

## Research Article

# Numerical Modeling for Engineering Analysis and Designing of Optimum Support Systems for Headrace Tunnel

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The empirical and numerical design approaches are considered very important in the viable and efficient design of support systems, stability analysis for tunnel, and underground excavations. In the present research work, the rock mass rating (RMR) and tunneling quality index (Q-system) were used as empirical methods for characterization of rock mass based on real-time geological and site geotechnical data and physical and strength properties of rock samples collected from the alignment of tunnel. The rock mass along the tunnel axis was classified into three geotechnical units (GU-1, GU-2, and GU-3). The support systems for each geotechnical unit were designed. The 2D elastoplastic finite-element method (FEM) was used for the analysis of rock mass behavior, in situ and redistribution stresses, plastic thickness around the tunnel, and performance of the design supports for the selection of optimum support system among RMR and Q supports for each geotechnical unit of tunnel. Based on results, Q support systems were found more effective for GU-1 and GU-2 as compared to RMR support systems and RMR support systems for GU-3 as compared to Q support systems.

## 1. Introduction

Modeling of rock mass is a very difficult job due to the presence of discontinuities, anisotropic, heterogeneous, and nonelastic nature of rock mass, using empirical and numerical methods [1, 2]. The complex nature and different formation make the rock masses a difficult material for empirical and numerical modeling.

During initial stages of excavation projects, the detailed data are not available about strength properties, deformation modulus, in situ stresses, and hydrological of rock masses [3]. To handle the nonavailability of the detailed project data, the empirical methods like rock mass classification systems are considered to be used for solving engineering problems [4]. The empirical methods used defined input parameters in designing of any underground structures, recommendation of support systems, and determination of input parameters

for numerical modeling [5]. The empirical methods classified the rock mass quantitatively into different classes having similar characteristics for easy understanding and construction of underground engineering structures [3]. Despite its wide applications, the empirical methods do not evaluate the performance of support systems, stress redistribution, and deformation around the tunnel [6]. Therefore, it is very important to consider these parameters in designing of optimum underground structure and support systems. This deficiency of empirical method is solved by numerical methods.

Numerical modeling is gaining more attention in the field of civil and rock engineering for prediction of rock mass response to various excavation activities [7]. The numerical methods are convenient, less costly, and less time-consuming for the analysis of redistribution stresses and their effects on the behavior of rock mass and designing of

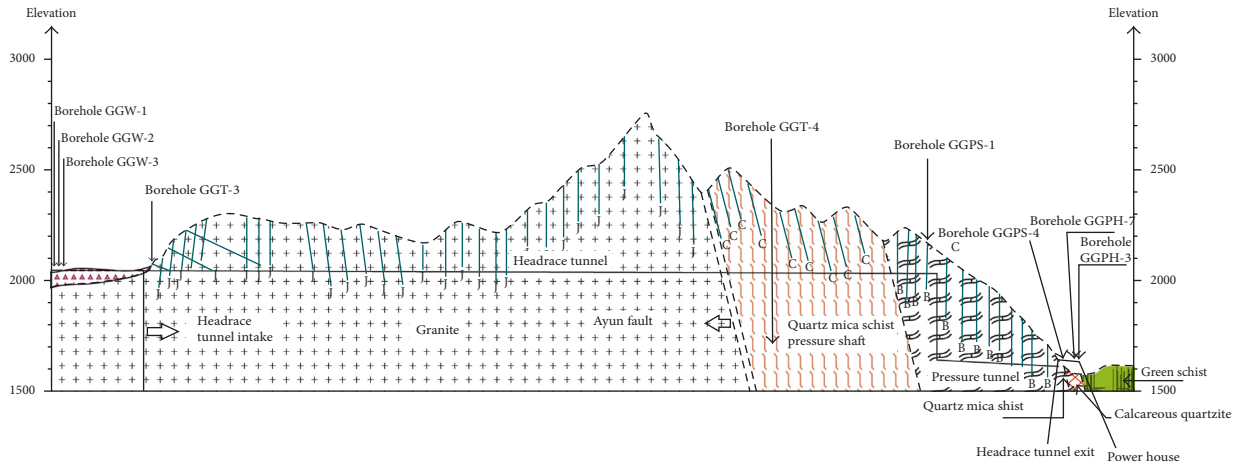


FIGURE 1: Geology and cross-sectional view of tunnel alignment [13].

structures within the rock mass environment. Numerical methods give the exact mathematical solution for the problem based on the engineering judgment and input parameters like physical and strength parameters of rock masses [8–12].

In this study, the rock mass along the tunnel axis was assessed using rock mass rating (RMR) and tunneling quality index (Q-system). The support system was recommended by these two classification systems. The rock mass behavior with the interaction of two different support systems was analyzed based on stresses, total deformation, and plastic yield thickness around the tunnel using finite-element method- (FEM-) based Phase<sup>2</sup> software for selection of an appropriate support system for tunnel, which is of great importance for the practicing engineers in the field.

## 2. Geology of Project

Golen Gol hydropower project is 106 MW. The project is to be developed on the river Golen Gol, Chitral District, Khyber Pakhtunkhwa, Pakistan. The tunnel of diameter 3.7 m of horseshoe shape is to be constructed for diversion of water from intake to power house. The surface and subsurface geology through sample collection from surface and subsurface was studied. After investigations of the tunnel geology, it is concluded that the surface and subsurface geology of the project is same, and the headrace tunnel is to be passed through granite, quartz mica schist, marble, and calcareous quartzite. The granite rock is separated from metamorphosed rocks by the unconformity/Ayun Fault, which is also very distinctly recorded. The geology and cross-sectional view of tunnel alignment is shown in Figure 1.

## 3. Rock Mass Classification

Various rock mass classification system has been developed based on civil and mining engineering case studies by different researchers, like rock mass rating (RMR), tunneling quality index (Q-system), geological strength index (GSI), new Austrian tunneling method (NATM), rock structure rating (RSR), rock quality designation (RQD), and so on for

assessment and classification of rock mass. In this research, RMR and Q systems were used due to its flexibility in terms of input parameters and widespread range for selection of support systems.

The latest version of RMR<sub>1989</sub> developed by Biniawski was used in this research [5]. This system has widespread applications in the field of mining and civil engineering. This system used uniaxial compressive strength (UCS), rock quality designation (RQD), discontinuities spacing, discontinuities condition, ground water condition, and discontinuities orientation as input parameters for characterization and classification of rock mass. The RMR is calculated by adding the rating of these six parameters.

The Q-system is developed by Bortan in 1974 at Norwegian Geotechnical Institute (NGI). The Q-system has wide applications in underground excavations and field mapping, and it depends on the underground opening and its geometry. The value of this system may be different for undisturbed and disturbed rock [14]. This system classifies the rock mass environment into different classes on the basis of the rock quality designation (RQD), joint number (J<sub>n</sub>), joint roughness number (J<sub>r</sub>), joint alteration (J<sub>a</sub>), joint water reduction factor (J<sub>w</sub>), and stress reduction factor (SRF). The values of this system indicate the quality of rock mass and give description about the stability of an excavation within the rock mass environment. The maximum value of Q-system indicates good quality of rock meaning good stability and the minimum value indicates poor quality of rock meaning poor stability. The value of Q-system is calculated by using the following formula:

$$Q = \frac{RQD}{J_n} \times \frac{J_r}{J_a} \times \frac{J_w}{SRF} \quad (1)$$

The RMR and Q classification systems were applied on bore hole data and physical and strength properties determined in laboratory of the collected rock samples along tunnel alignment. Based on the results obtained from RMR and Q system, the rock mass along the tunnel axis was divided into three geotechnical units. The results of RMR and Q classification system are presented in Table 1.

TABLE 1: Results of rock mass classification.

Average values of input parameters of RMR		1. Rock mass rating (RMR)						RMR
		Uniaxial compressive strength (UCS)	Rock quality designation (RQD)	Discontinuities spacing	Discontinuities condition	Ground water condition	Discontinuities orientation	
Bore hole	Rock type							
BH-1 at chainage 0+000 to 2+750	Granite	125.4	87%	200 to 600 mm	Rough, hard filling < 5 mm, slightly weathered, persistent < 1 mm, and aperture < 0.1 mm	Dry	Very favorable to fair	<b>75.67</b>
Rating		12	17	10	25	15	-3.33	
BH-2	Quartz mica schist	54.18	50%	<60 mm	Rough to slightly rough, hard filling < 5 mm, unweathered, persistent 1-3 m to 3-10 m, and aperture 0.1 to 1 mm	Dry	Very favorable to favorable	<b>59.00</b>
Rating		7	13	5	21	15	-2	
BH-3	Calcareous quartzite	106.7	80%	60 to 200 mm	Rough to slightly rough, hard filling < 5 mm to hard, filling > 5 mm, slightly weathered, persistent < 1 mm, and aperture 0.1 mm	Damp to dry	Very favorable to favorable	<b>71.50</b>
Rating		12	18	8	23	12.5	-2	
Average values of input parameters of RMR		2. Tunneling quality index (Q-system)						Q value
		Rock quality designation (RQD)	Joints number (Jn)	Joint roughness number (Jr)	Joint alteration (Ja)	Joint water reduction factor (Jw)	Stress reduction factor (SRF)	
Bore hole	Rock type							
BH-1 at chainage 0+000 to 2+750	Granite	88	Three joint sets plus random	Rough and irregular and undulating	Only surface staining with unaffected joint walls	Dry excavation	Low stress to medium stress	<b>12.57</b>
Rating		88	12	3	1	1	1.75	
BH-2	Quartz schist	50	Two joint sets	Rough to slickensided undulating	Unaltered joint walls and surface staining only	Dry excavation	Low stress to medium stress	<b>21.95</b>
Rating		50	4	2.11	1	1	1.23	
BH-3	Calcareous quartzite	80	One joint set plus random	Rough and irregular and undulating	Unaltered joint walls to slightly altered joint walls having nonsoftening to silty- or sandy-clay coatings	Damp to dry	Low stress to medium stress	<b>20.49</b>
Rating		80	3	3	2	0.83	1.62	

#### 4. In Situ Stresses

The in situ stresses are determined by direct and indirect methods. In direct methods, in situ stress determination methods like flat jack, overcoring and undercoring, and hydraulic fracturing are used. These methods are costly and time-consuming, the procedures used in determination of

these stresses are difficult, and the results may be questionable [9, 15, 16]. In direct methods, the developed empirical models were used for determination of vertical and horizontal stresses. In this study, the vertical stress was determined by

$$\sigma_v = \gamma H, \quad (2)$$

TABLE 2: Parameters for numerical modeling.

Geotechnical unit	Unit weight (g/cm <sup>3</sup> )	Modulus of elasticity (MPa)	Poisson's ratio ( $\nu$ )	Hoek and Brown constants			Vertical stress (MPa)	Horizontal stress (MPa)	RMR support	Q-system support
				mb	s	a				
GU-1	2.71	$3.41e^4$	0.188	7.669	0.0117	0.503	19.00	5.23	Locally, 3 m long bolts in crown of 20 mm diameter and fully grouted, spacing between bolts of 2.5 m with occasionally wire mesh, shotcrete with a thickness of 50 mm where necessary, and no steel set required	2 m long systematic bolting (fully grouted with 20 mm diameter) with a spacing of 2.32 m between bolts; fiber-reinforced sprayed concrete of 50–60 mm thickness at crown
GU-2	2.76	$3.42e^4$	0.051	1.934	0.0060	0.504	11.70	2.06	4 m long systematic bolts of 20 mm diameter and fully grouted, spacing range between bolts of 1.5–2 m in crown and walls with wire mesh in crown, shotcrete with a thickness range of between 50 mm and 100 mm in crown and 30 mm in the sides of tunnel, and no steel set required	2 m long systematic bolting (fully grouted with 20 mm diameter) with a spacing of 2.5 m between bolts; fiber-reinforced sprayed concrete of 50–60 mm thickness at crown
GU-3	2.69	$5e^4$	0.277	6.154	0.0256	0.502	11.60	3.02	Locally, 3 m long bolts in crown of 20 mm diameter and fully grouted, spacing between bolts of 2.5 m with occasionally wire mesh, shotcrete with a thickness of 50 mm where necessary, and no steel set required	2 m long systematic bolting (fully grouted with 20 mm diameter) with a spacing of 2.48 m between bolts; fiber-reinforced sprayed concrete of 50–60 mm thickness at crown

where  $\gamma$  is the unit weight of rock mass and  $H$  is the height of overburden.

The ratio between horizontal and vertical stress is  $K$ . However, it is convenient to use theoretical approach to determine horizontal stress from vertical stress. For horizontal stress determination, the following useful equation presented by [17] is used.

$$\sigma_h = \left( \frac{\nu}{1-\nu} \right) \sigma_v + \frac{\beta \text{Erm}G}{1-\nu} (H + 100), \quad (3)$$

where  $\nu$  is Poisson's ratio,  $\beta$  is the coefficient of thermal expansion and its value for rocks is  $8 \times 10^{-6}/^\circ\text{C}$  (Singh, Rao, and Ramamurthy, 2002),  $\text{Erm}$  is Young's modulus of intact rock in MPa,  $G$  is the thermal gradient of rock ( $^\circ\text{C}/\text{m}$ ).

However, the following simple relationship is adopted in this study for determination of horizontal stress:

$$\sigma_h = \left( \frac{\nu}{1-\nu} \right) \sigma_v. \quad (4)$$

The vertical and horizontal stresses were determined using (2) and (4) for each geotechnical unit. The results are presented in Table 2.

## 5. Numerical Methods

Numerical modeling in rock and civil engineering is used as a tool that facilitates the site engineers to evaluate the rock mass behavior and its effects on engineering structures and

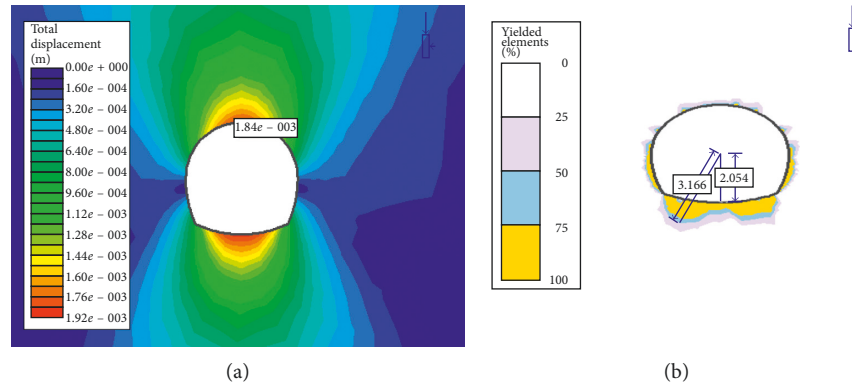


FIGURE 2: Maximum displacement and thickness of plastic zone before supports.

support systems. Numerical modeling gives a sound understanding for solving complex engineering problems related to the tunnel shape, size, mine layout, and design of roof support system to consent consistent and techno-economic feasible performance of mining structures throughout their planned life of operations [18]. The numerical modeling in rock engineering is the interesting field for research and innovations. Due to advancing of technology in the field of rock mechanics, different numerical methods like finite-difference method (FDM), finite-element method (FEM), and boundary element method (BEM) were developed by different researchers for solving engineering-related problems like design of underground openings or structures within the rock mass environment, support systems and evaluation of its performance, and analysis of stresses. Among these continuum numerical methods, the FEM is used mostly to solve rock engineering problems [19].

In FEM, the rock mass is modeled as a continuum and the discontinuities modeled discretely in the continuum model. The domain of representative model is discretized into defined elements that connect at certain points called nodes. By changing the surface/boundary conditions, the stress-strain and deformation can be analyzed. An appropriate constitutive model for material is used to define stress-strain relationship. In FEM, the models in multistage can easily be produced and analyzed quickly. It can handle material complexity and model a wide variety of support types. In finite-element analysis, liner elements are usually modeled as beam element and applied to model rock support, that is, steel sets, shotcrete, and concrete [19–21].

The numerical modeling in rock engineering is hot field for quality and innovative research [22, 23]. The FEM is used in solving the rock engineering problems such as characterization [21], design support assessment [9, 24–26], and back analysis of tunnels [27]. This method resolved complex engineering problem utilizing plane strain two-dimensional (2D) analysis, axisymmetric 2D analysis, and three-dimensional (3D) analysis.

## 6. Results and Discussions

**6.1. Input Parameters for Numerical Modeling.** FEM-based software Phase<sup>2</sup> was used for the analysis of the design

support system for the tunnel. The input parameters like physical and mechanical properties of rock mass, stresses (vertical and horizontal), deformation modulus of rock mass, and support systems recommended by RMR and Q-system as given in Table 2 were used in Phase<sup>2</sup> software. The Phase<sup>2</sup> software developed the simulated models for each defined geotechnical unit (GU). These simulated models were developed based on the following assumptions:

- (a) Supports were installed instantly after excavation.
- (b) Elastoplastic behavioral model using generalized Hoek–Brown criterion is used to simulate the models.
- (c) Tunnel model is 2D considering plane strain problem.

For numerical analysis, three-stage models were adopted to confirm the in situ ground stresses. In first stage of simulated model, ground stress distributions were examined. In the next stage, induced stress distributions, yield points, and the induced displacement were analyzed. In the final stage, behavior of the recommended support systems was investigated.

**6.2. Numerical Analysis for GU-1.** For this section, the simulated model of tunnel was developed using input parameters as given in Table 2 in Phase<sup>2</sup> software. The horizontal and vertical stresses are validated using gravity loading through simulating model before excavation. The virgin stress  $\sigma_1$  before excavation was 19.36 MPa, and  $\sigma_1$  at crown and sidewalls of tunnel is 0 MPa and 26 MPa, respectively, after excavation. The maximum virgin stress  $\sigma_3$  before was 5.35 MPa, and  $\sigma_3$  at crown and sidewalls of tunnel was 0.70 MPa and 0.70 MPa, respectively, after excavation.

For this section, the maximum stress concentration develops at sidewalls of the tunnel. The maximum deformation of 1.84 mm after excavation and before support was seen both at crown and base of the tunnel as shown in Figure 2(a). The thickness of plastic zone (yield zone of 50%) at crown and sidewalls is negligibly small; however, at the base, it is approximately 1112 mm as shown in Figure 2(b).

The recommended support systems by RMR and Q-system as discussed in Table 2 were installed in simulated models. The rock mass and support components both for

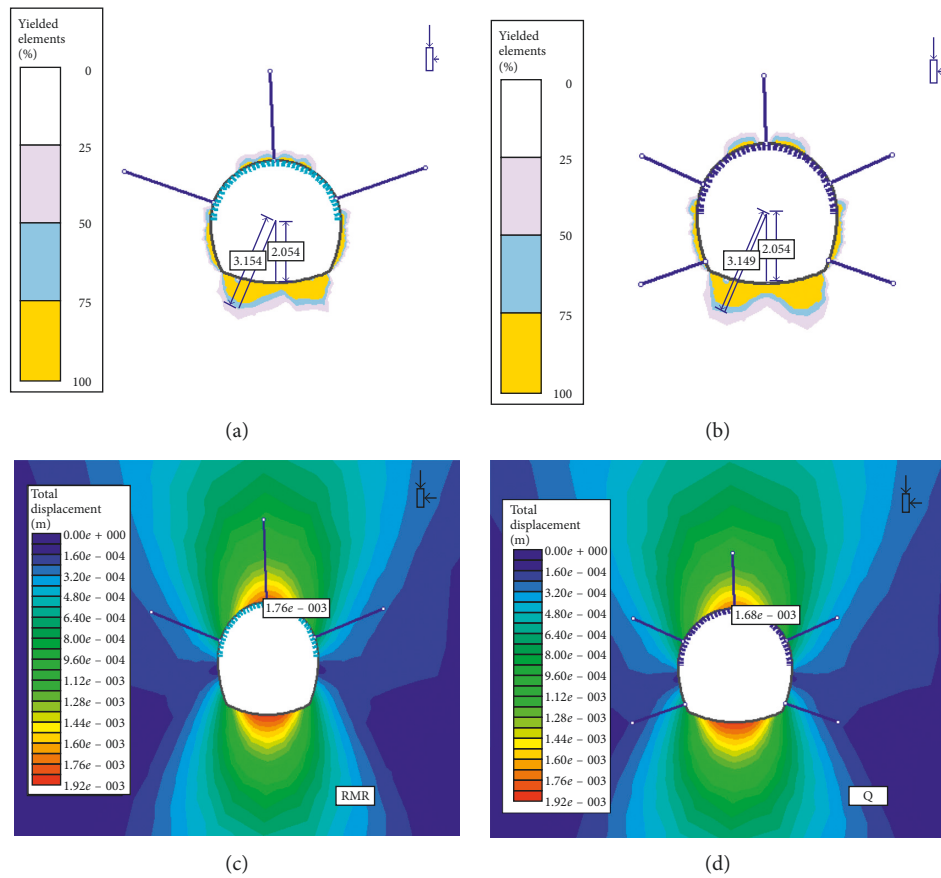


FIGURE 3: Yield elements and yield zone (50%) after RMR (a) and Q (b) supports. Deformation after RMR (c) and Q (d) supports.

RMR and Q support systems were in compression. For RMR support, the maximum axial stress in rock bolt and maximum axial force in shotcrete elements are 92.05 MPa and 0.972 MN, respectively. For Q support, the maximum axial stress in rock bolt and maximum axial force in shotcrete elements are 102.05 MPa and 4.35 MN, respectively. The total displacement in the tunnel after installation of RMR and Q recommended supports in simulated models was noted as before support, that is, 2.30 mm in case of RMR support and from 2.30 mm to 2.10 mm in case of Q support as shown in Figure 3.

After comparison and analysis of simulated models for RMR and Q supports, the maximum axial stress in rock bolts and maximum force in shotcrete for Q are greater than those for RMR support, the confining stress for Q support is greater than that for RMR support, the total displacement for Q support was found to be decreased as compared to RMR support from 1.84 mm to 1.68 mm, and the yield zone thickness decreased slightly greater in Q support than RMR support at the base of the tunnel as shown in Figure 3. Therefore, Q support seems to be more effective than RMR support for GU-1 section.

**6.3. Numerical Analysis for GU-2.** The input parameters used for simulation of models in Phase<sup>2</sup> software for this section are presented in Table 2. The horizontal and

vertical stresses are validated using gravity loading through simulating one model before excavation. The virgin stress  $\sigma_1$  before excavation is 11.84 MPa, and  $\sigma_1$  at crown and sidewalls of tunnel is 0.85 MPa and 4.25 MPa, respectively, after excavation. The virgin stress  $\sigma_3$  before excavation is 2.10 MPa, and  $\sigma_3$  at crown and sidewalls of tunnel is 0 MPa and 0 MPa, respectively, after excavation. The maximum stress concentration develops at sidewalls of the tunnel. The maximum deformation of 3.15 mm after excavation and before support is seen both at the crown and base of the tunnel as shown in Figure 4(a). The thickness of plastic zone (50%) at crown, sidewalls, and base is approximately 4638 mm, 1117 mm, and 5468 mm, respectively. The yield elements and plastic zone (50%) before supports are shown in Figure 4(b).

The recommended support systems by RMR and Q-system as discussed in Table 2 were installed in simulated models. The rock mass and support components both for RMR and Q support systems are in compression. The  $\sigma_1$ ,  $\sigma_3$ , yield elements, and plastic zone around the tunnel was found to be improved after installation of Q supports in simulated models as compared to RMR supports as shown in Figures 5(a) and 5(b). For RMR support, the maximum axial stress in rock bolt and maximum axial force in shotcrete elements are 193.24 MPa and 5.35 MN, respectively. For Q support, the maximum axial stress in

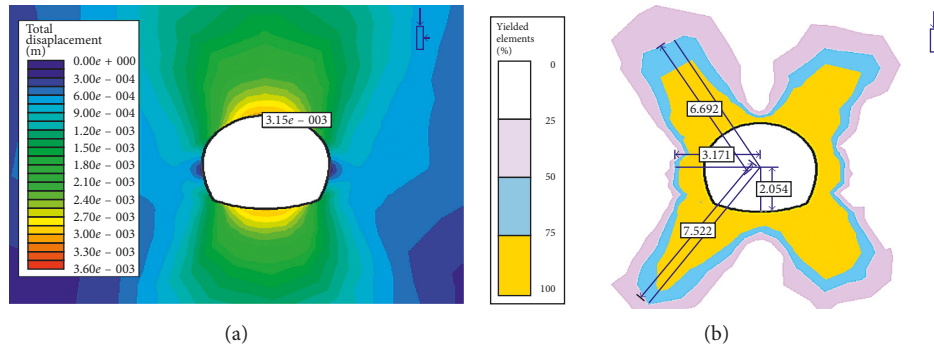


FIGURE 4: Total displacement (a) and yield elements (b) with plastic zone (50%) before supports.

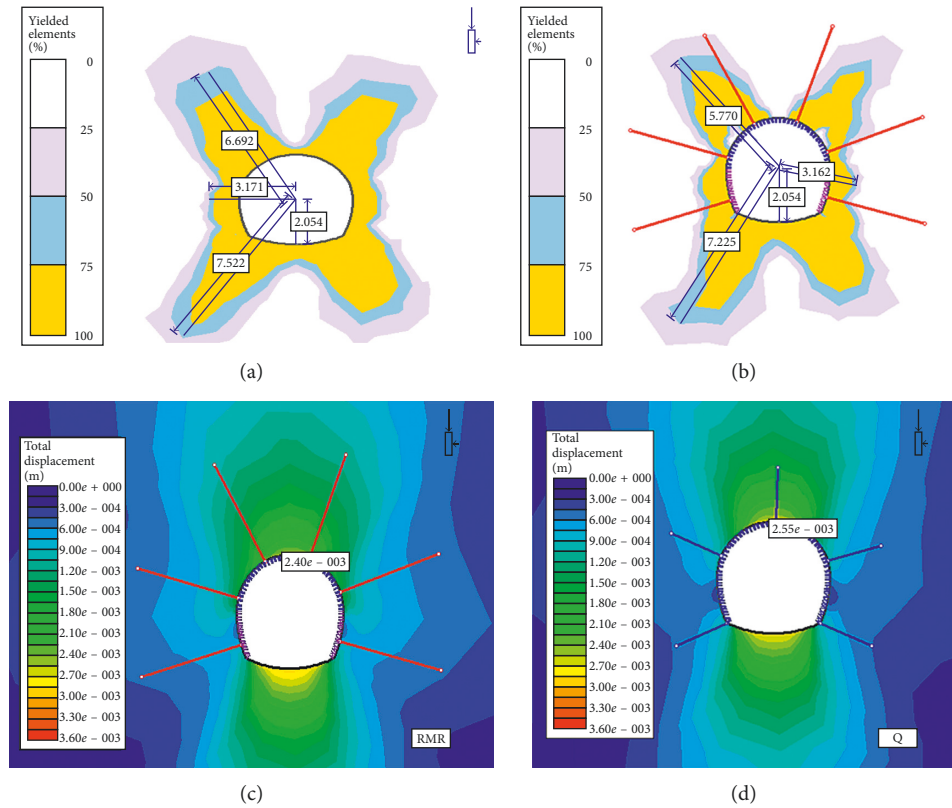


FIGURE 5: Yield elements and yield zone (50%) after RMR (a) and Q (b) supports. Total deformation after RMR (c) and Q (d) supports.

rock bolt and maximum axial force in shotcrete elements are 119.82 MPa and 3.06 MN, respectively. The total displacement in tunnel after installation of RMR and Q recommended supports in simulated models decreased from 3.15 mm to 2.40 mm in the case of RMR support and from 3.15 mm to 2.55 mm in the case of Q support as shown in Figure 5.

After comparison and analysis of simulated models for Q supports, the axial stress in rock bolts is less than RMR supports, the confining stress for Q support is greater than RMR support, the plastic zone for Q support is more improved than RMR support, and the total displacement decreases approximately same for RMR and Q supports. Therefore, the Q support seems to be more effective than RMR support for GU-2 section.

6.4. Numerical Analysis for GU-3. The input parameters used for simulation of models in Phase<sup>2</sup> software for this section are presented in Table 2. The horizontal and vertical stresses are validated using gravity loading through simulating one model before excavation. The virgin stress  $\sigma_1$  before excavation is 11.52 MPa, and  $\sigma_1$  at crown and sidewalls of tunnel is 0 MPa and 21 MPa, respectively, after excavation. The virgin stress  $\sigma_3$  before and  $\sigma_3$  at crown and sidewalls of tunnel are 0.20 MPa and 0.20 MPa, respectively, after excavation. For this section, the maximum stress concentration develops at sidewalls of the tunnel as shown in above figures. The maximum deformation of 0.990 mm after excavation and before support is seen both at crown and base of the tunnel as shown in Figure 6(a). The thickness of plastic zone (50%) at

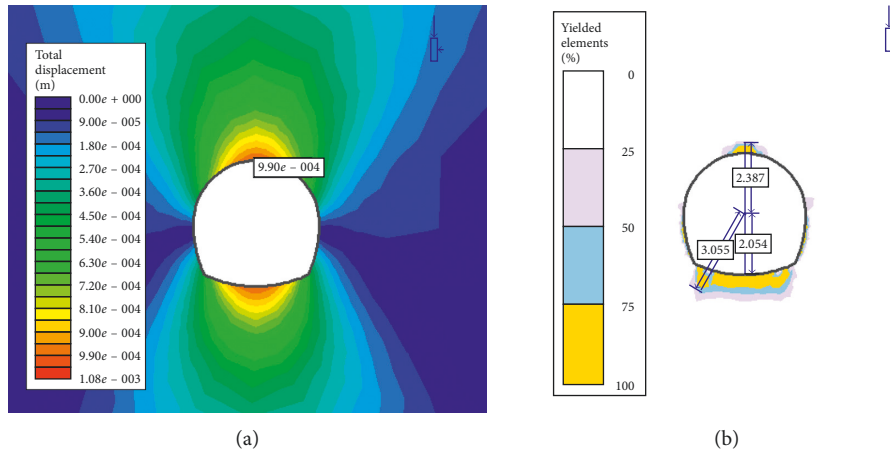


FIGURE 6: Total displacement (a) and yield elements (b) with plastic zone (50%) before supports.

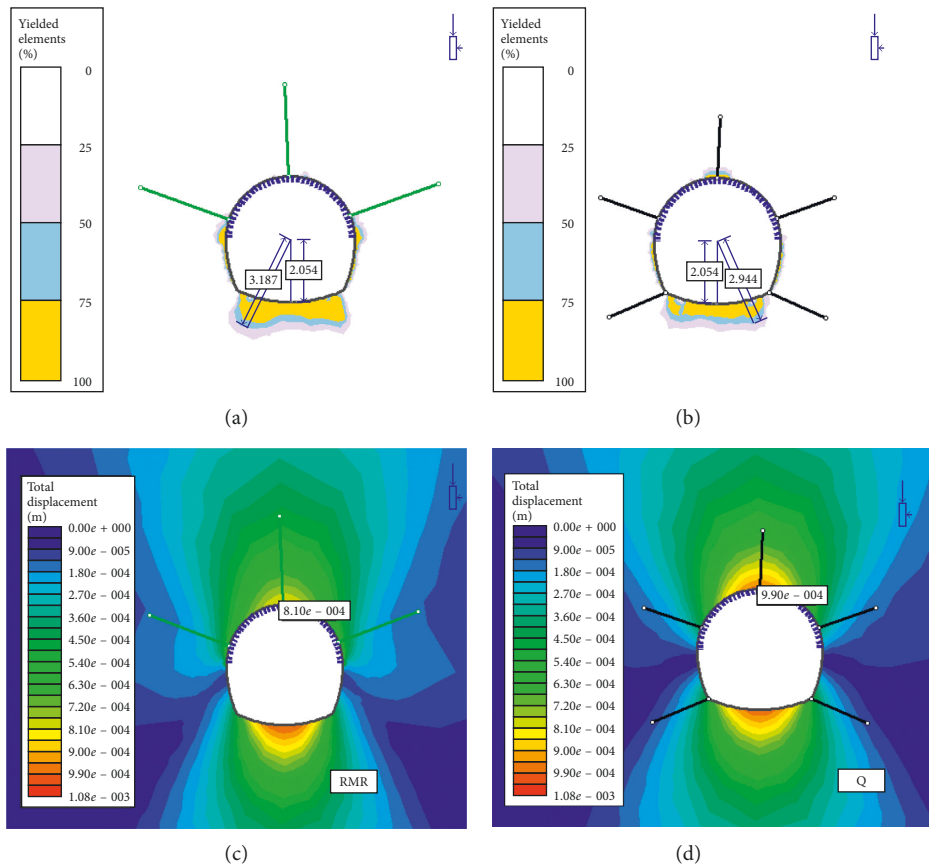


FIGURE 7: Yield elements and yield zone (50%) after RMR (a) and Q (b) supports. Total deformation after RMR (a) and Q (d) supports.

crown is approximately 333 mm, and sidewall is negligibly small; however, at the base is approximately 1001 mm. The yield zone and yield elements before supports are shown in Figure 6(b).

The recommended support systems by RMR and Q-system as discussed in Table 2 were installed in simulated models. The rock mass and support components both for RMR and Q support systems are in compression. For RMR support, the maximum axial stress in rock bolt and

maximum axial force in shotcrete elements are 34.50 MPa and 8.52 MN, respectively. For Q support, the maximum axial stresses in rock bolt and maximum axial force are 46.70 MPa and 1.13 MN, respectively. The thickness of plastic deformation was decreased after installation of RMR supports as compared to Q supports as shown in Figures 7(a) and 7(b), respectively. The total displacement in tunnel after installation of RMR and Q recommended supports in simulated models was found to be decreased from 0.990 mm



to 0.810 mm in the case of RMR support and not decreased in the case of Q support as shown in Figure 7.

## 7. Conclusions

In this research, the empirical and numerical methods were used to evaluate rock mass quality and estimate the support element required for headrace tunnel and stability analysis of tunnel before and after support system installation for selection of optimum support systems. The stability analysis of models developed for each geotechnical unit in Phase<sup>2</sup>, was carried out after installment of Q and RMR support systems. For Q support, the total displacement reduced from 1.84 mm to 1.68 mm and from 3.15 mm to 2.55 mm and did not reduce in GU-3, respectively; the maximum axial stresses in rock bolt and maximum axial force were observed as 102.05 MPa and 4.35 MN for GU-1, 119.82 MPa and 3.06 MN for GU-2, and 46.70 MPa and 1.13 MN, respectively, for GU-3; and 50% plastic zone thickness maximum reduced for GU-1 at base from 1112 mm to 1095 mm, for GU-2 at crown from 4638 mm to 3716 mm, and for GU-3 at base from 1001 mm to 894 mm. For RMR support systems, the total displacement reduced from 1.84 mm to 1.76 mm, from 3.15 mm to 2.40 mm, and from 0.990 mm to 0.810, respectively; the maximum axial stresses in rock bolt and maximum axial force were observed as 92.05 MPa and 0.972 MN for GU-1, 193.24 MPa and 5.35 MN for GU-2, and 34.50 MPa and 8.52 MN, respectively, for GU-3; and 50% plastic zone thickness maximum reduced for GU-1 at base from 1112 mm to 1100 mm, and for GU-2 and GU-3, it did not reduce. Based on analysis and comparison of results, it is concluded that Q support system seem to be good for GU-1 and GU-2 and RMR support system for GU-3.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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