

AN OBSERVATION FOR SIMPLE EXPANSIONS

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(Received March 26, 1991)

1980 MATHEMATICS SUBJECT CLASSIFICATION: 54A10, 54B05

If (X, τ) is a topological space and $A \subseteq X$, the simple expansion of τ by A is the topology $\tau(A) = \{\cup \cup (\bigvee \cap A) \mid U, V \in \tau\}$. Let us say that a topological property is weakly finitely additive if whenever closed subspaces A and B have the property then so does $A \cup B$. Recall that a property is weakly (open) hereditary if closed (open) subspaces inherit it. It is well known that certain weakly hereditary, weakly finitely additive properties such as normality, paracompactness, and the Lindelöf property, lift from (X, τ) to $(X, \tau(A))$ if and only if $(X - A, \tau|_{(X - A)})$ has the property, when (X, τ) , and $(X, \tau(A))$ are regular [6]. As in [2], a property is contagious if whenever a dense subspace has the property then so does the entire space. Without any need for regularity or any separation we have the following simple observation.

Theorem. If P is open hereditary, contagious, and weakly finitely additive, then $(X, \tau(A))$ has P if and only if both (X, τ) and $(A, \tau|_A)$ have P (where $\tau|_A$ is the subspace topology).

The proof is a consequence of the easy facts that $A \in \tau(A)$, $\text{Cl}_{\tau} A = \text{Cl}_{\tau(A)} A$, and $\tau(A)|_B = \tau|_B$ for $B = A$ or $X - A$. Among others, properties satisfying the hypotheses of the theorem include Baireness [4], resolvability, and separability. One consequence is that $(\mathbb{R}, \tau(I))$ is a Baire space and hence second category if (\mathbb{R}, τ) is the usual space of real numbers and I is the subset of irrationals. This fills a classification hole in [7].

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