Hindawi Publishing Corporation
Journal of Applied Mathematics
Volume 2013, Article ID 391056, 9 pages
http://dx.doi.org/10.1155/2013/391056

Research Article

Global Existence and Convergence Rates for the Strong Solutions in H^2 to the 3D Chemotaxis Model

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Received 5 September 2013; Accepted 7 November 2013

Academic Editor: K. S. Govinder

We are concerned with a 3D chemotaxis model arising from biology, which is a coupled hyperbolic-parabolic system. We prove the global existence of a strong solution when H^2-norm of the initial perturbation around a constant state is sufficiently small. Moreover, if additionally, L^1-norm of the initial perturbation is bounded; the optimal convergence rates are also obtained for such a solution. The proofs are obtained by combining spectral analysis with energy methods.

1. Introduction

In this paper, we investigate global existence and optimal convergence rates of strong solutions to the following 3D chemotaxis model:

$$\begin{aligned} v_t - \nabla f(u) &= 0, \quad x \in \mathbb{R}^3,\ t > 0, \\ u_t - \nabla \cdot (uv) &= D\Delta u, \quad x \in \mathbb{R}^3,\ t > 0, \end{aligned} \tag{1}$$

with initial data

$$(v, u)(x, 0) = (v_0, u_0)(x) \longrightarrow (0, \overline{u}), \quad \text{as } |x| \longrightarrow \infty, \tag{2}$$

where $\overline{u} > 0$ is a positive constant. The system (1) is one of the models describing the chemotaxis phenomenon in biology and is closely related to the following system:

$$\begin{aligned} \frac{\partial p}{\partial t} &= D\nabla \cdot \left(p\nabla \left(\ln \frac{p}{\Phi(w)} \right) \right), \\ \frac{\partial w}{\partial t} &= \Psi(p, w), \end{aligned} \tag{3}$$

which is motivated by biological considerations and numerical computations carried out by Othmer and Stevens in [1] and Levine and Sleeman in [2]. Here, $p(x,t)$ denotes the particle density and $w(x,t)$ is the concentration of chemicals. $D > 0$ is the diffusion rate of particles. The function Φ is commonly referred to as the chemotactic potential and Ψ denotes the chemical kinetics. Depending on the specific modeling goals, the kinetic function $\Psi(p,w)$ has a wide variability. In this paper, we consider a class of nonlinear kinetic functions $\Psi(p,w)$:

$$\Psi(p, w) = \beta f(p) w, \tag{4}$$

where β is a positive constant and f is a smooth function satisfying

$$f'(u) > 0, \tag{5}$$

for all u under consideration.

In fact, as in [3, 4], let $\Phi(w) = w^{-\alpha}$ with α being a positive constant and $\Psi(p,w)$ is defined in (4); the system (3) can be rewritten in the following form:

$$\begin{aligned} p_t &= D\Delta p + D\alpha\nabla \cdot \left(p\frac{\nabla w}{w} \right), \\ w_t &= \beta f(p) w. \end{aligned} \tag{6}$$

Furthermore, by setting

$$q = \nabla (\ln w) = \frac{\nabla w}{w}, \tag{7}$$

we can rewrite the system (6) as

$$p_t = D\Delta p + D\alpha\nabla\cdot(pq),$$
$$q_t = \beta\nabla f(p). \tag{8}$$

Finally, for positive constants A, B, and c_1 to be determined below, let $\tau = At$, $\xi = Bx$, $u = p$, and $v = c_1 q$; then the system (8) becomes

$$v_\tau = \frac{\beta B c_1}{A}\nabla_\xi f(u),$$
$$u_\tau = \frac{DB^2}{A}\Delta_\xi u + \frac{D\alpha B}{Ac_1}\nabla_\xi\cdot(uv). \tag{9}$$

If we choose

$$\frac{\beta B c_1}{A} = 1, \qquad \frac{B^2}{A} = 1,$$
$$\frac{D\alpha B}{Ac_1} = 1, \tag{10}$$

that is,

$$A = D\alpha\beta > 0, \qquad B = \sqrt{D\alpha\beta} > 0, \qquad c_1 = \sqrt{\frac{D\alpha}{\beta}} > 0, \tag{11}$$

then it is easy to see that u and v satisfy

$$v_\tau - \nabla_\xi f(u) = 0,$$
$$u_\tau - \nabla_\xi\cdot(uv) = D\Delta_\xi u. \tag{12}$$

If we replace the variables (τ, ξ) by (x, t), (12) is exactly (1).

To go directly to the theme of this paper, we now only review some former results which are closely related. For the one-dimensional version of the chemotaxis model (1), the existence and asymptotic behavior of smooth solutions have been studied by several authors. When the kinetic function is linear, that is, $f(u) = \lambda u - \mu$ with $\lambda(>0)$ and $\mu(\geq 0)$ being given constants, the corresponding system reads as follows:

$$v_t - u_x = 0,$$
$$u_t - (uv)_x = u_{xx}. \tag{13}$$

The initial boundary value problem and Cauchy problem for the system (13) were considered by [3, 5] and [4], respectively. In [3], they considered the initial boundary value problem for the system (13). When $\|u_0 - 1\|_{H^2}^2 + \|v_0\|_{H^2}^2$ is sufficiently small, they proved the global existence of smooth solutions to the system (13). The authors in [5] generalized the results of [3] to the arbitrarily large initial data case. In [4], the authors obtained the global existence of smooth solutions to the Cauchy problem for the system (13) with large initial data. Recently, the authors in [6–8] extended the results of [3–5] to the nonlinear kinetic function case, respectively. For high dimensions, the global well-posedness of smooth small solution to (1) with $f(u) = u$ was investigated in [5, 9] for initial-boundary value problem and Cauchy problem, respectively. For other related results, such as nonlinear stability of waves in one dimension and so on, please refer to [6–8, 10–31] and references therein.

However, to our knowledge, so far there is no result on the optimal convergence rates of the strong solutions to the Cauchy problem (1)-(2). The main motivation of this paper is to give a positive answer to this question. In particular, we prove the global existence of a strong solution when H^2-norm of the initial perturbation around a constant state is sufficiently small. Moreover, if in addition, L^1-norm of the initial perturbation is bounded, the optimal convergence rates are also obtained for such a solution. The proofs are based on energy methods and spectral analysis which have been developed in [32–35] and references therein.

Before stating our main results, we explain the notations and conventions used throughout this paper. We denote positive constants by C. Moreover, the character "C" may differ in different places. $L^p = L^p(\mathbb{R}^3)$ $(1 \leq p \leq \infty)$ denotes the usual Lebesgue space with the norm

$$\|g\|_{L^p} = \left(\int_{\mathbb{R}^3} |g(x)|^p dx\right)^{1/p}, \quad 1 \leq p < \infty,$$
$$\|g\|_{L^\infty} = \sup_{\mathbb{R}^3} |g(x)|. \tag{14}$$

$H^l(\mathbb{R}^3)$ $(l \geq 0)$ denotes the usual lth-order Sobolev space with the norm

$$\|g\|_l = \left(\sum_{j=0}^{l} \left\|\nabla^j g\right\|^2\right)^{1/2}, \tag{15}$$

where $\|\cdot\| = \|\cdot\|_0 = \|\cdot\|_{L^2}$.

Now, we are ready to state our main results.

Theorem 1. *Assume that $\nabla\times v_0 = 0$ and $\|(v_0, u_0 - \overline{u})\|_2$ is sufficiently small; then there exists a unique globally strong solution (v, u) of the Cauchy problem (1)-(2) such that for any $t \in [0, \infty)$, satisfying*

$$v \in C^0\left(0,\infty; H^2\left(\mathbb{R}^3\right)\right) \cap C^1\left(0,\infty; H^1\left(\mathbb{R}^3\right)\right),$$
$$u - \overline{u} \in C^0\left(0,\infty; H^2\left(\mathbb{R}^3\right)\right) \cap C^1\left(0,\infty; L^2\left(\mathbb{R}^3\right)\right),$$
$$\|(v, u - \overline{u})(t)\|_2^2 + \int_0^t \left(\|\nabla v(\tau)\|_1^2 + \|\nabla u(\tau)\|_2^2\right) d\tau \leq C_1\left\|(v_0, u_0 - \overline{u})\right\|_2^2, \tag{16}$$

where C_1 is a positive constant independent of t.

Moreover, if, in addition, $\|(v_0, u_0 - \overline{u})\|_{L^1}$ is bounded, then there is a positive constant C_2 independent of t such that for any $t \geq 0$, the solution (v, u) has the following decay properties:

$$\|\nabla(v, u)(t)\|_1 \leq C_2(1+t)^{-5/4}, \tag{17}$$

$$\|(v, u - \overline{u})(t)\|_{L^\infty} \leq C_2(1+t)^{-5/4}, \tag{18}$$

$$\|(v, u - \overline{u})(t)\|_{L^q} \leq C_2(1+t)^{(-3/2)(1-1/q)}, \quad 2 \leq q \leq 6, \tag{19}$$

$$\left\|\partial_t(v, u)(t)\right\| \leq C_2(1+t)^{-5/4}. \tag{20}$$

Remark 2. As compared to the classic results in [9, 17, 30], where smallness conditions on H^3-norm of the initial data were proposed, we are able to prove the global existence and convergence rates for the strong solutions to the Cauchy problem under only that H^2-norm of the initial data is sufficiently small.

Remark 3. Similar ideas can be applied to study the initial boundary value problem. This will be reported in a forthcoming paper.

Remark 4. Here and in what follows, (v, u) denotes the vector (v_1, v_2, v_3, u) with $v = (v_1, v_2, v_3)$ and $v \in C^0(0,\infty; H^2(\mathbb{R}^3)) \cap C^1(0,\infty; H^1(\mathbb{R}^3))$ represents each component v_i $(i = 1, 2, 3)$ of the vector v belonging to the function space $C^0(0,\infty; H^2(\mathbb{R}^3)) \cap C^1(0,\infty; H^1(\mathbb{R}^3))$.

The proofs of Theorem 1 are based on energy methods and spectral analysis. The key point for the proof of global existence is to obtain some *a priori* estimates independent of t. Then, we can combine spectral analysis with the uniform *a priori* estimates to prove our desired decay estimates (17)–(20).

The rest of this paper is organized as follows. In Section 2, we reformulate the problem. In Section 3, we give some elementary facts on the decay-in-time estimates on (v, u) for the linearized system. Sections 4 and 5 are dedicated to prove Theorem 1.

2. Reformulated System

In this section, we will first reformulate the problem. Set

$$\lambda = \sqrt{\frac{f'(\overline{u})}{\overline{u}}}, \qquad \lambda_1 = \sqrt{\overline{u} f'(\overline{u})}, \qquad \lambda_2 = \sqrt{\frac{\overline{u}}{f'(\overline{u})}}. \tag{21}$$

Taking change of variables $(v, u) \rightarrow (\lambda v, u + \overline{u})$ and linearizing the system around $(0, \overline{u})$, we can reformulate the Cauchy problem (1)-(2) as

$$\begin{gathered} v_t - \lambda_1 \nabla u = F_1, \\ u_t - \lambda_1 \nabla \cdot v - D\Delta u = F_2, \\ (v, u)(x, 0) = (v_0, u_0)(x) \longrightarrow (0, 0) \quad \text{as } |x| \longrightarrow \infty, \end{gathered} \tag{22}$$

where

$$\begin{gathered} F_1 = \lambda_2 \left(f'(u + \overline{u}) - f'(\overline{u}) \right) \nabla u, \\ F_2 = \lambda \nabla \cdot (uv). \end{gathered} \tag{23}$$

Here and in the sequel, for the notational simplicity, we still denote the reformulated variables by (v, u).

To prove the global existence of a solution to (22), we will combine the local existence result together with *a priori* estimates. To begin with, we state the following local existence, the proof of which can be found in [4, 36–38].

Proposition 5 (local existence). *Assume that $(v_0, u_0) \in H^2(\mathbb{R}^3)$. Then, there exists a sufficiently small positive constant t_0 depending only on $\|(v_0, u_0)\|_2$ such that the Cauchy problem (22) admits a unique solution $(v, u)(x, t) \in C([0, t_0], H^2(\mathbb{R}^3))$ satisfying*

$$\sup_{t \in [0, t_0]} \|(v, u)(\cdot, t)\|_2 \le 2\left\|(v_0, u_0)\right\|_2. \tag{24}$$

Proposition 6 (*a priori* estimate). *Let $\nabla \times v_0 = 0$ and $(v_0, u_0) \in H^2(\mathbb{R}^3)$. Assume that the Cauchy problem (22) has a solution $(v, u)(x, t)$ on $\mathbb{R}^3 \times [0, T]$ for some $T > 0$ in the same function class as in Proposition 5. Then, there exist a small constant $\delta > 0$ and a constant C_3, which are independent of T, such that if*

$$\sup_{0 \le t \le T} \|(v, u)(t)\|_2 \le \delta, \tag{25}$$

then for any $t \in [0, T]$, it holds that

$$\begin{aligned} &\|(v, u)(t)\|_2^2 + \int_0^t \left(\|\nabla v(\tau)\|_1^2 + \|\nabla u(\tau)\|_2^2 \right) d\tau \\ &\quad \le C_3 \left\|(v_0, u_0 - \overline{u})\right\|_2^2. \end{aligned} \tag{26}$$

Moreover, if, in addition, $\|(v_0, u_0)\|_{L^1}$ is bounded, then there is a positive constant C_4 independent of T such that for any $t \in [0, T]$, the solution (v, u) has the following decay properties:

$$\|\nabla (v, u)(t)\|_1 \le C_4 (1 + t)^{-5/4}, \tag{27}$$

$$\|(v, u)(t)\|_{L^\infty} \le C_4 (1 + t)^{-5/4}, \tag{28}$$

$$\|(v, u)(t)\|_{L^q} \le C_4 (1 + t)^{(-3/2)(1 - 1/q)}, \quad 2 \le q \le 6, \tag{29}$$

$$\left\|\partial_t (v, u)(t)\right\| \le C_4 (1 + t)^{-5/4}. \tag{30}$$

Theorem 1 follows from Propositions 5 and 6 and standard continuity arguments. The proof of Proposition 6 will be given in Sections 4 and 5.

3. Linear Decay Estimates

In this section, we consider the Cauchy problem for the linearized equations corresponding to $(22)_1$-$(22)_2$:

$$\begin{gathered} v_t - \lambda_1 \nabla u = 0, \\ u_t - \lambda_1 \nabla \cdot v - D\Delta u = 0, \\ (v, u)(x, 0) = (v_0, u_0)(x) \longrightarrow (0, 0) \quad \text{as } |x| \longrightarrow \infty. \end{gathered} \tag{31}$$

The solutions (v, u) of the linear system (31) can be expressed as

$$(v, u)^t (t) = G(t) * (v_0, u_0)^t, \quad t \ge 0. \tag{32}$$

Here, $G(t) := G(x, t)$ is the Green's matrix for the system (31).

To derive the large time behavior of the solutions, we first give an explicit expression for the Fourier transform $\widehat{G}(\xi, t)$ of Green's matrix $G(x, t)$.

Lemma 7. *The Fourier transform $\widehat{G}$ of Green's matrix for the linear system (31) is given by*

$$\widehat{G}(\xi,t)=\begin{bmatrix}\left(\dfrac{\lambda_+e^{\lambda_-t}-\lambda_-e^{\lambda_+t}}{\lambda_+-\lambda_-}\right)I & i\lambda_1\left(\dfrac{e^{\lambda_+t}-e^{\lambda_-t}}{\lambda_+-\lambda_-}\right)\xi^t\\ i\lambda_1\left(\dfrac{e^{\lambda_+t}-e^{\lambda_-t}}{\lambda_+-\lambda_-}\right)\xi & \dfrac{\lambda_+e^{\lambda_+t}-\lambda_-e^{\lambda_-t}}{\lambda_+-\lambda_-}\end{bmatrix}, \tag{33}$$

where

$$\lambda_\pm(|\xi|)=-\frac{D}{2}|\xi|^2\pm\frac{1}{2}\sqrt{D^2|\xi|^4-4\lambda_1^2|\xi|^2}. \tag{34}$$

The representation above holds for $|\xi|\neq\pm2\lambda_1/D$.

Proof. The proof is in spirit of Hoff and Zumbrun [39, 40]. Applying the Fourier transform to the system (31), we have

$$\widehat{v}_t=i\lambda_1\xi\widehat{u}, \tag{35}$$

$$\widehat{u}_t=-D|\xi|^2\widehat{u}+i\lambda_1\xi\cdot\widehat{v}. \tag{36}$$

From (35)-(36), we obtain the following initial value problem for $\widehat{u}(\xi,t)$:

$$\begin{gathered}\widehat{u}_{tt}+D|\xi|^2\widehat{u}_t+\lambda_1^2|\xi|^2\widehat{u}=0,\\ \widehat{u}(\xi,0)=\widehat{u}_0(\xi),\\ \widehat{u}_t(\xi,0)=-D|\xi|^2\widehat{u}_0(\xi)+i\lambda_1\xi\cdot\widehat{v}_0(\xi).\end{gathered} \tag{37}$$

The $\lambda_\pm(\xi)$ defined in (34) are exactly the eigenvalues of the ODE (37); hence for $\lambda_+\neq\lambda_-$,

$$\widehat{u}(\xi,t)=A(\xi)e^{\lambda_-(\xi)t}+B(\xi)e^{\lambda_+(\xi)t}. \tag{38}$$

The initial conditions in (37) give

$$A=\frac{-\lambda_-\widehat{u}_0-i\lambda_1\xi\cdot\widehat{v}_0}{\lambda_+-\lambda_-},\qquad B=\frac{\lambda_+\widehat{u}_0+i\lambda_1\xi\cdot\widehat{v}_0}{\lambda_+-\lambda_-}. \tag{39}$$

Substituting (39) into (38), by a straightforward computation, we obtain

$$\widehat{u}(\xi,t)=\frac{\lambda_+e^{\lambda_+t}-\lambda_-e^{\lambda_-t}}{\lambda_+-\lambda_-}\widehat{u}_0+i\lambda_1\left(\frac{e^{\lambda_+t}-e^{\lambda_-t}}{\lambda_+-\lambda_-}\right)\xi\cdot\widehat{v}_0. \tag{40}$$

To compute $\widehat{v}$, we substitute (40) into (35) to obtain

$$\widehat{v}(\xi,t)=\frac{\lambda_+e^{\lambda_-t}-\lambda_-e^{\lambda_+t}}{\lambda_+-\lambda_-}\widehat{v}_0+i\lambda_1\left(\frac{e^{\lambda_+t}-e^{\lambda_-t}}{\lambda_+-\lambda_-}\right)\xi\widehat{u}_0. \tag{41}$$

Hence, (40)-(41) give (33) and the proof of Lemma 7 is completed. □

To derive the long-time decay rate of solutions in L^2-framework, we need to verify the approximation of Green's function $\widehat{G}(\xi,t)$ for both lower frequency and high frequency. In terms of the definition of the eigenvalues (34), we are able to obtain that it holds for $|\xi|\ll1$ that

$$\begin{gathered}\frac{e^{\lambda_+t}-e^{\lambda_-t}}{\lambda_+-\lambda_-}\sim\frac{\sin(bt)}{b}e^{(-D/2)|\xi|^2t},\quad|\xi|\ll1,\\ \frac{\lambda_+e^{\lambda_+t}-\lambda_-e^{\lambda_-t}}{\lambda_+-\lambda_-}\sim\left[\cos(bt)+\frac{D}{2}\frac{\sin(bt)}{b}|\xi|^2\right]e^{(-D/2)|\xi|^2t},\quad|\xi|\ll1,\\ \frac{\lambda_+e^{\lambda_-t}-\lambda_-e^{\lambda_+t}}{\lambda_+-\lambda_-}\sim\left[\cos(bt)-\frac{D}{2}\frac{\sin(bt)}{b}|\xi|^2\right]e^{(-D/2)|\xi|^2t},\quad|\xi|\ll1,\end{gathered} \tag{42}$$

where

$$b=\frac{1}{2}\sqrt{4\lambda_1^2|\xi|^2-D^2|\xi|^4}\sim O(|\xi|),\quad|\xi|\ll1. \tag{43}$$

Next, we deal with the high frequency $|\xi|\gg1$. From the definitions of the eigenvalues (34), we can analyze the eigenvalues for $|\xi|\gg1$. Indeed, we have the leading orders of the eigenvalues for $|\xi|\gg1$ as

$$\begin{gathered}\lambda_+\sim-\frac{\lambda_1^2}{D}+O\left(|\xi|^{-1}\right),\\ \lambda_-\sim-D|\xi|^2+\frac{\lambda_1^2}{D}+O\left(|\xi|^{-1}\right).\end{gathered} \tag{44}$$

This approximation gives the leading order terms of the elements of Green's function as follows:

$$\begin{gathered}\frac{e^{\lambda_+t}-e^{\lambda_-t}}{\lambda_+-\lambda_-}=\frac{1}{\lambda_+-\lambda_-}e^{\lambda_+t}\times\left[1-e^{(\lambda_--\lambda_+)t}\right]\sim O(1)e^{-\vartheta t},\quad|\xi|\geq R,\\ \frac{\lambda_+e^{\lambda_+t}-\lambda_-e^{\lambda_-t}}{\lambda_+-\lambda_-}=\frac{\lambda_+}{\lambda_+-\lambda_-}e^{\lambda_+t}\left[1-e^{(\lambda_--\lambda_+)t}\right]+e^{\lambda_-t}\sim O(1)e^{-\vartheta t},\quad|\xi|\geq R,\\ \frac{\lambda_+e^{\lambda_-t}-\lambda_-e^{\lambda_+t}}{\lambda_+-\lambda_-}=\frac{1}{2}e^{\lambda_+t}\left[1+e^{(\lambda_--\lambda_+)t}\right]+\frac{D|\xi|^2}{\lambda_+-\lambda_-}e^{\lambda_+t}\times\left[1-e^{(\lambda_--\lambda_+)t}\right]\sim O(1)e^{-\vartheta t},\quad|\xi|\geq R,\end{gathered} \tag{45}$$

where ϑ and R are some positive constants.

With the help of the formula (33) for the Fourier transform $\widehat{G}$ of Green's matrix and the asymptotical analysis on its elements, we are able to establish the L^2 time decay rate. Indeed, we have the L^2 time decay rate of the global solution to the Cauchy problem for the linear problem (31) as follows.

Lemma 8. *Assume that* (v, u) *is the solution of the linear problem* (31) *with the initial data* $(v_0, u_0) \in H^2(\mathbb{R}^3) \cap L^1(\mathbb{R}^3)$*; then*

$$\left\|\nabla^k (v, u)(t)\right\| \le C(1+t)^{-3/4-k/2} \times \left(\left\|(v_0, u_0)\right\|_{L^1} + \left\|\nabla^k (v_0, u_0)\right\|\right), \tag{46}$$

for $0 \le k \le 2$.

Proof. From (33), (42), and (45), we have

$$\begin{aligned}\widehat{v}(\xi, t) &= \frac{\lambda_+ e^{\lambda_- t} - \lambda_- e^{\lambda_+ t}}{\lambda_+ - \lambda_-}\widehat{v}_0 + i\lambda_1\left(\frac{e^{\lambda_+ t} - e^{\lambda_- t}}{\lambda_+ - \lambda_-}\right)\xi\widehat{u}_0 \\ &\sim \begin{cases} O(1)\, e^{-(D/2)|\xi|^2 t}\left(|\widehat{v}_0| + |\widehat{u}_0|\right), & |\xi| \ll 1, \\ O(1)\, e^{-\vartheta t}\left(|\widehat{v}_0| + |\widehat{u}_0|\right), & |\xi| \gg 1,\end{cases}\end{aligned} \tag{47}$$

with $\vartheta > 0$ being a constant here and below, and

$$\begin{aligned}\widehat{u}(\xi, t) &= \frac{\lambda_+ e^{\lambda_+ t} - \lambda_- e^{\lambda_- t}}{\lambda_+ - \lambda_-}\widehat{u}_0 + i\lambda_1\left(\frac{e^{\lambda_+ t} - e^{\lambda_- t}}{\lambda_+ - \lambda_-}\right)\xi \cdot \widehat{v}_0 \\ &\sim \begin{cases} O(1)\, e^{-(D/2)|\xi|^2 t}\left(|\widehat{v}_0| + |\widehat{u}_0|\right), & |\xi| \ll 1, \\ O(1)\, e^{-\vartheta t}\left(|\widehat{v}_0| + |\widehat{u}_0|\right), & |\xi| \gg 1,\end{cases}\end{aligned} \tag{48}$$

where here and below $R > 0$ denotes a small but fixed constant.

Therefore, to prove (46), we only need to deal with the case for v since the case for u can be proved similarly. By virtue of the pointwise estimate (47), the Parseval theorem, and Hausdorff-Young's inequality, for $0 \le k \le 3$, we have

$$\begin{aligned}\left\|\nabla^k v(t)\right\|^2 &= \int_{|\xi|\le R} |\xi|^{2k} |\widehat{v}(\xi, t)|^2 d\xi + \int_{|\xi|\ge R} |\xi|^{2k} |\widehat{v}(\xi, t)|^2 d\xi \\ &\le C\int_{|\xi|\le R} e^{-D|\xi|^2 t} |\xi|^{2k}\left(|\widehat{v}_0(\xi)|^2 + |\widehat{u}_0(\xi)|^2\right) d\xi \\ &\quad + C\int_{|\xi|\ge R} e^{-\vartheta t} |\xi|^{2k}\left(|\widehat{v}_0(\xi)|^2 + |\widehat{u}_0(\xi)|^2\right) d\xi \\ &\le C(1+t)^{-3/2-k}\left\|(\widehat{v}_0, \widehat{u}_0)\right\|_{L^\infty}^2 + Ce^{-\vartheta t}\left\|\nabla^k (v_0, u_0)\right\|^2 \\ &\le C(1+t)^{-3/2-k}\left(\left\|(v_0, u_0)\right\|_{L^1}^2 + \left\|\nabla^k (v_0, u_0)\right\|^2\right).\end{aligned} \tag{49}$$

Thus, (49) gives (46) immediately, and the proof of Lemma 8 is completed. □

4. *A Priori* Estimates

Throughout this section and the next section, we assume that all conditions of Proposition 6 are satisfied. Moreover, we make *a priori* assumption:

$$\sup_{0\le t\le T} \|(v, u)(t)\|_2 \le \delta, \tag{50}$$

where δ is a sufficiently small positive constant.

In the following, a series of lemmas on the energy estimates are given. Firstly, we will obtain the lower-order energy estimate for (v, u) in the following lemma.

Lemma 9. *There exists a positive constant* D_1*, which is sufficiently large and independent of* δ*, such that*

$$\frac{d}{dt}\left(D_1\|(v, u)(t)\|^2 + \langle \nabla u, v\rangle(t)\right) + C\|\nabla(v, u)(t)\|^2 \le C\left\|\nabla^2 u(t)\right\|^2, \tag{51}$$

for any $0 \le t \le T$.

Proof. Multiplying $(22)_1$-$(22)_2$ by v, u, respectively, then summing up and integrating the resultant equation, we obtain

$$\frac{1}{2}\frac{d}{dt}\|(v, u)\|^2 + D\|\nabla u\|^2 = \langle v, F_1\rangle + \langle u, F_2\rangle. \tag{52}$$

Applying mean value theorem, Hölder's inequality, and Sobolev's inequality, it is clear that the two terms on the right-hand side of (52) can be estimated as follows:

$$\begin{aligned}\left|\langle v, F_1\rangle\right| + \left|\langle u, F_2\rangle\right| &\le C\int_{\mathbb{R}^3} |vu\nabla u|\, dx \\ &\le C\|v\|_{L^3}\|u\|_{L^6}\|\nabla u\| \\ &\le C\|v\|_1\|\nabla u\|^2 \\ &\le C\delta\|\nabla u\|^2.\end{aligned} \tag{53}$$

Combining (52) with (53) and using the fact that δ is sufficiently small, we have

$$\frac{d}{dt}\|(v, u)\|^2 + C\|\nabla u\|^2 \le 0. \tag{54}$$

Next, we will estimate $\|\nabla v\|^2$. Multiplying $(22)_2$ by $\nabla \cdot v$ and integrating the resulting equation over $\mathbb{R}^3$, we have

$$\lambda_1\|\nabla\cdot v\|^2 = \langle u_t, \nabla\cdot v\rangle + \langle -D\Delta u, \nabla\cdot v\rangle + \langle F_2, \nabla\cdot v\rangle, \tag{55}$$

where from $(22)_1$, the first term on the right-hand side can be written as

$$\begin{aligned}\langle u_t, \nabla\cdot v\rangle &= \frac{d}{dt}\langle u, \nabla\cdot v\rangle - \langle u, \nabla\cdot v_t\rangle \\ &= -\frac{d}{dt}\langle \nabla u, v\rangle + \langle \nabla u, v_t\rangle \\ &= -\frac{d}{dt}\langle \nabla u, v\rangle + \langle \nabla u, \lambda_1\nabla u + F_1\rangle.\end{aligned} \tag{56}$$

Then, it follows from (55)-(56), (23), Hölder's inequality, and Young's inequality that

$$\begin{aligned}\lambda_1\|\nabla\cdot v\|^2 + \frac{d}{dt}\langle\nabla u, v\rangle &= \langle\nabla u, \lambda_1\nabla u + F_1\rangle + \langle -D\Delta u, \nabla\cdot v\rangle + \langle F_2, \nabla\cdot v\rangle \\ &\le C\left\|\nabla^2 u\right\|^2 + \frac{\lambda_1}{4}\|\nabla\cdot v\|^2 + C\delta\|\nabla\cdot v\|^2 + C\|\nabla u\|^2.\end{aligned} \tag{57}$$

This together with the fact that δ is sufficiently small implies that

$$\frac{d}{dt}\langle\nabla u, v\rangle + \frac{\lambda_1}{2}\|\nabla\cdot v\|^2 \le C\left(\|\nabla u\|^2 + \left\|\nabla^2 u\right\|^2\right). \tag{58}$$

Taking the curl for $(22)_1$ and noting that $\nabla\times v_0 = 0$, we have

$$\nabla\times v = 0. \tag{59}$$

Since $\Delta v = \nabla(\nabla\cdot v) - \nabla\times(\nabla\times v)$, we have from (59) that

$$\Delta v = \nabla(\nabla\cdot v), \qquad \|\nabla v\| \le C\|\nabla\cdot v\|. \tag{60}$$

Combining (58) with (60) yields

$$\frac{d}{dt}\langle\nabla u, v\rangle + C\|\nabla v\|^2 \le C\left(\|\nabla u\|^2 + \left\|\nabla^2 u\right\|^2\right). \tag{61}$$

Finally, multiplying (54) by D_1 which is suitably large and adding it to (61), one has (51) since $\delta > 0$ is sufficiently small. This completes the proof of Lemma 9. □

The following lemma is concerned with the higher-order energy estimate on (v, u).

Lemma 10. *There exists a positive constant D_2, which is sufficiently large and independent of δ, such that*

$$\begin{aligned}&\frac{d}{dt}\left(D_2\|\nabla(v,u)(t)\|_1^2 + \langle\Delta u, \nabla\cdot v\rangle(t)\right) \\ &\quad + C\left(\left\|\nabla^2 v(t)\right\|^2 + \|\nabla^2 u(t)\|_1^2\right) \\ &\quad \le C\delta\|\nabla(v,u)(t)\|^2,\end{aligned} \tag{62}$$

for any $0 \le t \le T$.

Proof. Applying ∇ to $(22)_1$-$(22)_2$ and multiplying them by ∇v, ∇u, respectively, and then integrating them over $\mathbb{R}^3$, we obtain

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\|\nabla(v,u)\|^2 + D\left\|\nabla^2 u\right\|^2 \\ &\quad = \langle\nabla v, \nabla F_1\rangle + \langle\nabla u, \nabla F_2\rangle := J_1 + J_2.\end{aligned} \tag{63}$$

Next, we estimate the terms J_1 and J_2 one by one. To begin with, by using (23), (50), Hölder's inequality, Cauchy's inequality, and Sobolev's inequality, we can estimate the term J_1 as follows:

$$\begin{aligned}J_1 &\le C\left(\|\nabla u\|\|\nabla u\|_{L^6}\|\nabla v\|_{L^3} + \|u\|_{L^\infty}\left\|\nabla^2 u\right\|\|\nabla v\|\right) \\ &\le C\|\nabla u\|\left\|\nabla^2 u\right\|\|\nabla v\|_1 + \|u\|_2\left\|\nabla^2 u\right\|\|\nabla v\| \\ &\le C\delta\left(\left\|\nabla^2 u\right\|^2 + \|\nabla(v,u)\|^2\right).\end{aligned} \tag{64}$$

Using similar arguments, we also have the following estimate for the term J_2:

$$J_2 \le C\delta\left(\left\|\nabla^2 u\right\|^2 + \|\nabla(v,u)\|^2\right). \tag{65}$$

Substituting (64) and (65) into (63) and noting that δ is sufficiently small, we have

$$\frac{d}{dt}\|\nabla(v,u)\|^2 + C\left\|\nabla^2 u\right\|^2 \le C\delta\|\nabla(v,u)\|^2. \tag{66}$$

For higher-order derivatives of (v, u), we can apply the similar arguments used in obtaining (66) to get

$$\frac{d}{dt}\left\|\nabla^2(v,u)\right\|^2 + C\left\|\nabla^3 u\right\|^2 \le C\delta\left\|\nabla^2(v,u)\right\|^2. \tag{67}$$

Combining (66) and (67) and using the fact that δ is sufficiently small, we have

$$\frac{d}{dt}\|\nabla(v,u)\|_1^2 + C\left\|\nabla^2 u\right\|_1^2 \le C\delta\left(\|\nabla(v,u)\|^2 + \left\|\nabla^2 v\right\|^2\right). \tag{68}$$

Next, we estimate $\|\nabla^2 v\|^2$. To begin with, applying ∇ to $(22)_2$ and then multiplying by $\nabla(\nabla\cdot v)$, we have from Cauchy's inequality that

$$\frac{\lambda_1}{2}\|\nabla(\nabla\cdot v)\|^2 \le \langle\nabla u_t, \nabla(\nabla\cdot v)\rangle + C\left\|\nabla^3 u\right\|^2 + C\left\|\nabla F_2\right\|^2. \tag{69}$$

By integrating by parts several times, we estimate the first term on the right-hand side of (69) as follows:

$$\begin{aligned}\langle\nabla u_t, \nabla(\nabla\cdot v)\rangle &= -\frac{d}{dt}\langle\Delta u, \nabla\cdot v\rangle + \langle\Delta u, \nabla\cdot v_t\rangle \\ &= -\frac{d}{dt}\langle\Delta u, \nabla\cdot v\rangle + \langle\Delta u, \lambda_1\Delta u + \nabla\cdot F_1\rangle \\ &\le -\frac{d}{dt}\langle\Delta u, \nabla\cdot v\rangle + C\left\|\nabla^2 u\right\|^2.\end{aligned} \tag{70}$$

From (23), (50), Hölder's inequality, Cauchy's inequality, and Sobolev's inequality, we have

$$\begin{aligned}\left\|\nabla F_2\right\| &\le C\left\{\|\nabla v\|_{L^3}\|\nabla u\|_{L^6} + \|v\|_{L^\infty}\left\|\nabla^2 u\right\|\right. \\ &\quad \left. + \|u\|_{L^\infty}\left\|\nabla^2 v\right\|\right\} \\ &\le C\delta\left\|\nabla^2(v,u)\right\|.\end{aligned} \tag{71}$$

Substituting (70) and (71) into (69) yields

$$\frac{d}{dt}\langle \Delta u, \nabla\cdot v\rangle(t) + C\|\nabla(\nabla\cdot v)\|^2 \le C\left(\left\|\nabla^2 u\right\|_1^2 + \delta\left\|\nabla^2 v\right\|^2\right). \tag{72}$$

Due to (60), we have

$$\left\|\nabla^2 v\right\|^2 \le C\|\nabla(\nabla\cdot v)\|^2. \tag{73}$$

Substituting (73) into (72) and using the fact that δ is sufficiently small, we obtain

$$\frac{d}{dt}\langle \Delta u, \nabla\cdot v\rangle(t) + C\left\|\nabla^2 v\right\|^2 \le C\left\|\nabla^2 u\right\|_1^2. \tag{74}$$

Finally, since δ is sufficiently small, multiplying (68) by a suitably large positive constant D_2 and adding it to (74) give (62), and this completes the proof of Lemma 10. □

5. Global Existence and Convergence Rate

In this section, we devote ourselves to prove Proposition 6. To begin with, we give the following lemma which is concerned with the *a priori* decay-in-time estimates on (v, u).

Lemma 11. *Let* $K_0 = \|(v_0, u_0)\|_{H^2\cap L^1}$, *then*

$$\|\nabla(v,u)(t)\| \le CK_0(1+t)^{-5/4} + C\delta\int_0^t (1+t-\tau)^{-5/4}\|\nabla(v,u)(\tau)\|_1 d\tau, \tag{75}$$

for any $0 \le t \le T$.

Proof. From Duhamel's principle and Lemma 8, we have

$$\|\nabla(v,u)(t)\| \le CK_0(1+t)^{-5/4} + C\delta\int_0^t (1+t-\tau)^{-5/4} \times \left\|(F_1,F_2)(\tau)\right\|_{H^1\cap L^1} d\tau. \tag{76}$$

From (23), (50), Hölder's inequality, and Sobolev's inequality, we have

$$\left\|(F_1,F_2)(t)\right\|_{H^1\cap L^1} \le C\delta\|\nabla(v,u)(t)\|_1. \tag{77}$$

Therefore, (75) follows from (76) and (77) immediately, and this completes the proof of Lemma 11. □

Now, we are in position to prove Proposition 6.

5.1. Proof of Proposition 6. We will do it in two steps.

Step 1. Since $\delta > 0$ is suitably small, from Lemmas 9 and 10, we can choose a suitably large positive constant D_3 such that

$$\frac{d}{dt}\left(D_3\|(v,u)(t)\|_2^2 + \langle\nabla u, \nabla\cdot v\rangle + \langle\Delta u, \nabla\cdot v\rangle\right) + C\left(\|\nabla v(t)\|_2^2 + \|\nabla u(t)\|_2^2\right) \le 0, \tag{78}$$

for any $0 \le t \le T$. Notice that the expression under d/dt in (78) is equivalent to $\|(v,u)(t)\|_2^2$. Hence, by integrating (78) directly in time, we obtain (26).

Step 2. Define the temporal energy functional

$$\mathbb{E}(t) = D_2\|\nabla(v,u)(t)\|_1^2 + \langle\Delta u, \nabla\cdot v\rangle(t), \tag{79}$$

for any $0 \le t \le T$, where it should be mentioned that $\mathbb{E}(t)$ is equivalent to $\|\nabla(v,u)\|_1^2$ since D_2 is sufficiently large.

By virtue of Lemma 10, we have

$$\frac{d\mathbb{E}(t)}{dt} + C\left\|\nabla^2(v,u)(t)\right\|^2 \le C\delta\|\nabla(v,u)(t)\|^2. \tag{80}$$

Adding $\|\nabla(v,u)(t)\|^2$ to both sides of (80) yields

$$\frac{d\mathbb{E}(t)}{dt} + D_4\mathbb{E}(t) \le C\|\nabla(v,u)(t)\|^2, \tag{81}$$

where D_4 is a positive constant independent of δ. As [7, 17], we define

$$\mathbb{G}(t) = \sup_{0\le\tau\le t}(1+\tau)^{5/2}\mathbb{E}(\tau). \tag{82}$$

Notice that

$$\|\nabla(v,u)(\tau)\|_1 \le C\sqrt{\mathbb{E}(\tau)} \le C(1+\tau)^{-5/4}\sqrt{\mathbb{G}(t)}, \quad 0\le\tau\le t\le T. \tag{83}$$

Then, from (75) and (83), we have

$$\begin{aligned}\|\nabla(v,u)(t)\| &\le CK_0(1+t)^{-5/4} + \int_0^t (1+t-\tau)^{-5/4}(1+\tau)^{-5/4}\sqrt{\mathbb{G}(t)}d\tau \\ &\le C(1+t)^{-5/4}\left(K_0 + \delta\sqrt{\mathbb{G}(t)}\right),\end{aligned} \tag{84}$$

which together with Gronwall's inequality and (81) implies

$$\begin{aligned}\mathbb{E}(t) &\le \mathbb{E}(0)e^{-D_4 t} + C\int_0^t e^{-D_4(t-\tau)}\|\nabla(v,u)(\tau)\|^2 d\tau \\ &\le \mathbb{E}(0)e^{-D_4 t} + C\left(K_0^2 + \delta^2\mathbb{G}(t)\right) \times \int_0^t e^{-D_4(t-\tau)}(1+\tau)^{-5/2}d\tau \\ &\le C(1+t)^{-5/2}\left(\mathbb{E}(0) + K_0^2 + \delta^2\mathbb{G}(t)\right).\end{aligned} \tag{85}$$

Since $\mathbb{G}$ is nondecreasing, we have from (85) that

$$\mathbb{G}(t) \le C\left(\mathbb{E}(0) + K_0^2 + \delta^2\mathbb{G}(t)\right), \tag{86}$$

for any $0 \le t \le T$, which together with the fact that $\delta > 0$ is small enough implying that

$$\mathbb{G}(t) \le C\left(\mathbb{E}(0) + K_0^2\right) \le CK_0^2. \tag{87}$$

Thus, (82) and (87) yield

$$\|\nabla (v,u)(t)\|_1 \le CK_0(1+t)^{-5/4}, \quad 0 \le t \le T, \tag{88}$$

which together with Sobolev's inequality gives

$$\begin{aligned}\|(v,u)(t)\|_{L^\infty} &\le C\|\nabla (v,u)(t)\|_1 \\ &\le CK_0(1+t)^{-5/4}, \quad 0 \le t \le T.\end{aligned} \tag{89}$$

Therefore, we have proved (27) and (28).

Next, we turn to prove (29). To begin with, by using Sobolev's inequality and (27), we have

$$\|(v,u)(t)\|_{L^6} \le C\,\|\nabla (v,u)(t)\| \le CK_0(1+t)^{-5/4}. \tag{90}$$

On the other hand, applying Duhamel's principle and Lemma 8 again, we have from (26) and (28) that

$$\begin{aligned}\|(v,u)(t)\| &\le CK_0(1+t)^{-3/4} \\ &\quad + C\int_0^t (1+t-\tau)^{-3/4}\|(F_1,F_2)(\tau)\|_{L^2\cap L^1} d\tau \\ &\le CK_0(1+t)^{-3/4} \\ &\quad + C\delta\int_0^t (1+t-\tau)^{-3/4}\|\nabla (v,u)(\tau)\|_1 d\tau \\ &\le CK_0(1+t)^{-3/4} \\ &\quad + CK_0\delta\int_0^t (1+t-\tau)^{-3/4}(1+\tau)^{-5/4} d\tau \\ &\le CK_0(1+t)^{-3/4},\end{aligned} \tag{91}$$

for any $0 \le t \le T$.

Hence, by the interpolation, it holds that for any $2 \le q \le 6$,

$$\begin{aligned}\|(v,u)(t)\|_{L^q} &\le \|(v,u)(t)\|^{\theta}\|(v,u)(t)\|_{L^6}^{1-\theta} \\ &\le C_0(1+t)^{(-3/2)(1-1/q)},\end{aligned} \tag{92}$$

for any $0 \le t \le T$, where $\theta = (6-q)/2q$. Thus, we have proved (29). Therefore, (30) follows from (22)-(23) and (27)–(29) and this completes the proof of Proposition 6.

Acknowledgments

The second author was partially supported by the National Natural Science Foundation of China nos. 11301172 and 11226170, China Postdoctoral Science Foundation funded project no. 2012M511640, and Hunan Provincial Natural Science Foundation of China no. 13JJ4095.

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