Hindawi
Advances in High Energy Physics
Volume 2017, Article ID 8627918, 6 pages
https://doi.org/10.1155/2017/8627918

Research Article

Supersymmetry across the Hadronic Spectrum

Hans Günter Dosch

Institut für Theoretische Physik der Universität Heidelberg, Heidelberg, Germany

Correspondence should be addressed to Hans Günter Dosch; h.g.dosch@gmail.com

Received 16 May 2017; Accepted 20 June 2017; Published 30 August 2017

Academic Editor: Thierry Grandou

 The publication of this article was funded by SCOAP^3.

Semiclassical light-front bound-state equations for hadrons are presented and compared with experiment. The essential dynamical feature is the holographic approach; that is, the hadronic equations in four-dimensional Minkowski space are derived as holograms of classical equations in a 5-dimensional anti-de Sitter space. The form of the equations is constrained by the imposed superconformal algebra, which fixes the form of the light-front potential. If conformal symmetry is strongly broken by heavy quark masses, the combination of supersymmetry and the classical action in the 5-dimensional space still fixes the form of the potential. By heavy quark symmetry, the strength of the potential is related to the heavy quark mass. The contribution is based on several recent papers in collaboration with Stan Brodsky and Guy de Téramond.

1. Introduction

Semiclassical approximations play an essential role in quantum field theories. Think of atomic or molecular physics: in spite of all refinement of renormalized QED, the semiclassical Dirac and Schrödinger equations not only are of enormous practical advantage but also are essential in order to exhibit structural properties.

The search for semiclassical approximations even in strongly interacting quantum field theories received a significant advance through the Maldacena conjecture [1], which postulates the correspondence of a superconformal gauge quantum field theory in 4 dimensions with a classical gravitational theory in a 5-dimensional anti-de Sitter space (AdS_5). Here a light-front Hamiltonian for meson and boson wave functions is presented, which is based on the holographic AdS/CFT correspondence and receives essential dynamical constraints from an implemented superconformal algebra.

The kinematical frame is based on the light-front (LF) quantization at fixed LF time $x_+ = x^0 + x^3$; I refer for the essential features and the importance of this frame to the contribution of Stan Brodsky in the same volume. In the two constituent sectors, an essential variable is the boost invariant transverse separation $\zeta = \sqrt{x_1 x_2}|\vec{b}_{\perp,2} - \vec{b}_{\perp,1}|$, where x_i and $\vec{b}_{\perp,i}$, $i = 1, 2$, are the longitudinal momentum fractions and the transverse coordinates of the two constituents, respectively.

Since in the holographic correspondence there is only one variable to describe the internal structure of hadrons, namely, the coordinate of the 5th dimension, hadrons with more than two constituents have to be treated as clusters. In that case, one introduces the effective longitudinal momentum fraction ${x_a}^{\text{eff}} = \sum_{i=1}^{N_a} x_i$, where N_a is the number of constituents in the cluster a, and correspondingly one introduces an effective transverse coordinate $\vec{b}_{\perp,a}^{\text{eff}} = (\sum_{i=1}^{N_a} x_i \vec{b}_{\perp,i})/{x_a}^{\text{eff}}$. The effective boost invariant separation is then $\zeta = \sqrt{{x_a}^{\text{eff}} {x_b}^{\text{eff}}}|\vec{b}_{\perp,b}^{\text{eff}} - \vec{b}_{\perp,a}^{\text{eff}}|$. There is no theoretical limit for the number of the constituents in the cluster.

In the conformal limit (massless quarks), the LF Hamiltonian in the two-particle sector does not contain the longitudinal variable x_i or ${x_i}^{\text{eff}}$ explicitly:

$$H = -\frac{d^2}{d\zeta^2} + \frac{4L^2 - 1}{4\zeta^2} + U(\zeta); \tag{1}$$

here, L is the LF angular momentum. In this approach, the squared hadron masses are the eigenvalues of the LF Hamiltonian.

In order to get some insight into the LF potential $U(\zeta)$, which is in principle determined by the QCD Lagrangian, we turn to the dynamical scheme of AdS_5/CFT correspondence.

AdS_5 is a maximally symmetric 5-dimensional space with the metric

$$(ds)^2 = \frac{R^2}{z^2}\left(\left(dx^0\right)^2 - \left(dx^1\right)^2 - \left(dx^2\right)^2 - \left(dx^3\right)^2 - (dz)^2\right), \tag{2}$$

where the coordinate of the 5th dimension z is the holographic variable.

From the AdS_5 action, one derives the field equations of AdS, which are interpreted as bound-state wave equations. As noticed and exploited first by Brodsky and de Téramond [2], the structure of the field equations from the AdS_5 action is equal to the structure of the LF Hamilton, if one identifies the holographic variable z with the boost invariant separation ζ. From unperturbed AdS, one obtains a vanishing LF potential. This is no wonder: the space AdS_5 has maximal symmetry that in turn leads to a conformal symmetry in the holographically corresponding 4-dimensional quantum field theory and no scale can appear.

In order to introduce interaction, one has to break the maximal symmetry. This is done by introducing a dilaton term $e^{\varphi(z)}$ into the AdS action. Due to the occurrence of covariant derivatives in the AdS Lagrangian, the derivation of the Euler-Lagrange equations is rather tedious, but finally one obtains for a meson wave function [3] $\Phi_{\mu1\cdots\mu_J} = \epsilon^J_{\mu1\cdots\mu J}\phi_{J,L}(\zeta)$ the bound-state equation:

$$\begin{gathered}\left(-\frac{d^2}{d\zeta^2} + \frac{4L^2-1}{4\zeta^2} + U(\zeta)\right)\phi_J(\zeta) = M^2\phi_J(\zeta)\\ \text{with } U(\zeta) = \frac{1}{2}\varphi''(\zeta) + \frac{1}{4}\varphi'(z)^2 + \frac{2J-3}{2\zeta}\varphi'(\zeta);\end{gathered} \tag{3}$$

here, $\epsilon^J_{\mu1\cdots\mu J}$ is the polarization tensor of a meson with total angular momentum J; the LF angular momentum L is related to the AdS mass by $(mR)^2 = -(2-J)^2 + L^2$.; the holographic variable z is identified with the LF separation ζ.

For Baryons, a dilaton factor $e^{\varphi(\zeta)}$ does not lead to an interaction and there one has to introduce an additional Yukawa term $\overline{u}V(\zeta)u$ in the 5-dimensional fermionic AdS action. The wave function is split into a positive-chirality and a negative-chirality component:

$$\begin{aligned}u_{\mu_1\cdots\mu_{J-1/2}}(p) &= \frac{1+\gamma_5}{2}u_{\ldots}(p)\psi_{B+}(\zeta)\\ &\quad+\frac{1-\gamma_5}{2}u_{\ldots}(p)\psi_{B-}(\zeta)\end{aligned} \tag{4}$$

and one obtains the wave equations [3] with the potential $U_\pm = V^2 \pm V' + ((1+2L)/\zeta)V$:

$$\begin{aligned}&\left(-\frac{d^2}{d\zeta^2} + \frac{4(L+1)^2-1}{4\zeta^2} + U_-(\zeta)\right)\psi_{B-}(\zeta)\\ &\quad = M^2\psi_{B-}(\zeta),\end{aligned} \tag{5}$$

$$\left(-\frac{d^2}{d\zeta^2} + \frac{4L^2-1}{4\zeta^2} + U_+(\zeta)\right)\psi_{B+}(\zeta) = M^2\psi_{B+}(\zeta). \tag{6}$$

For phenomenological reasons one has chosen ad hoc [4–8]

$$e^{\varphi(\zeta)} = e^{\lambda_M z^2}, \quad V(\zeta) = \lambda_B z; \tag{7}$$

which leads to the potential for

$$\textit{mesons}: U(\zeta) = \lambda_M^2\zeta^2 + 2\lambda_M(J-1), \tag{8}$$

$$\textit{baryons}: U_\pm(\zeta) = \lambda_B^2\zeta^2 + 2L\lambda_F + \lambda_F\left(\frac{1}{2} \pm \frac{1}{2}\right). \tag{9}$$

These potentials lead to linear Regge trajectories and a massless pion. For the Delta trajectory, however, one has to choose formally half-integer twist (i.e., to add 1/2 to the angular momentum L) in order to get agreement with the data.

2. Constraints Given by the Superconformal Graded Algebra

Although one obtains in that way a satisfactory description of several essential features of hadron spectroscopy, there are some points that are not so satisfactory. First, and this is the most important shortcoming, the special choice of maximal symmetry breaking given in (7) is completely arbitrary and has no theoretical foundation. Second, the symmetry breaking for mesons and baryons is completely unrelated, and finally the remarkable fact that the Regge slopes of mesons and baryons are very similar, which implies that $\lambda_M \approx \lambda_B$ in (7), is in this context completely fortuitous.

In the search for further dynamical constraints, we go back to the original Maldacena correspondence [1] of a classical 5-dimensional theory and a 4-dimensional quantum field theory. There the latter is heavily constrained by symmetries: it is superconformal; that is, it obeys supersymmetry and conformal symmetry. Since our goal is to find a semiclassical approximation to QCD, it is not too far-fetched to investigate *superconformal* quantum mechanics (QM).

Supersymmetric QM is very simple [9]. It is built on a graded algebra consisting of two fermionic operators (supercharges), Q and $Q^\dagger$, and a bosonic operator (the Hamiltonian), H, with the anticommutation and commutation relations:

$$\begin{gathered}\left\{Q, Q^\dagger\right\} = H,\\ \{Q, Q\} = \left\{Q^\dagger, Q^\dagger\right\} = 0;\\ [Q, H] = \left[Q^\dagger, H\right] = 0.\end{gathered} \tag{10}$$

A matrix realization in a Hilbert space with two components is

$$\begin{gathered}Q = \begin{pmatrix}0 & q\\ 0 & 0\end{pmatrix},\\ Q^\dagger = \begin{pmatrix}0 & 0\\ q^\dagger & 0\end{pmatrix},\end{gathered} \tag{11}$$

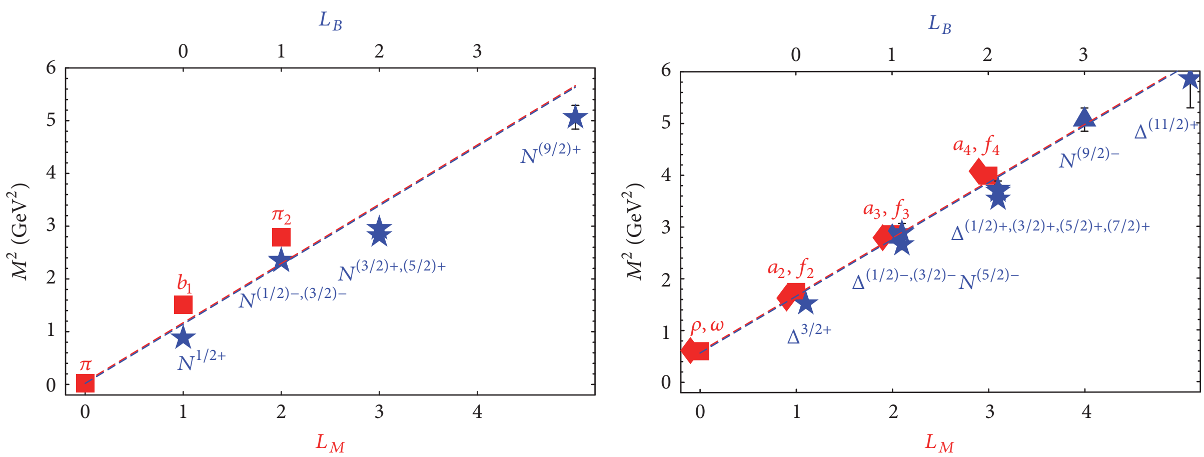

FIGURE 1: The $\pi - N$ trajectory and the $\rho - \Delta$ trajectory from [10].

$$q = -\frac{d}{dx} + \frac{f}{x} + V(x),$$
$$q^\dagger = \frac{d}{dx} + \frac{f}{x} + V(x), \tag{12}$$

where f is a dimensionless constant and V is the arbitrary superpotential.

If the superpotential $V = 0$, then there is no scale present and also conformal symmetry holds: one can extend the supersymmetric (graded) algebra 9 to a superconformal one by introducing the additional supercharges S and $S^\dagger$, which can be viewed as a square root of the special conformal transformation K, since $\{S, S^\dagger\} = K$; for more details, see [10, 11]. In matrix notation, one can write

$$S = \begin{pmatrix} 0 & x \\ 0 & 0 \end{pmatrix},$$
$$S^\dagger = \begin{pmatrix} 0 & 0 \\ x & 0 \end{pmatrix}. \tag{13}$$

Following the procedure introduced by de Alfaro et al. [12] for the conformal algebra and extended by and Fubini and Rabinovici [13] to the superconformal algebra, one can construct a new Hamiltonian G inside the superconformal algebra:

$$G = \left\{R_\lambda, R_\lambda^\dagger\right\} \quad \text{with } R_\lambda = Q + \lambda S,\; R_\lambda^\dagger = Q^\dagger + \lambda S^\dagger. \tag{14}$$

In this generalization of the Hamiltonian, a scale λ appears naturally, since the supercharges S and Q have different physical dimensions.

The new Hamiltonian is diagonal with

$$G_{11} = -\frac{d^2}{dx^2} + \frac{4(f+1/2)^2 - 1}{4x^2} + \lambda^2 x^2 + 2\lambda\left(f - \frac{1}{2}\right), \tag{15}$$

$$G_{22} = -\frac{d^2}{dx^2} + \frac{4(f-1/2)^2}{4x^2} + \lambda^2 x^2 + 2\lambda\left(f + \frac{1}{2}\right). \tag{16}$$

By construction, the eigenvalues of the two operators are identical and the operator $R_\lambda^\dagger$ transforms an eigenstate of G_{11} with eigenvalue $E \neq 0$ into an eigenfunction of G_{22}.

If we identify $f = L + 1/2$ and $x = \zeta$, it turns out that the potentials occurring in the new Hamiltonian in (15) and (16) are exactly the potentials following from the ad hoc introduced Yukawa term in (7) in the fermion action. Furthermore, the operator G_{11} is the Hamiltonian of a meson with $L = J = f + 1/2$ (see (8)). And G_{22} is the Hamiltonian of the baryon component ψ_{B+} with $L = f - 1/2$ (see (9)). That is, the meson eigenfunction of G_{11} with LF $L_M = f + 1/2$ has the same eigenvalue as a baryon with $L_B = f - 1/2$, eigenfunction of G_{22}. The supercharge $R_\lambda^\dagger$ transforms the meson wave function into a wave function of the mass-degenerate baryon. A meson with $L_M = f + 1/2$ is superpartner of baryon with $L_B = f - 1/2$; mesons with $L_M = 0$ (π, *e.g.*) have no superpartner, since the supercharge $R_\lambda^\dagger$ annihilates its wave function.

3. Spin Effects

In the superconformal quantum mechanical approach, no internal quark spin effects are foreseen, in contrast to the AdS$_5$ approach for mesons. The meson Hamiltonian derived in the frame of the superconformal QM, G_{11}, differs from the one derived from AdS by the constant λs, where s is the total quark spin of the meson. Supersymmetry demands the occurrence of the same term also for baryons; that is, we arrive at the following expression for the supersymmetric Hamiltonian for mesons and baryons:

$$G = \left\{R_\lambda, R_\lambda^\dagger\right\} + \lambda s \mathbf{I}, \tag{17}$$

where for π and its SUSY partner, namely, the nucleon, we have $s = 0$, and for ρ and its SUSY partner Δ, we have $s = 1$. Generally, for baryons, s is the lowest possible internal spin of the cluster. The results for the $\pi - N$ and the $\rho - \Delta$ trajectories are displayed in Figure 1; they show especially for Δ and ρ the nearly perfectly realized symmetry.

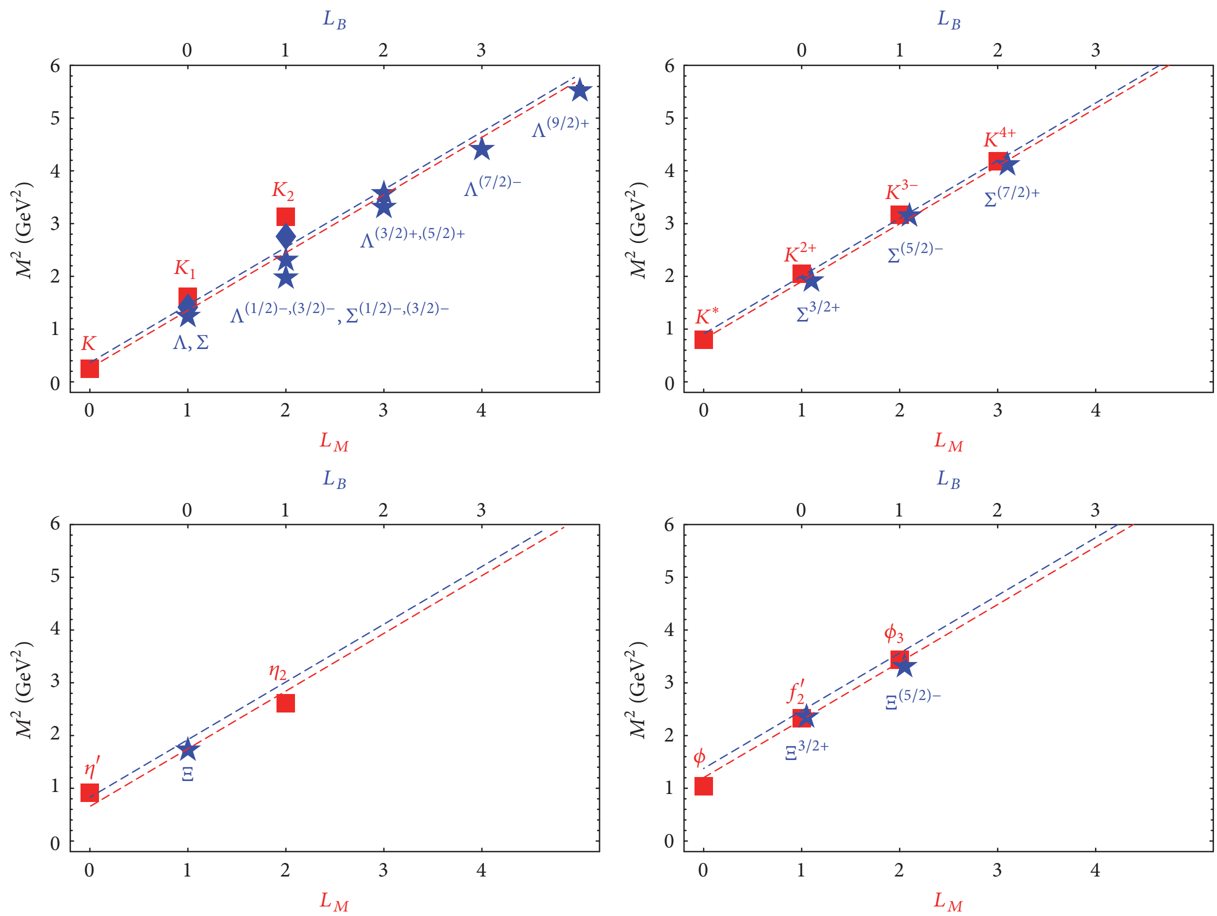

FIGURE 2: The K, Λ; K^*, Σ^*; η', Ξ; and ϕ, Ξ^* trajectories, from [14].

4. Completing the Supersymmetric Multiplet

The supersymmetric quadruplet containing the meson and the baryon is not yet complete, since the partner of negative chirality component of the baryon, ψ_{B-}, is missing. We can complete the supersymmetric multiplet by applying the fermion operator $R_\lambda^\dagger$ to that negative-chirality component baryon wave function and thus obtain a bosonic state. This operator, which transforms the meson wave function in the positive-chirality baryon wave function, can be interpreted as transforming a quark into an antidiquark cluster in the colour representation $\{3\}$. Therefore the operator $R_\lambda^\dagger$ applied to the negative-chirality component of a baryon will give a wave function $\phi_T = R_\lambda^\dagger \psi_{B-}$, which can be interpreted as that of a tetraquark consisting of a diquark cluster and an antidiquark cluster. The SUSY quadruplet can be arranged in a 2×2 matrix in the following way: $\begin{pmatrix} \phi_M(L_B+1) & \psi_{B-}(L_B+1) \\ \psi_{B+}(L_B) & \phi_T(L_B) \end{pmatrix}$.

Candidates for complete supermultiplets with fitting quantum numbers $I(J^P)$ are

> $b_1(1235)1(1^+)$ and $N_{+,-}(940)(1/2)(1/2^+)$ and as tetraquark $f_0(980)0(0^+)$,
>
> $a_2(1320)1(2^+)$ and $\Delta_{+,-}(1230)(3/2)(3/2^+)$ and as tetraquark $a_1(1260)1(1^+)$.

A special feature of tetraquarks is that states consisting of a spin-singlet and a spin-triplet cluster are degenerate in G parity.

5. Beyond the Chiral Limit

It is reasonable to assume that small quark masses do not affect superconformal dynamics; that is, the LF potentials remain unchanged and only the invariant mass is affected; that is, the chiral expression $\sum_i (k_{\perp i}^2/x_i)$ has to be replaced by $\sum_i ((k_{\perp i}^2 + m_i^2)/x_i)$. To first approximation, this gives an additive term to the squared hadron mass [14–16] for mesons $M_M^2 = 4\lambda(n + L_M) + 2\lambda s + \Delta M_2^2(m_1, m_2)$ and for baryons: $M_B^2 = 4\lambda(n + L_B + 1) + 2\lambda s + \Delta M_3^2(m_1, m_2, m_3)$, where $\Delta M^2[m_1, \ldots, m_n] = (\lambda^2/F)(dF/d\lambda)$ with $F[\lambda] = \int_0^1 \cdots \int dx_1 \cdots dx_n e^{-(1/\lambda)(\sum_i^n (m_i^2/x_i))} \delta(\sum_i^n x_i - 1)$.

From π and K mass, the light quark masses are determined to $m_q = 0.046$ GeV and $m_s = 0.357$ GeV; the resulting spectra are shown in Figure 2.

6. Strong Breaking of Conformal Symmetry by Heavy Quark Masses

In heavy-light hadrons, where one constituent is heavy and the other is light, we have still ultrarelativistic kinematics,

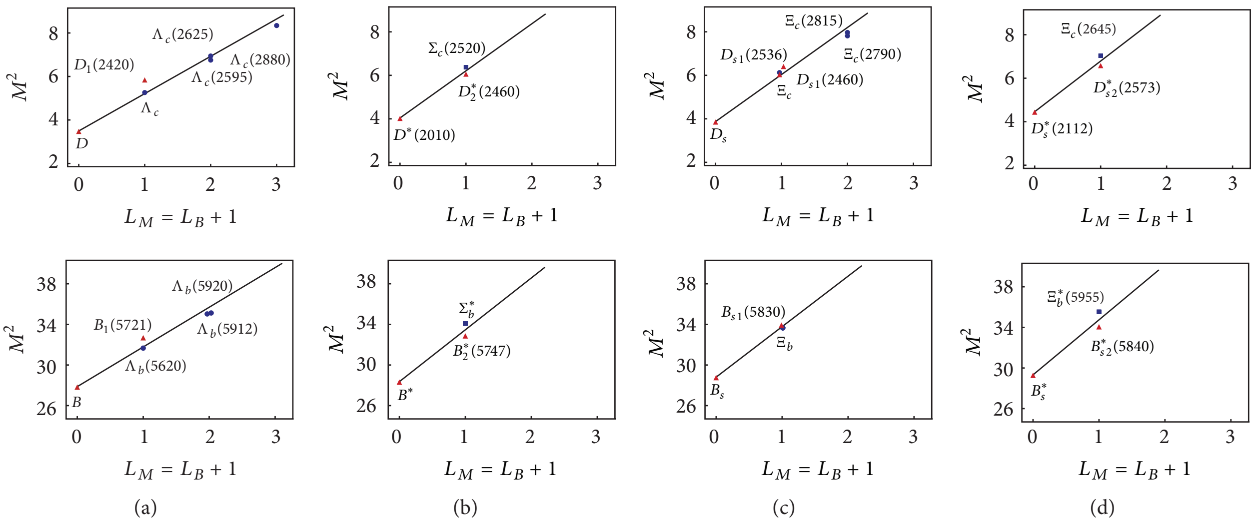

FIGURE 3: Trajectories for heavy-light hadrons with strongly broken conformal symmetry, from [14].

but conformal symmetry is strongly broken. Supersymmetry, however, can still survive and can give, together with AdS/CFT breaking, crucial dynamical constraints.

Let us go back to the SUSY algebra in (11) and (12). From $H = \{Q, Q^\dagger\}$, we can express the mesonic LF potential in terms of $V(x)$. For a meson with $J_M = L_M = L + 1$, we obtain

$$U_{\text{Susy}}(\zeta) = V^2 + \frac{2L+1}{\zeta} V - V'. \tag{18}$$

With a dilaton term $e^{\varphi(\zeta)}$, we obtain, from the AdS/CFT action for the same meson,

$$U_{\text{dil}}(\zeta) = \frac{1}{4}\left(\varphi'\right)^2 + \frac{2L_M - 3}{2\zeta}\varphi' + \frac{1}{2}\varphi''. \tag{19}$$

It has been shown in [14] that the two expressions are only compatible if

$$\begin{aligned} \varphi(\zeta) &= \frac{1}{2}\lambda A \zeta^2 + B, \\ V(\zeta) &= \frac{1}{2}\lambda A \zeta. \end{aligned} \tag{20}$$

Thus, the combination of SUSY with AdS constraints implies that the LF potential even for strongly broken conformal invariance is harmonic; the strength, however, is not determined. The resulting hadronic spectrum is for mesons $M^2 = 4\lambda_Q(n+L) + \mu_2^2$ and for baryons $M^2 = 4\lambda_Q(n+L+1) + \mu_3^2$.

In Figure 3, the heavy-light hadrons together with linear trajectories are displayed. In all cases where several recurrences are observed, the data are within the expected accuracy compatible with the predicted harmonic potential. The approximate masses of many states, which are still to be observed, can be predicted in that way [14]. The average values for the scale parameters for hadrons, which contain one c-quark or b-quark, are $\sqrt{\lambda_c} = 0.71 \pm 0.04\,\text{GeV}$ and $\sqrt{\lambda_b} = 1.06 \pm 0.1\,\text{GeV}$.

7. Constraints from Heavy Quark Symmetry

If the mass of the heavy quark M_Q goes to ∞, it decouples and heavy quark symmetry (HQS) holds [17]. This has as a consequence that the decay constant of the heavy-light pseudoscalar meson, f_M, scales in that limit with the inverse square root of the meson mass [17, 18]:

$$\sqrt{M_M} f_M \longrightarrow C. \tag{21}$$

With mass dependence of the wave functions modified according to the modification of the invariant mass discussed above, we can include the mass dependence into the wave function:

$$\psi_{n,L}^{(m)} = \frac{1}{N_m} e^{-(1/2\lambda)\sum_{i=1}^{n}(m_i^2/x_i)} \psi_{n,L}^{(0)}, \tag{22}$$

where $\psi_{n,L}^{(0)}$ is the wave function in the case of massless quarks.

From this wave function, one derives the decay constant:

$$\begin{aligned} f_M = {} & \frac{1}{\sqrt{\int_0^1 dx\, e^{-m_Q^2/\lambda(1-x)}}} \\ & \cdot \sqrt{\frac{\lambda}{\pi}} \int_0^1 dx\, e^{-m_Q^2/2\lambda(1-x)} \sqrt{x(1-x)}. \end{aligned} \tag{23}$$

Performing an integration by steepest descent, one obtains

$$f_M = \sqrt{\frac{6}{\pi e}} \left(1 + \operatorname{erf}\left(\frac{1}{2}\right)\right) \frac{\lambda^{3/2}}{m_Q^2}. \tag{24}$$

Taking into account the fact that, in the limit of $m_Q \to \infty$, the heavy quark mass and the meson mass become equal, comparison of this equation with (21) yields that $\lambda_Q \sim M_M$. Thus, HQS demands that, for heavy quark masses, the scale parameter λ increase. It relates λ_b with λ_c even in a quantitative way, as can be seen from Figure 4.

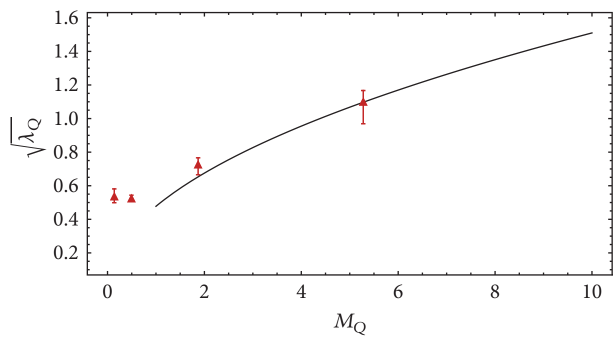

FIGURE 4: The scale parameter $\sqrt{\lambda}$ for different pseudoscalar mesons versus the meson mass. The solid line is the functional dependence $\sqrt{\lambda} \sim \sqrt{M_M}$ as predicted by HQS.

8. Conclusions

We were motivated by the Maldacena conjecture, which relates a classical gravitational theory in AdS_5 to a superconformal colour gauge QFT with infinitely many colours. Our approach, which aimed at a semiclassical approximation, led to superconformal quantum mechanics on the light front. The form of the light-front potential is determined by the superconformal algebra. This model describes essential features of light hadron spectra: linear trajectories in orbital and radial excitations and a massless pion. The observed similarity of meson and baryon spectra is a consequence of the supersymmetry between the wave functions. We also predict tetraquarks with definite masses and quantum numbers.

The supersymmetry seems to survive even in the case where conformal symmetry is strongly broken by heavy quark masses. Here, the supersymmetry, together with the embedding into AdS_5, implies a harmonic potential $\lambda\zeta^2$ also for heavy-light hadrons, but the scale parameter $\sqrt{\lambda}$ increases with heavy quark mass, in quantitative accordance with heavy quark symmetry.

Our model does not contain colour degrees of freedom, but the fact that all states have zero width is well compatible with the $N_C \to \infty$ limit of QCD; the accuracy, which is generally about 10%, is also compatible with the application of this limit to an $N_C = 3$ theory. The model can also be applied beyond spectroscopy [8, 19], but in this case additional assumptions and input are necessary.

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this paper.

References

[1] J. Maldacena, "The large N limit of superconformal field theories and supergravity," in *Proceedings of the Second Meeting on Trends in Theoretical Physics*, pp. 51–63, Buenos Aires, Argentina.

[2] S. J. Brodsky and G. F. de Téramond, "Light-front hadron dynamics and AdS/CFT correspondence," *Physics Letters B*, vol. 582, no. 3-4, pp. 211–221, 2004.

[3] G. F. de Téramond, H. G. Dosch, and S. J. Brodsky, "Kinematical and dynamical aspects of higher-spin bound-state equations in holographic QCD," *Physical Review D*, vol. 87, no. 7, 2013.

[4] A. Karch, E. Katz, D. T. Son, and M. A. Stephanov, "Linear confinement and AdS/QCD," *Physical Review D*, vol. 74, no. 1, Article ID 015005, 2006.

[5] S. J. Brodsky and G. F. de Téramond, "Hadronic spectra and light-front wavefunctions in holographic QCD," Tech. Rep. SLAC-PUB-11716, 2006.

[6] I. Kirsch, "Spectroscopy of fermionic operators in AdS/CFT," *Journal of High Energy Physics*, vol. 2006, no. 09, pp. 052-052, 2006.

[7] Z. Abidin and C. E. Carlson, "Gravitational form factors of vector mesons in an AdS/QCD model," *Physical Review D*, vol. 77, no. 9, 2008.

[8] S. J. Brodsky, G. F. de Téramond, H. . Dosch, and J. Erlich, "Light-front holographic QCD and emerging confinement," *Physics Reports. A Review Section of Physics Letters*, vol. 584, pp. 1–105, 2015.

[9] E. Witten, "Dynamical breaking of supersymmetry," *Nuclear Physics B*, vol. 188, no. 3, pp. 513–554, 1981.

[10] H. G. Dosch, G. F. de Téramond, and S. J. Brodsky, "Superconformal baryon-meson symmetry and light-front holographic QCD," *Physical Review D*, vol. 91, no. 8, 2015.

[11] G. F. de Téramond, H. G. Dosch, and S. J. Brodsky, "Baryon spectrum from superconformal quantum mechanics and its light-front holographic embedding," *Physical Review D*, vol. 91, no. 4, 2015.

[12] V. de Alfaro, S. Fubini, and G. Furlan, "Conformal invariance in quantum mechanics," *Il Nuovo Cimento A*, vol. 34, no. 4, pp. 569–612, 1976.

[13] S. Fubini and E. Rabinovici, "Superconformal quantum mechanics," *Nuclear Physics. B. Theoretical, Phenomenological, and Experimental High Energy Physics. Quantum Field Theory and Statistical Systems*, vol. 245, no. 1, pp. 17–44, 1984.

[14] H. G. Dosch, G. F. de Téramond, and S. J. Brodsky, "Supersymmetry across the light and heavy-light hadronic spectrum. II.," *Physical Review D*, vol. 95, no. 3, 2017.

[15] H. G. Dosch, G. F. de Téramond, and S. J. Brodsky, "Supersymmetry across the light and heavy-light hadronic spectrum," *Physical Review D*, vol. 92, no. 7, 2015.

[16] S. J. Brodsky, G. F. de Téramond, H. G. Dosch, and C. Lorcé, "Universal effective hadron dynamics from superconformal algebra," *Physics Letters B*, vol. 759, pp. 171–177, 2016.

[17] N. Isgur and M. B. Wise, "Spectroscopy with heavy-quark symmetry," *Physical Review Letters*, vol. 66, no. 9, pp. 1130–1133, 1991.

[18] E. Shuryak, "Hadrons containing a heavy quark and QCD sum rules," *Nuclear Physics B*, vol. 198, no. 1, pp. 83–101, 1982.

[19] R. S. Sufian, G. F. de Téramond, S. J. Brodsky, A. Deur, and H. G. Dosch, "Analysis of nucleon electromagnetic form factors from light-front holographic QCD: The spacelike region," *Physical Review D*, vol. 95, no. 1, 2017.