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Research Article

On Topological Structures of Fuzzy Parametrized Soft Sets

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We introduce the topological structure of fuzzy parametrized soft sets and fuzzy parametrized soft mappings. We define the notion of quasi-coincidence for fuzzy parametrized soft sets and investigated its basic properties. We study the closure, interior, base, continuity, and compactness and properties of these concepts in fuzzy parametrized soft topological spaces.

1. Introduction

In 1965, after Zadeh [1] generalized the usual notion of a set with the introduction of fuzzy set, the fuzzy set was carried out in the areas of general theories and applied to many real life problems in uncertain, ambiguous environment. In this manner, in 1968, Chang [2] gave the definition of fuzzy topology and introduced the many topological notions in fuzzy setting.

In 1999, Molodtsov [3] introduced the concept of soft set theory which is a completely new approach for modelling uncertainty and pointed out several directions for the applications of soft sets, such as game theory, perron integrations, and smoothness of functions. To improve this concept, many researchers applied this concept on topological spaces (e.g., [4–9]), group theory, ring theory (e.g., [10–14]), and also decision making problems (e.g., [15–18]).

Recently, researchers have combined fuzzy set and soft set to generalize the spaces and to solve more complicated problems. By this way, many interesting applications of soft set theory have been expanded. First combination of fuzzy set and soft set is fuzzy soft set and it was given by Maji et al. [19]. Then fuzzy soft set theory has been applied in several directions, such as topology (e.g., [20–23]), various algebraic structures (e.g., [24, 25]), and especially decision making (e.g., [26–29]). Another combination of fuzzy set and soft set was given by Çağman et al. [30] who called it fuzzy parametrized soft set (for short FP-soft set). In that paper, Çağman et al. defined operations on FP-soft sets and improved several results. After that, Çağman and Deli [31, 32] applied FP-soft sets to define some decision making methods and applied these methods to problems that contain uncertainties and fuzzy object.

In the present paper, we consider the topological structure of FP-soft sets. Firstly, we give some basic ideas of FP-soft sets and also studied results. We define FP-soft quasi-coincidence, as a generalization of quasi-coincidence in fuzzy manner [33], and use this notion to characterize concepts of FP-soft closure and FP-soft base in FP-soft topological spaces. We also introduce the notion of mapping on FP-soft classes and investigate the properties of FP-soft images and FP-soft inverse images of FP-soft sets. We define FP-soft topology in Chang's sense. We study the FP-soft closure and FP-soft interior operators and properties of these concepts. Lastly we define FP-soft continuous mappings and we show that image of a FP-soft compact space is also FP-soft compact.

2. Preliminaries

Throughout this paper X denotes initial universe, E denotes the set of all possible parameters which are attributes, characteristic, or properties of the objects in X, and the set of all subsets of X will be denoted by $P(X)$.

Definition 1 (see [1]). A fuzzy set A in X is a function defined as follows:

$$A = \left\{ \frac{\mu_A(x)}{x} : x \in X \right\}, \tag{1}$$

where $\mu_A : X \to [0, 1]$.

Here μ_A is called the membership function of A, and the value $\mu_A(x)$ is called the grade of membership of $x \in X$. This value represents the degree of x belonging to the fuzzy set A.

A fuzzy point in X, whose value is α $(0 < \alpha \leq 1)$ at the support $x \in X$, is denoted by x_α. A fuzzy point $x_\alpha \in A$, where A is fuzzy set in X if $\alpha \leq \mu_A(x)$.

Definition 2 (see [3]). A pair (F, E) is called a soft set over X if F is a mapping defined by $F : E \to P(X)$.

In other words, a soft set is a parametrized family of subsets of the set X. Each set $F(e)$, $e \in E$, from this family may be considered as the set of e-elements of the soft set (F, E).

Definition 3 (see [30]). Let A be a fuzzy set over E. A FP-soft set F_A on the universe X is defined as follows:

$$F_A = \left\{ \left(\frac{\mu_A(e)}{e}, f_A(e) \right) : e \in E, f_A(e) \in P(X), \mu_A(e) \in [0,1] \right\}, \tag{2}$$

where the function $f_A : E \to P(X)$ is called approximate function such that $f_A(e) = \varnothing$ if $\mu_A(e) = 0$, and the function $\mu_A : E \to [0,1]$ is called membership function of the set A.

From now on, the set of all FP-soft sets over X will be denoted by $\text{FPS}(X, E)$.

Definition 4 (see [30]). Let $F_A \in \text{FPS}(X, E)$.

(1) F_A is called the empty FP-soft set if $\mu_A(e) = 0$ for every $e \in E$, denoted by $F_\varnothing$.

(2) F_A is called A-universal FP-soft set if $\mu_A(e) = 1$ and $f_A(e) = X$ for all $e \in A$, denoted by $F_{\widetilde{A}}$.

If $A = E$, then A-universal FP-soft set is called universal FP-soft set, denoted by $F_{\widetilde{E}}$.

Definition 5 (see [30]). Let $F_A, F_B \in \text{FPS}(X, E)$.

(1) F_A is called a FP-soft subset of F_B if $A \leq B$ and $f_A(e) \subseteq f_B(e)$ for every $e \in E$ and one writes $F_A \tilde{\subseteq} F_B$.

(2) F_A and F_B are said to be equal, denoted by $F_A = F_B$ if $F_A \tilde{\subseteq} F_B$ and $F_B \tilde{\subseteq} F_A$.

(3) The union of F_A and F_B, denoted by $F_A \tilde{\cup} F_B$, is the FP-soft set, defined by the membership and approximate functions $\mu_{A\cup B}(e) = \max\{\mu_A(e), \mu_B(e)\}$ and $f_{A\cup B}(e) = f_A(e) \cup g_B(e)$ for every $e \in E$, respectively.

(4) The intersection of F_A and F_B, denoted by $F_A \tilde{\cap} G_B$, is the FP-soft set, defined by the membership and approximate functions $\mu_{A\cap B}(e) = \min\{\mu_A(e), \mu_B(e)\}$ and $f_{A\cap B}(e) = f_A(e) \cap g_B(e)$ for every $e \in E$, respectively.

Definition 6 (see [30]). Let $F_A \in \text{FPS}(X, E)$. Then the complement of F_A, denoted by F_A^c, is the FP-soft set, defined by the membership and approximate functions $\mu_{A^c}(e) = 1 - \mu_A(e)$ and $f_A^c(e) = X - f_A(e)$ for every $e \in E$, respectively.

Clearly $(F_A^c)^c = F_A$, $F_{\widetilde{E}}^c = F_\varnothing$, and $F_\varnothing^c = F_{\widetilde{E}}$.

Proposition 7 (see [30]). *Let F_A, F_B, and $F_C \in FPS(X, E)$. Then*

(1) $(F_A \tilde{\cup} F_B)^c = F_A^c \tilde{\cap} F_B^c$;

(2) $(F_A \tilde{\cap} F_B)^c = F_A^c \tilde{\cup} F_B^c$;

(3) $F_A \tilde{\cap} F_A = F_A$, $F_A \tilde{\cup} F_A = F_A$;

(4) $F_A \tilde{\cap} F_\varnothing = F_\varnothing$, $F_A \tilde{\cap} F_{\widetilde{E}} = F_A$;

(5) $F_A \tilde{\cap} F_B = F_B \tilde{\cap} F_A$, $F_A \tilde{\cup} F_B = F_B \tilde{\cup} F_A$;

(6) $F_A \tilde{\cap} (F_B \tilde{\cap} F_C) = (F_A \tilde{\cap} F_B) \tilde{\cap} F_C$, $F_A \tilde{\cup} (F_B \tilde{\cup} F_C) = (F_A \tilde{\cup} F_B) \tilde{\cup} F_C$;

(7) $F_A \tilde{\cup} F_\varnothing = F_A$, $F_A \tilde{\cup} F_{\widetilde{E}} = F_{\widetilde{E}}$.

3. Some Properties of FP-Soft Sets and FP-Soft Mappings

Definition 8. Let J be an arbitrary index set and $F_{A_i} \in \text{FPS}(X, E)$ for all $i \in J$.

(1) The union of F_{A_i}'s, denoted by $\tilde{\cup}_{i\in J} F_{A_i}$, is the FP-soft set, defined by the membership and approximate functions $\mu_{\cup_{i\in J} A_i}(e) = \sup_{i\in J}\{\mu_{A_i}(e)\}$ and $f_{\cup_{i\in J} A_i}(e) = \cup_{i\in J} f_{A_i}(e)$ for every $e \in E$, respectively.

(2) The intersection of F_{A_i}'s, denoted by $\tilde{\cap}_{i\in J} F_{A_i}$, is the FP-soft set, defined by the membership and approximate functions $\mu_{\cap_{i\in J} A_i}(e) = \inf_{i\in J}\{\mu_{A_i}(e)\}$ and $f_{\cap_{i\in J} A_i}(e) = \cap_{i\in J} f_{A_i}(e)$ for every $e \in E$, respectively.

Proposition 9. *Let J be an arbitrary index set and $F_{A_i} \in FPS(X, E)$ for all $i \in J$. Then*

(1) $(\tilde{\cup}_{i\in J} F_{A_i})^c = \tilde{\cap}_{i\in J} F_{A_i}^c$;

(2) $(\tilde{\cap}_{i\in J} F_{A_i})^c = \tilde{\cup}_{i\in J} F_{A_i}^c$.

Proof. (1) Put $F_B = (\tilde{\cup}_{i\in J} F_{A_i})^c$ and $F_C = \tilde{\cap}_{i\in J} F_{A_i}^c$. Then for all $e \in E$,

$$\begin{aligned}
\mu_B(e) &= 1 - \mu_{\cup_{i\in J} A_i}(e) = 1 - \sup_{i\in J}\{\mu_{A_i}(e)\} \\
&= \inf_{i\in J}\{1 - \mu_{A_i}(e)\} = \inf_{i\in J}\{\mu_{A_i^c}(e)\} = \mu_C(e), \\
f_B(e) &= X - f_{\cup_{i\in J} A_i}(e) = X - \bigcup_{i\in J} f_{A_i}(e) \\
&= \bigcap_{i\in J}\left(X - f_{A_i}(e)\right) = \bigcap_{i\in J} f_{A_i^c}(e) \\
&= f_{\cap_{i\in J} A_i^c}(e) = f_C(e).
\end{aligned} \tag{3}$$

This completes the proof. The other can be proved similarly. □

Definition 10. The FP-soft set $F_A \in \text{FPS}(X, E)$ is called FP-soft point if A is fuzzy singleton and $f_A(e) \in P(X)$ for $e \in \text{supp}\, A$. If $A = \{e\}$, $\mu_A(e) = \alpha \in (0, 1]$, then one denotes this FP-soft point by e_α^f.

Definition 11. Let e_α^f, $F_A \in \text{FPS}(X, E)$. One says that $e_\alpha^f \tilde{\in} F_A$ read as e_α^f belongs to the FP-soft set F_A if $\alpha \leq \mu_A(e)$ and $f(e) \subseteq f_A(e)$.

Proposition 12. *Every nonempty FP-soft set F_A can be expressed as the union of all the FP-soft points which belong to F_A.*

Proof. This follows from the fact that any fuzzy set is the union of fuzzy points which belong to it [33]. □

Definition 13. Let F_A, $F_B \in \text{FPS}(X, E)$. F_A is said to be FP-soft quasi-coincident with F_B, denoted by $F_A q F_B$, if there exists $e \in E$ such that $\mu_A(e) + \mu_B(e) > 1$ or $f_A(e)$ is not subset of $f_B^c(e)$. If F_A is not FP-soft quasi-coincident with F_B, then one writes $F_A \bar{q} F_B$.

Definition 14. Let e_α^f, $F_A \in \text{FPS}(X, E)$. e_α^f is said to be FP-soft quasi-coincident with F_A, denoted by $e_\alpha^f q F_A$, if $\alpha + \mu_A(e) > 1$ or $f(e)$ is not subset of $f_A^c(e)$. If e_α^f is not FP-soft quasi-coincident with F_A, then one writes $e_\alpha^f \bar{q} F_A$.

Proposition 15. *Let F_A, $F_B \in FPS(X, E)$. Then the following are true:*

(1) $F_A \tilde{\subseteq} F_B \Leftrightarrow F_A \bar{q} F_B^c$;

(2) $F_A q F_B \Rightarrow F_A \tilde{\cap} F_B \neq F_\varnothing$.

(3) $F_A \bar{q} F_A^c$;

(4) $F_A q F_B \Leftrightarrow$ *there exists an* $e_\alpha^f \tilde{\in} F_A$ *such that* $e_\alpha^f q F_B$;

(5) $e_\alpha^f \tilde{\in} F_A^c \Leftrightarrow e_\alpha^f \bar{q} F_A$;

(6) $F_A \tilde{\subseteq} F_B \Rightarrow$ *if* $e_\alpha^f q F_A$, *then* $e_\alpha^f q F_B$ *for all* $e_\alpha^f \in FPS(X, E)$;

Proof. (1) Consider

$$\begin{aligned} F_A \tilde{\subseteq} F_B &\Longleftrightarrow \forall e \in E, \\ &\quad \mu_A(e) \leq \mu_B(e), \quad f_A(e) \subseteq f_B(e) \\ &\Longleftrightarrow \forall e \in E, \\ &\quad \mu_A(e) - \mu_B(e) \leq 0, \quad f_A(e) \subseteq f_B(e) \\ &\Longleftrightarrow \forall e \in E, \\ &\quad \mu_A(e) + 1 - \mu_B(e) \leq 1, \quad f_A(e) \subseteq f_B(e) \\ &\Longleftrightarrow F_A \bar{q} F_B^c. \end{aligned} \tag{4}$$

(2) Let $F_A q F_B$. Then there exists an $e \in E$ such that $\mu_A(e) + \mu_B(e) > 1$ or $f_A(e)$ is not subset of $f_B^c(e)$. If $\mu_A(e) + \mu_B(e) > 1$, $A \wedge B \neq 0_E$ and the proof is easy. If $f_A(e)$ is not subset of $f_B^c(e)$, then $f_A(e) \cap f_B(e) \neq \varnothing$. Hence $F_A \tilde{\cap} F_B \neq F_\varnothing$.

(3) Suppose $F_A q F_A^c$. Then there exists $e \in E$ such that $\mu_A(e) + \mu_{A^c}(e) > 1$ or $f_A(e)$ is not subset of $(f_A^c(e))^c$. But this is impossible.

(4) If $F_A q F_B$, then there exists an $e \in E$ such that $\mu_A(e) + \mu_B(e) > 1$ or $f_A(e)$ is not subset of $f_B^c(e)$. Put $\alpha = \mu_A(e)$ and $f(e) = f_A(e)$. Then we have $e_\alpha^f \tilde{\in} F_A$ and $e_\alpha^f q F_B$.

Conversely, suppose $e_\alpha^f q F_B$ for some $e_\alpha^f \tilde{\in} F_A$. Then $\alpha + \mu_B(e) > 1$ or $f(e)$ is not subset of $f_B^c(e)$. Therefore, we have $\mu_A(e) + \mu_B(e) > 1$ or $f_A(e)$ is not subset of $f_B^c(e)$ for $e \in E$. This shows $F_A q F_B$.

(5) It is obvious from (1).

(6) Let e_α^f, $F_A \in \text{FPS}(X, E)$ and $e_\alpha^f q F_A$. Then $\alpha + \mu_A(e) > 1$ or $f(e)$ is not subset of $f_A^c(e)$. Since $F_A \tilde{\subseteq} F_B$, $\alpha + \mu_B(e) > 1$ or $f(e)$ is not subset of $f_B^c(e)$. Hence we have $e_\alpha^f q F_B$. □

Proposition 16. *Let $\{F_{A_i} : i \in J\}$ be a family of FP-soft sets in $FPS(X, E)$, where J is an index set. Then e_α^f is FP-soft quasi-coincident with $\tilde{\cup}_{i \in J} F_{A_i}$ if and only if there exists some $F_{A_i} \in \{F_{A_i} : i \in J\}$ such that $e_\alpha^f q F_{A_i}$.*

Proof. Obvious. □

Definition 17. Let $\text{FPS}(X, E)$ and $\text{FPS}(Y, K)$ be families of all FP-soft sets over X and Y, respectively. Let $u : X \to Y$ and $p : E \to K$ be two functions. Then a FP-soft mapping $f_{up} : \text{FPS}(X, E) \to \text{FPS}(Y, K)$ is defined as follows.

(1) For $F_A \in \text{FPS}(X, E)$, the image of F_A under the FP-soft mapping f_{up} is the FP-soft set G_S over Y defined by the approximate function, $\forall k \in K$,

$$g_S(k) = \begin{cases} \bigcup\limits_{e \in p^{-1}(k)} u(f_A(e)), & \text{if } p^{-1}(k) \neq \varnothing; \\ \varnothing, & \text{otherwise}, \end{cases} \tag{5}$$

where $p(A) = S$ is fuzzy set in K.

(2) For $G_S \in \text{FPS}(Y, K)$, then the preimage of G_S under the FP-soft mapping f_{up} is the FP-soft set F_A over X defined by the approximate function, $\forall e \in E$,

$$f_A(e) = u^{-1}(g_S(p(e))), \tag{6}$$

where $p^{-1}(S) = A$ is fuzzy set in E.

If u and p are injective, then the FP-soft mapping f_{up} is said to be injective. If u and p are surjective, then the FP-soft mapping f_{up} is said to be surjective. The FP-soft mapping f_{up} is called constant, if u and p are constant.

Theorem 18. *Let X and Y be crips sets F_A, $F_{A_i} \in FPS(X, E)$, G_S, $G_{S_i} \in FPS(Y, K)$ $\forall i \in J$, where J is an index set. Let $f_{up} : FPS(X, E) \to FPS(Y, K)$ be a FP-soft mapping. Then,*

(1) *if* $F_{A_1} \tilde{\subset} F_{A_2}$ *then* $f_{up}(F_{A_1}) \tilde{\subset} f_{up}(F_{A_2})$;

(2) *if* $G_{S_1} \tilde{\subset} G_{S_2}$ *then* $f_{up}^{-1}(G_{S_1}) \tilde{\subset} f_{up}^{-1}(G_{S_2})$;

(3) $F_A \tilde{\subset} f_{up}^{-1}(f_{up}(F_A))$; *the equality holds if* f_{up} *is injective;*

(4) $f_{up}(f_{up}^{-1}(G_S)) \tilde{\subset} G_S$; *the equality holds if* f_{up} *is surjective;*

(5) $f_{up}(\tilde{\cup}_{i \in J} F_{A_i}) = \tilde{\cup}_{i \in J} f_{up}(F_{A_i})$;

(6) $f_{up}(\tilde{\cap}_{i \in J} F_{A_i}) \tilde{\subset} \tilde{\cap}_{i \in J} f_{up}(F_{A_i})$; *the equality holds if* f_{up} *is injective;*

(7) $f_{up}^{-1}(\tilde{\cup}_{i \in J} G_{S_i}) = \tilde{\cup}_{i \in J} f_{up}^{-1}(G_{S_i})$;

(8) $f_{up}^{-1}(\widetilde{\cap}_{i\in J}G_{S_i}) = \widetilde{\cap}_{i\in J}f_{up}^{-1}(G_{S_i})$;

(9) $(f_{up}^{-1}(G_S))^c = f_{up}^{-1}(G_S^c)$;

(10) $(f_{up}(F_A))^c \widetilde{\subseteq} f_{up}(F_A^c)$;

(11) $f_{up}^{-1}(G_{\widetilde{K}}) = F_{\widetilde{E}}$;

(12) $f_{up}^{-1}(G_{\varnothing}) = F_{\varnothing}$;

(13) $f_{up}(F_{\widetilde{E}}) \widetilde{\subseteq} G_{\widetilde{K}}$; *the equality holds if* f_{up} *is surjective;*

(14) $f_{up}(F_{\varnothing}) = G_{\varnothing}$.

Proof. We only prove (3), (5), (7), (9), (11), and (12). The others can be proved similarly.

(3) Put $G_S = f_{up}(F_A)$ and $F_B = f_{up}^{-1}(G_S)$. Since $A \leq p^{-1}(p(A)) = p^{-1}(S) = B$, it is sufficient to show that $f_A(e) \subseteq f_B(e)$ for all $e \in E$,

$$\begin{aligned} f_B(e) &= u^{-1}(g_S(p(e))) \\ &= u^{-1}\left(\cup_{e\in p^{-1}(p(e))} u(f_A(e))\right) \\ &= \cup_{e\in p^{-1}(p(e))} u^{-1}(u(f_A(e))) \\ &\supseteq f_A(e). \end{aligned} \tag{7}$$

This completes the proof.

(5) Put $G_{S_i} = f_{up}(F_{A_i})$ and $G_S = f_{up}(\widetilde{\cup}_{i\in J}(F_{A_i}))$. Then $S = p(\vee A_i) = \vee p(A_i) = \vee S_i$ and for all $k \in K$,

$$\begin{aligned} g_S(k) &= \begin{cases} \underset{e\in p^{-1}(k)}{\cup} u\left(\underset{i\in J}{\cup} f_{A_i}(e)\right) & \text{if } p^{-1}(k) \neq \varnothing; \\ \varnothing & \text{otherwise} \end{cases} \\ &= \begin{cases} \underset{e\in p^{-1}(k)}{\cup}\ \underset{i\in J}{\cup} u(f_{A_i}(e)) & \text{if } p^{-1}(k) \neq \varnothing; \\ \varnothing & \text{otherwise} \end{cases} \\ &= \begin{cases} \underset{i\in J}{\cup}\ \underset{e\in p^{-1}(k)}{\cup} u(f_{A_i}(e)) & \text{if } p^{-1}(k) \neq \varnothing; \\ \varnothing & \text{otherwise} \end{cases} \\ &= \underset{i\in J}{\cup} \begin{cases} \underset{e\in p^{-1}(k)}{\cup} u(f_{A_i}(e)) & \text{if } p^{-1}(k) \neq \varnothing; \\ \varnothing & \text{otherwise} \end{cases} \\ &= \underset{i\in J}{\cup} g_{S_i}(k). \end{aligned} \tag{8}$$

This completes the proof.

(7) Put $F_{A_i} = f_{up}^{-1}(G_{S_i})$ and $F_A = f_{up}^{-1}(\widetilde{\cup}_{i\in J}G_{S_i})$. Then $A = p^{-1}(\vee S_i) = \vee p^{-1}(S_i) = \vee A_i$ and for all $e \in E$,

$$\begin{aligned} f_A(e) &= u^{-1}\left(\underset{i\in I}{\cup} g_{S_i}(p(e))\right) \\ &= \underset{i\in I}{\cup} u^{-1}\left(g_{S_i}(p(e))\right) \\ &= \underset{i\in J}{\cup} f_{A_i}(e). \end{aligned} \tag{9}$$

This completes the proof.

(9) Put $f_{up}^{-1}(G_S) = F_A$ and $f_{up}^{-1}(G_S^c) = F_B$. Then for all $e \in E$,

$$f_B(e) = f_{p^{-1}(S^c)}(e) = f_{(p^{-1}(S))^c}(e) = f_{A^c}(e), \tag{10}$$

where $p^{-1}(S)$ and $p^{-1}(S^c)$ are fuzzy sets over E. This shows that the approximate functions of F_B and F_A^c are equal. This completes the proof.

(11) Put $F_A = f_{up}^{-1}(G_{\widetilde{K}})$. Then for all $e \in E$,

$$f_A(e) = u^{-1}\left(G_{\widetilde{K}}(p(e))\right) = u^{-1}(Y) = X = f_E(e). \tag{11}$$

This shows that $F_A = F_{\widetilde{E}}$.

(12) Since $p^{-1}(K)$ is fuzzy empty set, that is, 0_E, the proof is clear. □

4. FP-Soft Topological Spaces

Definition 19. A FP-soft topological space is a pair (X, τ), where X is a nonempty set and τ is a family of FP-soft sets over X satisfying the following properties:

(T1) $F_{\varnothing}, F_{\widetilde{E}} \in \tau$;

(T2) if $F_A, F_B \in \tau$, then $F_A \widetilde{\cap} F_B \in \tau$;

(T3) if $F_{A_i} \in \tau$, $\forall i \in J$, then $\widetilde{\cup}_{i\in J}F_{A_i} \in \tau$.

τ is called a topology of FP-soft sets on X. Every member of τ is called FP-soft open in (X, τ). F_B is called FP-soft closed in (X, τ) if $F_B^c \in \tau$.

Example 20. $\tau_{\text{indiscrete}} = \{F_{\varnothing}, F_{\widetilde{E}}\}$ is a FP-soft topology on X.

$\tau_{\text{discrete}} = \text{FPS}(X, E)$ is a FP-soft topology on X.

Example 21. Assume that $X = \{x_1, x_2, x_3, x_4\}$ is a universal set and $E = \{e_1, e_2, e_3\}$ is a set of parameters. If

$$\begin{aligned} F_{A_1} &= \left\{\left((e_1)_{0,2}, \{x_1, x_3\}\right), \left((e_2)_{0,3}, \{x_1, x_4\}\right), \right. \\ &\qquad \left.\left((e_3)_{0,4}, \{x_2\}\right)\right\}, \\ F_{A_2} &= \left\{\left((e_1)_{0,2}, \{x_1, x_2, x_3\}\right), \left((e_2)_{0,5}, \{x_1, x_4\}\right), \right. \\ &\qquad \left.\left((e_3)_{0,4}, \{x_1, x_2\}\right)\right\}, \\ F_{A_3} &= \left\{\left((e_1)_{0,7}, \{x_1, x_3\}\right), \left((e_2)_{0,3}, X\right), \left((e_3)_{0,9}, \{x_2, x_3\}\right)\right\}, \\ F_{A_4} &= \left\{\left((e_1)_{0,7}, \{x_1, x_2, x_3\}\right), \left((e_2)_{0,5}, X\right), \right. \\ &\qquad \left.\left((e_3)_{0,9}, \{x_2, x_3\}\right)\right\}, \end{aligned} \tag{12}$$

then $\tau = \{F_{\varnothing}, F_{A_1}, F_{A_2}, F_{A_3}, F_{A_4}, F_{\widetilde{E}}\}$ is a FP-soft topology on X.

Theorem 22. *Let* (X, τ) *be a FP-soft topological space and let* τ' *denote family of all closed sets. Then,*

(1) $F_{\varnothing}, F_{\widetilde{E}} \in \tau'$;

(2) *if* $F_A, F_B \in \tau'$, *then* $F_A \widetilde{\cup} F_B \in \tau'$;

(3) *if* $F_{A_i} \in \tau'$, $\forall i \in J$, *then* $\widetilde{\cap}_{i\in J}F_{A_i} \in \tau'$.

Proof. Straightforward. □

Definition 23. Let (X, τ) be a FP-soft topological space and $F_A \in \mathrm{FPS}(X, E)$. The FP-soft closure of F_A in (X, τ), denoted by $\overline{F_A}$, is the intersection of all FP-soft closed supersets of F_A.

Clearly, $\overline{F_A}$ is the smallest FP-soft closed set over X which contains F_A, and $\overline{F_A}$ is closed.

Theorem 24. *Let (X, τ) be a FP-soft topological space and F_A, $F_B \in FPS(X, E)$. Then,*

(1) $\overline{F_\varnothing} = F_\varnothing$ *and* $\overline{F_{\tilde{E}}} = F_{\tilde{E}}$;

(2) $F_A \tilde{\subset} \overline{F_A}$;

(3) $\overline{\overline{F_A}} = \overline{F_A}$;

(4) *if* $F_A \tilde{\subset} F_B$, *then* $\overline{F_A} \tilde{\subset} \overline{F_B}$;

(5) F_A *is a FP-soft closed set if and only if* $F_A = \overline{F_A}$;

(6) $\overline{F_A \tilde{\cup} F_B} = \overline{F_A} \tilde{\cup} \overline{F_B}$.

Proof. (1), (2), (3), and (4) are obvious from the definition of FP-soft closure.

(5) Let F_A be a FP-soft closed set. Since $\overline{F_A}$ is the smallest FP-soft closed set which contains F_A, then $\overline{F_A} \tilde{\subset} F_A$. Therefore, $F_A = \overline{F_A}$.

(6) Since $F_A \tilde{\subset} F_A \tilde{\cup} F_B$ and $F_B \tilde{\subset} F_A \tilde{\cup} F_B$, then, by (4), $\overline{F_A} \tilde{\subset} \overline{F_A \tilde{\cup} F_B}$, $\overline{F_B} \tilde{\subset} \overline{F_A \tilde{\cup} F_B}$, and hence $\overline{F_A} \tilde{\cup} \overline{F_B} \tilde{\subset} \overline{F_A \tilde{\cup} F_B}$.

Conversely, since $\overline{F_A}$ and $\overline{F_B}$ are FP-soft closed sets, $\overline{F_A} \tilde{\cup} \overline{F_B}$ is a FP-soft closed set. Again since $F_A \tilde{\cup} F_B \tilde{\subset} \overline{F_A} \tilde{\cup} \overline{F_B}$, by (4), then $\overline{F_A \tilde{\cup} F_B} \tilde{\subset} \overline{F_A} \tilde{\cup} \overline{F_B}$. □

Definition 25. Let (X, τ) be a FP-soft topological space. A FP-soft set F_A in $\mathrm{FPS}(X, E)$ is called FP-Q-neighborhood (briefly, FP-Q-nbd) of a FP-soft set F_B if there exists a FP-soft open set F_C in τ such that $F_B q F_C$ and $F_C \tilde{\subseteq} F_A$.

Theorem 26. *Let e_α^f, $F_A \in FPS(X, E)$. Then $e_\alpha^f \tilde{\in} \overline{F_A}$ if and only if each FP-Q-nbd of e_α^f is FP-soft quasi-coincident with F_A.*

Proof. Let $e_\alpha^f \tilde{\in} \overline{F_A}$. Suppose that F_C is a FP-Q-nbd of e_α^f and $F_C \overline{q} F_A$. Then there exists a FP-soft open set F_B such that $e_\alpha^f q F_B \tilde{\subseteq} F_C$. Since $F_C \overline{q} F_A$, by Proposition 15(1), $F_A \tilde{\subseteq} F_C^c \tilde{\subseteq} F_B^c$. Again since $e_\alpha^f q F_B$, e_α^f does not belong to F_B^c. This is a contradiction with $\overline{F_A} \tilde{\subseteq} F_B^c$.

Conversely, let each Q-nbd of e_α^f be FP-soft quasi-coincident with F_A. Suppose that e_α^f does not belong to $\overline{F_A}$. Then there exists a FP-soft closed set F_B which is containing F_A such that e_α^f does not belong to F_B. By Proposition 15(5), we have $e_\alpha^f q F_B^c$. Then F_B^c is a FP-Q-nbd of e_α^f and, by Proposition 15(1), $F_A \overline{q} F_B^c$. This is a contradiction with the hypothesis. □

Definition 27. Let (X, τ) be a FP-soft topological space and $F_A \in \mathrm{FPS}(X, E)$. The FP-soft interior of F_A denoted by F_A° is the union of all FP-soft open subsets of F_A.

Clearly, F_A° is the largest fuzzy soft open set contained in F_A, and F_A° is FP-soft open.

Theorem 28. *Let (X, τ) be a FP-soft topological space and F_A, $F_B \in FPS(X, E)$. Then,*

(1) $(F_\varnothing)^\circ = F_\varnothing$ *and* $(F_{\tilde{E}})^\circ = F_{\tilde{E}}$;

(2) $F_A^\circ \tilde{\subset} F_A$;

(3) $(F_A^\circ)^\circ = F_A^\circ$;

(4) *if* $F_A \tilde{\subset} F_B$, *then* $F_A^\circ \tilde{\subset} F_B^\circ$;

(5) F_A *is a FP-soft open set if and only if* $F_A = F_A^\circ$;

(6) $(F_A \tilde{\cap} F_B)^\circ = F_A^\circ \tilde{\cap} F_B^\circ$.

Proof. Similar to that of Theorem 24. □

Theorem 29. *Let (X, τ) be a FP-soft topological space and $F_A \in FPS(X, E)$. Then,*

(1) $(F_A^\circ)^c = \overline{F_A^c}$;

(2) $(\overline{F_A})^c = (F_A^c)^\circ$.

Proof. We only prove (1). The other is similar. Consider

$$\begin{aligned}(F_A^\circ)^c &= \left(\tilde{\cup}\{F_B \mid F_B \in \tau, F_A \tilde{\subset} F_B\}\right)^c \\ &= \tilde{\cap}\{F_B^c \mid F_B \in \tau, F_A \tilde{\subset} F_B\} \\ &= \tilde{\cap}\left\{F_B^c \mid F_B^c \in \tau', F_B^c \tilde{\subset} F_A^c\right\} \\ &= \overline{F_A^c}.\end{aligned} \tag{13}$$

□

Theorem 30. *Let $c : FPS(X, E) \to FPS(X, E)$ be an operator satisfying the following:*

(c1) $c(F_\varnothing) = F_\varnothing$;

(c2) $F_A \tilde{\subseteq} c(F_A), \forall F_A \in FPS(X, E)$;

(c3) $c(F_A \tilde{\cup} F_B) = c(F_A) \tilde{\cup} c(F_B), \forall F_A, F_B \in FPS(X, E)$;

(c4) $c(c(F_A)) = c(F_A), \forall F_A \in FPS(X, E)$.

Then one can associate FP-soft topology in the following way:

$$\tau = \{F_A^c \in FPS(X, E) \mid c(F_A) = F_A\}. \tag{14}$$

Moreover with this FP-soft topology τ, $\overline{F_A} = c(F_A)$ for every $F_A \in FPS(X, E)$.

Proof. (T1) By (c1), $F_\varnothing^c = F_{\tilde{E}} \in \tau$. By (c2) $F_{\tilde{E}} \tilde{\subseteq} c(F_{\tilde{E}})$, so $c(F_{\tilde{E}}) = F_{\tilde{E}}$ and $F_\varnothing \in \tau$.

(T2) Let F_A, $F_B \in \tau$. By the definition of τ, $c(F_A^c) = F_A^c$ and $c(F_B^c) = F_B^c$. By (c3), $c((F_A \tilde{\cap} F_B)^c) = c(F_A^c \tilde{\cup} F_B^c) = c(F_A^c) \tilde{\cup} c(F_B^c) = F_A^c \tilde{\cup} F_B^c = (F_A \tilde{\cap} F_B)^c$. So $F_A \tilde{\cap} F_B \in \tau$.

(T3) Let $\{F_{A_i} \mid i \in J\} \subset \tau$. Since c is order preserving and $\tilde{\cap}_{i \in J} F_{A_i}^c \tilde{\subseteq} F_{A_k}^c, \forall k \in J$, then $c(\tilde{\cap}_{i \in J} F_{A_i}^c) \tilde{\subseteq} c(F_{A_k}^c) = F_{A_k}^c$. Then we have $c(\tilde{\cap}_{i \in J} F_{A_i}^c) \tilde{\subseteq} \tilde{\cap}_{i \in J} F_{A_i}^c$. Conversely, by (c2) we have $\tilde{\cap}_{i \in J} F_{A_i}^c \tilde{\subseteq} c(\tilde{\cap}_{i \in J} F_{A_i}^c)$. Hence, $c((\tilde{\cup}_{i \in J} F_{A_i})^c) = c(\tilde{\cap}_{i \in J} F_{A_i}^c) = \tilde{\cap}_{i \in J} F_{A_i}^c = (\tilde{\cup}_{i \in J} F_{A_i})^c$ and $\tilde{\cup}_{i \in J} F_{A_i} \in \tau$.

Now we will show that with this FP-soft topology τ, $\overline{F_A} = c(F_A)$ for every $F_A \in \mathrm{FPS}(X, E)$. Let $F_A \in \mathrm{FPS}(X, E)$. Since

$(\overline{F_A})^c \in \tau$, then $c(\overline{F_A}) = \overline{F_A}$. Since c is order preserving, $c(F_A)\widetilde{\subseteq}c(\overline{F_A}) = \overline{F_A}$. Conversely, by (c4) we have $(c(F_A))^c \in \tau$. Then since $F_A\widetilde{\subseteq}c(F_A)$ and $\overline{F_A}$ is the smallest FP-soft closed set over X which contains F_A, $\overline{F_A}\widetilde{\subseteq}c(F_A)$. □

The operator c is called the FP-soft closure operator.

Remark 31. By Theorem 24(1), (2), (3), and (6) and Theorem 30, we see that with a FP-soft closure operator we can associate a FP-soft topology and conversely with a given FP-soft topology we can associate a FP-soft closure operator.

Theorem 32. *Let $i : FPS(X,E) \to FPS(X,E)$ be an operator satisfying the following:*

(i1) $i(F_{\widetilde{E}}) = F_{\widetilde{E}}$;

(i2) $i(F_A)\widetilde{\subseteq}F_A$, $\forall F_A \in FPS(X,E)$;

(i3) $i(F_A \widetilde{\cap} F_B) = i(F_A)\widetilde{\cap}i(F_B)$, $\forall F_A, F_B \in FPS(X,E)$;

(i4) $i(i(F_A)) = i(F_A)$, $\forall F_A \in FPS(X,E)$.

Then one can associate a FP-soft topology in the following way:

$$\tau = \{F_A \in FPS(X,E) \mid i(F_A) = F_A\}. \quad (15)$$

Moreover, with this fuzzy soft topology τ, $(F_A)^\circ = i(F_A)$ for every $F_A \in FPS(X,E)$.

Proof. Similar to that of Theorem 30. □

The operator i is called the FP-soft interior operator.

Remark 33. By Theorem 28(1), (2), (3), and (6) and Theorem 32, we see that with a FP-soft interior operator we can associate a FP-soft topology and conversely with a given FP-soft topology we can associate a FP-soft interior operator.

Definition 34. Let (X,τ) be a FP-soft topological space. A subcollection $\mathscr{B}$ of τ is called a base for τ if every member of τ can be expressed as a union of members of $\mathscr{B}$.

Example 35. If we consider the FP-soft topology τ in Example 21, then one easily sees that the family $\mathscr{B} = \{F_\varnothing, F_{A_1}, F_{A_2}, F_{A_3}, F_{\widetilde{E}}\}$ is a basis for τ.

Proposition 36. *Let (X,τ) be a FP-soft topological space and $\mathscr{B}$ is subfamily of τ. $\mathscr{B}$ is a base for τ if and only if for each e_α^f in $FPS(X,E)$ and for each FP-soft open Q-nbd F_A of e_α^f, there exists a $F_B \in \mathscr{B}$ such that $e_\alpha^f q F_B \widetilde{\subseteq} F_A$.*

Proof. Let $\mathscr{B}$ be a base for τ, $e_\alpha^f \widetilde{\in} \mathrm{FPS}(X,E)$ and let F_A be a FP-soft open Q-nbd of e_α^f. Then there exists a subfamily $\mathscr{B}'$ of $\mathscr{B}$ such that $F_A = \widetilde{\cup}\{F_B \mid F_B \in \mathscr{B}'\}$. Suppose that $e_\alpha^f \overline{q} F_B$ for all $F_B \in \mathscr{B}'$. Then $\alpha + \mu_B(e) \le 1$ and $f(e) \subseteq f_B^c(e)$ for every $F_B \in \mathscr{B}'$. Therefore, we have $\alpha + \mu_A(e) \le 1$ and $f(e)$ is not subset of $f_A(e)$ since $\mu_A(e) = \sup\{\mu_B(e) \mid F_B \in \mathscr{B}'\}$ and $f_A(e) = \cup f_B(e)$. This is a contradiction.

Conversely, if $\mathscr{B}$ is not a base for τ, then there exists a $F_A \in \tau$ such that $F_C = \widetilde{\cup}\{F_B \in \mathscr{B} : F_B \widetilde{\subseteq} F_A\} \ne F_A$. Since $F_C \ne F_A$, there exists $e \in E$ such that $\mu_C(e) < \mu_A(e)$ or $f_C(e) \subset f_A(e)$. Put $\alpha = 1 - \mu_C(e)$ and $f(e) = f_C^c(e)$. Then in both cases, we obtain $e_\alpha^f q F_A$ and $e_\alpha^f \overline{q} F_C$. Therefore, we have $\mu_B(e) + \alpha \le \mu_C(e) + \alpha = 1$ and $f(e) \cap f_B(e) \subseteq f(e) \cap f_C(e) = \varnothing$, that is, $e_\alpha^f \overline{q} F_B$ for all $F_B \in \mathscr{B}$ which is contained in F_A. This is a contradiction. □

Definition 37. Let (X,τ_1) and (Y,τ_2) be two FP-soft topological spaces. A FP-soft mapping $f_{up} : (X,\tau_1) \to (Y,\tau_2)$ is called FP-soft continuous if $f_{up}^{-1}(G_S) \in \tau_1$, $\forall G_S \in \tau_2$.

Example 38. Assume that $X = \{x_1, x_2, x_3\}$, $Y = \{y_1, y_2, y_3\}$ are two universal sets, $E = \{e_1, e_2\}$, $K = \{k_1, k_2\}$ are two parameter sets, and $f_{up} : (X,\tau_1) \to (Y,\tau_2)$ is a FP-soft mapping, where $u(x_1) = y_2$, $u(x_2) = y_1$, $u(x_3) = y_3$, and $p(e_1) = k_2$ and $p(e_2) = k_1$. If we take $F_A = \{((e_1)_{0,3}, \{x_2, x_3\}), ((e_2)_{0,2}, \{x_1, x_2\})\}$, $G_S = \{((k_1)_{0,2}, \{y_1, y_2\}), ((k_2)_{0,3}, \{y_1, y_3\})\}$, $\tau_1 = \{F_\varnothing, F_{\widetilde{E}}, F_A\}$, and $\tau_2 = \{\widetilde{0}_K, \widetilde{1}_K, G_S\}$, then f_{up} is a FP-soft continuous mapping.

The constant mapping $f_{up} : (X,\tau_1) \to (Y,\tau_2)$ is not continuous in general.

Example 39. Assume that $X = \{x_1, x_2, x_3\}$, $Y = \{y_1, y_2\}$ are two universal sets, $E = \{e_1, e_2\}$, $K = \{k_1, k_2\}$ are two parameter sets, and $f_{up} : (X,\tau_1) \to (Y,\tau_2)$ is a constant FP-soft mapping, where $u(x_1) = u(x_2) = u(x_3) = y_2$ and $p(e_1) = p(e_2) = k_1$. If we take $G_S = \{((k_1)_{0,2}, \{y_1, y_2\}), ((k_2)_{0,5}, \{y_2, y_3\})\}$, $\tau_1 = \{F_\varnothing, F_{\widetilde{E}}\}$, and $\tau_2 = \{F_\varnothing, F_{\widetilde{K}}, G_S\}$, f_{up} is not a FP-soft continuous since $f_{up}^{-1}(G_S) \notin \tau_1$.

Let $\alpha \in [0,1]$. We denote by α_E the constant fuzzy set on E; that is, $\mu_{\alpha_E}(e) = \alpha$ for all $e \in E$ and $\alpha \in [0,1]$.

Definition 40. Let $F_A \in \mathrm{FPS}(X,E)$. F_A is called α-A-universal FP-soft set if $\mu_A(e) = \alpha$ and $f_A(e) = X$ for all $e \in A$, denoted by $F_{\widetilde{\alpha}_A}$.

Definition 41 (see [21]). A FP-soft topology is called enriched if it satisfies $F_{\widetilde{\alpha}_A} \in \tau$ and $F_{\widetilde{\alpha}_A}^c \in \tau$ for all $\alpha \in (0,1]$.

Theorem 42. *Let (X,τ_1) be an enriched FP-soft topological space, (Y,τ_2) a FP-soft topological space, and $f_{up} : FPS(X,E) \to FPS(Y,K)$ a constant FP-soft mapping. Then f_{up} is FP-soft continuous.*

Proof. Let $G_S \in \tau_2$. Put $f_{up}^{-1}(G_S) = F_A$. Then $A = p^{-1}(S) = \alpha_E$, where $\alpha = \sup_{k\in K}\{\mu_S(k)\}$ and

$$f_A(e) = u^{-1}(g_S(p(e))) = \begin{cases} X, & g_S(p(e)) \ne \varnothing \\ \varnothing, & \text{otherwise} \end{cases} \quad (16)$$

for all $e \in E$. Hence $F_A = F_{\widetilde{\alpha}_E} \in \tau_1$ or $F_A = F_{\widetilde{\alpha}_E}^c \in \tau_1$ and so $f_{up} : (X,\tau_1) \to (Y,\tau_2)$ is FP-soft continuous. □

Theorem 43. *Let (X,τ_1) and (Y,τ_2) be two FP-soft topological spaces and let $f_{up} : FPS(X,E) \to FPS(Y,K)$ be a FP-soft mapping. Then the following are equivalent:*

(1) f_{up} *is FP-soft continuous;*

(2) $f_{up}^{-1}(G_S)$ *is FP-soft closed for every FP-closed set* G_S *over* Y*;*

(3) $f_{up}(\overline{F_A})\tilde{\subset}\overline{f_{up}(F_A)}$, $\forall F_A \in FPS(X,E)$*;*

(4) $\overline{f_{up}^{-1}(G_S)}\tilde{\subset}f_{up}^{-1}(\overline{G_S})$, $\forall G_S \in FPS(Y,K)$*;*

(5) $f_{up}^{-1}(G_S^{\circ})\tilde{\subset}(f_{up}^{-1}(G_S))^{\circ}$, $\forall G_S \in FPS(Y,K)$.

Proof. (1)⇒(2) It is obvious from Theorem 18(9).

(2)⇒(3) Let $F_A \in \mathrm{FPS}(X,E)$. Since $F_A\tilde{\subset}f_{up}^{-1}(f_{up}(F_A))F_A\tilde{\subset}f_{up}^{-1}(\overline{f_{up}(F_A)}) \in \tau_1'$, therefore we have $\overline{F_A}\tilde{\subset}f_{up}^{-1}(\overline{f_{up}(F_A)})$. By Theorem 18(4), we get $f_{up}(\overline{F_A})\tilde{\subset}f_{up}(f_{up}^{-1}(\overline{f_{up}(F_A)}))\tilde{\subset}\overline{f_{up}(F_A)}$.

(3)⇒(4) Let $G_S \in \mathrm{FPS}(Y,K)$. If we choose $f_{up}^{-1}(G_S)$ instead of F_A in (3), then $f_{up}(\overline{f_{up}^{-1}(G_S)})\tilde{\subset}\overline{f_{up}(f_{up}^{-1}(G_S))}\tilde{\subset}\overline{G_S}$. Hence by Theorem 18(3), $\overline{f_{up}^{-1}(G_S)}\tilde{\subset}f_{up}^{-1}(f_{up}(\overline{f_{up}^{-1}(G_S)}))\tilde{\subset}f_{up}^{-1}(\overline{G_S})$.

(4)⇔(5) These follow from Theorem 18(9) and Theorem 29.

(5)⇒(1) Let $G_S \in \tau_2$. Since G_S is a FP-soft open set, then $f_{up}^{-1}(G_S) = f_{up}^{-1}(G_S^{\circ})\tilde{\subset}(f_{up}^{-1}(G_S))^{\circ}\tilde{\subset}f_{up}^{-1}(G_S)$. Consequently, $f_{up}^{-1}(G_S)$ is a FP-soft open and so f_{up} is FP-soft continuous. □

Theorem 44. *Let* $f_{up} : (X,\tau_1) \to (Y,\tau_2)$ *be a FP-soft mapping and let* $\mathscr{B}$ *be a base for* τ_2*. Then* f_{up} *is FP-soft continuous if and only if* $f_{up}^{-1}(G_S) \in \tau_1$, $\forall G_S \in \mathscr{B}$.

Proof. Straightforward. □

Definition 45. A family $\mathscr{C}$ of FP-soft sets is a cover of a FP-soft set F_A if $F_A\tilde{\subseteq}\tilde{\cup}\{F_{A_i} : F_{A_i} \in \mathscr{C}, i \in J\}$. It is a FP-soft open cover if each member of $\mathscr{C}$ is a FP-soft open set. A subcover of $\mathscr{C}$ is a subfamily of $\mathscr{C}$ which is also a cover.

Definition 46. A family $\mathscr{C}$ of FP-soft sets has the finite intersection property if the intersection of the members of each finite subfamily of $\mathscr{C}$ is not empty FP-soft set.

Definition 47. A FP-soft topological space (X,τ) is FP-compact if each FP-soft open cover of $F_{\tilde{E}}$ has a finite subcover.

Example 48. Let $X = \{x_1, x_2, \ldots\}$, $E = \{e_1, e_2, \ldots\}$, and $F_{A_n} = \{((e_i)_{1/n}, X - \{x_1, x_2, \ldots, x_n\}) : i = 1,2,\ldots\}$. Then $\tau = \{F_{A_n} : n = 1,2,\ldots\} \cup \{F_{\varnothing}, F_{\tilde{E}}\}$ is a FP-soft topology on X, and (X,τ) is FP-compact.

Theorem 49. *A FP-soft topological space is FP-soft compact if and only if each family of FP-soft closed sets with the finite intersection property has a nonempty FP-soft intersection.*

Proof. If $\mathscr{C}$ is a family of FP-soft sets in a FP-soft topological space (X,τ), then $\mathscr{C}$ is a cover of $F_{\tilde{E}}$ if and only if one of the following conditions hold:

(1) $\tilde{\cup}\{F_{A_i} : F_{A_i} \in \mathscr{C}, i \in J\} = F_{\tilde{E}}$;

(2) $(\tilde{\cup}\{F_{A_i} : F_{A_i} \in \mathscr{C}, i \in J\})^c = F_{\tilde{E}}^c = F_{\varnothing}$;

(3) $\tilde{\cap}\{F_{A_i}^c : F_{A_i} \in \mathscr{C}, i \in J\} = F_{\varnothing}$.

Hence the FP-soft topological space is FP-soft compact if and only if each family of FP-soft open sets over X such that no finite subfamily covers $F_{\tilde{E}}$ fails to be a cover, and this is true if and only if each family of FP-soft closed sets which has the finite intersection property has a nonempty FP-soft intersection. □

Theorem 50. *Let* (X,τ_1) *and* (Y,τ_2) *be FP-soft topological spaces and let* $f_{up} : FPS(X,E) \to FPS(Y,K)$ *be a FP-soft mapping. If* (X,τ_1) *is FP-soft compact and* f_{up} *is FP-soft continuous surjection, then* (Y,τ_2) *is FP-soft compact.*

Proof. Let $\mathscr{C} = \{G_{S_i} : i \in J\}$ be a cover of $G_{\tilde{K}}$ by FP-soft open sets. Then since f_{up} is FP-soft continuous, the family of all FP-soft sets of the form $f_{up}^{-1}(G_S)$, for $G_S \in \mathscr{C}$, is a FP-soft open cover of $F_{\tilde{E}}$ which has a finite subcover. However, since f_{up} is surjective, then $f_{up}(f_{up}^{-1}(G_S)) = G_S$ for any FP-soft set G_S over Y. Thus, the family of images of members of the subcover is a finite subfamily of $\mathscr{C}$ which covers $G_{\tilde{K}}$. Consequently, (Y,τ_2) is FP-soft compact. □

5. Conclusion

Topology is a branch of mathematics, whose concepts exist not only in almost all branches of mathematics, but also in many real life applications. In this paper, we introduce the topological structure of fuzzy parametrized soft sets and fuzzy parametrized soft mappings. We study some fundamental concepts in fuzzy parametrized soft topological spaces such as closures, interiors, bases, compactness, and continuity. Some basic properties of these concepts are also presented. This paper will form the basis for further applications of topology.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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