Research Article

Guaranteed Cost Finite-Time Control for Positive Switched Linear Systems with Time-Varying Delays

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This paper considers the guaranteed cost finite-time control for positive switched linear systems with time-varying delays. The definition of guaranteed cost finite-time boundedness is firstly given. Then, by using the mode-dependent average dwell time approach, a static output feedback law and a state feedback control law are constructed, respectively, and sufficient conditions are obtained to guarantee that the closed-loop system is guaranteed cost finite-time boundedness. Such conditions can be easily solved by linear programming. Finally, an example is given to illustrate the effectiveness of the proposed method.

1. Introduction

Positive switched systems are a class of hybrid systems consisting of a family of positive subsystems and a switching law that specifies which subsystem will be activated along the system trajectory at each instant of time. In recent years, many remarkable results about the problems of stability analysis and controller synthesis of positive switched systems have been presented [1–8]. Among these existing results, the average dwell time (ADT) approach and mode-dependent average dwell time (MDADT) technique are widely used; the latter allows that every subsystem has its own average dwell time to make the individual properties of each subsystem unneglected, which is more applicable and less conservative compared with ADT technique [9].

In fact, many studies focus on the asymptotic stability in the area of positive switched systems, which reflects the asymptotic behavior of the system in an infinite time interval. However, in many practical applications, one is more interested in what happens over a finite-time interval. Peter [10] has firstly defined finite-time stability (FTS) for linear deterministic systems, which means that, given a bound on the initial condition, the system state does not exceed a certain threshold during a specified time interval. Then Amato et al. [11] extended this definition to finite-time boundedness (FTB) when they dealt with the behavior of the

state in the presence of external disturbances. Then many related results have been presented [12–17]. Recently, [18] first extends the concept of FTS to positive switched systems and gives some FTS conditions of positive switched systems. In [19], the problem of finite-time L_1 control for a class of positive switched linear systems with time-varying delays is considered and the concept of finite-time L_1 boundedness is also proposed. In [20], the problem of output feedback L_1 finite-time control for switched positive systems with time-varying delays is investigated via MDADT approach. On the other hand, when controlling a real plant, it is also desirable to design a control system which not only is stable but also guarantees an adequate level of performance. One approach to this problem is the guaranteed cost control. It has the advantage of providing an upper bound on a given system performance index and thus the system performance degradation incurred by the uncertainties or time delays is guaranteed to be less than this bound [21]. So it is necessary to study the design problem of guaranteed cost finite-time controller. There are some results about this problem; see [22–29] and references therein. These results mainly focus on nonpositive systems, involved in fuzzy systems [22, 27], continuous-time nonlinear systems [23], interconnected systems [24], Markov switching systems [25], stochastic systems [26, 28], neural networks [29], and so on. However, to the best of our knowledge, there are few results available on guaranteed cost finite-time control

for positive switched linear systems with time-varying delays, which motivates our present study.

In this paper, the problem of the finite-time boundedness with guaranteed cost control for positive switched linear systems with time-varying delays is considered. Firstly, the definition of guaranteed cost finite-time boundedness is given, which is extended to positive switched linear systems with time-varying delays for the first time. Secondly, by using the MDADT approach and copositive Lyapunov-Krasovskii functional method, a static output feedback and a state feedback are designed, respectively, and sufficient conditions are obtained to guarantee that the closed-loop system is guaranteed cost finite-time boundedness (GCFTB). Such conditions can be easily solved by linear programming.

Notations. Throughout this paper, $A \geq 0$ (>0, ≤ 0, <0) means that all entries of matrix A are nonnegative (positive, nonpositive, negative). R_+^n is the n-dimensional nonnegative (positive) vector space, the n-dimensional Euclidean space is denoted by R^n and $R^{n \times n}$ represents the space of $n \times n$ matrices with real entries, and A^T is the transpose of a matrix A. $\|x\| = \sum_{k=1}^n |x_k|$, where x_k is the kth element of $x \in R^n$; if not explicitly stated, matrices are assumed to have compatible dimensions.

2. Problem Statements and Preliminaries

Consider the following positive switched linear systems with time-varying delays:

$$
\begin{aligned}
\dot{x}(t) &= A_{\sigma(t)} x(t) + A_{d\sigma(t)} x(t - d(t)) + G_{\sigma(t)} u(t) \\
&\quad + B_{\sigma(t)} w(t), \\
x(\theta) &= \varphi(\theta), \quad \theta \in [-\tau, 0], \\
y(t) &= C_{\sigma(t)} x(t),
\end{aligned}
\tag{1}
$$

where $x(t) \in R^n$, $u(t) \in R^m$, and $y(t) \in R^z$ represent the system state, control input, and control output, respectively. $w(t) \in R^l$ is the disturbance input, which satisfies

$$
\exists d > 0 : \int_0^{T_f} \|w(t)\| \, dt < d.
\tag{2}
$$

$\sigma(t) : [0, \infty) \to \underline{S} = \{1, 2, \ldots, S\}$ is the switching signal, where S is the number of subsystems; $\forall p \in \underline{S}$, A_p, A_{dp}, B_p, C_p, and G_p are constant matrices with appropriate dimensions, p denotes pth subsystems, and t_q denotes the qth switching instant; $\varphi(\theta)$ is the initial condition on $[-\tau, 0]$, $\tau > 0$, and $d(t)$ denotes the time-varying delay satisfying $0 \leq d(t) \leq \tau$, $\dot{d}(t) \leq h < 1$, where τ and h are known positive constants.

Then we will present some definitions and lemmas for the following positive switched systems:

$$
\begin{aligned}
\dot{x}(t) &= A_{\sigma(t)} x(t) + A_{d\sigma(t)} x(t - d(t)) + B_{\sigma(t)} w(t), \\
x(\theta) &= \varphi(\theta), \quad \theta \in [-\tau, 0], \\
y(t) &= C_{\sigma(t)} x(t).
\end{aligned}
\tag{3}
$$

The following definitions and lemmas are necessary for our further study.

Definition 1 (see [19]). System (3) is said to be positive if, for any initial conditions $\varphi(\theta) \geq 0$, $\theta \in [-\tau, 0]$, disturbance input $w(t) \geq 0$, and any switching signals $\sigma(t)$, the corresponding trajectory $x(t) \geq 0$ holds for all $t \geq 0$.

Definition 2 (see [19]). A is called a Metzler matrix if the off-diagonal entries of matrix A are nonnegative.

Lemma 3 (see [6]). *A matrix A is a Metzler matrix if and only if there exists a positive constant η such that $A + \eta I_n \geq 0$.*

Lemma 4 (see [19]). *System (3) is positive if and only if A_p, $\forall p \in \underline{S}$, are Metzler matrices and $\forall p \in \underline{S}$, $A_{dp} \geq 0$, $B_p \geq 0$, $G_p \geq 0$, and $C_p \geq 0$.*

Definition 5 (see [20]). For any switching signal $\sigma(t)$ and any $t_2 \geq t_1 \geq 0$, let $N_{\sigma p}(t_1, t_2)$ denote the switching numbers that the pth subsystem is activated over the interval $[t_1, t_2]$ and $T_p(t_1, t_2)$ denote the total running time of the pth subsystem over the interval $[t_1, t_2]$. If there exist $N_{0p} \geq 0$, $\tau_{\alpha p} > 0$, $\forall p \in \underline{S}$, such that

$$
N_{\sigma p}(t_1, t_2) \leq N_{0p} + \frac{T_p(t_1, t_2)}{\tau_{\alpha p}}, \quad \forall t_2 \geq t_1 \geq 0.
\tag{4}
$$

Then $\tau_{\alpha p}$ and N_{0p} are called MDADT and mode-dependent chatter bounds, respectively. Generally, we choose $N_{0p} = 0$.

Definition 6 (FTS). For a given time constant T_f and vectors $\delta \succ \varepsilon \succ 0$, positive switched system (3) with $w(t) \equiv 0$ is said to be finite-time stable with respect to $(\delta, \varepsilon, T_f, \sigma(t))$, if

$$
\begin{aligned}
\sup_{-\tau \leq t \leq 0} &\left\{ x^T(t) \delta \right\} \leq 1 \implies \\
&x^T(t) \varepsilon < 1, \\
&\forall t \in [0, T_f].
\end{aligned}
\tag{5}
$$

If the above condition is satisfied for any switching signals $\sigma(t)$, then system (3) is said to be uniformly finite-time stable with respect to $(\delta, \varepsilon, T_f)$.

Definition 7 (FTB). For a given time constant T_f and vectors $\delta \succ \varepsilon \succ 0$, positive switched system (3) is said to be finite-time boundedness with respect to $(\delta, \varepsilon, T_f, d, \sigma(t))$, where $w(t)$ satisfies (2), if (5) applies.

Now we give some definitions about GCFTB for the positive switched systems (1).

Definition 8. Define the cost function of positive switched system (1) as follows:

$$
J = \int_0^{T_f} \left[x^T(t) R_1 + u^T(t) R_2 \right] dt,
\tag{6}
$$

where $R_1 \succ 0$ and $R_2 \succ 0$ are two given vectors.

Remark 9. It should be noted that the proposed cost function is different from the existing literatures [21–24]; it is for the first time introduced in positive switched systems. This definition provides a more useful description, because it takes full advantage of the characteristics of nonnegative states of positive switched systems.

Definition 10 (GCFTB). For a given time constant T_f and vectors $\delta > \varepsilon > 0$, consider positive switched system (1) and cost function (6); if there exist a feedback control law $u(t)$ and a positive scalar J^* such that the closed-loop system is FTB with respect to $(\delta, \varepsilon, T_f, d, \sigma(t))$ and the cost function satisfies $J \leq J^*$, then the closed-loop system is called GCFTB, where J^* is a guaranteed cost value and $u(t)$ is a guaranteed cost finite-time controller.

The aim of this paper is to design a static output feedback controller $u(t) = K_{\sigma(t)} y(t)$, a state feedback controller $u(t) = \overline{K}_{\sigma(t)} x(t)$, and a class of switching signals $\sigma(t)$ for positive switched system (1) such that the corresponding closed-loop system is GCFTB.

3. Main Results

3.1. GCFTB Analysis. In this subsection, we will focus on the problem of GCFTB for positive switched system (3). The following theorem gives sufficient conditions of GCFTB for system (3) with the MDADT.

Theorem 11. *Consider the positive switched system (3). For given constants T_f, λ_p, γ and vectors $\delta > \varepsilon > 0$ and $R_1 > 0$, if there exist a set of positive vectors ν_p, υ_p, ϑ_p, $p \in \underline{S}$, and positive constants ξ_1, ξ_2, ξ_3, and ξ_4, the following inequalities hold:*

$$\Psi_p = \text{diag}\left\{\psi_{p1}, \psi_{p2}, \ldots, \psi_{pn}, \psi'_{p1}, \psi'_{p2}, \ldots, \psi'_{pn}\right\} \tag{7}$$
$$\leq 0,$$

$$\xi_1 \varepsilon < \nu_p < \xi_2 \delta,$$

$$\upsilon_p < \xi_3 \delta, \tag{8}$$

$$\vartheta_p < \xi_4 \delta,$$

$$\xi_2 b_{pr}^T \delta < \gamma, \tag{9}$$

$$\xi_2 + \tau e^{\lambda \tau} \xi_3 + \tau^2 e^{\lambda \tau} \xi_4 + \gamma d < \xi_1 e^{-\kappa T_f}, \tag{10}$$

where

$$\psi_{pr} = a_{pr}^T - \lambda_p \nu_{pr} + \upsilon_{pr} + \tau \vartheta_{pr} + R_1,$$

$$\psi'_{pr} = a_{dpr}^T \nu_p - (1 - h) \nu_{pr} \tag{11}$$

$$\lambda = \max_{p \in \underline{S}} \left\{\lambda_p\right\}, \ r \in \underline{n} = \{1, 2, \ldots, n\}.$$

$a_{pr}(a_{dpr})$ represents the rth column vector of the matrix $A_p(A_{dp})$, $\nu_p = [\nu_{p1}, \nu_{p2}, \ldots, \nu_{pn}]^T$, $\upsilon_p = [\upsilon_{p1}, \upsilon_{p2}, \ldots, \upsilon_{pn}]^T$,

and $\vartheta_p = [\vartheta_{p1}, \vartheta_{p2}, \ldots, \vartheta_{pn}]^T$, and ν_{pr}, υ_{pr}, and ϑ_{pr} represent the rth elements of the vectors ν_p, υ_p, and ϑ_p, respectively; then under the following MDADT scheme

$$\tau_{\alpha p} > \tau_{\alpha p}^*$$

$$= \max\left\{\frac{T_f \ln \mu_p}{\ln\left(\xi_1 e^{-\lambda_p T_f}\right) - \ln\left(\xi_2 + \tau e^{\lambda_p \tau} \xi_3 + \tau^2 e^{\lambda_p \tau} \xi_4 + \gamma d\right)},\right. \tag{12}$$

$$\left. \frac{\ln \mu_p}{\lambda}\right\},$$

system (3) is GCFTB with respect to $(\delta, \varepsilon, T_f, d, \sigma(t))$, where μ_p $(\mu_p \geq 1)$ satisfies

$$\nu_p \leq \mu_p \nu_q,$$

$$\upsilon_p \leq \mu_p \upsilon_q, \tag{13}$$

$$\vartheta_p \leq \mu_p \vartheta_q,$$

$$\forall p, q \in \underline{S}.$$

And the guaranteed cost value of system (3) is given by

$$J = \int_0^{T_f} x^T(s) R_1 ds \leq J^* \tag{14}$$

$$= e^{2\lambda T_f}\left(\xi_2 + \tau e^{\lambda \tau} \xi_3 + \tau^2 e^{\lambda \tau} \xi_4 + \gamma d\right).$$

Proof. Construct the following copositive Lyapunov-Krasovskii functional for system (3):

$$V_{\sigma(t)}(t, x(t)) = x^T(t) \nu_p + \int_{t-d(t)}^t e^{\lambda_p(t-s)} x^T(s) \upsilon_p ds$$
$$+ \int_{-\tau}^0 \int_{t+\theta}^t e^{\lambda_p(t-s)} x^T(s) \vartheta_p ds\, d\theta, \tag{15}$$

where ν_p, υ_p, and $\vartheta_p \in R_+^n$, $\forall p \in \underline{S}$.

For the sake of simplicity, $V_{\sigma(t)}(t, x(t))$ is written as $V_{\sigma(t)}(t)$ in this paper.

Along the trajectory of system (3), we have

$$\dot{V}_{\sigma(t)}(t) = x^T(t) A_p^T \nu_p + x^T(t - d(t)) A_{dp}^T \nu_p$$

$$+ w^T(t) B_p^T \nu_p$$

$$+ \lambda_p \int_{t-d(t)}^t e^{\lambda_p(t-s)} x^T(s) \upsilon_p ds + x^T(s) \upsilon_p$$

$$- \left(1 - \dot{d}(t)\right) e^{\lambda_p d(t)} x^T(t - d(t)) \upsilon_p$$

$$+ \lambda_p \int_{-\tau}^0 \int_{t+\theta}^t e^{\lambda_p(t-s)} x^T(s) \vartheta_p ds\, d\theta$$

$$+ \tau x^T(t) \vartheta_p - \int_{-\tau}^0 e^{-\lambda_p \theta} x^T(t + \theta) \vartheta_p d\theta$$

$$\leq x^T(t)A_p^T\nu_p + x^T(t-d(t))A_{dp}^T\nu_p$$
$$+ w^T(t)B_p^T\nu_p$$
$$+ \lambda_p \int_{t-d(t)}^{t} e^{\lambda_p(t-s)}x^T(s)\upsilon_p ds + x^T(s)\upsilon_p$$
$$- (1-h)x^T(t-d(t))\upsilon_p$$
$$+ \lambda_p \int_{-\tau}^{0}\int_{t+\theta}^{t} e^{\lambda_p(t-s)}x^T(s)\vartheta_p ds\,d\theta$$
$$+ \tau x^T(t)\vartheta_p - \int_{t-d(t)}^{t} x^T(s)\vartheta_p ds. \tag{16}$$

Combining (8), (9), (15), and (16) leads to

$$\dot{V}_{\sigma(t)}(t) - \lambda_p V_{\sigma(t)}(t) + x^T(t)R_1$$
$$= x^T(t)A_p^T\nu_p - \lambda_p x^T(t)\nu_p + x^T(s)\upsilon_p$$
$$+ \tau x^T(t)\vartheta_p + x^T(t-d(t))A_{dp}^T\nu_p$$
$$- (1-h)x^T(t-d(t))\upsilon_p + w^T(t)\xi_2 B_p^T\delta$$
$$- \int_{t-d(t)}^{t} x^T(s)\vartheta_p ds + x^T(t)R_1 \tag{17}$$
$$\leq x^T(t)\left(A_p^T\nu_p - \lambda_p\nu_p + \upsilon_p + \tau\vartheta_p + R_1\right)$$
$$+ x^T(t-d(t))\left(A_{dp}^T\nu_p - (1-h)\upsilon_p\right)$$
$$+ \gamma\|w(t)\|.$$

Substituting (7) into (17) yields

$$\dot{V}_{\sigma(t)}(t) - \lambda_p V_{\sigma(t)}(t) + x^T(t)R_1 - \gamma\|w(t)\| \leq 0. \tag{18}$$

It implies

$$\dot{V}_{\sigma(t)}(t) - \lambda_p V_{\sigma(t)}(t) - \gamma\|w(t)\| \leq 0. \tag{19}$$

Integrating both sides of (19) during the period $[t_k, t)$ for $t \in [t_k, t_{k+1})$ leads to

$$V_{\sigma(t)}(t) \leq e^{\lambda_{\sigma(t_k)}(t-t_k)}V_{\sigma(t_k)}(t_k)$$
$$+ \gamma \int_{t_k}^{t} e^{\lambda_{\sigma(t_k)}(t-s)}w^T(s)ds. \tag{20}$$

For any $T_f > 0$, let N be the switching number of $\sigma(t)$ over $[0, T_f)$, and denote $t_1, t_2, \ldots, t_k, \ldots, t_N$ as the switching instants over the interval $[0, T_f)$. Then, for $t \in$

$[0, T_f)$, $V_{\sigma(t_k)}(t_k) \leq \mu_{\sigma(t_k)}V_{\sigma(t_k^-)}(t_k^-)$ is easily obtained from (13) and (15). Combining with (19), we have

$$V_{\sigma(t)}(t) \leq e^{\lambda_{\sigma(t_k)}(t-t_k)}V_{\sigma(t_k)}(t_k)$$
$$+ \gamma \int_{t_k}^{t} e^{\lambda_{\sigma(t_k)}(t-s)}w^T(s)ds$$
$$\leq \mu_{\sigma(t_k)}e^{\lambda_{\sigma(t_k)}(t-t_k)}V_{\sigma(t_k^-)}(t_k^-)$$
$$+ \gamma \int_{t_k}^{t} e^{\lambda_{\sigma(t_k)}(t-s)}w^T(s)ds$$
$$\leq \mu_{\sigma(t_k)}e^{\lambda_{\sigma(t_k)}(t-t_k)}\left[e^{\lambda_{\sigma(t_{k-1})}(t_k-t_{k-1})}V_{\sigma(t_{k-1})}(t_{k-1})\right.$$
$$\left.+ \gamma\int_{t_{k-1}}^{t_k} e^{\lambda_{\sigma(t_{k-1})}(t_k-s)}w^T(s)ds\right]$$
$$+ \gamma\int_{t_k}^{t}e^{\lambda_{\sigma(t_k)}(t-s)}w^T(s)ds \leq \cdots \leq \left(\prod_{i=1}^{k}u_{\sigma(t_i)}\right)$$
$$\cdot e^{\lambda_{\sigma(t_k)}(t-t_k)}e^{\sum_{i=1}^{k-1}\lambda_{\sigma(t_i)}(t_i-t_{i-1})}V_{\sigma(0)}(0) + \left(\prod_{i=1}^{k}u_{\sigma(t_i)}\right)$$
$$\cdot \gamma\int_{0}^{t_1}e^{\lambda_{\sigma(0)}(t_1-s)}w^T(s)ds + \left(\prod_{i=2}^{k}u_{\sigma(t_i)}\right)$$
$$\cdot \gamma\int_{t_1}^{t_2}e^{\lambda_{\sigma(t_1)}(t_2-s)}w^T(s)ds + \cdots$$
$$+ \gamma\int_{t_k}^{t}e^{\lambda_{\sigma(t_k)}(t-s)}w^T(s)ds \leq \left(\prod_{p=1}^{S}u_p^{N_{\sigma p}(0,t)}\right)$$
$$\cdot e^{\sum_{p=1}^{S}\lambda_p T_p(0,t)}V_{\sigma(0)}(0)$$
$$+ \gamma\int_{0}^{t}\left(\prod_{p=1}^{S}u_p^{N_{\sigma p}(0,t)}\right)e^{\sum_{p=1}^{S}\lambda_p T_p(s,t)}w^T(s)ds. \tag{21}$$

According to the definition of MDADT, one can obtain from (21)

$$V_{\sigma(t)}(t)$$
$$\leq \left(\prod_{p=1}^{S}u_p^{T_p(0,t)/\tau_{\alpha p}}\right)e^{\sum_{p=1}^{S}\lambda_p T_p(0,t)}\left(V_{\sigma(0)}(0) + \gamma d\right) \tag{22}$$
$$= e^{\sum_{p=1}^{S}(\lambda_p + \ln\mu_p/\tau_{\alpha p})T_p(0,t)}\left(V_{\sigma(0)}(0) + \gamma d\right).$$

Choose $\kappa = \max_{p\in\underline{S}}(\lambda_p + \ln\mu_p/\tau_{\alpha p})$. Therefore, for $t \in [0, T_f)$

$$V_{\sigma(t)}(t) \leq e^{\kappa T_f}\left(V_{\sigma(0)}(0) + \gamma d\right). \tag{23}$$

Noting the definition of $V_{\sigma(t)}(t)$ and (8), we have

$$V_{\sigma(t)}(t) \geq \xi_1 x^T(t)\varepsilon, \tag{24}$$

$$V_{\sigma(0)}(0) \leq \xi_2 x^T(0)\delta + \tau e^{\lambda \tau}\xi_3 \sup_{-\tau \leq \theta \leq 0}\left\{x^T(\theta)\delta\right\}$$

$$+ \tau^2 e^{\lambda \tau}\xi_4 \sup_{-\tau \leq \theta \leq 0}\left\{x^T(\theta)\delta\right\}$$

$$\leq \left(\xi_2 + \tau e^{\lambda \tau}\xi_3 + \tau^2 e^{\lambda \tau}\xi_4\right)\sup_{-\tau \leq \theta \leq 0}\left\{x^T(\theta)\delta\right\} \quad (25)$$

$$\leq \xi_2 + \tau e^{\lambda \tau}\xi_3 + \tau^2 e^{\lambda \tau}\xi_4.$$

Combining (21)–(25), we obtain

$$x^T(t)\varepsilon \leq \frac{1}{\xi_1}e^{\kappa T_f}\left(\xi_2 + \tau e^{\lambda \tau}\xi_3 + \tau^2 e^{\lambda \tau}\xi_4 + \gamma d\right). \quad (26)$$

Substituting (10) into (26), one has

$$x^T(t)\varepsilon < 1. \quad (27)$$

From Definition 7, we conclude that system (3) is FTB with respect to $(\delta, \varepsilon, T_f, d, \sigma(t))$.

Next, we will give the guaranteed cost value of system (3).

According to (17), denoting $\nabla(t) = \gamma\|w(t)\| - x^T(t)R_1$ and integrating both sides of (18) from t_k to t for $t \in [t_k, t_{k+1}]$, it gives rise to

$$V_{\sigma(t)}(t) \leq e^{\lambda_p(t-t_k)}V_{\sigma(t_k)}(t_k) + \int_{t_k}^t e^{\lambda_p(t-s)}\nabla(s)\,ds. \quad (28)$$

Similar to the proof process of (21), for any $t \in [0, T_f]$, we can obtain

$$V_{\sigma(t)}(t) \leq \left(\prod_{p=1}^S u_p^{N_{\sigma p}(0,t)}\right)e^{\sum_{p=1}^S \lambda T_p(0,t)}V_{\sigma(0)}(0)$$

$$+ \int_0^t \left(\prod_{p=1}^S u_p^{N_{\sigma p}(s,t)}\right)e^{\sum_{p=1}^S \lambda T_p(s,t)}\nabla(s)\,ds, \quad (29)$$

where $\lambda = \max_{i \in \underline{s}}\lambda_p$. From (29), we can get

$$\int_0^t \left(\prod_{p=1}^S u_p^{N_{\sigma p}(s,t)}\right)e^{\sum_{p=1}^S \lambda T_p(s,t)}x^T(s)R_1\,ds$$

$$\leq \left(\prod_{p=1}^S u_p^{N_{\sigma p}(0,t)}\right)e^{\sum_{p=1}^S \lambda T_p(0,t)}V_{\sigma(0)}(0) \quad (30)$$

$$+ \gamma \int_0^t \left(\prod_{p=1}^S u_p^{N_{\sigma p}(s,t)}\right)e^{\sum_{p=1}^S \lambda T_p(s,t)}\|w(s)\|\,ds.$$

Multiplying both sides of (30) by $\prod_{p=1}^S \mu^{-N_{\sigma p}(0,t)}$ leads to

$$\int_0^t \left(\prod_{p=1}^S u_p^{-N_{\sigma p}(0,s)}\right)e^{\lambda(t-s)}x^T(s)R_1\,ds$$

$$\leq e^{\lambda T_f}V_{\sigma(0)}(0) \quad (31)$$

$$+ \gamma \int_0^t \left(\prod_{p=1}^S u_p^{-N_{\sigma p}(0,s)}\right)e^{\lambda(t-s)}\|w(s)\|\,ds.$$

Noting that $N_{\sigma p}(0,s) \leq T_p(0,s)/\tau_{\alpha p}$ and $\tau_{\alpha p} > \ln \mu_p/\lambda$, we obtain that $0 < N_{\sigma p}(0,s) \leq T_p(0,s)/\tau_{\alpha p} \leq \lambda s/\ln \mu_p$; that is, $e^{-\lambda s} \leq \mu_p^{-N_p(0,s)} < 1$.

Then (31) can be turned into

$$\int_0^t e^{-\lambda s}e^{\lambda(t-s)}x^T(s)R_1\,ds$$

$$\leq \int_0^t \mu^{-N_{\sigma p}(0,s)}e^{\lambda(t-s)}x^T(s)R_1\,ds \quad (32)$$

$$\leq e^{\lambda T_f}V_{\sigma(0)}(0) + \gamma \int_0^t e^{\lambda(t-s)}\|w(s)\|\,ds.$$

Let $t = T_f$; then multiplying both sides of (32) by $e^{-\lambda T_f}$ leads to

$$\int_0^{T_f} e^{-2\lambda s}x^T(s)R_1\,ds \leq V_{\sigma(0)}(0)$$

$$+ \gamma \int_0^{T_f} e^{-\lambda s}\|w(s)\|\,ds \quad (33)$$

$$\leq V_{\sigma(0)}(0) + \gamma \int_0^{T_f}\|w(s)\|\,ds.$$

Substituting (2) into (33) yields

$$e^{-2\lambda T_f}\int_0^{T_f} x^T(s)R_1\,ds \leq V_{\sigma(0)}(0) + \gamma d \quad (34)$$

which can be rewritten as

$$\int_0^{T_f} x^T(s)R_1\,ds \leq e^{2\lambda T_f}\left(V_{\sigma(0)}(0) + \gamma d\right). \quad (35)$$

Substituting (25) into (35), the guaranteed cost value of system (3) is given by

$$J = \int_0^{T_f} x^T(s)R_1\,ds \leq J^*$$

$$= e^{2\lambda T_f}\left(\xi_2 + \tau e^{\lambda \tau}\xi_3 + \tau^2 e^{\lambda \tau}\xi_4 + \gamma d\right). \quad (36)$$

Therefore, according to Definition 10, we can conclude that system (3) is GCFTB with respect to $(\delta, \varepsilon, T_f, d, \sigma(t))$. Thus, the proof is completed. □

Remark 12. In Theorem 11, if $\mu_p \equiv 1$ in (12), then one can obtain $\tau_{\alpha p}^* = 0$, which means that the switching signal can be arbitrary.

When the exogenous disturbance $w(t) = 0$, the result on guaranteed cost finite-time stability can be obtained as follows.

Corollary 13. *Consider switched positive system (3) with $w(t) = 0$. For given constants T_f, λ, and vectors $\delta \succ \varepsilon \succ 0$, $R_1 \succ 0$, if there exist a set of positive vectors v_p, u_p, ϑ_p, $p \in \underline{S}$, and positive constants ξ_1, ξ_2, ξ_3, and ξ_4, such that (7), (8), and the following inequality holds*

$$\xi_2 + \tau e^{\lambda \tau}\xi_3 + \tau^2 e^{\lambda \tau}\xi_4 < \xi_1 e^{-\kappa T_f} \quad (37)$$

then under the following MDADT scheme

$$\tau_{ap} > \tau_{ap}^{*}$$
$$= \frac{T_f \ln \mu_p}{\ln\left(\xi_1 e^{-\lambda_p T_f}\right) - \ln\left(\xi_2 + \tau e^{\lambda\tau}\xi_3 + \tau^2 e^{\lambda\tau}\xi_4\right)} \quad (38)$$

system (3) is guaranteed cost finite-time stable to $(\delta, \varepsilon, T_f, d, \sigma(t))$, where $\mu_p \geq 1$ satisfies (13); the corresponding guaranteed cost value is given by

$$J^* = e^{2\lambda T_f}\left(\xi_2 + \tau e^{\lambda\tau}\xi_3 + \tau^2 e^{\lambda\tau}\xi_4\right). \quad (39)$$

3.2. GCFTB Controller Design. In this section, we are concerned with the GCFTB controller design of positive switched system (1). The static output feedback control law and state feedback control law will be designed to ensure the close-loop system (1) is GCFTB, respectively.

3.2.1. Static Output Feedback Controller Design. Consider system (1); under the controller $u(t) = K_{\sigma(t)}y(t)$, the corresponding closed-loop system is given by

$$\dot{x}(t) = \left(A_{\sigma(t)} + G_{\sigma(t)}K_{\sigma(t)}C_{\sigma(t)}\right)x(t)$$
$$+ A_{d\sigma(t)}x(t - d(t)) + B_{\sigma(t)}w(t),$$
$$x(\theta) = \varphi(\theta), \quad \theta \in [-\tau, 0], \quad (40)$$
$$y(t) = C_{\sigma(t)}x(t).$$

By Lemma 3, to guarantee the positivity of system (40), $A_p + G_p K_p C_p$ should be Metzler matrices, $\forall p \in \underline{S}$. Theorem 14 gives some sufficient conditions to guarantee that the closed-loop system (1) is GCFTB.

Theorem 14. *Consider the positive switched system (1). For given constants T_f, λ_p, γ and vectors $\delta \succ \varepsilon \succ 0$, $R_1 \succ 0$, and $R_2 \succ 0$, if there exist a set of positive vectors v_p, υ_p, ϑ_p, $p \in \underline{S}$, and positive constants ξ_1, ξ_2, ξ_3, and ξ_4, (8)–(10) and the following conditions hold:*

$$A_p + G_p K_p C_p \text{ are Metzler matrices}, \quad (41)$$

$$\overline{\Psi}_p = \text{diag}\left\{\overline{\psi}_{p1}, \overline{\psi}_{p2}, \ldots, \overline{\psi}_{pn}, \overline{\psi}'_{p1}, \overline{\psi}'_{p2}, \ldots, \overline{\psi}'_{pn}\right\}$$
$$\leq 0, \quad (42)$$

where

$$\overline{\psi}_{pr} = a_{pr}^T - \lambda_p v_{pr} + \upsilon_{pr} + \tau\vartheta_{pr} + R_1 + g_{pr},$$
$$\overline{\psi}'_{pr} = a_{dpr}^T v_p - (1 - h)v_{pr} \quad (43)$$
$$\lambda = \max_{p \in \underline{S}}\left\{\lambda_p\right\}, \ r \in \underline{n} = \{1, 2, \ldots, n\}.$$

$g_p = C_p^T K_p^T (G_p^T v_p + R_2)$, where g_{pr} represents the rth elements of vector g_p, $a_{pr}(a_{dpr})$ represents the rth column vector of the matrix $A_p(A_{dp})$, and $v_p = [v_{p1}, v_{p2}, \ldots, v_{pn}]^T$, $\upsilon_p =$ $[\upsilon_{p1}, \upsilon_{p2}, \ldots, \upsilon_{pn}]^T$, and $\vartheta_p = [\vartheta_{p1}, \vartheta_{p2}, \ldots, \vartheta_{pn}]^T$, v_{pr}, υ_{pr}, and ϑ_{pr} represent the rth elements of the vectors v_p, υ_p, and ϑ_p, respectively, and μ_p $(\mu_p \geq 1)$ satisfies (13); then under the MDADT scheme (12), the resulting closed-loop system (40) is GCFTB with respect to $(\delta, \varepsilon, T_f, d, \sigma(t))$ and the guaranteed cost value of system (40) is given by

$$J = \int_0^{T_f}\left(x^T(s)R_1 + x^T(s)C_{\sigma(t)}^T K_{\sigma(t)}^T R_2\right)ds \leq J^*$$
$$= e^{2\lambda T_f}\left(\xi_2 + \tau e^{\lambda\tau}\xi_3 + \tau^2 e^{\lambda\tau}\xi_4 + \gamma d\right). \quad (44)$$

Proof. By Lemma 3, we know that $A_p + G_p K_p C_p$ is a Metzler matrix for each $p \in \underline{S}$. According to Lemma 4, system (40) is positive if A_{dp}, B_p, G_p, and C_p, $p \in \underline{S}$ are all nonnegative. Replacing A_p in (7) with $A_p + G_p K_p C_p$ and letting $g_p = C_p^T K_p^T (G_p^T v_p + R_2)$, similar to Theorem 11, we easily obtain that the resulting closed-loop system (40) is GCFTB with respect to $(\delta, \varepsilon, T_f, d, \sigma(t))$ and the guaranteed cost value is given by (44).

The proof is completed. □

3.2.2. State Feedback Controller Design. Consider system (1); under the controller $u(t) = \overline{K}_{\sigma(t)}x(t)$, the corresponding closed-loop system is given by

$$\dot{x}(t) = \left(A_{\sigma(t)} + G_{\sigma(t)}\overline{K}_{\sigma(t)}\right)x(t) + A_{d\sigma(t)}x(t - d(t))$$
$$+ B_{\sigma(t)}w(t),$$
$$x(\theta) = \varphi(\theta), \quad \theta \in [-\tau, 0], \quad (45)$$
$$y(t) = C_{\sigma(t)}x(t).$$

By Lemma 3, to guarantee the positivity of system (45), $A_p + G_p\overline{K}_p$ should be Metzler matrices, $\forall p \in \underline{S}$. The following Theorem 15 gives some sufficient conditions to guarantee that closed-loop system (1) is GCFTB.

Theorem 15. *Consider the positive switched system (1). For given constants T_f, λ_p, γ and vectors $\delta \succ \varepsilon \succ 0$, $R_1 \succ 0$, and $R_2 \succ 0$, if there exist a set of positive vectors v_p, υ_p, ϑ_p, $p \in \underline{S}$, and positive constants ξ_1, ξ_2, ξ_3, and ξ_4, (8)–(10) and the following conditions hold:*

$$A_p + G_p\overline{K}_p \text{ are Metzler matrices}, \quad (46)$$

$$\overline{\Psi}_p = \text{diag}\left\{\overline{\psi}_{p1}, \overline{\psi}_{p2}, \ldots, \overline{\psi}_{pn}, \overline{\psi}'_{p1}, \overline{\psi}'_{p2}, \ldots, \overline{\psi}'_{pn}\right\}$$
$$\leq 0, \quad (47)$$

where

$$\overline{\psi}_{pr} = a_{pr}^T - \lambda_p v_{pr} + \upsilon_{pr} + \tau\vartheta_{pr} + R_1 + g_{pr},$$
$$\overline{\psi}'_{pr} = a_{dpr}^T v_p - (1 - h)v_{pr}, \quad (48)$$
$$\lambda = \max_{p \in \underline{S}}\left\{\lambda_p\right\}, \ r \in \underline{n} = \{1, 2, \ldots, n\}.$$

$g_p = \overline{K}_p^T(G_p^T\nu_p + R_2)$, where g_{pr} represents the rth elements of vector g_p, $a_{pr}(a_{dpr})$ represents the rth column vector of the matrix $A_p(A_{dp})$, and $\nu_p = [\nu_{p1}, \nu_{p2}, \ldots, \nu_{pn}]^T$, $\upsilon_p = [\upsilon_{p1}, \upsilon_{p2}, \ldots, \upsilon_{pn}]^T$, and $\vartheta_p = [\vartheta_{p1}, \vartheta_{p2}, \ldots, \vartheta_{pn}]^T$, ν_{pr}, υ_{pr}, and ϑ_{pr} represent the rth elements of the vectors ν_p, υ_p, and ϑ_p, respectively, and μ_p ($\mu_p \geq 1$) satisfies (13); then under the MDADT scheme (12), the resulting closed-loop system (45) is GCFTB with respect to $(\delta, \varepsilon, T_f, d, \sigma(t))$ and the guaranteed cost value of system (45) is given by

$$J = \int_0^{T_f} \left(x^T(s)R_1 + x^T(s)\overline{K}_{\sigma(s)}^T R_2\right)ds \leq J^* \tag{49}$$
$$= e^{2\lambda T_f}\left(\xi_2 + \tau e^{\lambda\tau}\xi_3 + \tau^2 e^{\lambda\tau}\xi_4 + \gamma d\right).$$

Proof. By Lemma 3, we know that $A_p + G_p\overline{K}_p$ is a Metzler matrix for each $p \in \underline{S}$. According to Lemma 4, system (45) is positive if A_{dp}, $\overline{B}_p$, G_p, and C_p are all nonnegative. Replacing A_p in (7) with $A_p + G_p\overline{K}_p$ and letting $g_p = \overline{K}_p^T(G_p^T\nu_p + R_2)$, similar to Theorem 11, we easily obtain that the resulting closed-loop system (45) is GCFTB with respect to $(\delta, \varepsilon, T_f, d, \sigma(t))$ and the guaranteed cost value is given by (49).

The proof is completed. □

Next, a procedure is presented to obtain the feedback gain matrices $K_p(\overline{K}_p)$, $p \in \underline{S}$.

Step 1. By adjusting the parameters λ_p and solving (8)–(10), (13), (42), and (47) via linear programming, positive vectors ν_p, υ_p, ϑ_p, and g_p can be obtained.

Step 2. Substituting ν_p and g_p into $g_p = C_p^T K_p^T(G_p^T\nu_p + R_2)$ (or $g_p = \overline{K}_p^T(G_p^T\nu_p + R_2)$), $K_p(\overline{K}_p)$ can be obtained.

Step 3. The gain $K_p(\overline{K}_p)$ is substituted into $A_p + G_pK_pC_p$ (or $A_p + G_p\overline{K}_p$). If $A_p + G_pK_pC_p$ (or $A_p + G_p\overline{K}_p$) are Metzler matrices, then $K_p(\overline{K}_p)$ are admissible. Otherwise, return to Step 1.

4. Numerical Example

In this section, a numerical example in [6] is provided to show the effectiveness of the proposed approach. Consider positive switched system (1) with the parameters as follows:

$$A_1 = \begin{bmatrix} -3.5 & 0.1 \\ 0.3 & -3 \end{bmatrix},$$

$$A_{d1} = \begin{bmatrix} 0.2 & 0.5 \\ 0.8 & 0.6 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.3 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.1 & 0.2 \\ 0.4 & 0.1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -5 & 0.1 \\ 0.1 & -2.5 \end{bmatrix},$$

$$A_{d2} = \begin{bmatrix} 0.6 & 0.3 \\ 0.8 & 0.5 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0.3 & 0.1 \\ 0.2 & 0.1 \end{bmatrix},$$

$$G_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0.2 & 0.5 \\ 0.3 & 0.1 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix},$$

$$R_2 = \begin{bmatrix} 0.3 \\ 0.2 \end{bmatrix},$$

$$\varepsilon = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},$$

$$\delta = \begin{bmatrix} 1 \\ 2 \end{bmatrix}. \tag{50}$$

Assuming that $d(t) = 0.2 + 0.2\sin(t)$, we can get $\tau = 0.4$ and $h = 0.2$.

Let

$$w(t) = \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix}, \tag{51}$$

$$f_1(t) = f_2(t) = \begin{cases} \dfrac{1 + \sin(t)}{t} & 3 \leq t \leq 10, \\ 0 & \text{others.} \end{cases}$$

Choosing the parameters $T_f = 10$, $\lambda_1 = 0.2$, $\lambda_2 = 0.14$, $\gamma = 1$, and $\mu_1 = \mu_2 = 1.4$ and solving the inequalities in Theorem 14 by linear programming, we get

$$\nu_1 = \begin{bmatrix} 1.2590 \\ 2.1813 \end{bmatrix},$$

$$\upsilon_1 = \begin{bmatrix} 1.8096 \\ 3.1278 \end{bmatrix},$$

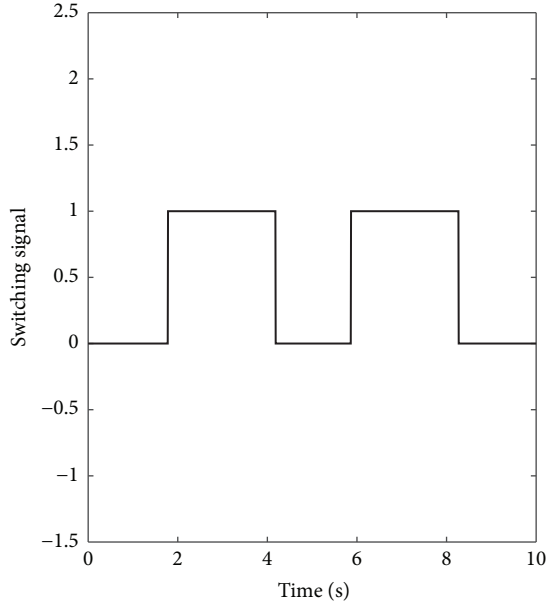$$\vartheta_1 = \begin{bmatrix} 1.4848 \\ 1.9703 \end{bmatrix},$$

FIGURE 1: Switching signal of system (1).

x1

x2

FIGURE 2: State trajectories of the closed-loop system (1) under static output feedback law.

$$\nu_2 = \begin{bmatrix} 1.1825 \\ 2.2998 \end{bmatrix},$$

$$\upsilon_2 = \begin{bmatrix} 1.8191 \\ 3.0665 \end{bmatrix},$$

$$\vartheta_2 = \begin{bmatrix} 1.4945 \\ 1.9646 \end{bmatrix},$$

$$g_1 = \begin{bmatrix} 0.7149 \\ 0.7630 \end{bmatrix},$$

$$g_2 = \begin{bmatrix} 0.8474 \\ 0.7229 \end{bmatrix}.$$

$$(52)$$

(I) By $g_p = C_p^T K_p^T (G_p^T \nu_p + R_2)$, we obtain the static output feedback controller:

$$K_1 = \begin{bmatrix} 0.1900 & 0.4910 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} 0.2650 & 0.1050 \end{bmatrix}.$$

$$(53)$$

(II) By $g_p = \overline{K}_p^T (G_p^T \nu_p + R_2)$, we obtain the state feedback controller:

$$\overline{K}_1 = \begin{bmatrix} 0.2881 & 0.1998 \end{bmatrix},$$
$$\overline{K}_2 = \begin{bmatrix} 0.1275 & 0.4291 \end{bmatrix}.$$

$$(54)$$

It is easy to verify that (41) and (46) are satisfied. Then, according to (12), we get $\tau_{\alpha1}{}^* = 1.6824$ and $\tau_{\alpha2}{}^* = 2.4034$.

The simulation results are shown in Figures 1–5; the initial conditions of system (1) are $x(0) = \begin{bmatrix} 0.5 & 0.2 \end{bmatrix}^T$

FIGURE 3: The evolution of $x^T(t)\varepsilon$ of system (1) under static output feedback law.

and $x(\theta) = 0$, $\theta \in [-\tau, 0)$, which satisfy the condition $\sup_{-\tau \leq t \leq 0} \{x^T(t)\delta\} \leq 1$. Figure 1 depicts the switching signal with MDADT. Under the static output feedback control law, Figures 2 and 3 are obtained to show the state trajectories and the evolution of $x^T(t)\varepsilon$ of the closed-loop system (1). Under the state feedback control law, Figures 4 and 5 depict the
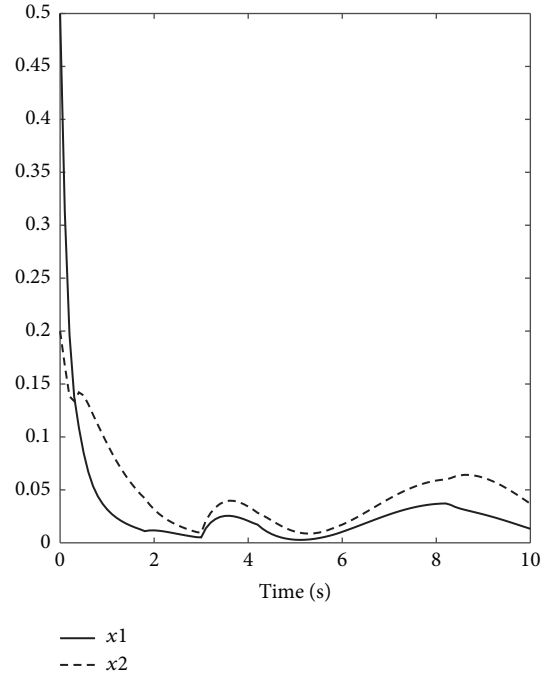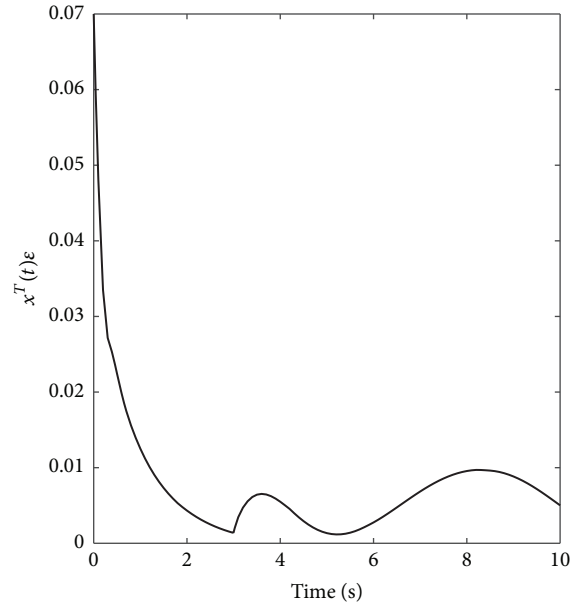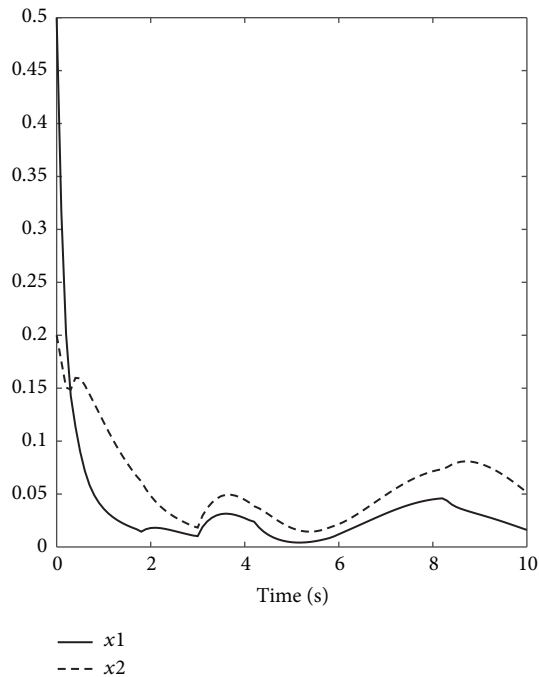
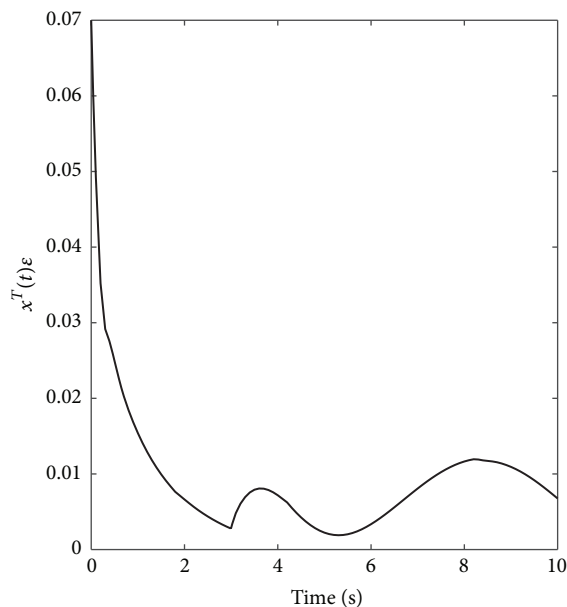FIGURE 4: State trajectories of the closed-loop system (1) under state feedback law.

FIGURE 5: The evolution of $x^T(t)\varepsilon$ of system (1) under state feedback law.

state trajectories and the evolution of $x^T(t)\varepsilon$ of the closed-loop system (1). The cost value is $J^* = 50.9407$, which can be obtained by (44) or (49).

Remark 16. Reference [6] considered the problem of exponential L_1 output tracking control for positive switched linear systems with ADT, which focuses on the asymptotic stability and the exponential L_1 performance index. There are some differences between [6] and this paper. Firstly, the finite-time stability in this paper and the asymptotic stability in [6] are two absolutely different concepts. Secondly, the performance indexes are also different. Finally, this paper uses MDADT approach instead of ADT method in [6]. So we do not compare the simulation results.

5. Conclusions

This paper has dealt with the problem of guaranteed cost finite-time control for positive switched systems with tine-varying delays. A novel guaranteed cost performance index is introduced. Based on the MDADT approach, some sufficient conditions and two kinds of controllers are obtained to guarantee that the closed-loop system is GCFTB. Finally, a numerical example is provided to show the effectiveness of the proposed method.

Competing Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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