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Research Article

Fast Method for DOA Estimation with Circular and Noncircular Signals Mixed Together

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Most of the existing algorithms to estimate the direction of arrival (DOA) of signals deal with the situation that all signals are circular. However, it is quite often in practical engineering that circular and noncircular signals appear in the same time. To effectively detect DOA of signals in such circumstances, we propose a novel algorithm. Firstly, using received data and its conjugate, we can detect more signals because of the doubled array aperture. Secondly, through unitary transform and multistage Wiener filter (MSWF) technology, we can obtain the noise subspace of array without performing eigendecomposition. Finally, by employing the improved MUSIC algorithm,we can acquire the DOA of the mixed circular and noncircular signals through two-stage search. Simulation results clearly demonstrate the effectiveness of the proposed algorithm.

1. Introduction

In the array signal processing, as a kernel technology, direction of arrival (DOA) estimation for narrowband plane wave has received a significant amount of attention and it has been extensively applied in the field of radar, mobile communication, sonar, and seismology. As the typical subspace high-resolution DOA estimation algorithms, multiple signal classification (MUSIC) algorithm and estimation of signal parameters via rotational invariance technique (ESPRIT) algorithm [1] have become popular and have received significant amount of attention due to their high-resolution performance over the last several decades. Recently, some novel methods for DOA estimation have been proposed in [2–5]. In [2], based on modified covariance matching criterion method, Si et al. proposed a novel algorithm which has more precise estimation even with low SNR. Using projection spectrum and eigenspectrum, Huang [3] presented a fast DOA estimation algorithm. Chen et al. [4] proposed a new approach which can be used for direction of departure (DOD) estimation and DOA estimation. In [5], Jančovič et al. proposed an algorithm which is useful when there are more sources than sensors. These algorithms above mentioned enlarge the scope range of DOA estimation; however, all these algorithms usually assume that the incoming signals are circular, but there are many noncircular signals, such as BPSK and GMSK modulated signals in the applications of intelligent antenna systems. People often use BPSK and GMSK signals instead of normal narrow circular signals to conduct DOA estimation in such systems. Utilizing received noncircular signals and their conjugate, the array aperture is doubled and it is possible to estimate more signals than array sensors. Noncircular signals have acquired a lot of attention because of the noncircularity properties. More and more researchers have shown their concerns on using the feature of noncircular to improve the performance of parameters estimation, for example, noncircular root-MUSIC [6], enhancing unitary ESPRIT [7], and MUSIC-like algorithm [8, 9] for noncircular signals. In [10], Abeida and Delmas analyzed explicitly a MUSIC-like algorithm for noncircular signals. In [11], Hassen et al. proposed a new algorithm which can be used for spatially correlated noncircular signals. Abamovich et al. [12] expanded noncircular to 2D space and Yang et al. [13] proposed a low-complexity 2D noncircular algorithm. More recently, the emerging domain of sparse representations has given further interest to the issue of DOA estimation, using sparse representations; Liu et al. [14] proposed a high-precision DOA estimation algorithm for noncircular sources. All these algorithms for noncircular signals above mentioned enlarge the array aperture and can detect more signals. Also, these algorithms have higher resolution than those algorithms corresponding to using circular signals. However, it is more realistic in practical engineering that circular and noncircular signals appear together. It is a realistic problem to estimate signals DOA under such circumstances. The author in [15] brought forward an improved MUSIC algorithm that has good performance when circular and noncircular signals appear together, but it needs eigendecomposition to obtain signal subspace. Because performing eigendecomposition results in heavy computation load to the systems, it is not appropriate to use the improved MUSIC algorithm generated in [15] in actual applications. Recently, Goldstein et al. [16] introduced a multistage Wiener filter (MSWF), adopting the MSWF technique; without the estimation of covariance matrix and its eigendecomposition, the signal subspace can be estimated. Based on MSWF theory, the authors in [17] proposed a fast subspace algorithm for narrowband circular signals; however, this method fails when there are noncircular signals in the received data.

In this paper, we propose a fast DOA estimation algorithm at the circumstance of mixed circular and noncircular signals. The paper is organized as follows. In Section 2, we introduce the system model that will be used throughout the paper. In Section 3, we introduce improved MUSIC algorithm for mixing circular and noncircular signals. In Section 4, MSWF technique is introduced. In Section 5, we formulate the proposed fast improved MUSIC algorithm for mixed circular and noncircular signals. In Section 6, simulation results confirm the good performance of our proposed fast algorithm. Finally, our conclusion is drawn in Section 7.

2. Array and Data Model

Consider unifrom linear array (ULA) composed of M sensors, regarding first array sonser as the reference; array interspace is d which is equal to $\lambda/2$ and λ is signal wavelength. Suppose that there are L signals including L_r noncircular signals with direction θ_i $(i = 1, 2, \dots, L_r)$ and L_c circular signals with direction θ_i $(i = 1, 2, \dots, L_c)$ impinging on the array, where $L = L_r + L_c$. The vector of received signal at time t can be modeled as follows:

$$\mathbf{X}(t) = \mathbf{A}(\theta)\mathbf{S}(t) + \mathbf{N}(t), \tag{1}$$

where $\mathbf{X}(t)$ is array output vector. $\mathbf{A}(\theta) = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \dots, \mathbf{a}(\theta_L)]$ is the $M \times L$ array manifold matrix and $\mathbf{a}(\theta_i) = [1, e^{j\pi \sin(\theta_i)}, \dots, e^{j\pi(M-1)\sin(\theta_i)}]^T$ is steering vector; the operators $(\cdot)^T$ denote transpose. Consider $\mathbf{S}(t) = \left[\mathbf{S}_c^T(t)\ \ \mathbf{S}_{nc}^T(t)\right]^T$, where $\mathbf{S}_c(t) = [\mathbf{s}_{c_1}(t), \mathbf{s}_{c_2}(t), \dots, \mathbf{s}_{c_{L_c}}(t)]^T$ is circular signal vector and $\mathbf{S}_{nc}(t) = [\mathbf{s}_{nc_1}(t), \mathbf{s}_{nc_2}(t), \dots, \mathbf{s}_{nc_{L_r}}(t)]^T$ is noncircular signal vector. Using the nature of noncircular signals, $\mathbf{s}_{nc_i}(t)$ can be expressed as $\mathbf{s}_{nc_i}(t) = e^{j\phi_i}\overline{\mathbf{s}}_{nc_i}(t)$, in which $\overline{\mathbf{s}}_{nc_i}(t)$ refer to the real of noncircular signal, ϕ_i is initial phase of noncircular signal, and we can get $\mathbf{S}_{nc}(t) = \Gamma\overline{\mathbf{S}}_{nc}(t)$, where $\mathbf{\Gamma} = \operatorname{diag}\{e^{j\phi_i}\}$, $\overline{\mathbf{S}}_{nc}(t) = [\overline{\mathbf{s}}_{nc_1}(t), \overline{\mathbf{s}}_{nc_2}(t), \dots, \overline{\mathbf{s}}_{nc_{L_r}}(t)]^T$. Define $\mathbf{S}_0(t) = [\overline{\mathbf{S}}_{nc}^T(t), \mathbf{S}_c^T(t)]^T$, and signal vector $\mathbf{S}(t)$ can be denoted as

$$\mathbf{S}(t) = \mathbf{B}\mathbf{S}_0(t) \tag{2}$$

with $\mathbf{B} = \left[\begin{smallmatrix}\mathbf{\Gamma} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}\end{smallmatrix}\right]$, and $\mathbf{I}$ is $\mathbf{L}_c \times \mathbf{L}_c$ identity matrix. Substituting formula (2) into formula (1), we can get

$$\mathbf{X}(t) = \mathbf{A}(\theta)\mathbf{B}\mathbf{S}_0(t) + \mathbf{N}(t) \tag{3}$$

in which $\mathbf{N}(t) = [\mathbf{n}_1(t), \dots, \mathbf{n}_M(t)]^T$ is noise vector; here additional noise is Gaussian white noise.

3. Improved MUSIC Algorithm

The key technique of MUSIC algorithm is the orthogonal between with signal subspace and noise subspace, and the steering vector $\mathbf{a}(\theta)$ belongs to signal subspace which is orthogonal to noise subspace. Using the orthogonal feature, through angle search from $0^\circ\sim180^\circ$, we can obtain the DOA estimation of signals. MUSIC algorithm has good performance when all the signals are circular sources, but it is more realistic that some users transmit circular signals while others send out noncircular signals in practical engineering; in this circumstance, MUSIC algorithm cannot work well. In order to correctly estimate the signals DOA, Gao et al. [15] proposed the improved MUSIC algorithm to cope with a more general scenario where both circular and noncircular signals coexist.

Using observed signal vector and its complex conjugate counterpart, get a new vector

$$\begin{aligned}\mathbf{y}_0(t) &= \begin{bmatrix}\mathbf{X}\\ \mathbf{X}^*\end{bmatrix}\\ &= \begin{bmatrix}\mathbf{A}(\theta)\mathbf{B}\mathbf{S}_0(t)\\ (\mathbf{A}(\theta)\mathbf{B}\mathbf{S}_0(t))^*\end{bmatrix} + \begin{bmatrix}\mathbf{N}(t)\\ \mathbf{N}(t)^*\end{bmatrix}\\ &= \widehat{\mathbf{A}}\widehat{\mathbf{S}}(t) + \widehat{\mathbf{N}}(t)\end{aligned} \tag{4}$$

from which

$$\widehat{\mathbf{A}} = \begin{bmatrix}\mathbf{A}_{nc}\Gamma & \mathbf{A}_c & \mathbf{0}\\ (\mathbf{A}_{nc}\Gamma)^* & \mathbf{0} & {\mathbf{A}_c}^*\end{bmatrix}, \tag{5}$$

where $\mathbf{A}_{nc} = [\mathbf{a}(\theta_{nc_1}), \mathbf{a}(\theta_{nc_2}), \dots, \mathbf{a}(\theta_{nc_{L_r}})]$ is manifold matrix about noncircular signals, and $\mathbf{A}_c = [\mathbf{a}(\theta_{c_1}), \mathbf{a}(\theta_{c_2}), \dots, \mathbf{a}(\theta_{c_{L_c}})]$ is manifold matrix for circular signals. Consider $\widehat{\mathbf{S}}(t) = \left[\overline{\mathbf{S}}_{nc}^T(t)\ \ \mathbf{S}_c^T(t)\ \ \mathbf{S}_c^H(t)\right]^T$; $\mathbf{0}$ represents the $M \times L_c$ zeros matrix. When $L_r + 2L_c \le 2M$, $\widehat{\mathbf{A}}$ is full column rank for any θ. Supposing that N snapshots are collected, utilizing $\mathbf{y}_0(t)$, we can acquire the array autocovariance matrix. Consider

$$\mathbf{R}_x = E\left[\mathbf{y}_0(t)\mathbf{y}_0(t)^H\right] = \widehat{\mathbf{A}}\widehat{\mathbf{R}}_s\widehat{\mathbf{A}}^H + \sigma_n^2\mathbf{I}_{2M}. \tag{6}$$

The operators $E\{\cdot\}$ and $(\cdot)^H$ denote expectation and conjugate transpose, respectively; $\widehat{\mathbf{R}}_s = E\{\widehat{\mathbf{S}}(t)\widehat{\mathbf{S}}^H(t)\}$. $\widehat{\mathbf{R}}_s$ is signal autocovariance matrix which is full rank when received signals are not correlated.

The eigendecomposition of the positive definite Hermitian matrix $\mathbf{R}_x$ can be written as

$$\mathbf{R}_x = \mathbf{U}_s \mathbf{\Lambda} \mathbf{U}_s^H + \sigma^2 \mathbf{U}_n \mathbf{U}_n^H, \quad (7)$$

where $\mathbf{\Lambda}$ is $(L_r + 2L_c) \times (L_r + 2L_c)$ diagonal matrix. According to the knowledge of space spectrum, we know that $\mathbf{R}_x$ has $(L_r + 2L_c)$ larger eigenvalues and $(2M - L_r - 2L_c)$ smaller eigenvalues if the received signals are uncorrelated. $\mathbf{U}_s = [\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_{L_r+2L_c}]$, $\mathbf{U}_n = [\mathbf{u}_{L_r+2L_c+1}, \ldots, \mathbf{u}_{2M}]$, the columns of $\mathbf{U}_s$ contain the signal subspace eigenvectors of $\mathbf{R}_x$, and the columns of $\mathbf{U}_n$ contain the noise subspace eigenvectors f $\mathbf{R}_x$.

Since $\widehat{\mathbf{A}}$ and $\mathbf{U}_s$ span the signal subspace, both of them are orthogonal to the noise subspace spanned by the matrix $\mathbf{U}_n$. We derive DOA estimation by the criteria as follows.

For noncircular signals, let us define

$$\mathbf{G}(\theta) = \mathbf{Z}^H(\theta) \mathbf{U}_n \mathbf{U}_n^H \mathbf{Z}(\theta), \quad (8)$$

where $\mathbf{Z}(\theta) = \left[\begin{smallmatrix} \mathbf{a}(\theta) & \mathbf{0} \\ \mathbf{0} & \mathbf{a}^*(\theta) \end{smallmatrix}\right]$. When received signals come from the direction of θ_{nc_i} and $\mathbf{G}(\theta)$ is rank deficient, so we can use the following formula to estimate noncircular sources DOA:

$$\mathbf{P}_{nc}(\theta) = \frac{1}{\det\{\mathbf{G}(\theta)\}} = \frac{1}{\det\{\mathbf{Z}^H(\theta) \mathbf{U}_n \mathbf{U}_n^H \mathbf{Z}(\theta)\}}. \quad (9)$$

The prerequisite for using formula (9) is that the number of columns of $\mathbf{U}_n$ should be no less than 2; otherwise, $\mathbf{G}(\theta)$ is rank deficient whatever the θ is. Therefore, correctly using formula (9) is $2M - L_r - 2L_c \geq 2$.

For any direction coming from $\{\theta_{c1}, \theta_{c2}, \ldots, \theta_{cL_c}\}$, the $\mathbf{U}_n$ and $\widehat{\mathbf{A}}$ are orthogonal, and we can get formula as follows:

$$\mathbf{U}_n^H \begin{bmatrix} \mathbf{a}(\theta_{c_i}) \\ \mathbf{0} \end{bmatrix} = \mathbf{0}, \qquad \mathbf{U}_n^H \begin{bmatrix} \mathbf{0} \\ \mathbf{a}^*(\theta_{c_i}) \end{bmatrix} = \mathbf{0}. \quad (10)$$

Consequently, we can apply the following estimator to estimate DOA for circular signals:

$$\mathbf{P}_c(\theta) = \frac{1}{\mathbf{a}^H(\theta) \mathbf{U}_{n1} \mathbf{U}_{n1}^H \mathbf{a}(\theta)}, \quad (11)$$

where $\mathbf{U}_{n1} = \mathbf{U}_n(1:M,:)$.

4. Subspace Estimation by Multistage Wiener Filter

The MSWF technology presented by Goldstein et al. is to find an approximate solution to the Wiener-Hopf equation which does not need the inverse of array covariance matrix. Using multiple decomposition of MSWF, we can obtain estimated signal subspace fast. Figure 1 shows the structure of two-stage MSWF.

Using the operator $\mathbf{T}_1 = [\mathbf{h}_1, \mathbf{B}_1]$, we can decompose $\mathbf{y}_0(t)$ into two subspaces through orthogonal projection. One subspace is parallel to $\mathbf{h}_1$, and the other subspace named $\mathbf{B}_1$ is orthogonal to $\mathbf{h}_1$. The definition of $\mathbf{h}_1$ can be expressed as follows:

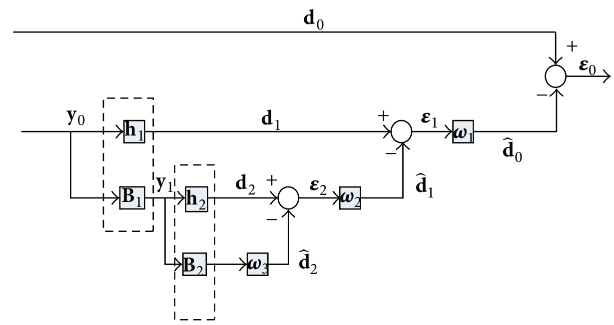

Figure 1: Two-stage orthogonal projection decomposition based on MSWF technique.

$$\mathbf{h}_1 = \frac{\mathbf{r}_{\mathbf{y}_0\mathbf{d}_0}}{\sqrt{(\mathbf{r}_{\mathbf{y}_0\mathbf{d}_0})^H \mathbf{r}_{\mathbf{y}_0\mathbf{d}_0}}}, \quad (12)$$

where $\mathbf{r}_{\mathbf{y}_0\mathbf{d}_0} = E[\mathbf{y}_0(t)\mathbf{d}_0^H(t)]$ is cross-correlation function and $\mathbf{B}_1$ is block matrix and it is the null space of $\mathbf{r}_{\mathbf{y}_0\mathbf{d}_0}$. Using $\mathbf{T}_2 = [\mathbf{h}_2, \mathbf{B}_2]$, we can deal with $\mathbf{y}_1(t)$ by the same method, and then new MSWF has appeared. If the stage of MWSF is big enough, the dimesion of the cross-correlation vector and the row of input data could be declined, untill droping to 1 finally. Using MSWF forward decline, we can attain signal subspace of $\mathbf{R}_x$ fast. Through $L_r + 2L_c$ stage recursive decomposition, signal subspace $\mathbf{U}_s$ could be achieved, which avoid $2M$ stage recursive decomposition of the noise subspace, and the whole algorithm computational complexity is decreased.

5. Fast Improved MUSIC Algorithm Based on Unitary Transform and MSWF

5.1. Unitary Transform for Received Data. Compared with real multiplication, the computational complexity of the complex multiplication is about fourfold. Usually, the array autocovariance matrix $\mathbf{R}_x$ is a complex matrix, and the operand of complex matrix is bigger than that of real matrix. If $\mathbf{R}_x$ is complex centro-Hermitian, through unitary transform, we can change it into a real matrix.

For a given arbitrarily matrix $\mathbf{Q}$, we denote $\widetilde{\mathbf{O}}_P$ as a $P \times P$ dimension exchange matrix with ones on its antidiagonal and zeros in other places. We say $\mathbf{Q}$ is a left-Π-real matrix if it satisfies $\widetilde{\mathbf{O}}_\mathbf{p}\mathbf{Q}^* = \mathbf{Q}$. Here we define a unitary matrix

$$\mathbf{Q}_{2n+1} = \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbf{I}_n & 0 & j\mathbf{I}_n \\ \mathbf{0}^T & \sqrt{2} & \mathbf{0}^T \\ \widetilde{\mathbf{O}}_n & 0 & -j\widetilde{\mathbf{O}}_n \end{bmatrix}, \quad (13)$$

where $\mathbf{Q}_{2n+1}$ is a left-Π-real matrix when the dimension of matrix is odd. If it is even, we can get a unitary left-Π-real matrix by dropping its center row and center column.

Suppose $\mathbf{T}_p$ and $\mathbf{U}_q$ are $p \times p$ and $q \times q$ dimension left-Π-real matrices. Considering the transformation of ϕ, we can obtain

$$\phi(\mathbf{M}) : \mathbf{M} \longmapsto \mathbf{T}_p^{-1}\mathbf{M}\mathbf{U}_q. \tag{14}$$

For an arbitrarily complex centro-Hermitian matrix $\mathbf{M}$, under the condition that the dimensions are kept the same, we can transform it into a real matrix by (14).

In order to exploit the feature of the noncircular signal completely, we construct a novel data vector

$$\mathbf{Z} = \left[\mathbf{y}_0 \quad \widetilde{\mathbf{O}}_{2M}\mathbf{y}_0^*\widetilde{\mathbf{O}}_N\right]. \tag{15}$$

Using $\phi(\mathbf{Z}) = \mathbf{Q}_{2M}\mathbf{Z}\mathbf{Q}_{2N}$, we can obtain the corresponding real matrix of $\mathbf{Z}$.

5.2. Fast Improved MUSIC Algorithm. Although we can utilize improved MUSIC algorithm to conduct DOA estimation in the circumstance that circular and noncircular signals coexist, it is difficult to use it in practical engineering because of the heavy computational burden of the algorithm. The improved MUSIC algorithm can get the noise subspace through eigendecomposition, while it is time-consuming to compute the eigendecomposition, so the improved MUSIC algorithm is not suitable. If we want to use it in practice, we must reduce the computational load of the algorithm. Using unitary transform and MSWF, we can get a fast improved MUSIC algorithm. Here we suppose that the stage decline number of signal subspace is $L_r + 2L_c$, the stage decline number of noise subspace is $2M$, and $L_r + 2L_c$ is far less than $2M$. The detailed procedures of the proposed fast improved MUSIC algorithm are given as follows.

(1) Choose initial reference signal $\mathbf{d}_0(k) = \mathbf{e} * \mathbf{Q}_{2M}\mathbf{Z}\mathbf{Q}_{2N}$, and $\mathbf{x}_0 = \mathbf{Q}_{2M}\mathbf{Z}\mathbf{Q}_{2N}$ is the input data of MSWF, where $\mathbf{e} = [1, 0, \ldots, 0]$.

(2) Using forward decline for any i, compute cross-correlation $\mathbf{h}_i$ $(i = 1, 2, \ldots, L_r + 2L_c)$. Consider

$$\begin{aligned}\mathbf{h}_i &= \frac{E\left[\mathbf{x}_{i-1}(t)\,\mathbf{d}_{i-1}^H(t)\right]}{\left\|E\left[\mathbf{x}_{i-1}(t)\,\mathbf{d}_{i-1}^H(t)\right]\right\|_2},\\ \mathbf{d}_i(t) &= \mathbf{h}_i^H\mathbf{x}_{i-1}(t),\\ \mathbf{B}_i &= \operatorname{null}\{\mathbf{h}_i\} = \mathbf{I} - \mathbf{h}_i\mathbf{h}_i^H,\\ \mathbf{x}_i(t) &= \mathbf{B}_i^H\mathbf{x}_{i-1}(t).\end{aligned} \tag{16}$$

(3) Using estimated signal number $L_r + 2L_c$ to estimate signal subspace

$$\mathbf{U}_s = \left[\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_{L_r+2L_c}\right]. \tag{17}$$

(4) Using formula (18),we can get noncircular signals and circular signals DOA

$$\begin{aligned}\mathbf{P}_{nc}(\theta) &= \frac{1}{\det\{\mathbf{K}(\theta)\}}\\ &= \frac{1}{\det\left\{(\mathbf{Q}_{2M}\mathbf{Z}(\theta))^H(\mathbf{I}_{2M} - \mathbf{U}_s\mathbf{U}_s^H)(\mathbf{Q}_{2M}\mathbf{Z}(\theta))\right\}},\\ \mathbf{P}_c(\theta) &= \frac{1}{\left[\mathbf{Q}_M\mathbf{a}(\theta)\right]^H\mathbf{U}_{n1}\mathbf{U}_{n1}^H\mathbf{Q}_M\mathbf{a}(\theta)},\end{aligned} \tag{18}$$

where $\mathbf{U}_{n1}\mathbf{U}_{n1}^H$ is a block matrix which is equal to $\mathbf{V}(1 : M, 1 : M)$, and $\mathbf{V} = \mathbf{I} - \mathbf{U}_s\mathbf{U}_s^H$.

5.3. Fast Improved MUSIC Algorithm Computational Complexity Analysis. We know that the columns of $\mathbf{K}(\theta)$ should be greater than or equal to 2, and $\mathbf{K}(\theta)$ does not decrease rank only at the condition of $2M - L_r - 2L_c \geq 2$. So the maximum estimated signal number of our fast improved MUSIC algorithm is $2M - 2$.

Computational complexity of subspace estimation for MUSIC algorithm mainly consists of two parts: one is correlation matrix operand which is equal to $O(NM^2)$ and the other is eigendecomposition which is equal to $O(M^3)$. The sum of the computational complexity is $O(NM^2 + M^3)$. For NC-MUSIC algorithm, the computational complexity is $O(4NM^2 + 8M^3)$. Using forward decline of MSWF based on correlation subtraction construction, if the number of mixed signals is known in advance, the computational load of estimation subspace is only $O(2MNL_r + 4MNL_c)$. Because we utilize the uniform transform before using MSWF technique, the computational load for acquiring estimated signal subspace of our fast improved MUSIC algorithm is only $O(MNL_r/2 + MNL_c)$. Normally, source number $L = L_r + L_c$ is less than M; therefore, the computational complexity of our fast algorithm is decreased fast.

6. Simulation Results

In this section, we will restrict our discussion to 1-D ULA consisting of 6 sensors with interelement space $d = \lambda/2$ and noncircular and circular signals come from far field at the same time. Consider the application in communication; here noncircular signals are binary phase shift keying (BPSK) signals. We use $N = 600$ snapshots to estimate the array covariance matrix, and the additional noise is ideal Gaussian white noise. Independent Monte-Carlo research number is 100, and the root mean square error (RMSE) for circular and noncircular signals is defined as

$$\begin{aligned}\text{RMSE}_1 &= \sqrt{\frac{1}{L_cN}\sum_{i=1}^{L_c}\sum_{k=1}^{N}\left(\widehat{\theta}_i - \theta_i\right)^2},\\ \text{RMSE}_2 &= \sqrt{\frac{1}{L_rN}\sum_{i=1}^{L_r}\sum_{k=1}^{N}\left(\widehat{\theta}_i - \theta_i\right)^2}.\end{aligned} \tag{19}$$

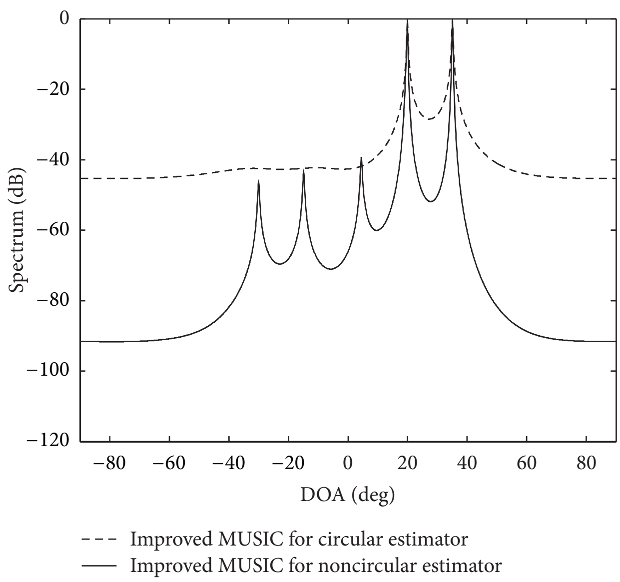

Figure 2: Improved MUSIC spectrum with 10 SNR.

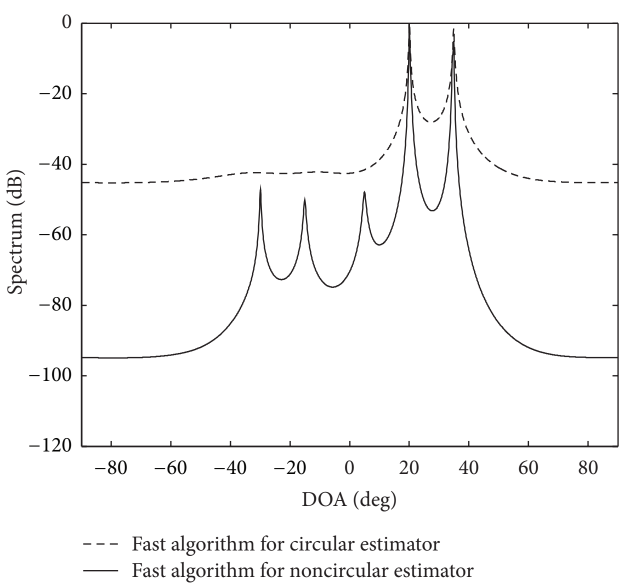

Figure 3: Fast improved MUSIC spectrum with 10 SNR.

Experiment 1. Three BPSK noncircular signals and two circular signals are impinging on the array; the incidence angles are −30°, −15°, 5° and 20°, and 35°, respectively. Simulation results of the improved MUSIC algorithm and the fast improved MUSIC algorithm are shown in Figures 2 and 3, respectively. From the simulation results, we know that both algorithms can estimate five signals DOA correctly and estimated effection are perfect. It is noticed that although the peak of the improved MUSIC algorithm is sharper than the fast improved MUSIC algorithm, the difference is not obvious, especially when the SNR is high. The computational load for signal subspace of the fast improved MUSIC algorithm is $O(12600)$, while the improved MUSIC algorithm computational burden is $O(88128)$. The former computational load is far less than the latter. Because of the low computational complexity, the fast improved MUSIC algorithm has more wonderful prospect than the improved MUSIC algorithm in practical engineering.

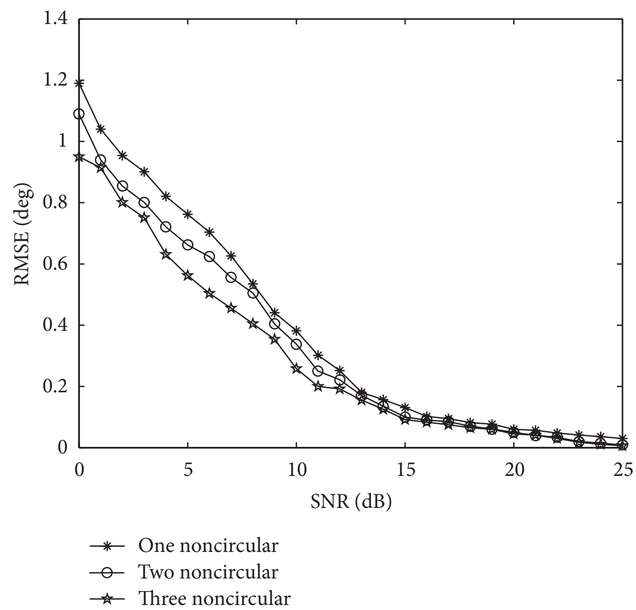

Figure 4: RMSE versus SNR.

Experiment 2. The number of noncircular and circular signals is seven; the incidence angles are −40°, −30°, −15°, −5°, 10°, 20°, and 35°. Consider three cases where there are one, two, and three noncircular sources. The first case with one noncircular signal is coming from angle −40°; the angles of the second case with two noncircular signals are −40° and −30°; the third case with three noncircular signals is coming from −40°, −30°, and −15°, respectively. From simulation results shown in Figures 4 and 5, we know that the fast improved MUSIC algorithm can estimate 7-signal DOA, and the estimated number is larger than the number of array sensors. It is clear that the performance of the fast improved MUSIC algorithm becomes better with the increasement of noncircular signals due to the increasement in the dimension of noise subspace.

Experiment 3. The maximum detected number for the fast improved MUSIC algorithm is $2M-2$ under the circumstance of mixed noncircular and circular signals. For 6-sensor ULA, we know the largest detected number is 10 which is equal to $L_r + 2L_c$, so the maximum detected number for noncircular signals is 8 (eight noncircular signals and one circular signal are impinging at the same time) and the maximum detected number for circular signals is 4 (two noncircular signals and four circular signals are impinging at the same time). The simulation results (see Figures 6 and 7) confirm the effectiveness of theoretical analysis.

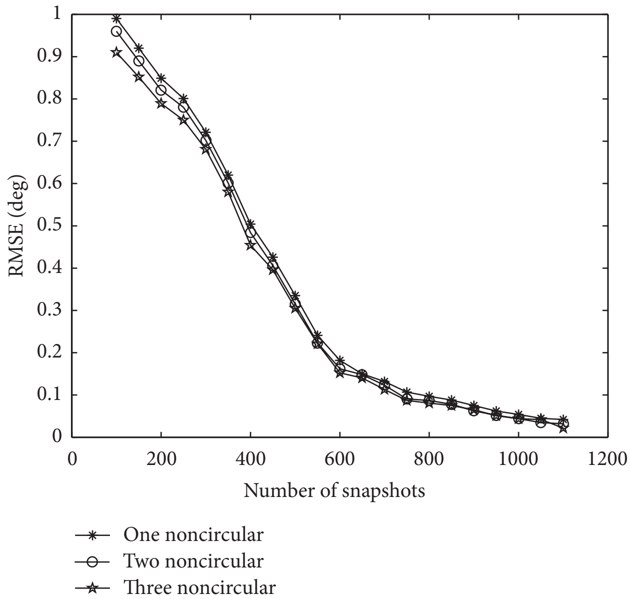

Figure 5: RMSE versus number of snapshots.

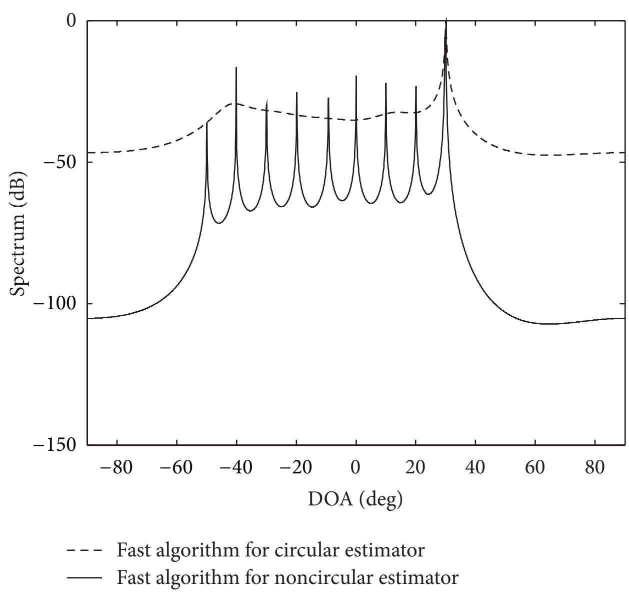

Figure 6: Spectrum with eight noncircular signals under 10 SNR.

7. Conclusion

In this paper, we presented a fast DOA estimation algorithm for the situation where circular and noncircular signals coexist. The proposed algorithm has high estimation accuracy and can estimate both circular and noncircular signals for ULA. It has two important advantages: firstly, because the number of sources resolved by our method can be greater than the number of array sensors, it is very suitable when there are multiple signals needing to be detected with small array. Secondly, it has low computational complexity compared to the improved MUSIC algorithm, so it has a wider range of prospective application in real-time DOA estimation. The computer simulations validate the effectiveness of our new method.

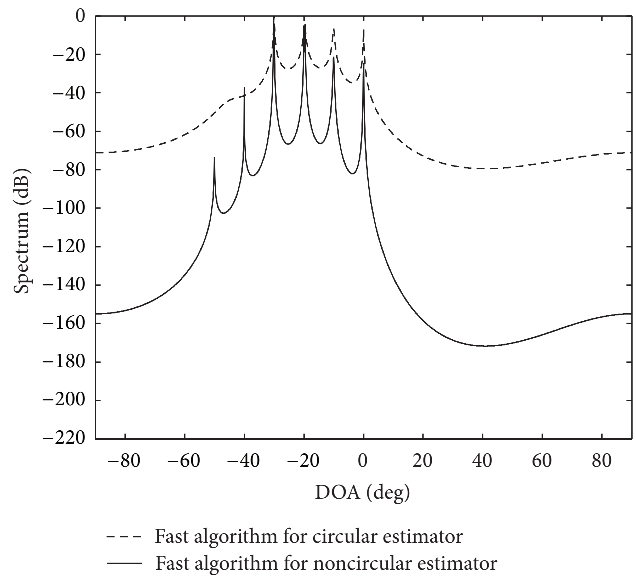

Figure 7: Spectrum with four circular signals under 15 SNR.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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