 in the Cir/Con/2/9 G system.

p_9	I_9	Optimal permutation
0.9500	0.0008900	2-8-7-5-9-3-4-6-1-2
0.9484	0.0008571	2-8-7-5-9-4-3-6-1-2
0.9467	0.0008623	2-8-7-5-9-4-3-6-1-2
0.9451	0.0008661	2-8-7-5-9-4-3-6-1-2
0.9434	0.0008709	2-8-7-4-3-9-5-6-1-2
0.9418	0.0008762	2-8-7-5-9-3-4-6-1-2
0.9401	0.0008802	2-8-7-5-9-3-4-6-1-2
0.9385	0.0008509	2-8-7-4-3-9-5-6-1-2
0.9368	0.0008693	2-8-7-5-9-3-4-6-1-2
0.9352	0.0008889	1-6-5-4-3-9-7-8-2-1
0.9335	0.0009091	1-6-5-4-3-9-7-8-2-1
0.9319	0.0008952	1-6-5-4-3-9-7-8-2-1
0.9302	0.0009128	1-6-5-4-3-9-7-8-2-1

Cir/Con/2/9 G system, the system reliability with $p_9 = 0.9401$ is lower than that of $p_9 = 0.9385$. As shown in Table 6, when $p_9 = 0.9401$, the permutation 2-8-7-5-9-3-4-6-1-2 is not the optimal assignment. Therefore, the permutation at the abnormal floating point is not necessarily the optimal assignment.

Case 3 (Cir/Con/k/15 system). Considering all kinds of Cir/Con/k/15 systems, the value of k is 2, 6, 10, 13, and 14. From Figure 11, I_{15} has some obvious fluctuation, when the value of p_{15} is in the interval $[0.75, 0.8]$ or $[0.85, 0.99]$. In order to analyze the change of I_{15} in the Cir/Con/2/15 G system clearly, we choose that p_{15} is in the intervals $[0.75, 0.8]$, $[0.85, 0.9]$, and $[0.9, 0.99]$. From Figure 12, the change of I_{15} does not show the monotonicity. The value of p_{15} is especially in the interval $[0.75, 0.8]$, the floating of I_{15} is more significant, which reflects that the invariance of the optimal permutation is not fixed.

When p_{15} is decreasing from 0.7804 to 0.7639 in the Cir/Con/2/15:G system, the I_{15} and the optimal permutation are shown in Table 7. When the system has the same optimal permutation, the change of I_{15} is very small. However, when the optimal permutation changes, the change of I_{15} will be larger. Therefore, the BI of component is influenced by the optimal permutation.

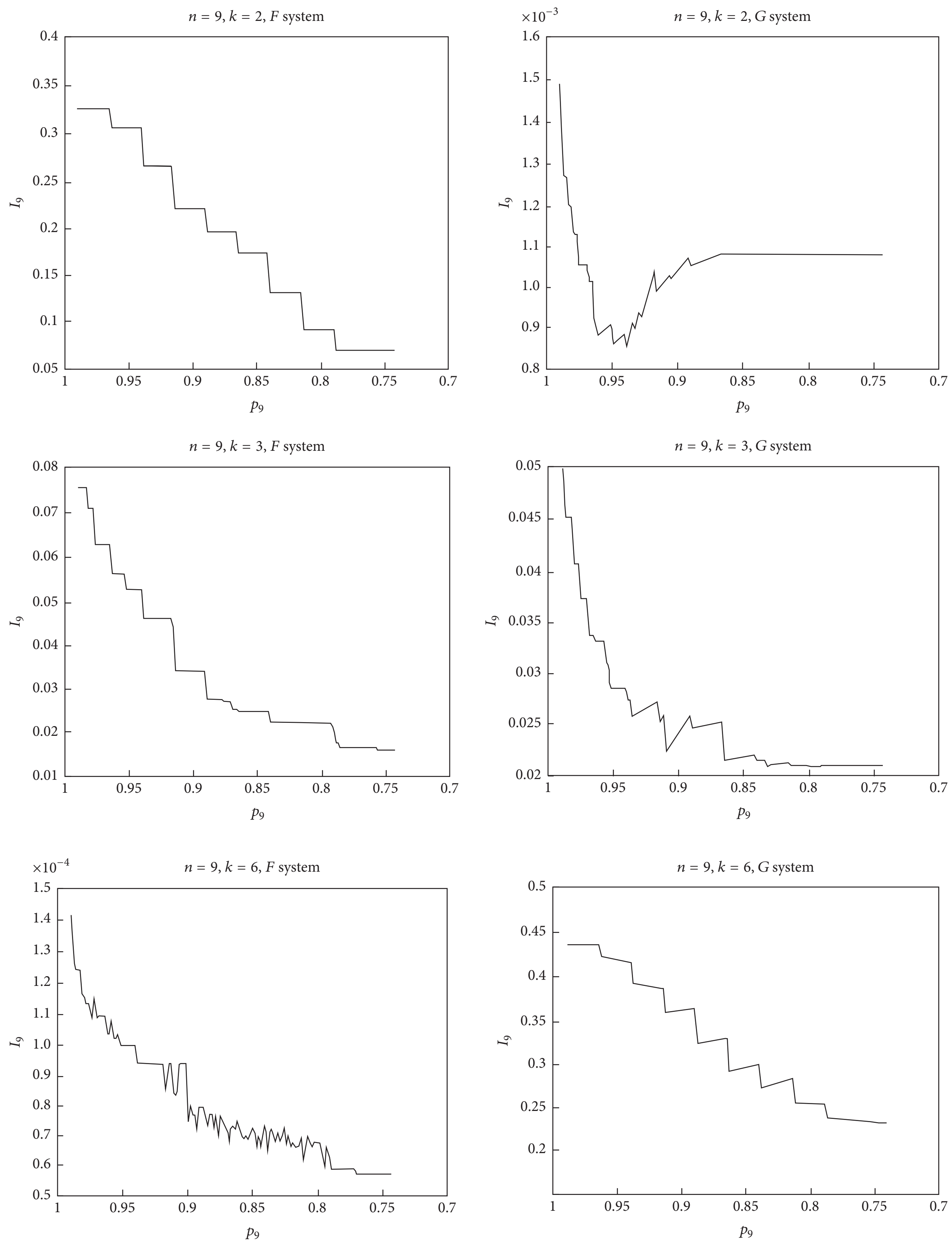


Figure 7: Continued.

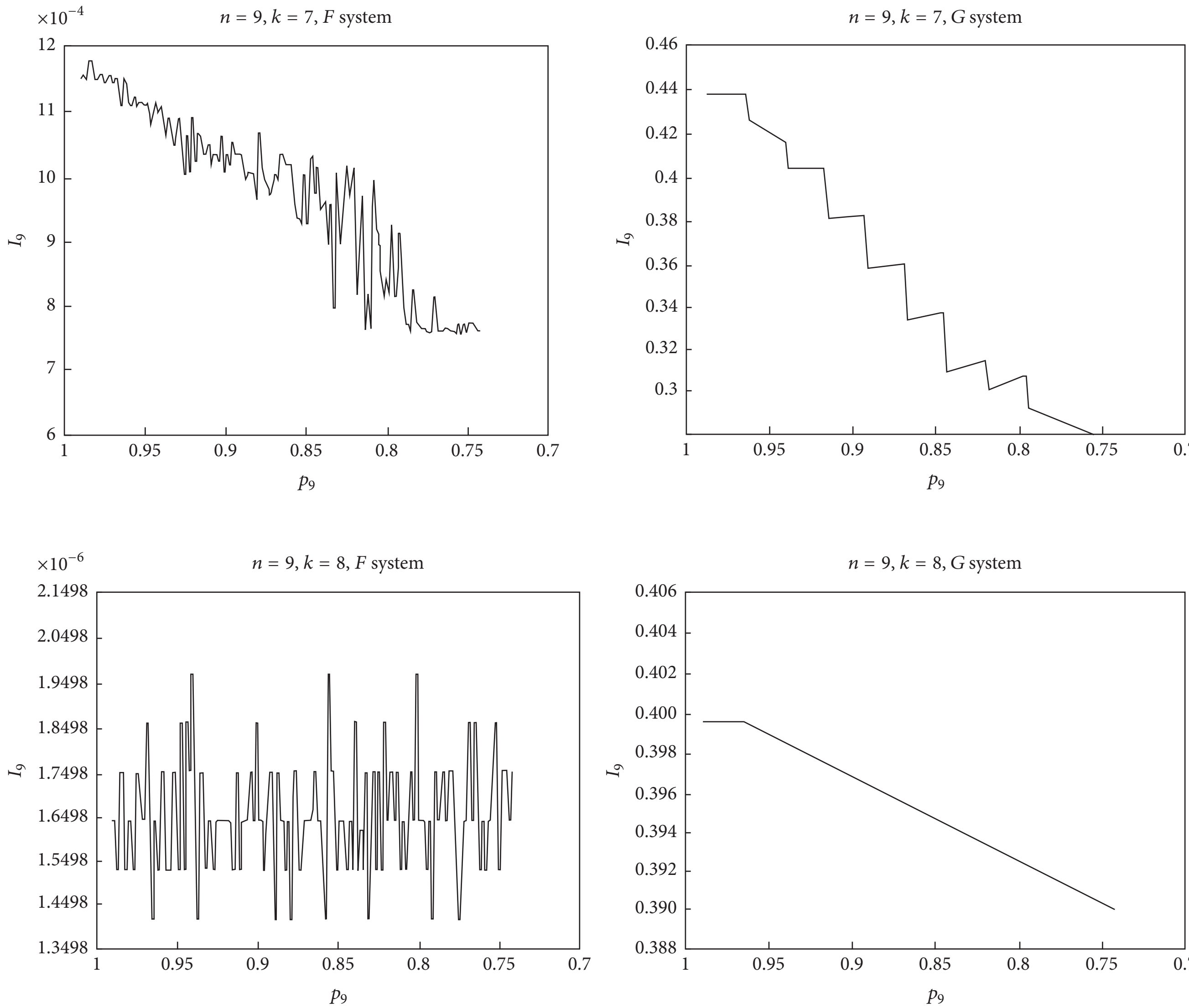


Figure 7: Changes of I_9 with different p_9 in the Cir/Con/k/9 systems.

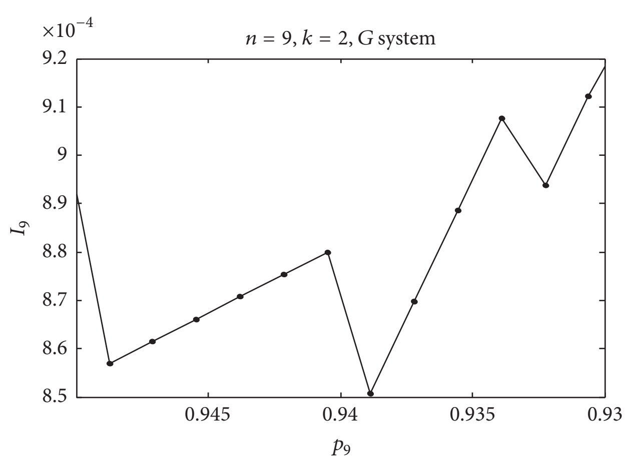


Figure 8: Changes of I_9 with p_9 in the interval [0.9302, 0.9500] for Cir/Con/2/9 G systems.

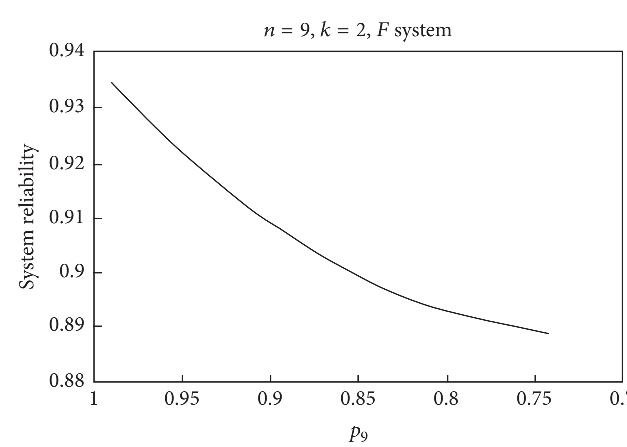


Figure 9: Changes of system reliability with different p_9 for Cir/Con/2/9 F systems.

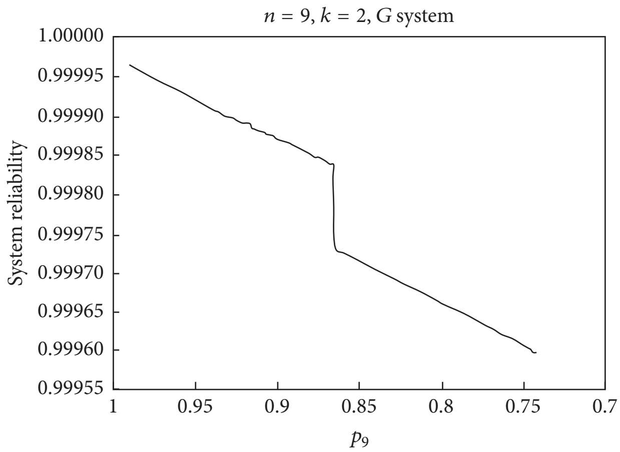


Figure 10: Changes of system reliability with different p_9 for Cir/Con/2/9 G systems.

Table 7: Optimal permutation of component 15 with different reliabilities of p_{15}.

p_{15}	Birnbaum importance	Optimal arrangement
0.7804	8.348×10^{-5}	1-5-3-4-6-15-7-8-9-10-12-14-13-11-2-1
0.7788	8.354×10^{-5}	1-5-3-4-6-15-7-8-9-10-12-14-13-11-2-1
0.7772	7.948×10^{-5}	1-10-13-14-12-8-15-5-3-4-6-7-9-11-2-1
0.7755	8.366×10^{-5}	1-5-3-4-6-15-7-8-9-10-12-14-13-11-2-1
0.7738	8.598×10^{-5}	2-10-11-13-14-12-15-7-9-8-5-3-4-6-1-2
0.7722	8.378×10^{-5}	1-5-3-4-6-15-7-8-9-10-12-14-13-11-2-1
0.7705	8.384×10^{-5}	1-5-3-4-6-15-7-8-9-10-12-14-13-11-2-1
0.7689	8.060×10^{-5}	1-12-14-13-11-10-9-15-3-4-7-6-5-8-2-1
0.7672	8.342×10^{-5}	1-8-6-15-7-5-3-4-9-11-10-12-14-13-2-1
0.7656	8.358×10^{-5}	1-8-6-4-15-10-11-13-14-12-7-3-5-9-2-1
0.7639	8.728×10^{-5}	2-11-10-9-13-14-12-15-3-4-5-6-7-8-1-2

Considering Figures 11 and 12, in the Cir/Con/2/15 system, the volatility of I_{15} for G system is clearer than that of F system. We can learn from Table 5 that the Cir/Con/2/15:G system does not have the invariant permutation because of $2 \le k < (n/2)$, and we can find the approximate optimal assignment based on the BIGA. The change of system reliability for Cir/Con/2/15 $F(G)$ system with decreasing of p_{15} is shown in Figures 13 and 14, respectively. With the p_{15} decreasing, the system reliability of F system and G system will decrease, but changes of the system reliability for G system are more fluctuant. The system reliability with $p_{15} = 0.7689$ is lower than $p_{15} = 0.7672$ in the G system. Therefore, the permutation at the abnormal floating point is not the optimal permutation.

From Table 5, there is no invariant permutation for Cir/Con/6/15:F system. So the system optimal permutation depends on the component reliability, and the I_{15} is connected with the optimal permutation. We can find that I_{15} has the larger fluctuation in Figure 11, when p_{15} is in the intervals $[0.8, 0.99]$, $[0.7, 0.75]$, and $[0.6, 0.65]$. In order to analyze the change of I_{15} obviously, we choose I_{15} in the intervals [0.95, 0.9], [0.9, 0.95], [0.85, 0.9], [0.8, 0.85], [0.7, 0.75], and [0.6, 0.65]. From Figure 15, the change of I_{15} does not show monotonicity, and I_{15} is influenced by the change of the optimal permutation.

Figure 16 demonstrates the reliability of Cir/Con/6/15 system with changing of p_{15}. The system reliability with $p_{15} = 0.7474$ is lower than that of $p_{15} = 0.7458$, which illustrates that the assignment at the floating point $p_{15} = 0.7474$ is not the optimal assignment.

With regard to the Cir/Con/k/n systems, from the above, we can get the relationship of component reliability, importance measure, and the system optimal assignment as follows:

(1) If the component reliability changes, the initial optimal permutation of system may not be the optimal.

(2) With the change of the component reliability, the optimal assignment will be changed, and the Birnbaum importance measure of component and the optimal permutation have the stronger relevance.

(3) The volatility of component Birnbaum importance measure does not show the monotonicity, and the fluctuation of importance measure illustrates the change of the optimal assignment.

4. Conclusions

This paper solves the reliability optimization problem for Cir/Con/k/n system by introducing the BIGA method, which combines the genetic algorithm with Birnbaum importance. The numerical experiments are implemented and comparison results with BIGLS verify the superiority of BIGA. By analyzing three typical cases with different n, k, and p in Cir/Con/k/n systems, the relationships among the Birnbaum importance and the optimal assignment are discussed. With the change of the component reliability, the optimal assignment will be changed, while the importance measure of component has strong relevance with the optimal permutation. The volatility of importance measure does not show the monotonicity with the decrease of component reliability, and the fluctuation of importance measure illustrates the change of the optimal assignment.

Notations

n :	The number of components in the system
p_i:	The reliability of component i $(i = 1, 2, \ldots, n)$
q_i:	The unreliability of component i $(i = 1, 2, \ldots, n)$, $p_i + q_i = 1$
p :	The vector of n components' reliabilities for a sequence, $p = (p_1, p_2, \ldots, p_n)$
Cir/Con/k/n system:	Circular-consecutive k-out-of-n system
Lin/Con/k/n system:	Linear-consecutive k-out-of-n system

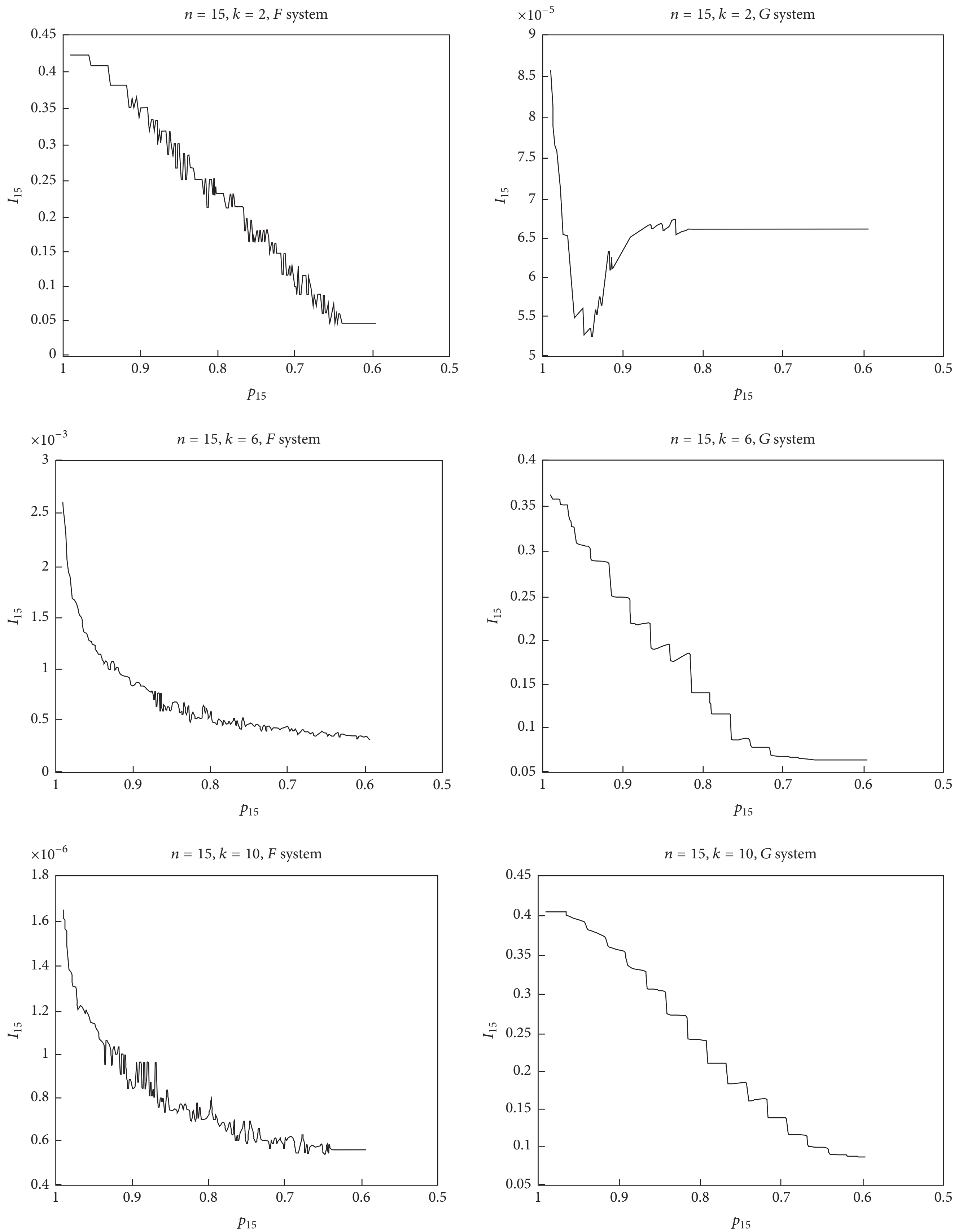


Figure 11: Continued.

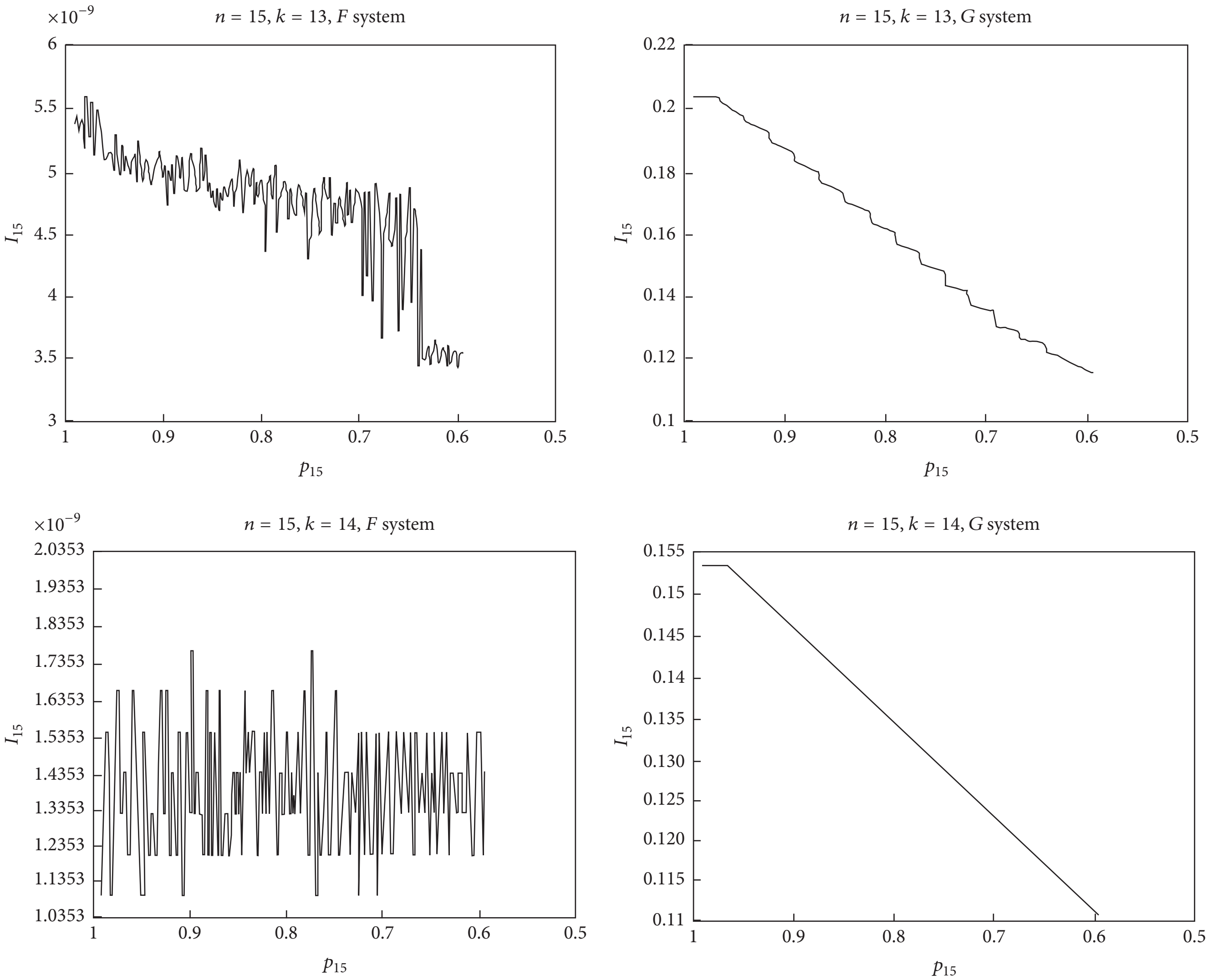


Figure 11: Changes of I_{15} with different p_{15} in the Cir/Con/k/15 system.

$p(t)$: The population of genetic algorithm when generation is t

M : The number of individuals in a population

$f_{\max}, f_{\min}, f_{\text{avg}}$: The maximum, minimum, and average fitness of chromosomes, respectively

S_f: The conversion factor of termination condition 1

α : A very small positive number

$(\cdot_i, p)$: The vector of n components' reliabilities with the special value of position i, $(p_1, \ldots, p_{i-1}, \cdot_i, p_{i+1}, \ldots, p_n)$

$(\cdot_i, \cdot_j, p)$: The vector of n components' reliabilities with the special value of position i and position j, $(p_1, \ldots, p_{i-1}, \cdot_i, p_{i+1}, \ldots, p_{j-1}, \cdot_j, p_{j+1}, \ldots, p_n)(i < j)$

τ : The sequence of n components from position 1 to position n, $\tau = (\tau(1), \tau(2), \ldots, \tau(n))$

$\varphi(\tau, p)$: The system reliability with the sequence τ

τ_{ij}: A new sequence after exchanging the position i and position j, $\tau_{ij} = (\tau(1), \ldots, \tau(i-1), \tau(j), \tau(i+1), \ldots, \tau(j-1), \tau(i), \tau(j+1), \ldots, \tau(n))$

$p_{\tau(i)}$: The reliability of component in position $\tau(i)$

S : The set of all the sequences

$R(p)$: The system reliability with the vector p

$R(\cdot_i, p)$: The system reliability of vector p if the reliability of component i is the special value

$R(n; k)$: The reliability of the Cir/Con/k/n system

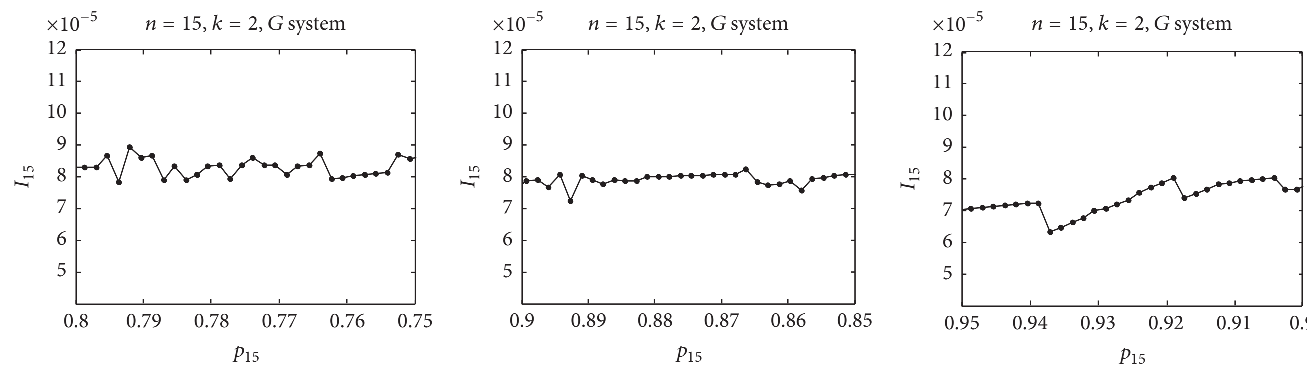


FIGURE 12: Changes of I_{15} with different p_{15} in the Cir/Con/2/15:G system.

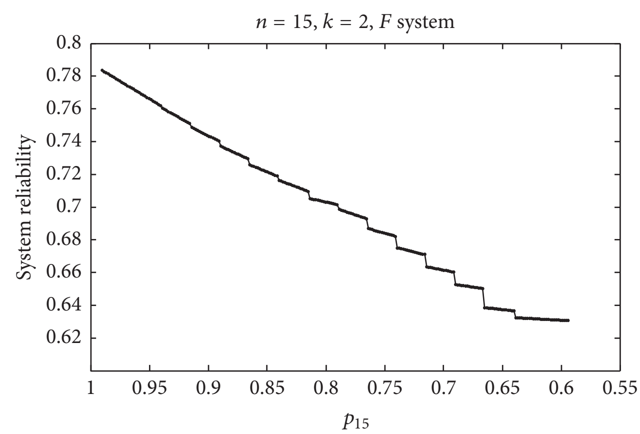


FIGURE 13: System reliability of Cir/Con/2/15:F system with different p_{15}.

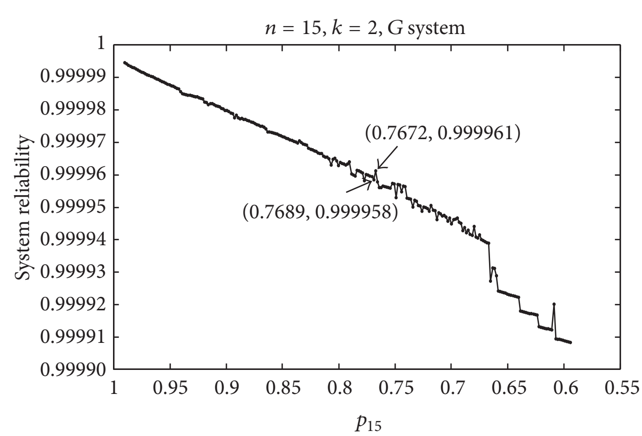


FIGURE 14: System reliability of Cir/Con/2/15:G system with different p_{15}.

$Q(n;k)$: The unreliability of the Cir/Con/k/n system

$R'(j;k)$: The reliability of the Cir/Con/k/n subsystem, which consists of component $(n-j+1, n-j+2, \ldots, n)$

I_i: The importance of component i

I_i^F: The importance of component i in the Cir/Con/k/n F system

I_i^G: The importance of component i in the Cir/Con/k/n G system.

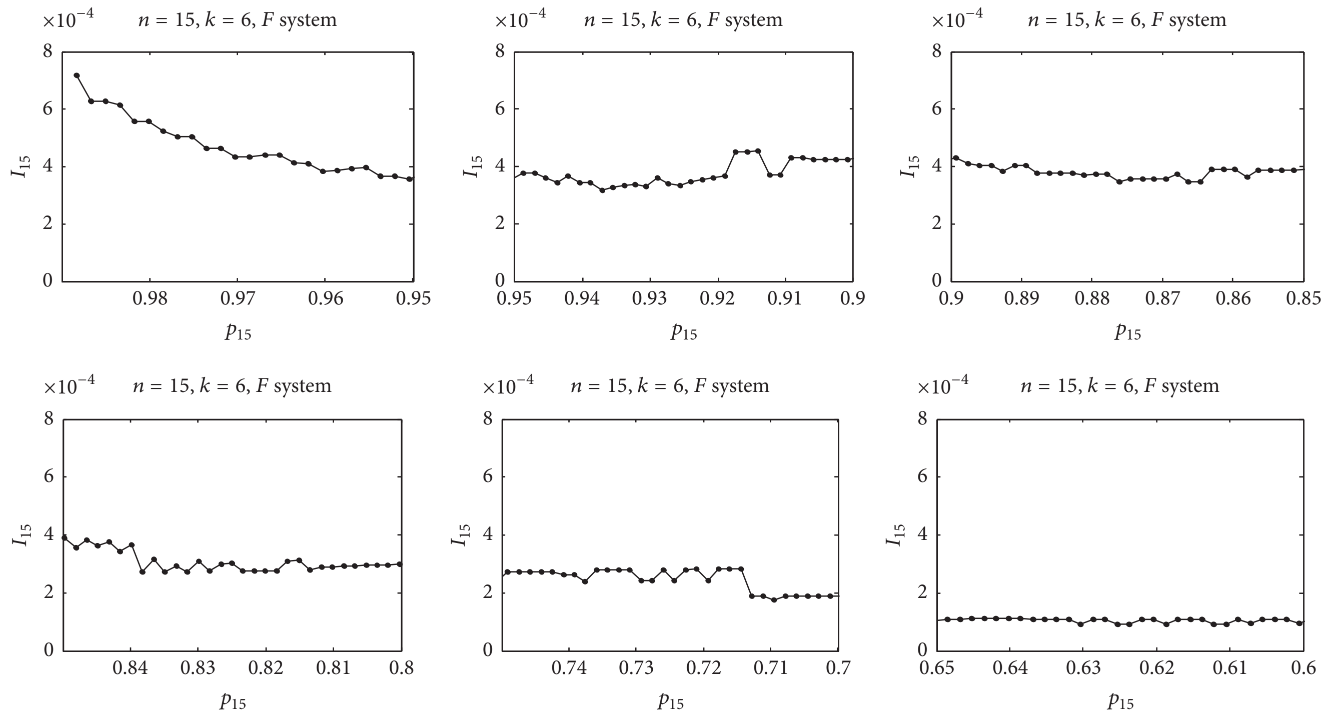


Figure 15: Changes of I_{15} with different p_{15} for the Cir/Con/6/15:F system.

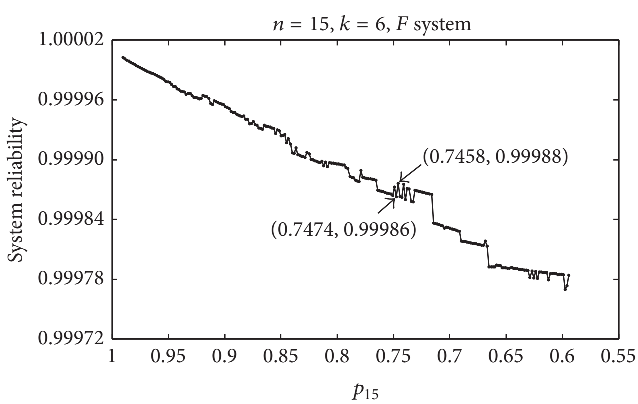


Figure 16: System reliability of Cir/Con/6/15:F system with different p_{15}.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

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