Research Article

# Low-Speed Control of Heavy-Load Transfer Robot with Long Telescopic Boom Based on Stribeck Friction Model

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The severe low-speed creep phenomenon occurs in the telescopic boom system of a heavy-load transfer robot with a long telescopic boom as a result of nonlinear friction. In order to improve control precision and operation performance at low speeds, we built a three-loop control nonlinear model of an AC servo motor with Stribeck friction disturbance. Traditional proportional-integral-derivative controller (PID) and fuzzy PID controls were, respectively, adopted in the position loop, and the control performance was simulated. The results showed that a system with fuzzy PID control eliminates "flat top" position tracking and "dead zone" speed tracking, which are generated by traditional PID, and thereby decreases the effect of friction on the performance of the servo system. This elimination also improved the tracking accuracy and robustness of the system.

## **1. Introduction**

The heavy-load transfer robot with a long telescopic boom is an automatic equipment used to replace manual labour. It is widely used in situations where a small entrance leads into a large inner space, such as in installations in airplanes, space capsules, and bullet trains. The accuracy of the assembly and safety of the high-tech products installed depend on the positioning accuracy and stability of the robot when moving at low speeds. In an extensive system that bears heavy loads like this one, friction is a key factor that cannot be neglected.

Many research of adopting fuzzy control to solve the problem of stability of robot have been done, as example of [1–5]. Friction is a physical phenomenon that is complex, nonlinear, and probabilistic. It is generated between contacting surfaces that are in relative motion or tending to such motions [6, 7]. Friction significantly contributes to the effect that

low-speed creep and steady-state cyclic oscillations have on the dynamic and static properties of the system [8, 9]. In a heavy-load transfer robot system with a long telescopic boom, the quivering caused by the low-speed creep of the telescopic boom affects not only the response of the system but also its mechanical structure. Hence, eliminating the constraint of the nonlinear friction element in the mechanical system is the key to improving the performance of the control system.

The key to solving the problem of nonlinear friction is the development of an applicable friction model and the adoption of the dynamic compensation and advanced control algorithm [10]. Many scholars have done extensive research on this subject. According to nonlinear systems, Qiu et al. do some simulation studies about subsection  $H\infty$  static output feedback control [11] based on fuzzy control and asynchronous output feedback control [12] based on fuzzy affine model, and the results indicate the effectiveness of this method presented. Li et al. do some simulation studies about reliable fuzzy control [13] with run time delay and fault active suspension system and adaptive sliding mode control of the nonlinear vehicle active suspension system [14] based on T-S fuzzy approaches, and the results indicate the effectiveness of control technology design. Xu and Yao put forward nonlinear dynamic friction compensation [15] and achieved self-adaption compensation control of friction. However, this method requires the foreknowledge of the structures and characteristic parameters of the friction model, and it is difficult to realise in practise. Morel proposed that adopting torque feedback control restrains friction interference [16]. This method is effective but is not widely used because the sensor that it requires is expensive and difficult to install. Besides, the flexibility of the system is increased by the installation. Xiao et al. simulated the flight simulation turntable servo system. He found that adopting the traditional PID (proportional-integral-derivative) control method was more effective when there was no friction. Highaccuracy tracking can only be achieved in a friction element by the addition of advanced PID control to traditional PID control [17].

For solving the nonlinear friction problem happening with heavy-duty servo system running slowly to improving control precision and performance of control system, in this paper, a fuzzy PID control based on a Stribeck friction model is proposed for controlling the telescopic boom of the heavy-load transfer robot with a long telescopic boom to achieve low-speed and high-precision control. We also present our three-loop control nonlinear model of an AC servo motor with Stribeck friction. Traditional PID and fuzzy PID controls were, respectively, adopted for the positioning loop, and the control performance was simulated. The results showed that traditional PID cannot be easily used to achieve the desired control performance, but adopting fuzzy PID control can yield better positioning and speed-tracking accuracy. Fuzzy-PID control algorithm fuses fuzzy control and PID control, and make up for the deficiency of the other party with their respective advantages. On the one hand, it makes PID control have the intellectuality of fuzzy control. On the other hand, it makes fuzzy control have certain structure [18]. Compared with past control algorithm, it has the advantages such as conciser algorithm, stronger robustness, wider subject range, higher control precision, overcoming nonlinear, and so on [19–21].

This paper partly removes the nonlinear rub problem of big arms-outstretched heavyduty servo system, improving control precision and performance of control system, providing valuable reference for removing the influence of the nonlinear rub when heavy-duty servo system runs at a low speed, providing a powerful support for successfully using Fuzzy-PID control on engineering control system, and providing an another way to solve nonlinear rub problem in the control system. Mathematical Problems in Engineering

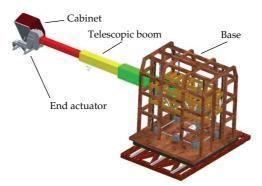


Figure 1: Three-dimensional map of a heavy loading and unloading robot with outstretched arm.

## 2. Control System Design of Heavy-Load Transfer Robot with Long Telescopic Boom

The structure of the heavy-load transfer robot with long telescopic boom is shown in Figure 1. It includes the base, telescopic boom, and end actuator. The telescopic boom, which has a length of 6.6 m, is moveable and bears the loads. It is an extensive and heavy-load system, so the influence of friction on its performance cannot be neglected. This is especially so when it is running at a super-low speed and bearing heavy load, which would result in a severe case of the creep phenomenon.

The control system of the robot is shown in Figure 2. It primarily consists of the motion controller, servo motor, and industrial computer. Its architecture comprises a host and lower computer with different CPUs, with the industrial computer and PMAC constituting a strong functional opening motion control system. The industrial computer serves as a user interface and medium for system state feedback, while the PMAC is used for motion and logical control. The PMAC2A-PC104 communicates in real-time with the host computer through the RS232 port. The host computer converts the operator's commands into control parameters and downloads the parameters to the PMAC through the serial port. The PMAC completes the operation and logical control and uploads the status of the machine to the display of the host computer. VC++6.0 is used in programming the host computer of the control system. The host computer, which uses a WINDOWS operating system, completes the settings of the control parameters and the status display. The control software of the lower computer includes two parts: one to write the PLC program that executes the I/O signal disposition for PMAC and the instructions of the schedule of motion control program and the other to write the motion control program for the servo motor. With these two programs, the robot can be conveniently controlled.

#### **3. Stribeck Friction Model**

Owing to its complexity, it is difficult to directly test friction. Considering that the output torsion of a motor is equal to the sum of the motor and load torque, rotor and moment of inertia of the load, friction torque and disturbance. In the experiment, the controller operated the motor smoothly and eliminated various moments of inertia. The total friction torque generated by the load and rotor could be measured approximately.

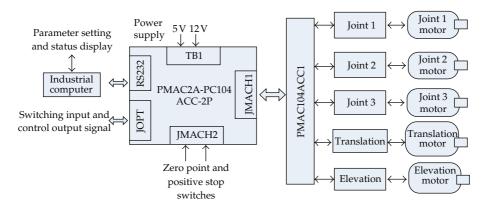


Figure 2: Control system structure of the telescopic boom.

Obviously, it is difficult to maintain uniform motion in a telescopic boom's low-speed zone in an actual system. To facilitate the development of the friction model of the telescopic boom system, we designed a test-bed of the strong stiffness [22] shrink ratio telescopic boom strove system, which included an experimental stent, servo, ball screw, and load block. They corresponded to the outriggers, servo motor, ball screw, and transportation load of an actual telescopic boom system. The friction between the load block and the test-bed corresponded to the friction between the lead screw and the ball, while the friction between the load block and the ball screw corresponded to the friction between outrigger of an actual system.

In this system, the servo motor powered the load module through a ball screw. The weight of the load was appropriately selected to maintain a constant speed for the load block in the low-speed zone. By inputting the appropriate parameters to the servo motor driver, several types of experimental data could be collected by adjusting the value of the torsion output through the monitoring interface and the value of the actual speed of the servo drive. Table 1 lists the friction torque data for different speeds.

The curve fitting of the friction torque was done with Matlab, as shown in Figure 3.

As can be seen from the relationship between the friction torque and the speed, it is more appropriate to select the Stribeck friction model for this system. According to the research of scholars, the Stribeck friction model most appropriately describes the behaviour of friction [6] in the low-speed zone. Thus, the model was adopted in our study of the influence of friction on the long telescopic boom of a heavy-load transfer robot. Figure 4 shows the Stribeck curve [10, 23].

The Stribeck friction model can be demonstrated with the following.

When  $|v(t)| < \alpha$ , the static friction is

$$F_{f}(t) = \begin{cases} F_{m} & F(t) > F_{m}, \\ F(t) & -F_{m} < F(t) < F_{m}, \\ -F_{m} & F(t) < -F_{m}. \end{cases}$$
(3.1)

| Speed (rad/s)         | 0.35  | 0.56  | 1.00  | 3.30  | 6.00  | 7.50  | 10.00 |
|-----------------------|-------|-------|-------|-------|-------|-------|-------|
| Friction torque (N·m) | -0.82 | -0.88 | -0.74 | -0.64 | -0.47 | -0.53 | -0.42 |
| Speed (rad/s)         | 15.00 | 17.50 | 20.00 | 25.00 | 30.00 | 34.70 | 40.00 |
| Friction torque (N·m) | -0.56 | -0.54 | -0.51 | -0.64 | -0.58 | -0.56 | -0.63 |

Table 1: Correspondence between speed and friction torque.

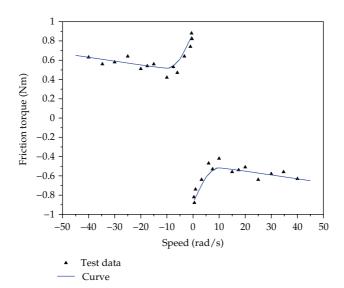


Figure 3: Friction torque curve-fitting.

When  $|v(t)| > \alpha$ , the dynamic friction is

$$F_{f}(t) = \left[F_{c} + (F_{m} - F_{c})e^{-\alpha_{1}|v(t)|}\right] \operatorname{sgn}(v(t)) + K_{v}v(t),$$
  

$$F(t) = -Jv(t),$$
(3.2)

where F(t) is the driving force,  $F_c$  is the coulomb friction,  $F_m$  is the maximum static friction,  $K_v$  is the viscous friction coefficient, and  $\alpha$  and  $\alpha_1$  are the titchy integers.

### 4. Design of Fuzzy PID Controller

PID control is widely used in conventional control systems and has the advantages of simplicity of principle, ease of realisation, and high precision. It can be designed both analytically on the basis of mathematical models and by experiment and trial-and-error. However, nonlinear friction is known to increase the difficulty of control with a telescopic boom AC servo control system. It is therefore difficult to guarantee the control performance when a traditional PID control algorithm is adopted. Fuzzy control is more adaptive and performs robustly in complex nonlinear systems. However, a static error exists in fuzzy control, and it is not suitable for precision control [24–26]. The fuzzy PID control combines the advantages of fuzzy logic control and traditional PID control; it applies the control experience of human

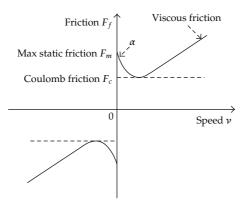


Figure 4: Friction-speed relationship curve (Stribeck curve).

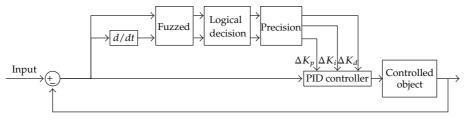


Figure 5: Structure of fuzzy adaptive PID controller.

experts, has robust performance, and is precise in handling a nonlinear complex control system.

Fuzzy PID control adopts fuzzy regulations in modifying three parameters of PID online to constitute the fuzzy adaptive PID controller, shown in Figure 5.

The process involves considering the error e and the error changing rate ec as inputs, and then blurring them. Fuzzy regulations are adopted in the blurring and deduction and eventually produce more accurate inference results, which are used to adjust the parameters of the PID controller by consulting the fuzzy matrix table [27]. The final parameters of the fuzzy PID are determined by

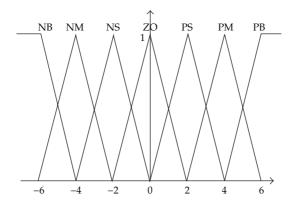
$$K_{p} = K_{po} + \Delta K_{p},$$

$$K_{i} = K_{io} + \Delta K_{i},$$

$$K_{d} = K_{do} + \Delta K_{d},$$
(4.1)

where  $K_{po}$ ,  $K_{io}$ , and  $K_{do}$  are the initial PID values.

The universe of the variables  $K_p$ ,  $K_i$ , and  $K_d$  are even fuzzy subsets of {NB, NM, NS, ZO, PS, PM, PB}, the membership function of which obeys the triangle disturbance. Figure 6 shows the distribution map of the membership function.



**Figure 6:** Distribution map of the membership function.

### 5. System Model Development

A three-loop control model was adopted for the AC servo system of the telescopic boom. The current loop and speed loop were achieved by a traditional PID integrated in the servo, while the position loop was achieved by a fuzzy PID integrated in the *PMAC* motion controller. The position loop was a semiclosed loop, and the feedback was the angular displacement of the servo motor shaft. The triphase AC permanent magnetic synchronous machine (PMSM) was used, and the effect of the spatial harmonics was ignored. It was assumed that the three-phase winding was symmetrical and the magnetic motive force (MMF) was distributed sinusoidally along the circumference. The magnetic saturation, eddy currents, and magnetoresistive effect were ignored. It was also assumed that the power supply voltages of the three phases were equal. Under this circumstance, the inductance parameters could be considered to be approximately equal (i.e.,  $L_d = L_q = L$ ) and the friction coefficient B to be equal to 0. With the adoption of the field orientation vector control tactics ( $i_d = 0$ ), the linear state equation and electromagnetic torque equation were obtained.

The linear state equation was determined to be

$$\begin{bmatrix} \frac{di_q}{dt} \\ \frac{d\omega}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-R}{L} & \frac{-p_n \varphi_f}{L} \\ \frac{3p_n \varphi_f}{2J} & 0 \end{bmatrix} \begin{bmatrix} i_q \\ \omega \end{bmatrix} + \begin{bmatrix} \frac{u_q}{L} \\ \frac{-T_l}{L} \end{bmatrix}.$$
(5.1)

The electromagnetic torque equation was determined to be

$$T_e = \frac{3p_n \varphi_f i_q}{2},\tag{5.2}$$

where *R* is the equivalent resistance ( $\Omega$ ),  $L_d = L_q = L$  is the equivalent inductance (*H*),  $p_n$  is the number of pole pairs,  $\omega$  is the palstance of the rotor (rad/s),  $\varphi_f$  is the equivalent magnetic linkage (*Wb*),  $T_l$  is the load moment (N·m),  $i_q$  is the current in the *q* shaft (*A*), and *J* is the moment of inertia (kg·m<sup>2</sup>).

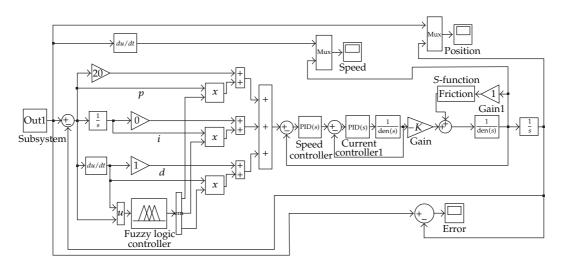


Figure 7: Simulink block diagram of the telescopic boom.

The mechanical transmission rig of the telescopic boom system comprised the driving motor, speed reducer, ball screw pairs, screw steady bearings, and moving parts. When the angular displacement of the servo motor  $\theta(t)$  was the input of the mechanical transmission, the motion of the boom X(t) was the output. The connection between the motor and the speed reducer was equivalent to a fixed joint. All the loads of the boom were converted to the moment of inertia  $J_l$  of the motor shaft. The friction torques and sticky connections among other parts of the boom are described by the Stribeck friction model and were added to the system model in perturbation of the speed loop. The parameters of the Stribeck friction model were adjusted. The Simulink block diagram of the telescopic boom system using fuzzy PID control with a nonlinear friction element is shown in Figure 7.

#### 6. Results and Analysis

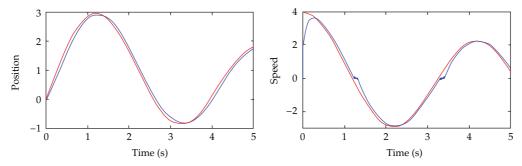
The sinusoidal superimposed signal was chosen as the input to the system, and is as follows:

$$r(t) = A\sin(2\pi Ft) + 0.5A\sin(1.0\pi Ft) + 0.25A\sin(0.5\pi Ft).$$
(6.1)

The position loop is controlled by traditional PID control and Fuzzy-PID control. Simulation results from position, speed, and error scope are shown below.

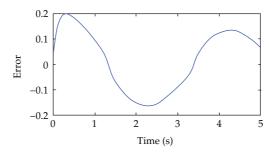
The position trailing curve, speed trailing curve, and position trailing error curve of the position loop controlled by traditional PID are shown in Figure 8. In Figure 8(a), red curve is input signal curve, blue curve is output signal curve. From the curves, output signal has obvious position trailing "flat top" phenomena and has more error following input signal, as is shown in Figure 8(c). From the curve in Figure 8(b), the wave form of output curve distorts, and speed trailing "dead zone" phenomena happens. This meant that the robustness of the low-speed servo system with friction was poor when traditional PID control was adopted, for which reason high-precision tracking could not be achieved.

The position trailing curve, speed trailing curve, and position trailing error curve of the position loop controlled with fuzzy PID control are shown in Figure 9. In Figure 9(a),



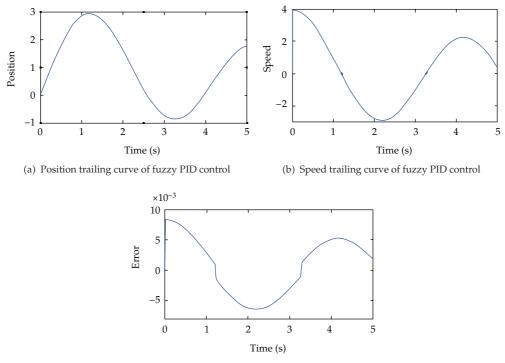
(a) Position trailing curve of traditional PID control

(b) Speed trailing curve of traditional PID control



(c) Position tracking error curve of traditional PID control

Figure 8: The simulation curve of traditional PID control.



(c) Position tracking error curve of fuzzy PID control

Figure 9: The simulation curve of fuzzy PID control.

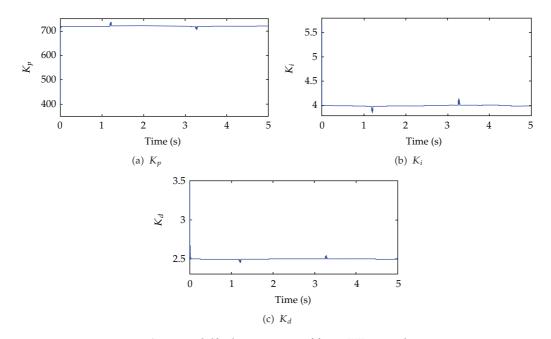


Figure 10: Self-adapting curve of fuzzy PID control.

the output curve mainly covers input curve, so using Fuzzy-PID control partly removed the position trailing "flat top" phenomena, and mainly induces position trailing error, as is shown in Figure 9(c). From the curve in Figure 9(b), output signal speed trailing "dead zone" phenomena is almost removed. It indicates that Fuzzy-PID control could achieve higher control precision and performance in slow servo system with rub.

Figure 10 shows the self-adapting curve of three parameters of fuzzy PID control. In this paper, Fuzzy-PID control means that  $K_p$ ,  $K_i$ , and  $K_d$  of position loop change on line in real time to realize higher trailing precision and control performance.

### 7. Conclusions

The low-speed performance of the telescopic boom AC servo system of the heavy-load transfer robot with a long telescopic boom was investigated. First, we designed a scale model telescopic boom servo system experiment table with high stiffness and used it to develop a friction model of the system. Secondly, the three-loop control nonlinear model of the AC servo system was developed based on the Stribeck friction model of the actual telescopic boom. The position loops with traditional PID and fuzzy PID controls were simulated. The results showed that it was difficult to track the input signal by traditional PID control, and that the position trailing "flat top" and speed trailing "dead zone" phenomena were basically eliminated by fuzzy PID, which also produced a significant improvement in the low-speed performance and tracking precision of the servo system. Research results in this paper provide valuable control method for the accuracy control of heavy-duty servo system in low running speed.

With increasing and growing since technology, intelligent Fuzzy-PID control will become a very good development direction. Fuzzy-PID control need rules formulated by

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humans expert on earth so that one certain control rule can not achieve ideal control effect in an unpredictable system running process. So more intelligent control algorithm should be input in the Fuzzy-PID control algorithm, such as neural network to form intelligent Fuzzy-PID control algorithm and remove the influence of human will in control process, to really realize the intelligent control algorithm with automatic adaptation, perfection and adjustment in system running process.

### Acknowledgments

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#### References

- L. L. Liu, H. Z. Liu, Z. Y. Wu, and Z. M. Wang, "An overview of friction models in mechanical systems," *Advances in Mechanics*, vol. 38, no. 2, pp. 201–213, 2008.
- [2] J. Huang and S. H. Ye, "Analysis and control offset research of servo system with rub," Machine Science and Technology, vol. 18, no. 1, pp. 1–4, 1999.
- [3] B. Armstrong, P. Dupont, and C. C. De Wit, "A survey of models, analysis tools and compensation methods for the control of machines with friction," *Automatica*, vol. 30, no. 7, pp. 1083–1138, 1994.
- [4] L. J. Ma, Y. Z. Tang, and X. B. Zhi, "Synovium variable structure control research of low-speed friction servo system," *Learned Journal of Shot and Arrow Guide*, vol. 27, no. 3, pp. 77–79, 2007.
- [5] J. S. Peng, Q. Y. Su, and S. L. Song, "Application of the servo system with fuzzy PID control that base on stribeck friction model," *Electrical Engineering*, vol. 1, pp. 15–17, 2007.
- [6] J. Qiu, G. Feng, and H. Gao, "Fuzzy-model-based piecewise *I*∞ static-output-feedback controller design for networked nonlinear systems," *IEEE Transactions on Fuzzy Systems*, vol. 18, no. 5, pp. 919– 934, 2010.
- [7] J. Qiu, G. Feng, and H. Gao, "Asynchronous output feedback control of networked nonlinear systems with multiple packet dropouts: T-S fuzzy affine model based approach," *IEEE Transactions on Fuzzy Systems*, vol. 19, no. 6, pp. 1014–1030, 2011.
- [8] H. Li, H. Liu, H. Gao, and P. Shi, "Reliable fuzzy control for active suspension systems with actuator delay and fault," *IEEE Transactions on Fuzzy Systems*, vol. 20, no. 2, pp. 342–357, 2012.
- [9] H. Li, J. Yu, C. Hilton, and H. Liu, "Adaptive sliding mode control for nonlinear active suspension systems using T-S fuzzy model," *IEEE Transactions on Industrial Electronics*. In press.
- [10] L. Xu and B. Yao, "Adaptive robust control of mechanical systems with nonlinear dynamic friction compensation," in *Proceedings of the American Control Conference*, pp. 2595–2599, Chicago, Ill, USA, June 2000.
- [11] G. Morel, K. Iagnemma, and S. Dubowsky, "Precise control of manipulators with high joint-friction using base force/torque sensing," *Automatica*, vol. 36, no. 7, pp. 931–941, 2000.
- [12] B. N. Xiao, L. H. Gong, and S. Zhang, "Simulation research on friction model of a flight simulation table servo system," *Lubrication Engineering*, vol. 34, no. 3, pp. 76–80, 2009.
- [13] S. Y. Li, Neural Control and Intelligent Control Theory, Harbin Institute of Technology Press, Harbin, China, 1996.
- [14] G. Y. Li, "New fuzzy PID controller," Journal of System Simulation, vol. 15, no. 10, pp. 1492–1493, 2003.
- [15] A. Haj-Ali and H. Ying, "Structural analysis of fuzzy controllers with nonlinear input fuzzy sets in relation to nonlinear PID control with variable gains," *Automatica*, vol. 40, no. 9, pp. 1551–1559, 2004.
- [16] H. B. Zhou, H. Deng, and J. A. Duan, "Fuzzy-PID control for the motion platform of thermosonic flip-chip bonding machines," *Journal of System Simulation*, vol. 19, no. 13, pp. 2944–2954, 2007.
- [17] M. Wang and L. Y. Yang, "The study of research machine tool guide crawl appearance," in *Coal Mine Machinery*, vol. 3, pp. 35–37, Harbin North Pressure Container Limited Company, Harbin, China, 2004.

- [18] D. D. Guo, X. D. Liu, and Y. H. Zhang, "Repetitive control strategy of servo system with fiction offset," in *Proceedings of the 21st Chinese Control Conference*, pp. 555–558, Zhe Jiang University Press, Zhe Jiang, China, 2002.
- [19] J. Zhu, Fuzzy Control Theory and Application, Mechanical Industry Press, 1999.
- [20] Y. F. Wang and T. Y. Chai, "Adaptive fuzzy output feedback control for a class of nonlinear MIMO systems," *Journal of System Simulation*, vol. 17, no. 10, pp. 2442–2446, 2005.
- [21] Y. Zhang and X. X. Yang, "Design for fuzzy immune controller based on genetic algorithm," *Journal of System Simulation*, vol. 16, no. 7, pp. 1548–1551, 2004.
- [22] S. W. Yang and M. Q. Zheng, "Simulation of nonlinear friction with modeling methodology," Journal of System Simulation, vol. 14, no. 10, pp. 1365–1368, 2002.
- [23] R. J. Lian and C. N. Huang, "Self-organizing fuzzy radial basis function neural-network controller for robotic motion control," *International Journal of Innovative Computing, Information and Control*, vol. 8, no. 3, pp. 1955–1971, 2012.
- [24] M. R. Soltanpour, J. Khalilpour, and M. Soltani, "Robust nonlinear control of robot manipulator with uncertainties in kinematics. Dynamics and actuator models," *International Journal of Innovative Computing, Information and Control*, vol. 8, no. 8, pp. 5487–5498, 2012.
- [25] A. Pallegedara, Y. Matsuda, T. Matsumoto, K. Tsukamoto, N. Egashira, and S. Goto, "Teleoperation of robot arms using force-free control and template matching," *International Journal of Innovative Computing, Information and Control*, vol. 8, no. 10, pp. 6869–6884, 2012.
- [26] S. E. Shafiei and M. R. Soltanpour, "Neural network sliding-mode-PID controller design for electrically driven robot manipulators," *International Journal of Innovative Computing, Information and Control*, vol. 7, no. 2, pp. 511–524, 2011.
- [27] M. Yue, W. Sun, and P. Hu, "Sliding mode robust control for two-wheeled mobile robot with lower center of gravity," *International Journal of Innovative Computing, Information and Control*, vol. 7, no. 2, pp. 637–646, 2011.



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