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DOUBT FUZZY BCI-ALGEBRAS

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The aim of this note is to introduce the notion of doubt fuzzy p-ideals in BCI-algebras and to study their properties. We also solve the problem of classifying doubt fuzzy p-ideals and study fuzzy relations on BCI-algebras.

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1. Introduction and preliminaries. The concept of a fuzzy set is applied to generalize some of the basic concepts of general topology [2]. Rosenfeld [6] constituted a similar application to the elementary theory of groupoids and groups. Xi [7] applied the concept of fuzzy set to BCK-algebras. Jun [4] defined a doubt fuzzy subalgebra, doubt fuzzy ideal, doubt fuzzy implicative ideal, and doubt fuzzy prime ideal in BCI-algebras, and got some results about it. In this note, we define a doubt fuzzy p-ideal of a BCI-algebra and investigate its properties.

A mapping $f : X \to Y$ of BCI-algebras is called homomorphism if $f(x*y) = f(x)*f(y)$ for all $x,y \in X$. A nonempty subset I of a BCI-algebra X is called an ideal of X if (i) $0 \in I$, (ii) $x*y \in I$ and $y \in I$ imply that $x \in I$. We recall that a fuzzy subset μ of a set X is a function μ from X into $[0,1]$. Let $\operatorname{Im}\mu$ denote the image set of μ. We will write $a \wedge b$ for $\min\{a,b\}$, and $a \vee b$ for $\max\{a,b\}$, where a and b are any real numbers.

Given a fuzzy set μ and $t \in [0,1]$, let $\mu_t = \{x \in X \mid \mu(x) \geq t\}$ and $\mu^t = \{x \in X \mid \mu(x) \leq t\}$. These could be empty sets. The set $\mu_t \neq \varnothing$ (resp., $\mu^t \neq \varnothing$) is called the t-confidence (resp., t-doubt) set of μ (see [3]).

Definition 1.1 (see [7]). For any x, y in a BCI-algebra X,

(i) if $\mu(x*y) \geq \mu(x) \wedge \mu(y)$, then μ is called a fuzzy subalgebra of X;

(ii) if $\mu(0) \geq \mu(x)$ and $\mu(x) \geq \mu(x*y) \wedge \mu(y)$, then μ is called a fuzzy ideal of X.

Definition 1.2 (see [4]). Let X be a BCI-algebra. A fuzzy set μ in X is called (i) a doubt fuzzy subalgebra (briefly, DF-subalgebra) of X if $\mu(x*y) \leq \mu(x) \vee \mu(y)$ for all $x,y \in X$; and (ii) a doubt fuzzy ideal (briefly, DF-ideal) of X if $\mu(0) \leq \mu(x)$ and $\mu(x) \leq \mu(x*y) \vee \mu(y)$ for all $x,y \in X$.

Definition 1.3 (see [5]). A nonempty subset I of BCI-algebra X is called p-ideal if

(i) $0 \in I$;

(ii) $(x*z)*(y*z) \in I$ and $y \in I$ imply that $x \in I$ for all $x,y,z \in X$.

Definition 1.4 (see [5]). A fuzzy subset μ of a BCI-algebra X is called a fuzzy p-ideal of X if

(i) $\mu(0) \geq \mu(x)$ for any $x \in X$;

(ii) $\mu(x) \geq \mu((x*z)*(y*z)) \wedge \mu(y)$ for any $x,y,z \in X$.

2. Doubt fuzzy p-ideals

Definition 2.1. A fuzzy subset μ of a BCI-algebra X is called a doubt fuzzy p-ideal (briefly, DF p-ideal) of X if

(i) $\mu(0) \le \mu(x)$ for any $x \in X$;

(ii) $\mu(x) \le \mu((x*z)*(y*z)) \vee \mu(y)$ for any $x,y,z \in X$.

Example 2.2. Let $X = \{0,a,b,c\}$ in which $*$ is defined by

$*$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	c	b	0

Then $(X;*,0)$ is a BCI-algebra. Let $t_0,t_1,t_2 \in [0,1]$ be such that $t_0 < t_1 < t_2$. Define $\mu : X \to [0,1]$ by $\mu(0) = t_0$, $\mu(a) = t_1$, and $\mu(b) = \mu(c) = t_2$. Routine calculations give that μ is a DF p-ideal of X.

Proposition 2.3. *If μ is a DF p-ideal of a BCI-algebra X, then $\mu(x) \le \mu(0*(0*x))$ for all $x \in X$.*

Proof. Since μ is a DF p-ideal of X, we have $\mu(x) \le \mu((x*x)*(0*x)) \vee \mu(0) = \mu(0*(0*x)) \vee \mu(0) = \mu(0*(0*x))$. □

Proposition 2.4. *Every DF p-ideal is a DF-ideal.*

Proof. Let μ be a DF p-ideal of X. We have $\mu(x) \ge \mu((x*0)*(y*0)) \vee \mu(y) = \mu(x*y) \vee \mu(y)$ for all $x,y \in X$. Hence μ is a DF-ideal. □

Remark 2.5. The converse of Proposition 2.4 is not true in general as shown in the following example.

Example 2.6. Let $X = \{0,a,1,2,3\}$ in which $*$ is defined by

$*$	0	a	1	2	3
0	0	0	3	2	1
a	a	0	3	2	1
1	1	1	0	3	2
2	2	2	1	0	3
3	3	3	2	1	0

Then X is a BCI-algebra. Let $t_0,t_1,t_2 \in [0,1]$ be such that $t_0 < t_1 < t_2$. Define $\mu : X \to [0,1]$ by $\mu(0) = t_0$, $\mu(a) = t_1$, and $\mu(1) = \mu(2) = \mu(3) = t_3$. Routine calculations give that μ is a DF-ideal of X. But μ is not a DF p-ideal of X, because $\mu(a) = t_1$, and $\mu((a*1)*(0*1)) \vee \mu(0) = \mu(0) = t_0$, that is, $\mu(a) > \mu((a*1)*(0*1)) \vee \mu(0)$.

Proposition 2.7. *If μ is a DF p-ideal of a BCI-algebra X, then $\mu(x*y) \ge \mu((x*z)*(y*z))$ for all $x,y,z \in X$.*

Proof. Note that in a BCI-algebra X the inequality $(x*z)*(y*z) \le x*y$ holds. It follows that $((x*z)*(y*z))*(x*y) = 0$. Since μ is a DF-ideal by Proposition 2.4.

We have $\mu((x*z)*(y*z)) \geq \mu(((x*z)*(y*z))*(x*y)) \vee \mu(x*y) = \mu(0) \vee \mu(x*y) = \mu(x*y)$. This completes the proof. □

Proposition 2.8. *Let μ be a DF-ideal of a BCI-algebra X. If μ satisfies $\mu(x*y) \leq \mu((x*z)*(y*z))$ for any $x,y,z \in X$, then μ is a DF p-ideal of X.*

Proof. Let μ be a DF-ideal of X satisfying $\mu(x*y) \leq \mu((x*z)*(y*z))$ for all $x,y,z \in X$. Then $\mu((x*z)*(y*z)) \vee \mu(y) \geq \mu(x)$. This completes the proof. □

Proposition 2.9. *Let μ be a DF-ideal of a BCI-algebra X. Then $\mu(0*(0*x)) \leq \mu(x)$ for all $x \in X$.*

Proof. We have that $\mu(0*(0*x)) \leq \mu((0*(0*x))*x) \vee \mu(x) = \mu(0) \vee \mu(x) = \mu(x)$ for all $x \in X$. □

Proposition 2.10. *Let μ be a DF-ideal of a BCI-algebra X satisfying $\mu(0*(0*x)) \geq \mu(x)$ for all $x \in X$.*

Proof. Let $x,y,z \in X$. Then

$$\begin{aligned}\mu((x*z)*(y*z)) &\geq \mu(0*(0*((x*z)*(y*z)))) \\ &= \mu((0*y)*(0*x)) \\ &= \mu(0*(0*(x*y))) \\ &\geq \mu(x*y).\end{aligned} \tag{2.1}$$

It follows from Proposition 2.8 that μ is a DF p-ideal of X. □

Theorem 2.11. *Let μ be a fuzzy subset of a BCI-algebra X. If μ is a DF p-ideal of X, then the set $I = \{x \in X \mid \mu(x) = \mu(0)\}$ is a p-ideal of X.*

Proof. Assume that μ is a DF p-ideal of X. Clearly $0 \in I$. Let $(x*z)*(y*z) \in I$ and $y \in I$. Then $\mu(x) \leq \mu((x*z)*(y*z)) \vee \mu(y) = \mu(0)$. But $\mu(0) \leq \mu(x)$ for all $x \in X$. Thus $\mu(0) = \mu(x)$. Hence $x \in I$. This completes the proof. □

Definition 2.12 (see [6]). Let f be a mapping defined on a set X. If μ is a fuzzy subset of X, then the fuzzy subset v of $f(x)$, defined by

$$v(y) = \inf_{x \in f^{-1}(y)} \mu(x) \tag{2.2}$$

for all $y \in f(x)$, is called the image of μ under f. Similarly, if v is a fuzzy subset of $f(x)$, then the fuzzy subset $\mu = v \circ f$ in X (i.e., the fuzzy subset defined by $\mu(x) = v(f(x))$ for all $x \in X$) is called the preimage of v under f.

Theorem 2.13. *An onto homomorphic preimage of a DF p-ideal is also a DF p-ideal.*

Proof. Let $f : X \to X'$ be an onto homomorphism of BCI-algebras, v a DF p-ideal of X', and μ the preimage of v under f. Then $v(f(x)) = \mu(x)$ for all $x \in X$. Since $f(x) \in X'$ and v is a DF p-ideal of X', it follows that $v(0') \leq v(f(x)) = \mu(x)$ for all $x \in X$, where $0'$ is the zero element of X'. But $v(0') = v(f(0) = \mu(0))$, and so $\mu(0) \leq \mu(x)$ for all $x \in X$.

Since υ is a DF p-ideal, we have $\mu(x) = \upsilon(f(x)) \le \upsilon((f(x)*z')*(y'*z')) \vee \upsilon(y')$ for any $y', z' \in X'$. Since f is onto, there exist $y, z \in X$ such that $f(y) = y'$ and $f(z) = z'$. Then

$$\begin{aligned} \mu(x) &\le \upsilon((f(x)*z')*(y'*z')) \vee \upsilon(y') \\ &= \upsilon((f(x)*f(z))*(f(y)*f(z))) \\ &= \upsilon(f(x*z)*f(y*z)) \vee \upsilon(f(y)) \\ &= \upsilon(f(x*z)*(y*z)) \vee \upsilon(f(y)) \\ &= \mu((x*z)*(y*z)) \vee \mu(y). \end{aligned} \tag{2.3}$$

Since y' and z' are arbitrary elements of X', the above result is true for all $y, z \in X$, that is, $\mu(x) \le \mu((x*z)*(y*z)) \vee \mu(y)$ for all $x, y, z \in X$. This completes the proof. □

Definition 2.14 (see [6]). A fuzzy subset μ of X has inf property if for any subset T of X, there exists $t_0 \in T$ such that

$$\mu(t_0) = \inf_{t \in T} \mu(t). \tag{2.4}$$

Theorem 2.15. *An onto homomorphic image of a DF p-ideal with* inf *property is a DF p-ideal.*

Proof. Let $f : X \to X'$ be an onto homomorphism of BCI-algebras, μ a DF p-ideal of X with inf property, and υ the image of μ under f. Since μ is a DF p-ideal of X, we have $\mu(0) \le \mu(x)$ for all $x \in X$. Note that $0 \in f^{-1}(0')$, where 0 and $0'$ are the zero elements of X and X', respectively. Thus $\upsilon(0') = \inf_{t \in f^{-1}(0')} \mu(t) = \mu(0) \le \mu(x)$ for all $x \in X$, which implies that $\upsilon(0') \le \inf_{t \in f^{-1}(x')} \mu(t) = \upsilon(x')$ for any $x' \in X'$. For any $x', y', z' \in X'$, let $x_0 \in f^{-1}(x')$, $y_0 \in f^{-1}(y')$, and $z_0 \in f^{-1}(z')$ be such that

$$\begin{gathered} \mu(x_0) = \inf_{t \in f^{-1}(x')} \mu(t), \qquad \mu(y_0) = \inf_{t \in f^{-1}(y')} \mu(t), \\ \mu((x_0 * z_0)*(y_0 * z_0)) = \inf_{t \in f^{-1}((x'*z')*(y'*z'))} \mu(t). \end{gathered} \tag{2.5}$$

Then

$$\begin{aligned} \upsilon(x') &= \inf_{t \in f-1(x')} \mu(t) \\ &= \mu(x_0) \le \mu((x_0 * z_0)*(y_0*z_0)) \vee \mu(y_0) \\ &= \inf_{t \in f^{-1}((x'*z')*(y'*z'))} \mu(t) \vee \inf_{t \in f^{-1}(y')} \mu(t) \\ &= \upsilon((x'*z')*(y'*z')) \vee \upsilon(y'). \end{aligned} \tag{2.6}$$

Hence υ is a DF p-ideal of X'. □

Theorem 2.16. *A fuzzy subset μ of a BCI-algebra X is a DF p-ideal if and only if, for every $t \in [0,1]$, μ^t is a p-ideal of X, when $\mu^t \ne \varnothing$.*

Proof. Assume that μ is a DF p-ideal of X. By Definition 2.1, we have $\mu(0) \le \mu(x)$ for any $x \in X$. Therefore, $\mu(0) \le \mu(x) \le t$ for $x \in \mu^t$, and so $0 \in \mu^t$. Let $(x*z)*(y*z) \in \mu^t$ and $y \in \mu^t$. Since μ is a DF p-ideal, it follows that $\mu(x) \le \mu((x*z)*(y*z)) \vee \mu(y) \le t$, and that $x \in \mu^t$. Hence μ^t is a p-ideal of X. Conversely, we only need to show that μ is a DF p-ideal of X. If Definition 2.1(i) is not true, then there exists $x' \in X$ such that $\mu(0) > \mu(x')$. If we take $t' = (\mu(x') + \mu(0))/2$, then $\mu(0) > t'$ and $0 \le \mu(x') < t' \le 1$. Thus $x' \in \mu^{t'}$ and $\mu^{t'} \ne \emptyset$. As $\mu^{t'}$ is a p-ideal of X, we have $0 \in \mu^{t'}$, and so $\mu(0) \le t'$. This is a contradiction. Now assume that Definition 2.1(ii) is not true. Suppose that there exist $x', y', z' \in X$ such that $\mu(x') > \mu((x'*z')*(y'*z')) \vee \mu(y')$. Putting $t' = (\mu(x') + \mu((x'*z')*(y'*z')) \vee \mu(y'))/2$, then $\mu(x') > t'$ and $0 \le \mu((x'*z')*(y'*z')) \vee \mu(y') \le 1$. Hence, $\mu((x'*z')*(y'*z')) < t'$ and $\mu(y') < t'$, which imply that $(x'*z')*(y'*z') \in \mu^{t'}$ and $y' \in \mu^{t'}$, since $\mu^{t'}$ is a p-ideal, it follows that $x' \in \mu^{t'}$, and $\mu(x') \le t'$. This is also a contradiction. Hence, μ is a DF p-ideal of X. □

Corollary 2.17. *If a fuzzy subset μ of a BCI-algebra X is a DF p-ideal, then for every $t \in \operatorname{Im}\mu$, μ^t is a p-ideal of X, when $\mu^t \ne \emptyset$.*

3. Doubt Cartesian product of doubt fuzzy p-ideals

Definition 3.1 (see [1]). A fuzzy relation on any set S is a fuzzy subset $\mu : S \times S \to [0,1]$.

Definition 3.2. If μ is a fuzzy relation on a set S and v is a fuzzy subset of S, then μ is a doubt fuzzy relation on v if $\mu(x,y) \ge \mu(x) \vee \mu(y)$ for all $x, y \in S$.

Definition 3.3. Let μ and v be fuzzy subsets of a set S. The doubt Cartesian product of μ and v is defined by $(\mu \times v)(x,y) = \mu(x) \vee v(y)$ for all $x, y \in S$.

Lemma 3.4. *Let μ and v be fuzzy subsets of a set S. Then* (i) *$\mu \times v$ is a fuzzy relation on S;* (ii) *$(\mu \times v)_t = \mu_t \times v_t$ for all $t \in [0,1]$.*

Definition 3.5. If v is a fuzzy subset of a set S, the smallest doubt fuzzy relation on S that is a doubt fuzzy relation on v is μ_v, given by $\mu_v(x,y) = v(x) \vee v(y)$ for all $x, y \in S$.

Lemma 3.6. *For a given fuzzy subset v of a set S, let μ_v be the smallest doubt fuzzy relation on a set S. Then for $t \in [0,1]$, $(\mu_v)_t = v_t \times v_t$.*

Proposition 3.7. *For a given fuzzy subset v of a BCI-algebra X, let μ_v be the smallest doubt fuzzy relation on X. If μ_v is a DF p-ideal of $X \times X$, then $v(x) \ge v(0)$ for all $x \in X$.*

Proof. Since μ_v is a DF p-ideal of $X \times X$, it follows that $\mu_v(x,x) \ge \mu_v(0,0)$, where $(0,0)$ is the zero element of $X \times X$. But this means that $v(x) \vee v(x) \ge v(0) \vee v(0)$, which implies that $v(x) \ge v(0)$. □

Theorem 3.8. *Let μ and v be DF p-ideal of a BCI-algebra X. Then $\mu \times v$ is a DF p-ideal of $X \times X$.*

Proof. Note first that for every $(x,y) \in X\times X$, $(\mu\times v)(0,0) = \mu(0)\vee v(0) \le v(x)\vee v(y) = (\mu\times v)(x,y)$. Now let $(x_1,x_2),(y_1,y_2),(z_1,z_2) \in X\times X$. Then

$$\begin{aligned}
&(\mu\times v)(((x_1,x_2)*(z_1,z_2))*((y_1,y_2)*(z_1,z_2)))\vee(\mu\times v)(y_1,y_2)\\
&\quad= (\mu\times v)((x_1*z_1,x_2*z_2)*(y_1*z_1,y_2*z_2))\vee(\mu\times v)(y_1,y_2)\\
&\quad= (\mu\times v)((x_1*z_1)*(y_1*z_1),(x_2*z_2)*(y_2*z_2))\vee(\mu\times v)(y_1,y_2)\\
&\quad= (\mu((x_1*z_1)*(y_1*z_1))\vee v((x_2*z_2)*(y_2*z_2)))\vee(\mu(y_1)\vee v(y_2)) \qquad (3.1)\\
&\quad= (\mu((x_1*z_1)*(y_1*z_1))\vee\mu(y_1))\vee(v((x_2*z_2)*(y_2*x_2))\vee v(y_2))\\
&\quad\le \mu(x_1)\vee v(x_2)\\
&\quad= (\mu\times v)(x_1,x_2).
\end{aligned}$$

This completes the proof. □

Theorem 3.9. *Let μ and v be fuzzy subsets of a BCI-algebra X such that $\mu\times v$ is a DF p-ideal of $X\times X$. Then*

(i) *either $\mu(x)\ge\mu(0)$ or $v(x)\ge v(0)$ for all $x\in X$;*
(ii) *if $\mu(x)\ge\mu(0)$ for all $x\in X$, then either $\mu(x)\ge v(0)$ or $v(x)\ge v(0)$;*
(iii) *if $v(x)\ge v(0)$ for all $x\in X$, then either $\mu(x)\ge\mu(0)$ or $v(x)\ge\mu(0)$;*
(iv) *either μ or v is a DF p-ideal of X.*

Proof. (i) Suppose that $\mu(x)<\mu(0)$ and $v(y)<v(0)$ for some $x,y\in X$. Then

$$(\mu\times v)(x,y) = \mu(x)\vee v(y) < \mu(0)\vee v(0) = (\mu\times v)(0,0). \qquad (3.2)$$

This is a contradiction and we obtain (i).

(ii) Assume that there exist $x,y\in X$ such that $\mu(x)<v(0)$ and $v(y)<v(0)$. Then $(\mu\times v)(0,0)=\mu(0)\vee v(0)=v(0)$. It follows that $(\mu\times v)(x,y)=\mu(x)\vee v(y)<v(0)=(\mu\times v)(0,0)$, which is a contradiction. Hence (ii) holds.

(iii) Its proof follows by a similar method to (ii).

(iv) Since by (i) either $\mu(x)\ge\mu(0)$ or $v(x)\ge v(0)$ for all $x\in X$; without loss of generality, we may assume that $v(x)\ge v(0)$ for all $x\in X$. From (iii) it follows that either $\mu(x)\ge\mu(0)$ or $v(x)\ge\mu(0)$. If $v(x)\ge\mu(0)$ for any $x\in X$, then $(\mu\times v)(0,x)=\mu(0)\vee v(x)=v(x)$. Let $(x_1,x_2),(y_1,y_2),(z_1,z_2)\in X\times X$, since $\mu\times v$ is a DF p-ideal of $X\times X$. We have $(\mu\times v)(x_1,x_2)\le(\mu\times v)(((x_1,x_2)*(z_1,z_2))*((y_1,y_2)*(z_1,z_2)))\vee(\mu\times v)(y_1,y_2)=(\mu\times v)((x_1*z_1)*(y_1,z_1),(x_2*z_2)*(y_2*x_2))\vee(\mu\times v)(y_1,y_2)$. If we take $x_1=y_1=z_1=0$, then $v(x_2)=(\mu\times v)(0,x_2)\le(\mu\times v)(0,(x_2*z_2)*(y_2*z_2))\vee(\mu\times v)(0,y_2)=(\mu(0)\vee v((x_2*z_2)*(y_2*z_2)))\vee(\mu(0)\vee v(y_2))=v((x_2*z_2)*(y_2*z_2))\vee v(y_2)$. This proves that v is a DF p-ideal of X. Now we consider the case $\mu(x)\ge\mu(0)$ for all $x\in X$, suppose that $v(y)<\mu(0)$ for some $y\in X$. Then $v(0)\le v(y)<\mu(0)$, since $\mu(x)\ge\mu(0)$. It follows that $\mu(x)>v(0)$ for any $x\in X$. Hence $(\mu\times v)(x,0)=\mu(x)\vee v(0)=\mu(x)$. Taking $x_2=y_2=x_2=0$, then $\mu(x_1)=$

$(\mu\times v)(x_1,0) \le (\mu\times v)((x_1*z_1)*(y_1*z_1),0)\vee(\mu\times v)(y_1,0) = \mu((x_1*z_1)*(y_1*z_1))\vee\mu(y_1)$, which proves that μ is a DF p-ideal of X. Hence either μ or v is a DF p-ideal of X. □

Theorem 3.10. *Let v be a fuzzy subset of a BCI-algebra X and let μ_v be the smallest doubt fuzzy relation on X. Then v is a DF p-ideal of X if and only if μ_v is a DF p-ideal of $X\times X$.*

Proof. Assume that v is a DF p-ideal of X, we note that $\mu_v(0,0) = v(0)\vee v(0) \le v(x)\vee v(y)$ for all $(x,y)\in X\times X$

$$\begin{aligned}\mu_v(x_1,x_2) &= v(x_1)\vee v(x_2)\\ &\le \big(v((x_1*z_1)*(y_1*z_1))\vee v(y_1)\big)\vee\big(v((x_2*z_2)*(y_1*z_2))\vee v(y_2)\big)\\ &= \big(v((x_1*z_1)*(y_1*z_1))\vee v((x_2*z_2)*(y_2*z_2))\big)\vee\big(v(y_2)\vee v(y_2)\big)\\ &= \mu_v((x_1*z_1)*(y_1*z_1),(x_2*z_2)*(y_2*z_2))\vee\mu_v(y_1,y_2)\\ &= \mu_v((x_1*z_1,x_2*z_2)*(y_1*z_1,y_2*x_2))\vee\mu_v(y_1,y_2)\\ &= \mu_v(((x_1,x_2)*(z_1,z_2))*((y_1,y_2)*(z_1,z_2)))\vee\mu_v(y_1,y_2),\end{aligned} \tag{3.3}$$

for all $(x_1,x_2),(y_1,y_2),(z_1,z_2)\in X\times X$. Hence μ_v is a DF p-ideal of $X\times X$.

Conversely, suppose that μ_v is a DF p-ideal of $X\times X$. Then for all $(x_1,x_2)\in X\times X$, $v(0)\vee v(0) = \mu_v(0,0) \le \mu_v(x,x) = v(x)\vee v(x)$. It follows that $v(0)\le v(x)$ for all $x\in X$. Now let $(x_1,x_2),(y_1,y_2),(z_1,z_2)\in X\times X$. Then

$$\begin{aligned}v(x_1)\vee v(x_2) &= \mu_v(x_1,x_2)\\ &\le \mu_v(((x_1,x_2)*(z_1,z_2))*((y_1,y_2)*(z_1,z_2)))\vee\mu_v(y_1,y_2)\\ &= \mu_v((x_1*x_2,x_2*z_2)*(y_1*z_1,y_2*z_2))\vee\mu_v(y_1,y_2)\\ &= \mu_v((x_1,z_1)*(y_1*z_1),(x_2*z_2)*(y_2*z_2))\vee\mu_v(y_1,y_2)\\ &= \big(v((x_1*z_1)*(y_1*z_1))\vee v(y_1)\big)\vee\big(v((x_2*z_2)*(y_2*x_2))\vee v(y_2)\big).\end{aligned} \tag{3.4}$$

In particular, if we take $x_2 = y_2 = z_2 = 0$ (resp., $x_1 = y_1 = z_1 = 0$) then $v(x_1) \le v((x_1*z_1)*(y_1*z_1))\vee v(y_1)$ (resp., $v(x_2) \le ((x_2*z_2)*(y_2*z_2))\vee v(y_2)$). This completes the proof. □

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