Hindawi Publishing Corporation
Discrete Dynamics in Nature and Society
Volume 2016, Article ID 4958217, 19 pages
http://dx.doi.org/10.1155/2016/4958217

Research Article

Attractor and Boundedness of Switched Stochastic Cohen-Grossberg Neural Networks

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Received 15 February 2016; Accepted 5 April 2016

Academic Editor: Guoqiang Hu

We address the problem of stochastic attractor and boundedness of a class of switched Cohen-Grossberg neural networks (CGNN) with discrete and infinitely distributed delays. With the help of stochastic analysis technology, the Lyapunov-Krasovskii functional method, linear matrix inequalities technique (LMI), and the average dwell time approach (ADT), some novel sufficient conditions regarding the issues of mean-square uniformly ultimate boundedness, the existence of a stochastic attractor, and the mean-square exponential stability for the switched Cohen-Grossberg neural networks are established. Finally, illustrative examples and their simulations are provided to illustrate the effectiveness of the proposed results.

1. Introduction

In the last few decades, theoretical and applied researches of artificial neural networks have been the new worldwide focus. Some of the reasons for this are due to the successful hardware implementations and their various applications, such as classification, associative memories, parallel computation, optimization, and signal processing [1, 2]. It is recognized that such applications of neural networks depend heavily on some dynamic behaviors, such as stability properties, periodic oscillatory behavior, and attractor and boundedness (see [3–16] and references therein).

Since the seminal work by Cohen and Grossberg [17], Cohen-Grossberg neural networks have been intensively studied [2, 18–22]. During hardware implementation, time delays do exist due to the finite switching speed of the amplifiers and communication time; it is important to incorporate delays into the neural networks. Generally speaking, there are two kinds of delays, discrete delays and distributed delays [2, 16, 23]. The utilization of discrete delays in models of delayed feedback provides a good approximation in simple circuits consisting of a small number of cells. When the neural networks have a spatial extent due to the presence of a multitude of parallel pathways with a variety of axon sizes and lengths, it is necessary to incorporate continuously distributed delays. The distributed delay includes finite delay and infinite delay [2, 5, 18, 24].

In real nervous systems, synaptic transmission is a noisy process brought about by random fluctuations from the release of neurotransmitters and other probabilistic causes [19, 25–27]. It is well known that for stochastic neural networks, it is rather difficult to analyze their dynamic properties due to the introduction of noise. Such studies are however important for understanding the dynamic characteristics of neuron behavior in stochastic environments. For instance, during the implementation of Kalman filter training, stochastic neural networks characterized as zero-mean white noise have been successfully employed [2].

On the other hand, neural networks are complex and large-scale nonlinear dynamics; during hardware implementation, the connection topology of networks may change very quickly and link failures or new creations in networks often bring about switching connection topology [2].

To obtain a deep and clear understanding of the dynamics of this complex system, one of the usual ways is to investigate the switched neural network. As a special class of hybrid systems, switched neural network systems are composed of a family of continuous-time or discrete time subsystems and a rule that orchestrates the switching among the subsystems [28]. In general, the switched rule is a piecewise constant function dependent on the state or time. The logical rule that orchestrates switching between these subsystems generates switching signals [29]. Recently, switched systems have numerous applications in the control of mechanical systems, the automotive industry, aircraft and air traffic control, switching power converters, and many other fields [30]. In [28], Huang et al. are the first to investigate the robust stability of switched Hopfield neural networks with time-varying delays by an arbitrary switched rule. The average dwell time approach provided an effective tool to study the stability of switched systems. Wu et al. used average dwell time approach to analyze the exponential stability of continuous-time switched delayed neural networks in [31]. In [27], the average dwell time and LMI method have been utilized to discuss the exponential synchronization of switched stochastic competitive neural networks with mixed delays. In addition, [32] has focused on the delay-dependent global robust asymptotic stability problem of uncertain switched Hopfield neural networks (USHNNs) with discrete interval and distributed time-varying delays and time delay in the leakage term. Moreover, parametric uncertainty which often breaks the stability of systems can be commonly encountered due to modeling inaccuracies or changes in the environment of the model. To deal with the difficulties brought about by uncertainty, exponential stability analysis and H_∞ control of different uncertain systems have received great research attention [33]. Moreover, the parametric uncertainty is assumed to be norm-bounded in [34]. Unfortunately, up to now, few researchers have considered the mean-square uniformly ultimate boundedness and stochastic attractor for switched SCGNN with discrete delays and infinite distributed delays.

However, these available literatures mainly consider the stability property of switching neural networks. In fact, except for the stability property, boundedness and attractor are also the foundational concepts of dynamical neural networks, which play important roles in the investigation of the uniqueness of the equilibrium point (periodic solutions), global asymptotic stability, global exponentially stability, and the synchronization [35]. To the best of the author's knowledge, few researchers have considered the uniformly ultimate boundedness and attractors for switched CGNN with discrete delays and distributed delays.

Inspired by the above discussions, the objects of this paper are to study the mean-square uniformly ultimate boundedness and stochastic attractor for switched SCGNN with discrete delays and infinitely distributed delays by employing stochastic analysis technology, the Lyapunov-Krasovskii functional method, the linear matrix inequalities (LMI) technique, and the average dwell time approach (ADT). In addition, the parametric uncertainty is considered and assumed to be norm-bounded.

As is well known, mean-square uniformly ultimate boundedness (MSUUB) conditions are derived in terms of linear matrix inequalities (LMIs), which can be easily calculated by the MATLAB LMI control toolbox. All of the above mentioned reasons motivate us to investigate the problems of the MSUUB and stochastic attractor for switched SCGNN in this paper. Numerical examples are provided to demonstrate the feasibility and effectiveness of the proposed criteria.

The rest of this paper is organized as follows. Some preliminaries are given in Section 2. We present some basic definitions and notations, as well as some lemmas needed in later sections. In Section 3, we present some sufficient conditions of MSUUB and stochastic attractor for switched stochastic CGNN. In Section 4, an example is presented to illustrate the effectiveness of the proposed approach. The conclusions are summarized in Section 5.

Notations. The superscript "T" stands for matrix transposition; R^n denotes the n-dimensional Euclidean space; the notation $P > 0$ means that P is real symmetric and positive-definite; I and O represent the identity matrix and a zero matrix, respectively; $\mathrm{diag}\{\cdots\}$ stands for a block-diagonal matrix; and $\lambda_{\min}(P)$ ($\lambda_{\max}(P)$) denotes the minimum (maximum) eigenvalue of symmetric matrix P. In symmetric block matrices or long matrix expressions, a $(*)$ is used to represent a term that is induced by symmetry.

2. Preliminaries and Problem Formulation

The *Itô*'s formula plays a key role in the dynamic analysis of stochastic systems. To facilitate understanding, some related results are cited here (see [36] for details). For a general stochastic system $dx(t) = f(x(t),t)dt + g(x(t),t)dw(t)$ on $t \geq t_0$ with initial value $x(t_0) = x_0 \in R^n$, where $w(t)$ is m-dimensional Brownian motion defined on $(\Omega, \mathscr{F}, \mathscr{P})$, $f : R^n \times R^+ \to R^n$, and $g : R^n \times R^+ \to R^{n\times m}$. Let $\mathscr{C}^{1,2}(R^n \times R^+; R^+)$ be the family of all nonnegative functions which are continuous once differentiable in t and twice differentiable in x. For $V \in \mathscr{C}^{1,2}(R^n \times R^+; R^+)$, define an operator $\mathscr{L}V$ from $(R^n \times R^+; R^+)$ to R by

$$\begin{aligned} dV &= \mathscr{L}V dt + \frac{\partial V}{\partial x} g(x(t),t)\,d\omega(t), \\ \mathscr{L}V(x(t),t) &= V_t(x(t),t) + V_x(x(t),t) f(x(t),t) \\ &\quad + \frac{1}{2}\mathrm{tra}\left[g^T(x(t),t) V_{xx}(x(t),t) g(x(t),t)\right], \end{aligned} \tag{1}$$

and $V_t(x(t),t) = \partial V(x(t),t)/\partial t$, $V_{xx}(x(t),t) = (\partial^2 V(x(t),t)/\partial x_i \partial x_j)_{n\times n}$, and $V_x(x(t),t) = (\partial V(x(t),t)/\partial x_1, \ldots, \partial V(x(t),t)/\partial x_n)$.

In practical systems, the neural network models are disturbed by environmental noises. Therefore, in this paper, we will consider the stochastic Cohen-Grossberg neural networks with mixed time delay described by the following stochastic nonlinear integrodifferential equations:

$$\begin{aligned} dx_i(t) = & -\widehat{\alpha}_i(x_i(t))\left[\widehat{\beta}_i(x_i(t)) - \sum_{j=1}^{n} a_{ij} f_j(x_j(t))\right. \\ & - \sum_{j=1}^{n} b_{ij} f_j(x_j(t-\tau_j(t))) \\ & \left. - \sum_{j=1}^{n} c_{ij} \int_{-\infty}^{t} k_{ij}(t-s) f_j(x_j(s))\, ds - J_i\right] dt \\ & + \sum_{j=1}^{n} \sigma_{ij}(t, x_i(t), x_i(t-\tau_j(t)))\, dw_j(t), \\ & \qquad i = 1, \ldots, n. \end{aligned} \tag{2}$$

System (2) for convenience can be rewritten as the following vector form:

$$\begin{aligned} dx(t) = & -\widehat{\alpha}(x(t))\left[\widehat{\beta}(x(t)) - AF(x(t))\right. \\ & - BF(x(t-\tau(t))) \\ & \left. - C\int_{-\infty}^{t} K(t-s) F(x(s))\, ds - J\right] dt \\ & + \sigma(t, x(t), x(t-\tau(t)))\, dw(t), \end{aligned} \tag{3}$$

where A, B, C are known constant matrices with appropriate dimensions, $x(t) = [x_1(t), \ldots, x_n(t)]^T \in R^n$ is the neural state vector associated with the n neurons at time t, $\widehat{\alpha}(x(t)) = \text{diag}[\widehat{\alpha}_1(x_1(t)), \ldots, \widehat{\alpha}_n(x_n(t))] \in R^{n\times n}$ represents an amplification function, $\widehat{\beta}(x(t)) = [\widehat{\beta}_1(x_1(t)), \ldots, \widehat{\beta}_n(x_n(t))] \in R^{n\times n}$ denotes the behavior function, $F(x(t)) = [F_1(x_1(t)), \ldots, F_n(x_n(t))]^T$ is the neuron activation functions, $K(t-s) = \text{diag}[k_1(t-s), k_2(t-s), \ldots, k_n(t-s)]$, $\tau(t) = [\tau_1(t), \ldots, \tau_n(t)]^T$, $J = [J_1, \ldots, J_n]^T$ is the constant external input vector, $\sigma(t) = (\sigma_{ij}(t))_{n\times n}$ is the diffusion coefficient matrix, and $w(x(t)) = [w_1(x_1(t)), \ldots, w_n(x_n(t))]^T$ is an n-dimension Brownian motion satisfying $E\{d\omega(t)\} = 0$, $E\{d\omega^2(t)\} = dt$ and defined on a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$ with a natural filtration $\{\mathcal{F}_t\}_{t\geq 0}$ generated by $\{w(s) : 0 \leq s \leq t\}$, where we associate Ω with the canonical space generated by $w(t)$ and denote by $\mathcal{F}$ the associated σ-algebra generated by $\{w(t)\}$ with the probability measure $\mathcal{P}$.

The discrete time-varying delays $\tau(t)$ satisfy

$$\begin{aligned} & 0 \leq \tau_m \leq \tau(t) \leq \tau_M < +\infty, \\ & \dot{\tau}(t) < \mu, \end{aligned} \tag{4}$$

and the delay kernel k_j is a real valued continuous function defined on $[0, +\infty]$ and satisfies; for each j,

$$\int_0^{\infty} k_j(s)\, ds = 1. \tag{5}$$

Moreover, there exist $\iota > 0$ and matrix $\underline{K} = \text{diag}\{\underline{k}_1(\iota), \underline{k}_2(\iota), \ldots, \underline{k}_n(\iota)\} > 0$,

$$\int_0^{\infty} k_j(s)\, e^{\iota s} ds = \underline{k}_j(\iota). \tag{6}$$

As usual, the initial conditions associated with system (3) are given in the form

$$x(t) = \varphi(t), \quad -\infty < t \leq 0, \tag{7}$$

where the initial value function $\varphi \in L^b_{\mathcal{F}_0}([-\infty, 0], R^n)$ is the family of all $\mathcal{F}_0$-measurable $C([-\infty, 0]; R^n)$-valued random variables satisfying $\sup_{-\infty\leq s\leq 0} E\|\varphi(s)\|^2 < \infty$, in which E denotes expectations with respect to $\mathcal{P}$ and $C([-\infty, 0])$ denotes the family of all continuous R^n-valued functions $\varphi(s)$ on $[-\infty, 0]$.

We can describe the switched stochastic Cohen-Grossberg neural networks as follows:

$$\begin{aligned} dx(t) = & -\widehat{\alpha}(x(t))\left[\widehat{\beta}(x(t)) - A_{\sigma(t)} F(x(t))\right. \\ & - B_{\sigma(t)} F(x(t-\tau(t))) \\ & \left. - C_{\sigma(t)} \int_{-\infty}^{t} K(t-s) F(x(s))\, ds - J\right] dt \\ & + \sigma(t, x(t), x(t-\tau(t)))\, d\omega(t), \end{aligned} \tag{8}$$

where the function $\sigma(t) : [0, +\infty) \to \underline{N} = \{1, 2, \ldots, N\}$ is the switching signal, which is deterministic, piecewise constant, and right continuous.

To continue our discussion, we give the following basic assumptions.

(H1) We assume there exist constants δ_j^- and δ_j^+, $i = 1, 2, \ldots, n$, such that

$$\delta_j^- \leq \frac{f_j(x) - f_j(y)}{x - y} \leq \delta_j^+, \quad \forall x, y \in R,\ x \neq y. \tag{9}$$

(H2) There exist positive constants $\underline{\alpha}_i$, $\overline{\alpha}_i$, for all $i = 1, 2, \ldots, n$, such that

$$\underline{\alpha}_i \leq a_i(x_i(t)) \leq \overline{\alpha}_i. \tag{10}$$

(H3) There exist positive constants b_j, such that

$$x_j(t)\, \widehat{\beta}_j(x_j(t)) \geq b_j x_j^2(t). \tag{11}$$

(H4) We assume that the stochastic term satisfies $\sigma(t, \cdot, \cdot) : R^+ \times R^n \times R^n \to R^{n\times m}(\sigma(t, 0, 0) = 0)$ which is locally Lipschitz continuous and satisfies the linear growth condition as well. Moreover, $\sigma(t, \cdot, \cdot)$ satisfies the following condition:

$$\begin{aligned} & \text{trace}\left[\sigma^T(t, x, y)\, \sigma(t, x, y)\right] \\ & \quad \leq x^T \Pi_1^T \Pi_1 x + y^T \Pi_2^T \Pi_2 y, \end{aligned} \tag{12}$$

where Π_i, $(i = 1, 2)$ are known constant matrices with appropriate dimensions.

(H5) The $\Delta A_i(t)$, $\Delta B_i(t)$, and $\Delta C_i(t)$ are unknown matrices that represent the time-varying parameter uncertainties and are assumed to be of the following form:

$$\begin{aligned}\Delta A_i(t) &= H_{A_i} G_{A_i}(t) E_{A_i},\\ \Delta B_i(t) &= H_{B_i} G_{B_i}(t) E_{B_i},\\ \Delta C_i(t) &= H_{C_i} G_{C_i}(t) E_{C_i},\end{aligned} \tag{13}$$

where H_{A_i}, H_{B_i}, H_{C_i}, E_{A_i}, E_{B_i}, and E_{C_i} are known real constant matrices with appropriate dimensions. G_{A_i}, G_{B_i}, and G_{C_i} may be time-varying matrices with Lebesgue measurable elements bounded by

$$\begin{aligned}G_{A_i}^T(t) G_{A_i} &\leq I,\\ G_{B_i}^T(t) G_{B_i} &\leq I,\\ G_{C_i}^T(t) G_{C_i} &\leq I.\end{aligned} \tag{14}$$

Remark 1. The constants l_j and L_j can be positive, negative, or zero. Therefore, the activation functions $f(\cdot)$ are more general than the forms $|f_j(u)| \leq K_j|u|$, $K_j > 0$, $j = 1, 2, \ldots, n$.

Remark 2. It is worth mentioning that the structures of the parametric uncertainties with the form (13) and (14) are more general than those in previous literature in [28, 34, 37]. However, $H_{A_i} = H_{B_i} = H_{C_i}$ has been discussed [28, 34, 37]. Recently, in [35], the attractor and boundedness of stochastic Cohen-Grossberg neural networks without parametric uncertainties were investigated.

Definition 3 (see [38]). The system (3) is mean-square uniformly ultimately bounded, if there exists a constant vector $\widetilde{B} > 0$, for any constant $\varrho > 0$; there is $t' = t'(\varrho) > 0$, for all $t \geq t_0 + t'$, $t_0 > 0$, $\|\varphi\| < \varrho$; the solution $x(t, t_0, \varphi)$ of system (3) satisfies such that

$$E\left\|x(t, t_0, \varphi)\right\| < \widetilde{B}, \tag{15}$$

where $E\|x(t, t_0, \varphi)\| = \sup_{-\infty \leq s \leq 0} E\|x_i(t+s, t_0, \varphi)\|$ and $\varphi \in L^b_{\mathscr{F}_0}([-\infty, 0]; R^n)$.

In this case, the set $\mathbb{A} = \{\varphi \in L^b_{\mathscr{F}_0} \mid E\|\varphi(s)\|_\infty \leq \widetilde{B}\}$ is called the attractor for the solution $x(t; \varphi)$ of system (3) in the mean-square sense. Clearly, the proposition above is equal to

$$\lim_{t\to\infty} \sup \inf_{y \in \mathbb{A}} \|x - y\| = 0. \tag{16}$$

Definition 4 (see [39]). For the switching signal $\sigma(t)$, construct a switching sequence $\{(i_0, \sigma(t_0)), \ldots, (i_k, \sigma(t_k)), \ldots\}$, $\sigma(i_k) \in \underline{N}$, $k = 0, 1, \ldots, N$, where $t_0 = 0$ is the initial time and t_k denotes the i_kth switching instant. Moreover, $\sigma(t) = i$ means that the ith subsystem is activated. N denotes the number of the subsystems. For each $T > t \geq 0$, let $N_\sigma(t, T)$ denote the number of discontinuities of $\sigma(t)$ in the interval (t, T). If there exist $N_0 > 0$ and $T_a > 0$ such that $N_\sigma(t, T) \leq N_0 + (T-t)/T_a$ holds, then T_a is called the average dwell time. N_0 is the chatter bound.

Remark 5. It should be pointed out that for the chatter bound N_0, in our work, we take $N_0 \geq 1$, which is more preferable than those previously reported in [31, 37]. If $T_a = 0$ is equivalent to the existence of a common function for all subsystems, this implies that switching signals can be arbitrary. Hence, the results reported in this paper are more effective than the arbitrary switching signals reported in the previous literatures [28, 37].

So, to obtain the main results of this paper, we introduce the following lemmas.

Lemma 6 (see [40]). *For any positive-definite constant matrix $Z \in R^{n\times n}$, scalar $\tau > 0$, and vector function $u(t) : [t-r, t] \to R^n$, $t \geq 0$, then*

(1) *Jensen's inequality*

$$\begin{aligned}&-\int_{t-\tau}^{t} \varrho^T(s) Z \varrho(s)\, ds\\ &\quad \leq -\frac{1}{\tau}\left(\int_{t-\tau}^{t} \varrho(s)\, ds\right)^T Z \left(\int_{t-\tau}^{t} \varrho(s)\, ds\right),\end{aligned} \tag{17}$$

(2)

$$\begin{aligned}&-\int_{\beta}^{\alpha}\int_{t+\theta}^{t} \varrho^T(s) Z \varrho(s)\, ds\, d\theta\\ &\quad \leq -\frac{2}{\alpha^2 - \beta^2}\left(\int_{\beta}^{\alpha}\int_{t+\theta}^{t} \varrho(s)\, ds\, d\theta\right)^T\\ &\quad\quad \cdot Z\left(\int_{\beta}^{\alpha}\int_{t+\theta}^{t} \varrho(s)\, ds\, d\theta\right).\end{aligned} \tag{18}$$

Lemma 7 (see [41]). *Let $\mathscr{L}$, $\mathscr{M}_1$, and $\mathscr{M}_2$ be real matrices of appropriate dimension such that $\mathscr{L}^T\mathscr{L} \leq I$. Then for any scalar $\varepsilon > 0$ and vectors x and y with appropriate dimensions, the following inequality is true:*

$$2\mathscr{M}_1^T \mathscr{L} \mathscr{M}_2 y \leq \varepsilon^{-1} x^T \mathscr{M}_1 \mathscr{M}_1^T x + \varepsilon y^T \mathscr{M}_2^T \mathscr{M} y. \tag{19}$$

Lemma 8 (see [41]). *For any real matrix X, Y and one positive-definite matrix G, the following matrix inequality holds:*

$$2X^T G Y \leq \varepsilon X^T G X + \varepsilon^{-1} Y^T G Y. \tag{20}$$

Lemma 9 (see [42], Schur's complement). *The LMI*

$$\Omega = \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ * & \Omega_{22} \end{pmatrix} < 0 \tag{21}$$

with $\Omega_{11} = \Omega_{11}^T$, $\Omega_{22} = \Omega_{22}^T$ is equivalent to

$$\begin{aligned}\Omega_{22} &< 0,\\ \Omega_{11} - \Omega_{12}\Omega_{22}^{-1}\Omega_{12}^T &< 0.\end{aligned} \tag{22}$$

3. Main Results

Theorem 10. *For given constants $\beta > 0$, $\varepsilon > 0$, $\mu < 1$, if there exist positive scalars λ and positive-definite matrices $P = \mathrm{diag}(p_1, p_2, \ldots, p_n)$, $L = \mathrm{diag}(l_1, l_2, \ldots, l_n)$, $D_i = \mathrm{diag}(D_{i1}, D_{i2}, \ldots, D_{in})$, $(i = 1, 2)$, Q_1, Q_2, Q_3, Q_4, R, S, $\widetilde{Z}$, Z, U, such that the following conditions hold:*

$$\Delta_1 = \begin{bmatrix} \phi_{11} & 0 & 0 & 0 & \phi_{15} & 0 & 0 & \phi_{18} & \phi_{19} & 0 & 0 & 0 \\ * & \phi_{22} & 0 & 0 & 0 & \phi_{26} & 0 & \phi_{28} & 0 & 0 & 0 & 0 \\ * & * & \phi_{33} & 0 & 0 & 0 & \phi_{37} & 0 & \phi_{39} & 0 & 0 & 0 \\ * & * & * & \phi_{44} & \phi_{45} & 0 & 0 & \phi_{48} & \phi_{49} & 0 & 0 & 0 \\ * & * & * & * & \phi_{55} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \phi_{66} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \phi_{77} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \phi_{88} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \phi_{99} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \phi_{10} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & \overline{\phi}_{11} & 0 \\ * & * & * & * & * & * & * & * & * & * & * & \overline{\phi}_{12} \end{bmatrix} < 0,$$
$$P < \lambda I, \tag{23}$$

where

$$\phi_{11} = \frac{1}{a^2}\Big[\beta P - 2\Omega_1 P \Omega_2 + \lambda \Pi_1^T \Pi_1 + \varepsilon \Omega_4 D_1 + P + \frac{1}{4\beta^2} I + \tau_M e^{\beta\tau_M} Q_1 + \tau_m e^{\beta\tau_m} Q_2 + e^{\beta\tau_M} Q_3 + \tau_M R + (\tau_M - \tau_m) S + \frac{\tau_M^2}{4} Z + \frac{\tau_M^2}{4} \widetilde{Z} - \Omega_3 D_1\Big],$$
$$\phi_{15} = PA,$$
$$\phi_{18} = PB,$$
$$\phi_{19} = PC,$$
$$\phi_{22} = -\tau_M Q_1,$$
$$\phi_{26} = \phi_{28} = 0,$$
$$\phi_{33} = -\tau_m Q_2,$$
$$\phi_{37} = \phi_{39} = 0,$$
$$\phi_{44} = \lambda \Pi_2^T \Pi_2 - \Omega_3 D_2 + \frac{1}{4\beta^2} I - (1-\mu) Q_3,$$
$$\phi_{45} = 0,$$
$$\phi_{48} = \Omega_4 D_2,$$
$$\phi_{49} = 0,$$
$$\phi_{55} = e^{\beta\tau_M} Q_4 + LK(\iota) + \varepsilon^{-1} \Omega_4 D_1 + \tau_M^2 U - D_1 + \frac{1}{\beta^2} \cdot I,$$
$$\phi_{66} = -\frac{e^{-\beta\tau_M}}{\tau_M} R,$$
$$\phi_{77} = -\frac{e^{-\beta\tau_M}}{\tau_M - \tau_m} S,$$
$$\phi_{88} = -(1-\mu) Q_4 - D_2 + \frac{1}{\beta^2} I,$$
$$\phi_{99} = -L,$$
$$\phi_{10} = -(1-\mu) e^{-\beta\tau_M} U,$$
$$\overline{\phi}_{11} = -e^{-\beta\tau_M} Z,$$
$$\overline{\phi}_{12} = -e^{-\beta\tau_M} \widetilde{Z},$$
$$D_i > 0, \quad i = 1, 2,$$
$$\Omega_3 = \mathrm{diag}\{\delta_1^- \delta_1^+, \delta_2^- \delta_2^+, \ldots, \delta_n^- \delta_n^+\},$$
$$\Omega_4 = \mathrm{diag}\left\{\frac{\delta_1^- + \delta_1^+}{2}, \frac{\delta_2^- + \delta_2^+}{2}, \ldots, \frac{\delta_n^- + \delta_n^+}{2}\right\}, \tag{24}$$

then system (3) is mean-square uniformly ultimately bounded.

Proof. Choose the following Lyapunov-Krasovskii functional:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t), \tag{25}$$

where

$$V_1(t) = e^{\beta t} x^T(t) P x(t),$$
$$V_2(t) = \tau_M \int_{t-\tau_M}^{t} e^{\beta(s+\tau_M)} x^T(s) Q_1 x(s)\, ds + \tau_m \int_{t-\tau_m}^{t} e^{\beta(s+\tau_m)} x^T(s) Q_2 x(s)\, ds + \int_{t-\tau(t)}^{t} e^{\beta(s+\tau_M)} x^T(s) Q_3 x(s)\, ds + \int_{t-\tau(t)}^{t} e^{\beta(s+\tau_M)} F^T(x(s)) Q_4 F(x(s))\, ds,$$
$$V_3(t) = \int_{-\tau_M}^{0} \int_{t+\theta}^{t} e^{\beta s} x^T(s) R x(s)\, ds\, d\theta + \int_{-\tau_M}^{-\tau_m} \int_{t+\theta}^{t} e^{\beta s} x^T(s) S x(s)\, ds\, d\theta + \tau_M \int_{-\tau(t)}^{0} \int_{t+\theta}^{t} e^{\beta s} F^T(x(s)) U F(x(s))\, ds\, d\theta,$$

$$V_4(t) = \frac{\tau_M^2}{2}\int_{-\tau_M}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t} e^{\beta s}x^T(s)Zx(s)\,ds\,d\lambda\,d\theta + \frac{\left(\tau_M^2-\tau_m^2\right)}{2} \cdot \int_{-\tau_M}^{-\tau_m}\int_{\theta}^{0}\int_{t+\lambda}^{t} e^{\beta s}x^T(s)\widetilde{Z}X(s)\,ds\,d\lambda\,d\theta,$$
$$V_5(t) = \sum_{j=1}^{n} l_j \int_0^{\infty} k_j(\delta)\int_{t-\delta}^{t} e^{\beta(\gamma+\delta)}F_j^2\left(x_j(\gamma)\right)d\gamma\,d\delta. \tag{26}$$

By applying the *Itô* differential formula, the stochastic derivative of $V(t)$ along the trajectory of system (3) is

$$dV(t) = \mathscr{L}V dt + 2e^{\beta t}x^T(t)P\sigma(t,x(t),x(t-\tau(t)))\,d\omega(t). \tag{27}$$

With the infinitesimal-operator, we can deduce that

$$\begin{aligned}\mathscr{L}V_1 &= \beta e^{\beta t}x^T(t)Px(t) - 2e^{\beta t}x^T(t)P\alpha(x(t)) \cdot \Big[\beta(x(t)) - AF(x(t)) \\ &\quad - C\int_{-\infty}^{t}K(t-s)F(x(s))\,ds - BF(x(t-\tau(t))) \\ &\quad - J\Big] + e^{\beta t}\frac{1}{2}\mathrm{tra}\left[\sigma(t,x(t),x(t-\tau(t)))^T V_{1xx}(x,t) \cdot \sigma(t,x(t),x(t-\tau(t)))\right].\end{aligned} \tag{28}$$

Denote $\Omega_1 = \mathrm{diag}\{\underline{\alpha}_1,\underline{\alpha}_2,\ldots,\underline{\alpha}_n\}$, $\Omega_2 = \mathrm{diag}\{b_1,b_2,\ldots,b_n\}$, according to assumptions (H2) and (H3). Then, we obtain the following inequalities:

$$\begin{aligned}&-2e^{\beta t}x^T(t)P\alpha(x(t))\beta(x(t)) \\ &\quad = -2e^{\beta t}\sum_{j=1}^{n}x_j(t)p_j\alpha_j\left(x_j(t)\right)\beta_j\left(x_j(t)\right) \\ &\quad \le -2e^{\beta t}\sum_{j=1}^{n}\underline{\alpha}_j p_j b_j x_j^2(t) = -2e^{\beta t}x^T(t)\Omega_1 P\Omega_2 x(t).\end{aligned} \tag{29}$$

By assumption (H4), we have

$$\begin{aligned}&\frac{1}{2}e^{\beta t}\mathrm{tra}\left[\sigma(t,x(t),x(t-\tau(t)))^T V_{1xx}(x,t) \cdot \sigma(t,x(t),x(t-\tau(t)))\right] = e^{\beta t}\lambda_{\max}(P) \\ &\quad \cdot \mathrm{tra}\left[\sigma(t,x(t),x(t-\tau(t)))^T \cdot \sigma(t,x(t),x(t-\tau(t)))\right] \le \lambda e^{\beta t}x^T(t)\Pi_1^T\Pi_1 x(t) \\ &\quad + \lambda e^{\beta t}x^T(t-\tau(t))\Pi_2^T\Pi_2 x(t-\tau(t)).\end{aligned} \tag{30}$$

Combining inequalities (29) and (30), we derive

$$\begin{aligned}\mathscr{L}V_1 &= \beta e^{\beta t}x^T(t)Px(t) - 2e^{\beta t}x^T(t)\Omega_1 P\Omega_2 x(t) \\ &\quad + e^{\beta t}\Big[2x^T(t)P\alpha(x(t))AF(x(t)) \\ &\quad + 2x^T(t)P\alpha(x(t))BF(x(t-\tau(t))) \\ &\quad + 2x^T(t)P\alpha(x(t))C\int_{-\infty}^{t}K(t-s)F(x(s)) \\ &\quad + 2x^T(t)P\alpha(x(t))J\Big] + \rho_1\left[x^T(t)\Pi_1^T P\Pi_1 x(t)\right] \\ &\quad + \rho_2 e^{\beta t}x^T(t-\tau(t))\Pi_2^T P\Pi_2 x(t-\tau(t)) \\ &\le e^{\beta t}\left[\beta x^T(t)Px(t) - 2x^T(t)\Omega_1 P\Omega_2 x(t)\right] \\ &\quad + \Big[2x^T(t)P\alpha(x(t))AF(x(t)) \\ &\quad + 2x^T(t)P\alpha(x(t))BF(x(t-\tau(t))) \\ &\quad + 2x^T(t)P\alpha(x(t))C\int_{-\infty}^{t}K(t-s)F(x(s)) \\ &\quad + x^T(t)Px(t) + \overline{\alpha}^2 J^T PJ\Big]e^{\beta t} \\ &\quad + e^{\beta t}\left[\lambda x^T(t)\Pi_1^T\Pi_1 x(t) + \lambda x^T(t-\tau(t))\Pi_2^T\Pi_2 x(t-\tau(t))\right],\end{aligned} \tag{31}$$

where $\overline{\alpha} = \max\{\overline{\alpha}_1,\overline{\alpha}_2,\ldots,\overline{\alpha}_n\}$.

Then, we easily derive

$$\begin{aligned}\mathscr{L}V_2 &= \tau_M e^{\beta(t+\tau_M)}x^T(t)Q_1x(t) - \tau_M e^{\beta t}x^T(t-\tau_M) \cdot Q_1x(t-\tau_M) + \tau_m e^{\beta(t+\tau_m)}x^T(t)Q_2x(t) \\ &\quad - \tau_m e^{\beta t}x^T(t-\tau_m)Q_2x(t-\tau_m) \\ &\quad + e^{\beta(t+\tau_M)}\left[x^T(t)Q_3x(t) + F^T(x(t))Q_4F(x(t))\right] \\ &\quad - (1-\dot{\tau}(t))e^{\beta(t-\tau(t)+\tau_M)}\Big[x^T(t-\tau(t))Q_3x(t\tau(t)) \\ &\quad + F^T(x(t-\tau(t)))Q_4F(x(t-\tau(t)))\Big] \\ &\le \tau_M e^{\beta(t+\tau_M)}x^T(t)Q_1x(t) - \tau_M e^{\beta t}x^T(t-\tau_M) \cdot Q_1x(t-\tau_M) + \tau_m e^{\beta(t+\tau_m)}x^T(t)Q_2x(t) \\ &\quad - \tau_m e^{\beta t}x^T(t-\tau_m)Q_2x(t-\tau_m) \\ &\quad + e^{\beta(t+\tau_M)}\left[x^T(t)Q_3x(t) + F^T(x(t))Q_4F(x(t))\right] \\ &\quad - (1-\mu)e^{\beta t}\Big[x^T(t-\tau(t))Q_3x(t\tau(t)) \\ &\quad + F^T(x(t-\tau(t)))Q_4F(x(t-\tau(t)))\Big].\end{aligned} \tag{32}$$

Similarly, calculating the operation of $\mathscr{L}V_3$ along the trajectory of system (3), one can get

$$\begin{aligned}\mathscr{L}V_3 &= \tau_M e^{\beta t}x^T(t)Rx(t) - \int_{t-\tau_M}^{t} e^{\beta s}x^T(s)Rx(s)\,ds \\ &\quad + (\tau_M-\tau_m)e^{\beta t}x^T(t)Rx(t) \\ &\quad - \int_{t-\tau_M}^{t-\tau_m} e^{\beta s}x^T(s)Rx(s)\,ds + \tau_M\tau(t)e^{\beta t}F^T(x(t)) \\ &\quad \cdot UF(x(t)) - \tau_M(1-\dot{\tau}(t)) \\ &\quad \cdot \int_{t-\tau(t)}^{t} e^{\beta(t-\tau(t))}F^T(x(s))UF(x(s))\,ds \\ &\le \tau_M e^{\beta t}x^T(t)Rx(t) \\ &\quad - e^{\beta(t-\tau_M)}\int_{t-\tau_M}^{t} x^T(s)Rx(s)\,ds + (\tau_M-\tau_m) \\ &\quad \cdot e^{\beta t}x^T(t)Rx(t) - e^{\beta(t-\tau_M)}\int_{t-\tau_M}^{t-\tau_m} x^T(s)Rs(t)\,ds \\ &\quad + \tau_M^2 e^{\beta t}F^T(x(t))UF(x(t)) - \tau_M(1-\mu) \\ &\quad \cdot e^{\beta(t-\tau_M)}\int_{t-\tau(t)}^{t} F^T(x(s))UF(x(s))\,ds.\end{aligned} \tag{33}$$

By Lemma 6 (1), it follows that

$$\begin{aligned}&-e^{-\beta\tau_M}\int_{t-\tau_M}^{t} x^T(s)Rx(s)\,ds \\ &\quad \le -\frac{e^{-\beta\tau_M}}{\tau_M}\left(\int_{t-\tau_M}^{t} x(s)\,ds\right)^T R\left(\int_{t-\tau_M}^{t} x(s)\,ds\right) \\ &\quad - e^{-\beta\tau_M}\int_{t-\tau_M}^{t-\tau_m} x^T(s)Sx(s)\,ds \\ &\quad \le -\frac{e^{-\beta\tau_M}}{\tau_M-\tau_m}\left(\int_{t-\tau_M}^{t-\tau_m} x(s)\,ds\right)^T S\left(\int_{t-\tau_M}^{t-\tau_m} x(s)\,ds\right) \\ &\quad - \tau_M e^{-\beta\tau_M}\int_{t-\tau(t)}^{t} F^T(x(s))UF(x(s))\,ds \\ &\quad \le -e^{-\beta\tau_M}\left(\int_{t-\tau(t)}^{t} F(x(s))\,ds\right)^T \\ &\quad \cdot U\left(\int_{t-\tau(t)}^{t} F(x(s))\,ds\right).\end{aligned} \tag{34}$$

Similarly, one can derive

$$\begin{aligned}\mathscr{L}V_4 &= \frac{\tau_M^2}{4}e^{\beta t}x^T(t)Zx(t) - \frac{\tau_M^2}{2} \\ &\quad \cdot \int_{-\tau_M}^{0}\int_{t+\theta}^{t} e^{\beta s}x^T(t)Zx(t)\,ds\,d\theta + \frac{(\tau_M^2-\tau_m^2)}{4} \\ &\quad \cdot e^{\beta t}x^T(t)\widetilde{Z}x(t) - \frac{(\tau_M^2-\tau_m^2)}{2} \\ &\quad \cdot \int_{-\tau_M}^{-\tau_m}\int_{t+\theta}^{t} e^{\beta s}x^T(t)\widetilde{Z}x(t)\,ds\,d\theta \\ &\le e^{\beta t}\left[\frac{\tau_M^2}{4}x^T(t)Zx(t)\right. \\ &\quad - \frac{e^{-\beta\tau_M}\tau_M^2}{2}\int_{-\tau_M}^{0}\int_{t+\theta}^{t} x^T(t)Zx(t)\,ds\,d\theta \\ &\quad + \frac{(\tau_M^2-\tau_m^2)}{4}x^T(t)\widetilde{Z}x(t) \\ &\quad \left. - \frac{e^{-\beta\tau_M}(\tau_M^2-\tau_m^2)}{2}\int_{-\tau_M}^{-\tau_m}\int_{t+\theta}^{t} x^T(t)\widetilde{Z}x(t)\,ds\,d\theta\right].\end{aligned} \tag{35}$$

Using Lemma 6 (2), we have

$$\begin{aligned}&-\frac{e^{-\beta\tau_M}\tau_M^2}{2}\int_{-\tau_M}^{0}\int_{t+\theta}^{t} x^T(t)Zx(t)\,ds\,d\theta \\ &\quad \le -e^{-\beta\tau_M}\left(\int_{-\tau_M}^{0}\int_{t+\theta}^{t} x(s)\,ds\,d\theta\right)^T \\ &\quad \cdot Z\left(\int_{-\tau_M}^{0}\int_{t+\theta}^{t} x(s)\,ds\,d\theta\right).\end{aligned} \tag{36}$$

Similarly, one can obtain

$$\begin{aligned}&-\frac{e^{-\beta\tau_M}(\tau_M^2-\tau_m^2)}{2}\int_{-\tau_M}^{-\tau_m}\int_{t+\theta}^{t} x^T(t)\widetilde{Z}x(t)\,ds\,d\theta \\ &\quad \le -e^{-\beta\tau_M}\left(\int_{-\tau_M}^{-\tau_m}\int_{t+\theta}^{t} x(t)\,ds\,d\theta\right)^T \\ &\quad \cdot \widetilde{Z}\left(\int_{-\tau_M}^{-\tau_m}\int_{t+\theta}^{t} x(t)\,ds\,d\theta\right).\end{aligned} \tag{37}$$

Then, noting the condition $\int_0^\infty k_j(s)ds = 1$ and the Cauchy-Schwarz inequality

$$\left(\int p(s)q(s)\,ds\right)^2 \le \left(\int p^2(s)\,ds\right)\left(\int q^2(s)\,ds\right), \tag{38}$$

it can yield

$$\begin{aligned}\mathscr{L}V_5 &= \sum_{j=1}^{n} l_j\int_0^\infty k_j(\delta)e^{\beta(t+\delta)}F_j^2(x_j(t))\,d\delta - \sum_{j=1}^{n} l_j \\ &\quad \cdot \int_0^\infty k_j(\delta)e^{\beta t}F_j^2(x_j(t-\delta))\,d\delta\end{aligned}$$

$$
\begin{aligned}
&= e^{\beta t}\sum_{j=1}^{n} l_j \bar{k}_j(\iota) F_j^2\left(x_j(t)\right) d\delta - e^{\beta t}\sum_{j=1}^{n} l_j \\
&\cdot \int_0^\infty k_j(\delta)\, d\delta \int_0^\infty k_j(\delta) F_j^2\left(x_j(t-\delta)\right) d\delta \\
&\le e^{\beta t} F^T(x(t)) LK(\iota) F(x(t)) - \sum_{j=1}^{n} l_j \\
&\cdot \left(\int_0^\infty k_j(\delta) F_j\left(x_j(t-\delta)\right) d\delta\right)^2 = e^{\beta t} F^T(x(t)) \\
&\cdot LK(\iota) F(x(t)) - \left(\int_{-\infty}^{t} K(t-s) F(x(s))\, ds\right)^T \\
&\cdot L\left(\int_{-\infty}^{t} K(t-s) F(x(s))\, ds\right).
\end{aligned} \tag{39}
$$

Then, based on assumption (H1), it easy to see that, for $j = 1, 2, \ldots, n$,

$$
\begin{aligned}
&\left[F_j\left(x_j(t)\right) - F_j(0) - \delta_j^- x_j(t)\right] \\
&\quad\cdot \left[F_j\left(x_j(t)\right) - F_j(0) - \delta_j^+ x_j(t)\right] \le 0, \\
&\left[F_j\left(x_j(t-\tau(t))\right) - F_j(0) - \delta_j^- x_j(t-\tau(t))\right] \\
&\quad\cdot \left[F_j\left(x_j(t-\tau(t))\right) - F_j(0) - \delta_j^+ x_j(t-\tau(t))\right] \\
&\quad \le 0.
\end{aligned} \tag{40}
$$

Then, let

$$
\begin{aligned}
\Upsilon_1 &= -\sum_{j=1}^{n} D_{1j}\left[F_j\left(x_j(t)\right) - F_j(0) - \delta_j^- x_j(t)\right] \\
&\quad\cdot \left[F_j\left(x_j(t)\right) - F_j(0) - \delta_j^+ x_j(t)\right] \ge 0, \\
\Upsilon_2 &= -\sum_{j=1}^{n} D_{2j}\left[F_j\left(x_j(t-\tau(t))\right) - F_j(0)\right. \\
&\quad \left. - \delta_j^- x_j(t-\tau(t))\right]\left[F_j\left(x_j(t-\tau(t))\right) - F_j(0)\right. \\
&\quad \left. - \delta_j^+ x_j(t-\tau(t))\right] \ge 0.
\end{aligned} \tag{41}
$$

Therefore,

$$
\begin{aligned}
\Upsilon_1 + \Upsilon_2 &= -\sum_{j=1}^{n} D_{1j}\left[F_j\left(x_j(t)\right) - F_j(0) - \delta_j^- x_j(t)\right] \\
&\quad \cdot \left[F_j\left(x_j(t)\right) - F_j(0) - \delta_j^+ x_j(t)\right] \\
&\quad - \sum_{j=1}^{n} D_{2j}\left[F_j\left(x_j(t-\tau(t))\right) - F_j(0)\right. \\
&\quad \left. - \delta_j^- x_j(t-\tau(t))\right]\left[F_j\left(x_j(t-\tau(t))\right) - F_j(0)\right. \\
&\quad \left. - \delta_j^+ x_j(t-\tau(t))\right] = -\sum_{j=1}^{n} D_{1j}\left[F_j\left(x_j(t)\right)\right. \\
&\quad \left. - \delta_j^- x_j(t)\right]\left[F_j\left(x_j(t)\right) - \delta_j^+ x_j(t)\right] - \sum_{j=1}^{n} D_{2j} \\
&\quad \cdot \left[F_j\left(x_j(t-\tau(t))\right) - \delta_j^- x_j(t-\tau(t))\right] \\
&\quad \cdot \left[F_j\left(x_j(t-\tau)\right) - \delta_j^+ x_j(t-\tau(t))\right] - \sum_{j=1}^{n} D_{1j} F_j^2(0) \\
&\quad + \sum_{j=1}^{n} D_{1j} F_j(0)\left[2F_j\left(x_j(t)\right) - \left(\delta_j^+ + \delta_j^-\right) x_j(t)\right] \\
&\quad - \sum_{j=1}^{n} D_{2j} F_j^2(0) + \sum_{j=1}^{n} D_{2j} F_j(0)\left[2F_j\left(x_j(t-\tau(t))\right)\right. \\
&\quad \left. - \left(\delta_j^+ + \delta_j^-\right) x_j(t-\tau(t))\right] \le -\sum_{j=1}^{n} D_{1j}\left[F_j\left(x_j(t)\right)\right. \\
&\quad \left. - \delta_j^- x_j(t)\right]\left[F_j\left(x_j(t)\right) - \delta_j^+ x_j(t)\right] - \sum_{j=1}^{n} D_{2j} \\
&\quad \cdot \left[F_j\left(x_j(t-\tau(t))\right) - \delta_j^- x_j(t-\tau(t))\right] \\
&\quad \cdot \left[F_j\left(x_j(t-\tau(t))\right) - \delta_j^+ x_j(t-\tau(t))\right] \\
&\quad + \sum_{j=1}^{n}\left[\frac{1}{\beta^2} F_j^2\left(x_j(t)\right) + \beta^2 D_{1j}^2 F_j^2(0) + \frac{1}{4\beta^2} x_j^2(t)\right. \\
&\quad \left. + \beta^2 D_{1j}^2 F_j^2(0)\left(\delta_j^+ + \delta_j^-\right)^2\right] \\
&\quad + \sum_{j=1}^{n}\left[\frac{1}{\beta^2} F_j^2\left(x_j(t-\tau(t))\right) + \beta^2 D_{2j}^2 F_j^2(0)\right. \\
&\quad \left. + \frac{1}{4\beta^2} x_j^2(t-\tau(t)) + \beta^2 D_{2j}^2 F_j^2(0)\left(\delta_j^+ + \delta_j^-\right)^2\right].
\end{aligned} \tag{42}
$$

From Lemma 8, the following inequality holds true:

$$
\begin{aligned}
&2x^T(t)\,\Omega_4 D_1 F(x(t)) \\
&\quad \le \varepsilon x^T(t)\,\Omega_4 D_1 x(t) \\
&\qquad + \varepsilon^{-1} F^T(x(t))\,\Omega_4 D_1 F(x(t)).
\end{aligned} \tag{43}
$$

Denote

$$
\begin{aligned}
\zeta^T(t) = &\left[x^T(t), x^T\left(t-\tau_M\right), x^T\left(t-\tau_m\right),\right. \\
&x^T(t-\tau(t)), F^T(x(t)), \left(\int_{t-\tau_M}^{t} x(s)\, ds\right)^T, \\
&\left(\int_{t-\tau_M}^{t-\tau_m} x(s)\, ds\right)^T, F^T(x(t-\tau(t))),
\end{aligned}
$$

$$\left(\int_{-\infty}^{t} K(t-s)F(x(s))\,ds\right)^T, \left(\int_{t-\tau(t)}^{t} F(x(s))\,ds\right)^T, y_1^T(t), y_2^T(t)\Bigg], \tag{44}$$

where

$$y_1(t) = \left(\int_{-\tau_M}^{0}\int_{t+\theta}^{t} x^T(s)Zx(s)\,ds\,d\theta\right), \quad y_2(t) = \left(\int_{-\tau_M}^{-\tau_m}\int_{t+\theta}^{t} x^T(s)\widetilde{Z}x(s)\,ds\,d\theta\right). \tag{45}$$

Using (27)–(43), we can derive

$$\begin{aligned} dV(t) &\le \mathscr{L}Vdt + 2e^{\beta t}x^T(t)P\sigma(t, x(t), x(t - \tau(t)))\,d\omega(t) + e^{\beta t}(\Upsilon_1 + \Upsilon_2) \le e^{\beta t}\zeta^T(t) \\ &\cdot \Delta\zeta(t) + e^{\beta t}\overline{\alpha}^2 J^T PJ + 2e^{\beta t}x^T(t)P\sigma(t, x(t), x(t - \tau(t)))\,d\omega(t) + e^{\beta t}\sum_{j=1}^{n}\Big[\beta^2 D_{1j}^2 F_j^2(0) \\ &+ \beta^2 D_{1j}^2 F_j^2(0)\left(\delta_j^+ + \delta_j^-\right)^2 + \beta^2 D_{2j}^2 F_j^2(0) + \beta^2 D_{2j}^2 F_j^2(0)\left(\delta_j^+ + \delta_j^-\right)^2\Big], \end{aligned} \tag{46}$$

where

$$\Delta = \begin{bmatrix} \Phi_{11} & 0 & 0 & 0 & \Phi_{15} & 0 & 0 & \Phi_{18} & \Phi_{19} & 0 & 0 & 0 \\ * & \Phi_{22} & 0 & 0 & 0 & \Phi_{26} & 0 & \Phi_{28} & 0 & 0 & 0 & 0 \\ * & * & \Phi_{33} & 0 & 0 & 0 & \Phi_{37} & 0 & \Phi_{39} & 0 & 0 & 0 \\ * & * & * & \Phi_{44} & \Phi_{45} & 0 & 0 & \Phi_{48} & \Phi_{49} & 0 & 0 & 0 \\ * & * & * & * & \Phi_{55} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Phi_{66} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Phi_{77} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Phi_{88} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \Phi_{99} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \Phi_{10} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & \overline{\Phi}_{11} & 0 \\ * & * & * & * & * & * & * & * & * & * & * & \overline{\Phi}_{12} \end{bmatrix},$$

$$\Phi_{11} = \beta P - 2\Omega_1 P\Omega_2 + \lambda\Pi_1^T\Pi_1 + \varepsilon\Omega_4 D_1 + P + \frac{1}{4\beta^2}I + \tau_M e^{\beta\tau_M}Q_1 + \tau_m e^{\beta\tau_m}Q_2 + e^{\beta\tau_M}Q_3 + \tau_M R + (\tau_M - \tau_m)S + \frac{\tau_M^2}{4}Z + \frac{\tau_M^2}{4}\widetilde{Z} - \Omega_3 D_1,$$

$$\Phi_{15} = P\alpha(x(t))A,$$

$$\Phi_{18} = P\alpha(x(t))B,$$

$$\Phi_{19} = P\alpha(x(t))C,$$

$$\Phi_{22} = -\tau_M Q_1,$$

$$\Phi_{26} = \phi_{28} = 0,$$

$$\Phi_{33} = -\tau_m Q_2,$$

$$\Phi_{37} = \Phi_{39} = 0,$$

$$\Phi_{44} = \lambda\Pi_2^T\Pi_2 - \Omega_3 D_2 + \frac{1}{4\beta^2}I - (1-\mu)Q_3,$$

$$\Phi_{45} = 0,$$

$$\begin{aligned}
&\Phi_{48} = \Omega_4 D_2,\\
&\Phi_{49} = 0,\\
&\Phi_{55} = e^{\beta\tau_M} Q_4 + LK(\iota) + \varepsilon^{-1}\Omega_4 D_1 + \tau_M^2 U - D_1 + \frac{1}{\beta^2} I,\\
&\Phi_{66} = -\frac{e^{-\beta\tau_M}}{\tau_M} R,\\
&\Phi_{77} = -\frac{e^{-\beta\tau_M}}{\tau_M - \tau_m} S,\\
&\Phi_{88} = -(1-\mu) Q_4 - D_2 + \frac{1}{\beta^2} I,\\
&\Phi_{99} = -L,\\
&\Phi_{10} = -(1-\mu) e^{-\beta\tau_M} U,\\
&\overline{\Phi}_{11} = -e^{-\beta\tau_M} Z,\\
&\overline{\Phi}_{12} = -e^{-\beta\tau_M} \widetilde{Z}.
\end{aligned} \tag{47}$$

By integrating both sides of (46) in time interval $t \in [t_0, t]$ and taking expectation results in

$$\begin{aligned}
E\{V(x(t))\} \le{}& E\{V(x(t_0))\} + E\{\beta^{-1} e^{\beta t} \overline{\alpha}^2 J^T P J\}\\
&+ E\left\{ e^{\beta t} \sum_{j=1}^{n} \left[\beta D_{1j}^2 F_j^2(0) + \beta D_{1j}^2 F_j^2(0) \left(\delta_j^+ + \delta_j^-\right)^2 \right.\right.\\
&\left.\left. + \beta D_{2j}^2 F_j^2(0) + \beta D_{2j}^2 F_j^2(0) \left(\delta_j^+ + \delta_j^-\right)^2 \right] \right\}.
\end{aligned} \tag{48}$$

Note from (25) that

$$E\{V(x(t))\} \ge \Upsilon E\|x(t)\|^2 e^{\beta t}, \tag{49}$$

which implies

$$E\|x(t)\|^2 \le \frac{e^{-\beta t} E\{V(x(t_0))\} + \beta^{-1}\overline{\alpha}^2 J^T P J + H}{\Upsilon}, \tag{50}$$

where $\Upsilon = \lambda_{\min}(P)$ and

$$\begin{aligned}
H = \sum_{j=1}^{n} & \left[\beta D_{1j}^2 F_j^2(0) + \beta D_{1j}^2 F_j^2(0) \left(\delta_j^+ + \delta_j^-\right)^2 \right.\\
&\left. + \beta D_{2j}^2 F_j^2(0) + \beta D_{2j}^2 F_j^2(0) \left(\delta_j^+ + \delta_j^-\right)^2 \right].
\end{aligned} \tag{51}$$

If one chooses $\widetilde{B} = \sqrt{(1 + \beta^{-1}\overline{\alpha}^2 J^T P J + H)/\Upsilon} > 0$, then for any constant $\varrho > 0$ and $\|\varphi\| < \varrho$, there is $t' = t'(\varrho) > 0$, such that $e^{-\beta t} E\{V(x(t_0))\} < 1$ for all $t \ge t'$. According to Definition 3, we have $E\|x(t, t_0, \varphi)\| < \widetilde{B}$ for all $t \ge t'$. In other words, system (3) is mean-square uniformly ultimately bounded. This completes the proof. □

Note from (25), we know that there exists a constant vector $\mathfrak{B}$, such that

$$E\{V(x(t_0))\} \le \mathfrak{B} E\|x(t_0)\|^2 e^{-\beta t_0}. \tag{52}$$

Thus, combining (50) and (52) leads to

$$\begin{aligned}
E\|x(t)\|^2 &\le \frac{e^{-\beta(t-t_0)} \mathfrak{B} E\|x(t_0)\|^2 + \beta^{-1}\overline{\alpha}^2 J^T P J + H}{\Upsilon}\\
&= \frac{e^{-\beta(t-t_0)} \mathfrak{B} E\|x(t_0)\|^2}{\Upsilon} + \frac{\beta^{-1}\overline{\alpha}^2 J^T P J + H}{\Upsilon}\\
&\le \frac{e^{-\beta(t-t_0)} \mathfrak{B} E\|x(t_0)\|^2}{\Upsilon} + N,
\end{aligned} \tag{53}$$

where $N = (\beta^{-1}\overline{\alpha}^2 J^T P J + H)/\Upsilon$.

Theorem 11. *If all of the conditions of Theorem 10 hold, then there exists an attractor $\mathbb{A}_{\widetilde{B}}$ for the solutions of system (3), where $\mathbb{A}_{\widetilde{B}} = \{\varphi \in L^b_{\mathcal{F}_0} \mid E\|\varphi(s)\|_\infty \le \widetilde{B},\ t \ge t_0\}$.*

Proof. If one choose $\widetilde{B} = \sqrt{(1 + \beta^{-1}\overline{\alpha}^2 J^T P J)/\Upsilon} > 0$, Theorem 10 shows that for any ϕ, there is $t' > 0$, such that $E\|x(t, t_0, \phi)\| < \widetilde{B}$ for all $t \ge t'$. Let $\mathbb{A}_{\widetilde{B}} = \{\varphi \in L^b_{\mathcal{F}_0} \mid E\|\varphi(s)\|_\infty \le \widetilde{B},\ t \ge t_0\}$. Clearly, $\mathbb{A}_{\widetilde{B}}$ is closed, bounded, and invariant. Furthermore, $\lim_{t\to\infty} \sup \inf_{y \in \mathbb{A}_{\widetilde{B}}} \|x(t; 0, \phi) - y\| = 0$. Therefore, $\mathbb{A}_{\widetilde{B}}$ is a stochastic attractor for the solutions of system (3). □

Corollary 12. *In addition to all of the conditions of Theorem 10 that hold, if $J = 0$ and $F_j(0) = 0$, then system (3) has a trivial solution $x(t) \equiv 0$, and the trivial solution of system (3) is mean-square exponentially stable.*

Proof. If $J = 0$ and $F_j(0) = 0$, then it is obvious that system (3) has a trivial solution $x(t) \equiv 0$. From Theorem 10, one has

$$E\left\|x(t;0,\phi)\right\|^2 \le Ke^{-\beta t}, \quad \forall\phi, \tag{54}$$

where $K = E\|V(x(0))\|^2/\Upsilon$. Therefore, the trivial solution of system (3) is mean-square exponentially stable. This completes the proof. □

In practice, parameter uncertainties in neural networks are always unavoidable, in order to explain such a phenomenon. In this section, we will investigate the mean-square uniform ultimate boundedness of the switching stochastic systems with uncertainties by applying the average dwell time.

Now, we consider the switched stochastic Cohen-Grossberg neural networks with unknown parameters as follows:

$$\begin{aligned} dx(t) = {} & -\widehat{\alpha}(x(t))\Big[\widehat{\beta}(x(t)) - (A_\sigma(t) + \Delta A_\sigma(t)) \\ & \cdot F(x(t)) - (B_\sigma(t) + \Delta B_\sigma(t))F(x(t-\tau(t))) \\ & - (C_\sigma(t) + \Delta C_\sigma(t)) \\ & \cdot \int_{-\infty}^{t} K(t-s)F(x(s))\,ds - J\Big]dt \\ & + \sigma(t, x(t), x(t-\tau(t)))\,d\omega(t). \end{aligned} \tag{55}$$

Theorem 13. *For a given constant $\beta > 0$, $\varepsilon > 0$, $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\varepsilon_3 > 0$, if there exist positive scalars λ_i, positive-definite matrix $P_i = \operatorname{diag}(p_{i1}, p_{i2}, \dots, p_{in})$, $L = \operatorname{diag}(l_{i1}, l_{i2}, \dots, l_{in})$, $D_i = \operatorname{diag}(D_{i1}, D_{i2}, \dots, D_{in})$, $(i = 1, 2)$, Q_{i1}, Q_{i2}, Q_{i3}, Q_{i4}, R_i, S_i, $\widetilde{Z}_i$, Z_i, U_i, and any matrices $\overline{M}_{i1}$, $\overline{M}_{i2}$, $\overline{M}_{i3}$ with appropriate dimensions such that the following condition holds:*

$$\begin{bmatrix} \Delta_{1i} & \overline{M}_{i1} & \overline{M}_{i2} & \overline{M}_{i3} \\ * & -\varepsilon_1 I & 0 & 0 \\ * & * & -\varepsilon_2 I & 0 \\ * & * & * & -\varepsilon_3 I \end{bmatrix} < 0, \tag{56}$$

$$P_i < \lambda_i I, \tag{57}$$

where

$$\Delta_{1i} = \begin{bmatrix} \phi_{i11} & 0 & 0 & 0 & \phi_{i15} & 0 & 0 & \phi_{i18} & \phi_{i19} & 0 & 0 & 0 \\ * & \phi_{i22} & 0 & 0 & 0 & \phi_{i26} & 0 & \phi_{i28} & 0 & 0 & 0 & 0 \\ * & * & \phi_{i33} & 0 & 0 & 0 & \phi_{i37} & 0 & \phi_{i39} & 0 & 0 & 0 \\ * & * & * & \phi_{i44} & \phi_{i45} & 0 & 0 & \phi_{i48} & \phi_{i49} & 0 & 0 & 0 \\ * & * & * & * & \widetilde{\phi}_{i55} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \phi_{i66} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \phi_{i77} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \widetilde{\phi}_{i88} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \widetilde{\phi}_{i99} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \phi_{i10} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & \overline{\phi}_{i11} & 0 \\ * & * & * & * & * & * & * & * & * & * & * & \overline{\phi}_{i12} \end{bmatrix} < 0,$$

$$\begin{aligned} \phi_{i11} = {} & \frac{1}{\underline{a}^2}\Big[\beta P_i - 2\Omega_1 P_i \Omega_2 + \lambda \Pi_1^T \Pi_1 + \varepsilon \Omega_4 D_1 + P_i + \frac{1}{4\beta^2} I + \tau_M e^{\beta\tau_M} Q_{i1} + \tau_m e^{\beta\tau_m} Q_{i2} + e^{\beta\tau_M} Q_{i3} + \tau_M R_i + (\tau_M - \tau_m) S_i \\ & + \frac{\tau_M^2}{4} Z_i + \frac{\tau_M^2}{4}\widetilde{Z}_i - \Omega_3 D_1\Big], \\ \phi_{i15} = {} & P_i A_i, \\ \phi_{i18} = {} & P_i B_i, \\ \phi_{i19} = {} & P_i C_i, \\ \phi_{i22} = {} & -\tau_M Q_{i1}, \\ \phi_{i26} = {} & \phi_{i28} = 0, \\ \phi_{i33} = {} & -\tau_m Q_{i2}, \end{aligned}$$

$$\begin{aligned}
\phi_{i37} &= \phi_{i39} = 0,\\
\phi_{i44} &= \lambda\Pi_2^T\Pi_2 - \Omega_3 D_2 + \frac{1}{4\beta^2}I - (1-\mu)Q_{i3},\\
\phi_{i45} &= 0,\\
\phi_{i48} &= \Omega_4 D_2,\\
\phi_{i49} &= 0,\\
\widetilde{\phi}_{i55} &= e^{\beta\tau_M}Q_{i4} + LK(\iota) + \varepsilon^{-1}\Omega_4 D_1 + \tau_M^2 U_i - D_1 + \frac{1}{\beta^2}I + \varepsilon_1 E_{A_i}^T E_{A_i},\\
\phi_{i66} &= -\frac{e^{-\beta\tau_M}}{\tau_M}R,\\
\phi_{i77} &= -\frac{e^{-\beta\tau_M}}{\tau_M - \tau_m}S_i,\\
\widetilde{\phi}_{i88} &= -(1-\mu)Q_{i4} - D_2 + \frac{1}{\beta^2}I + \varepsilon_2 E_{B_i}^T E_{B_i},\\
\widetilde{\phi}_{i99} &= -L_i + \varepsilon_3 E_{C_i}^T E_{C_i},\\
\phi_{i10} &= -(1-\mu)e^{-\beta\tau_M}U_i,\\
\overline{\phi}_{i11} &= -e^{-\beta\tau_M}Z_i,\\
\overline{\phi}_{i12} &= -e^{-\beta\tau_M}\widetilde{Z}_i,\\
\overline{M}_{i1} &= \begin{bmatrix} P_iH_{A_i} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T,\\
\overline{M}_{i2} &= \begin{bmatrix} P_iH_{B_i} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T,\\
\overline{M}_{i3} &= \begin{bmatrix} P_iH_{C_i} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T,
\end{aligned} \tag{58}$$

then system (55) is mean-square uniformly ultimately bounded for any switching signal with average dwell time satisfying

$$\tau_a > \tau_a^* = \frac{\ln M_{\max}}{\beta}, \tag{59}$$

where $M_{\max} = \mathfrak{B}_{\max}/\Upsilon_{\min}$, $\mathfrak{B}_{\max} = \max_{k\in\overline{N},1\le i\le n}\{\mathfrak{B}_{i_k}\}$, $\Upsilon_{\min} = \min_{i_k}\{\Upsilon_{i_k}\}$.

Proof. Let us consider the same Lyapunov functional candidate

$$\begin{aligned}
V_\sigma(t) &= e^{\beta t}x^T(t)P_{\sigma(t)}x(t)\\
&+ \tau_M\int_{t-\tau_M}^{t} e^{\beta(s+\tau_M)}x^T(s)Q_{1_{\sigma(t)}}x(s)\,ds\\
&+ \tau_m\int_{t-\tau_m}^{t} e^{\beta(s+\tau_m)}x^T(s)Q_{2_{\sigma(t)}}x(s)\,ds\\
&+ \int_{t-\tau(t)}^{t} e^{\beta(s+\tau_M)}x^T(s)Q_{3_{\sigma(t)}}x(s)\,ds\\
&+ \int_{t-\tau(t)}^{t} e^{\beta(s+\tau_M)}F^T(x(s))Q_{4_{\sigma(t)}}F(x(s))\,ds\\
&+ \int_{-\tau_M}^{0}\int_{t+\theta}^{t} e^{\beta s}x^T(s)R_{\sigma(t)}x(s)\,ds\,d\theta\\
&+ \int_{-\tau_M}^{-\tau_m}\int_{t+\theta}^{t} e^{\beta s}x^T(s)S_{\sigma(t)}x(s)\,ds\,d\theta\\
&+ \tau_M\int_{-\tau(t)}^{0}\int_{t+\theta}^{t} e^{\beta s}F^T(x(s))U_{\sigma(t)}F(x(s))\,ds\,d\theta\\
&+ \frac{\tau_M^2}{2}\int_{-\tau_M}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t} e^{\beta s}x^T(s)Z_{\sigma(t)}x(s)\,ds\,d\lambda\,d\theta\\
&+ \frac{(\tau_M^2-\tau_m^2)}{2}\\
&\cdot \int_{-\tau_M}^{-\tau_m}\int_{\theta}^{0}\int_{t+\lambda}^{t} e^{\beta s}x^T(s)\widetilde{Z}_{\sigma(t)}x(s)\,ds\,d\lambda\,d\theta\\
&+ \sum_{j=1}^{n} l_{j_{\sigma(t)}}\int_0^\infty k_j(\delta)\int_{t-\delta}^{t} e^{\beta(\gamma+\delta)}F_j^2(x_j(\gamma))\,d\gamma\,d\delta.
\end{aligned} \tag{60}$$

Then, we will compute the stochastic derivative of $V(t)$ along the trajectory of system (55). Therefore, from

Theorem 10, we merely need to obtain the idea of the following equalities:

$$\begin{aligned}
&2e^{\beta t}x^T(t)P_i\alpha(x(t))\Delta A_i(t)F(x(t)) \\
&\quad = 2e^{\beta t}P_i\alpha(x(t))H_{A_i}G_{A_i}(t)E_{A_i}F(x(t)), \\
&2e^{\beta t}x^T(t)P_i\alpha(x(t))\Delta B_i(t)F(x(t-\tau(t))) \\
&\quad = 2e^{\beta t}P_i\alpha(x(t))H_{B_i}G_{B_i}(t)E_{B_i}F(x(t-\tau(t))), \\
&2e^{\beta t}x^T(t)P_i\alpha(x(t))\Delta C_i(t)\int_{-\infty}^{t}K(t-s)F(x(s))\,ds \\
&\quad = 2e^{\beta t}P_i\alpha(x(t))H_{C_i}G_{C_i}(t) \\
&\quad\cdot E_{C_i}\int_{-\infty}^{t}K(t-s)F(x(s))\,ds.
\end{aligned} \tag{61}$$

Moreover, by assumption (H5) and Lemma 7, we obtain

$$\begin{aligned}
&2e^{\beta t}x^T(t)P_i\alpha(x(t))H_{A_i}G_{A_i}(t)E_{A_i}F(x(t)) \\
&\quad \le e^{\beta t}\varepsilon_1^{-1}\overline{\alpha}^2x^T(t)P_iH_{A_i}H_{A_i}^TP_ix(t) + e^{\beta t}\varepsilon_1F^T(x(t)) \\
&\quad\cdot E_{A_i}^TE_{A_i}F(x(t)), \\
&2e^{\beta t}x^T(t)P_i\alpha(x(t))H_{B_i}G_{B_i}(t)E_{B_i}F(x(t-\tau(t))) \\
&\quad \le e^{\beta t}\varepsilon_2^{-1}\overline{\alpha}^2x^T(t)P_iH_{B_i}H_{B_i}^TP_ix(t) \\
&\quad + e^{\beta t}\varepsilon_1F^T\left(x(t-\tau(t))E_{B_i}^TE_{B_i}F(x(t-\tau(t)))\right), \\
&2e^{\beta t}P_i\alpha(x(t))H_{C_i}G_{C_i}(t)E_{C_i}\int_{-\infty}^{t}K(t-s)F(x(s))\,ds \\
&\quad \le e^{\beta t}\varepsilon_3^{-1}\overline{\alpha}^2x^T(t)P_iH_{C_i}H_{C_i}^TP_ix(t) \\
&\quad + e^{\beta t}\varepsilon_3\left(\int_{-\infty}^{t}K(t-s)F(x(s))\,ds\right)^T \\
&\quad\cdot E_{B_i}^TE_{B_i}\int_{-\infty}^{t}K(t-s)F(x(s))\,ds.
\end{aligned} \tag{62}$$

Then, along a similar way to Theorem 10, it can be deduced that

$$\widetilde{\Delta}_{1i} = \begin{bmatrix}
\widetilde{\phi}_{i11} & 0 & 0 & 0 & \widetilde{\phi}_{i15} & 0 & 0 & \widetilde{\phi}_{i18} & \widetilde{\phi}_{i19} & 0 & 0 & 0 \\
* & \phi_{i22} & 0 & 0 & 0 & \phi_{i26} & 0 & \phi_{i28} & 0 & 0 & 0 & 0 \\
* & * & \phi_{i33} & 0 & 0 & 0 & \phi_{i37} & 0 & \phi_{i39} & 0 & 0 & 0 \\
* & * & * & \phi_{i44} & \phi_{i45} & 0 & 0 & \phi_{i48} & \phi_{i49} & 0 & 0 & 0 \\
* & * & * & * & \widetilde{\phi}_{i55} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & \phi_{i66} & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & \phi_{i77} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & \widetilde{\phi}_{i88} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & \widetilde{\phi}_{i99} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & \phi_{i10} & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & \overline{\phi}_{i11} & 0 \\
* & * & * & * & * & * & * & * & * & * & * & \overline{\phi}_{i12}
\end{bmatrix},$$

$$\begin{aligned}
&P_i < \lambda_iI, \\
&\widetilde{\phi}_{i11} = \beta P_i - 2\Omega_1P_i\Omega_2 + \lambda\Pi_1^T\Pi_1 + \varepsilon\Omega_4D_1 + P_i + \frac{1}{4\beta^2}I + \tau_Me^{\beta\tau_M}Q_{i1} + \tau_me^{\beta\tau_m}Q_{i2} + e^{\beta\tau_M}Q_{i3} + \tau_MR_i + (\tau_M-\tau_m)S_i \\
&\quad + \frac{\tau_M^2}{4}Z_i + \frac{\tau_M^2}{4}\widetilde{Z}_i - \Omega_3D_1 + \varepsilon_1^{-1}\overline{\alpha}^2P_iH_{A_i}H_{A_i}^TP_i + \varepsilon_2^{-1}\overline{\alpha}^2P_iH_{B_i}H_{B_i}^TP_i + \varepsilon_3^{-1}\overline{\alpha}^2P_iH_{C_i}H_{C_i}^TP_i, \\
&\widetilde{\phi}_{i15} = P_i\alpha(x(t))A_i, \\
&\widetilde{\phi}_{i18} = P_i\alpha(x(t))B_i, \\
&\widetilde{\phi}_{i19} = P_i\alpha(x(t))C_i.
\end{aligned} \tag{63}$$

By using Schur's complement lemma, the LMI (56) is equivalent to

$$\underline{\Delta}_{1i} = \begin{bmatrix} \underline{\phi}_{i11} & 0 & 0 & 0 & \phi_{i15} & 0 & 0 & \phi_{i18} & \phi_{i19} & 0 & 0 & 0 \\ * & \phi_{i22} & 0 & 0 & 0 & \phi_{i26} & 0 & \phi_{i28} & 0 & 0 & 0 & 0 \\ * & * & \phi_{i33} & 0 & 0 & 0 & \phi_{i37} & 0 & \phi_{i39} & 0 & 0 & 0 \\ * & * & * & \phi_{i44} & \phi_{i45} & 0 & 0 & \phi_{i48} & \phi_{i49} & 0 & 0 & 0 \\ * & * & * & * & \widetilde{\phi}_{i55} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \phi_{i66} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \phi_{i77} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \widetilde{\phi}_{i88} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \widetilde{\phi}_{i99} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \phi_{i10} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & \overline{\phi}_{i11} & 0 \\ * & * & * & * & * & * & * & * & * & * & * & \overline{\phi}_{i12} \end{bmatrix} < 0, \tag{64}$$

where

$$\begin{aligned} \underline{\phi}_{i11} &= \frac{1}{\overline{\alpha}^2}\Big[\beta P_i - 2\Omega_1 P_i \Omega_2 + \lambda \Pi_1^T \Pi_1 + \varepsilon \Omega_4 D_1 + P_i \\ &\quad + \frac{1}{4\beta^2} I + \tau_M e^{\beta \tau_M} Q_{i1} + \tau_m e^{\beta \tau_m} Q_{i2} + e^{\beta \tau_M} Q_{i3} \\ &\quad + \tau_M R_i + (\tau_M - \tau_m) S_i + \frac{\tau_M^2}{4} Z_i + \frac{\tau_M^2}{4} \widetilde{Z}_i - \Omega_3 D_1 \Big] \\ &\quad + \varepsilon_1^{-1} P_i H_{A_i} H_{A_i}^T P_i + \varepsilon_2^{-1} P_i H_{B_i} H_{B_i}^T P_i \\ &\quad + \varepsilon_3^{-1} P_i H_{C_i} H_{C_i}^T P_i, \\ \phi_{i15} &= PA, \\ \phi_{i18} &= PB, \\ \phi_{i19} &= PC. \end{aligned} \tag{65}$$

Supposing $\alpha(x(t))\alpha(x(t)) \leq \overline{\alpha}^2 I$ and multiplying both sides of LMI (64) by

$$\operatorname{diag}\{\alpha(x(t)), I, I, I, I, I, I, I, I, I, I, I\}, \tag{66}$$

we derive the LMI $\widetilde{\Delta}_{1i} < 0$.

Therefore, when $t \in [t_k, t_{k+1})$, the i_kth subsystem is activated. From the proof of Theorem 10 and (53), there exists a positive constant vector $\mathfrak{B}_{i_k}$, such that

$$\begin{aligned} &E\|x(t)\|^2 \\ &\quad \leq \frac{\mathfrak{B}_{i_k} E\|x(t_k)\|^2 e^{-\beta(t-t_k)} + \beta^{-1}\overline{\alpha}^2 J^T P J + H}{\Upsilon_{i_k}} \\ &\quad = M_{i_k} E\|x(t_k)\|^2 e^{-\beta(t-t_k)} + N_{i_k}, \end{aligned} \tag{67}$$

where $M_{i_k} = \mathfrak{B}_{i_k}/\Upsilon_{i_k}$, $\Upsilon_{i_k} = \min_{k\in\overline{N}, 1\leq i\leq n}\{\lambda_{\min}(P_i)\}$, $N_{i_k} = (\beta^{-1}\overline{\alpha}^2 J^T P J + H)/\Upsilon_{i_k}$.

As the system state is continuous, it follows from (67) that

$$\begin{aligned} &E\|x(t)\|^2 \\ &\quad \leq \frac{\mathfrak{B}_{i_k} E\|x(t_k)\|^2 e^{-\beta(t-t_k)} + \beta^{-1}\overline{\alpha}^2 J^T P J + H}{\Upsilon_{i_k}} \\ &\quad = M_{i_k} E\|x(t_k)\|^2 e^{-\beta(t-t_k)} + N_{i_k} \leq \cdots \\ &\quad \leq e^{\sum_{v=0}^{k} \ln M_{i_v} - \beta(t-t_0)} E\|x(t_0)\|^2 + \Big[M_{i_k} e^{-\beta(t-t_k)} N_{i_k} \\ &\quad + M_{i_k} M_{i_{k-1}} e^{-\beta(t-t_{k-1})} N_{i_{k-1}} \\ &\quad + M_{i_k} M_{i_{k-1}} M_{i_{k-2}} e^{-\beta(t-t_{k-2})} N_{i_{k-2}} + \cdots \\ &\quad + M_{i_k} M_{i_{k-1}} M_{i_{k-2}} \cdots M_{i_1} e^{-\beta(t-t_1)} N_{i_1} + N_{i_k}\Big] \\ &\quad \leq e^{(k+1)\ln M_{\max} - \beta(t-t_0)} E\|x(t_0)\|^2 + \Big[M_{\max}^k N_{\max} \\ &\quad + M_{\max}^{(k-1)} N_{\max} + M_{\max}^{(k-2)} N_{\max} + \cdots + M_{\max}^2 N_{\max} \\ &\quad + M_{\max} N_{\max} + N_{\max}\Big] \\ &\quad \leq M_{\max} e^{k \ln M_{\max} - \beta(t-t_0)} E\|x(t_0)\|^2 \\ &\quad + \frac{N_{\max}\left(1 - M_{\max}^{(k+1)}\right)}{1 - M_{\max}} \\ &\quad \leq M_{\max} e^{\ln M_{\max} N_\sigma(t-t_0) - \beta(t-t_0)} E\|x(t_0)\|^2 \\ &\quad + \frac{N_{\max}\left(1 - M_{\max}^{(k+1)}\right)}{1 - M_{\max}} \end{aligned}$$

$$\begin{aligned}&\leq M_{\max} e^{N_0 \ln M_{\max} - (\beta - \ln M_{\max}/T_\alpha)(t-t_0)} E \left\| x(t_0) \right\|^2 \\ &+ \frac{N_{\max}\left(1 - M_{\max}^{(k+1)}\right)}{1 - M_{\max}} \\ &\leq \frac{\mathfrak{B}_{\max} e^{N_0 \ln M_{\max} - (\beta - \ln M_{\max}/T_\alpha)(t-t_0)} E \left\| x(t_0) \right\|^2}{\Upsilon_{\min}} \\ &+ \frac{\left(\beta^{-1}\overline{\alpha}^2 J^T P J + H\right)\left(1 - \mathfrak{B}_{\max}^{(k+1)}/\Upsilon_{\min}^{(k+1)}\right)}{\Upsilon_{\min} - \mathfrak{B}_{\max}}.\end{aligned} \tag{68}$$

If one chooses

$$\widetilde{B} = \sqrt{\frac{1}{\Upsilon_{\min}} + \frac{\left(\beta^{-1}\overline{\alpha}^2 J^T P J + H\right)\left(1 - \mathfrak{B}_{\max}^{(k+1)}/\Upsilon_{\min}^{(k+1)}\right)}{\Upsilon_{\min} - \mathfrak{B}_{\max}}} > 0, \tag{69}$$

then for any constant $\varrho > 0$ and $\|\varphi\| < \varrho$ there is $t' = t'(\varrho) > 0$, such that $\mathfrak{B}_{\max} e^{N_0 \ln M_{\max} - (\beta - \ln M_{\max}/T_\alpha)(t-t_0)} E\|x(t_0)\|^2 < 1$ for all $t \geq t'$. According to Definition 3, we have $E\|x(t, t_0, \varphi)\| < \widetilde{B}$ for all $t \geq t'$. In other words, the switched stochastic Cohen-Grossberg neural networks system (55) is mean-square uniformly ultimately bounded. This completes the proof. □

Theorem 14. *If all of the conditions of Theorem 10 hold, then there exists an attractor $\mathbb{A}_{\widetilde{B}}$ for the solutions of system (55), where $\widetilde{\mathbb{A}}_{\widetilde{B}} = \{\varphi \in L^b_{\mathcal{F}_0} \mid E\|\varphi(s)\|_\infty \leq \widetilde{B},\ t \geq t_0\}$.*

Proof. If one chooses

$$\widetilde{B} = \sqrt{\frac{1}{\Upsilon_{\min}} + \frac{\left(\beta^{-1} J^T P J + H\right)\left(1 - \mathfrak{B}_{\max}^{(k+1)}/\Upsilon_{\min}^{(k+1)}\right)}{\Upsilon_{\min} - \mathfrak{B}_{\max}}} > 0, \tag{70}$$

then Theorem 10 shows that, for any ϕ, there is $t' > 0$; we have $E\|x(t, t_0, \phi)\| < \widetilde{B}$ for all $t \geq t'$. Let $\widetilde{\mathbb{A}}_{\widetilde{B}} = \{\varphi \in L^b_{\mathcal{F}_0} \mid E\|\varphi(s)\|_\infty \leq \widetilde{B},\ t \geq t_0\}$. Clearly, $\widetilde{\mathbb{A}}_{\widetilde{B}}$ is closed, bounded, and invariant. Furthermore, $\lim_{t\to\infty} \sup \inf_{y \in \widetilde{\mathbb{A}}_{\widetilde{B}}} \|x(t; t_0, \phi) - y\| = 0$. Therefore, $\widetilde{\mathbb{A}}_{\widetilde{B}}$ is an attractor for the solutions of system (55). □

Corollary 15. *In addition to all of the conditions of Theorem 13 that hold, if $J = 0$, $F_j(0) = 0$, then system (55) has a trivial solution $x(t) \equiv 0$, and the trivial solution of system (55) is mean-square exponentially stable.*

Proof. If $J = 0$, $F_j(0) = 0$, then it is obvious that system (55) has a trivial solution $x(t) \equiv 0$. From Theorem 10, one has

$$E\left\|x(t; t_0, \phi)\right\|^2 \leq \widetilde{\Upsilon} e^{-\beta(t-t_0)}, \quad \forall \phi, \tag{71}$$

where $\widetilde{\Upsilon} = (\mathfrak{B}_{\max} e^{N_0 \ln M_{\max} + (\ln M_{\max}/T_\alpha)(t-t_0)}/\Upsilon_{\min}) E\|x(t_0)\|^2$.

Therefore, the trivial solution of system (55) is mean-square exponentially stable. This completes the proof. □

Remark 16. It is noteworthy that the time-varying delay $\tau(t)$ restricts the interval $[\tau_m, \tau_M]$ and the lower bound of time delay τ_m may not be equal to 0. In previous work, such as [19, 24, 26, 37], the well-used Lyapunov functional, in which the time delay information is from 0 to an upper bound τ_M, is of the form $\int_{-\tau_M}^{0} \int_{t+\theta}^{t}$. In this paper, a new Lyapunov functional is constructed, which contains the information of the lower bound of time delay τ_m, and is of the form $\int_{-\tau_M}^{-\tau_m} \int_{t+\theta}^{t}$. Thus the methods in the paper can be adopted to discuss the dynamic behaviors of interval stochastic switched Cohen-Grosberg neural networks with time delays. Therefore, the time-varying from τ_m to τ_M is more general and less conserving of the neural networks models. If the lower bound of time delay $\tau_m = 0$, then our results will turn into the traditional time delay results.

Remark 17. It is known that noise disturbance is a major source of instability and poor performances in neural networks in real neural networks. If $\sigma(t, x(t), x(t - \tau(t))) = 0$, the switched stochastic Cohen-Grossberg neural networks (8) degenerate into the ordinary switched Cohen-Grossberg neural networks, which have been studied for exponential stability in [22] and robust stability in [37]. In addition, when $\alpha_i(x_i(t)) = 1$, $i = 1, 2, \ldots, n$, the switched Cohen-Grosberg neural networks will turn into the famous switched Hopfield neural networks; this has been investigated in [28] without distributed time delay and for global robust asymptotic stability in [32] with finite distributed time delay. However, the infinite distributed time delay was not taken into account in neural networks. Therefore, our developed results in this paper are more comfortable and general than those reported in [28, 32, 37].

Remark 18. If $N = 1$, then the switched stochastic Cohen-Grossberg neural networks (8) degenerate into the ordinary stochastic Cohen-Grossberg neural networks without being switched. The attractor and boundedness for stochastic Cohen-Grossberg neural networks with delays have been discussed in [35] by LaSalle-type theorem and stability has been studied in [2, 18, 19]. So our results generalize these previous results.

Remark 19. The triple integral terms

$$\begin{gathered}\int_{-\tau_M}^{0} \int_{\theta}^{0} \int_{t+\lambda}^{t} e^{\beta s} x^T(s) Z x(s)\, ds\, d\lambda\, d\theta, \\ \int_{-\tau_M}^{-\tau_m} \int_{\theta}^{0} \int_{t+\lambda}^{t} e^{\beta s} x^T(s) \widetilde{Z} x(s)\, ds\, d\lambda\, d\theta\end{gathered} \tag{72}$$

considered in this paper lead to new dynamic criteria. We make full use of Lemma 6 and do not ignore any terms, which can reduce some conservatism of proposed method. This can be verified from the numerical examples discussed in Section 5.

Remark 20. It should be mentioned that the nonlinear output function in [2, 18, 24, 35, 37, 38] is required to satisfy $F_j(0) = 0$; however, in our paper, the assumption condition was deleted. In assumption (H1), the constants δ_i^- and δ_i^+ are allowed to be positive, negative, or zero, whereas the constant δ_i^- is restricted to be zero or positive in [2, 18, 26, 35]. Moreover, assumption (H2) is weaker than those given in [26, 35] since they are required to be differentiable of the amplification function $\widehat{\alpha}(x(t))$. The usual condition (H3) required to the behaved function $\widehat{\beta}(x(t))$ is differentiable in [18, 37] or satisfies $(\beta_j(u) - \beta_j(v))/(u-v) \geq \gamma_i$, $(u, v \in R)$ or $\beta_i(0) = 0$ in [26]. If we take $\widehat{\beta}_i(x(t)) = x(t)e^{|x(t)/2|}$, obviously, the assumption (H3) in this paper holds; yet, the conditions in [26] can not be achieved.

4. Illustrative Examples

In this section, we present examples to show the effectiveness of the proposed method. Let $N = 2$ and consider the switched stochastic neural networks with two subsystems.

Example 1. Consider the following switched stochastic Cohen-Grossberg neural network (73) with unknown parameters:

$$\begin{aligned} dx(t) = &-\widehat{\alpha}(x(t))\Big[\widehat{\beta}(x(t)) - (A_i + \Delta A_i(t))F(x(t)) \\ &- (B_i + \Delta B_i(t))F(x(t-\tau(t))) - (C_i + \Delta C_i(t)) \\ &\cdot \int_{-\infty}^{t} K(t-s)F(x(s))\,ds - J\Big]dt \\ &+ \sigma(t, x(t), x(t-\tau(t)))\,d\omega(t), \end{aligned} \tag{73}$$

where $\Delta A_i(t)$, $\Delta B_i(t)$, $\Delta C_i(t)$ are uncertainties, satisfying $\Delta A_i(t) = H_{A_i}G_{A_i}(t)E_{A_i}$, $\Delta B_i(t) = H_{B_i}G_{B_i}(t)E_{B_i}$, $\Delta C_i(t) = H_{C_i}G_{C_i}(t)E_{C_i}$, where G_{A_i}, G_{B_i}, G_{C_i} are normal bounded matrices. Let $\widehat{\alpha}(x(t)) = \operatorname{diag}(3+\sin(x_1), 3+\cos(x_2))$, $\widehat{\beta}_i(x_i(t)) = 2x_i(t)$, $f_i(x_i(t)) = \tanh(x_i(t))$ $(i = 1, 2)$, $K(t) = \operatorname{diag}(2e^{-(3/2)t}, (1/2)e^{-2t})$, $J = (2, 3)^T$ and the connection weight matrices are as follows.

Subsystems 1. Consider

$$\begin{aligned} A_1 &= \begin{pmatrix} 5 & -2 \\ -1 & 3 \end{pmatrix}, \\ B_1 &= \begin{pmatrix} 6 & -2 \\ 3 & 4 \end{pmatrix}, \\ C_1 &= \begin{pmatrix} 8 & 2 \\ -5 & 2 \end{pmatrix}, \\ E_{A_1} &= \begin{pmatrix} 0.5 & 0 \\ 0 & 0.4 \end{pmatrix}, \\ E_{B_1} &= \begin{pmatrix} 0.1 & 0.3 \\ 0 & 0.4 \end{pmatrix}, \\ E_{C_1} &= \begin{pmatrix} 0.1 & 0 \\ 0 & 0.7 \end{pmatrix}. \end{aligned} \tag{74}$$

Subsystems 2. Consider

$$\begin{aligned} A_2 &= \begin{pmatrix} 2 & 3 \\ -2 & 7 \end{pmatrix}, \\ B_2 &= \begin{pmatrix} 9 & 3 \\ -3 & 4 \end{pmatrix}, \\ C_2 &= \begin{pmatrix} 2 & 2 \\ -1 & 5 \end{pmatrix}, \\ E_{A_2} &= \begin{pmatrix} 0.1 & 0 \\ 0 & 0.2 \end{pmatrix}, \\ E_{B_2} &= \begin{pmatrix} 0.1 & 0 \\ 0.3 & 0.2 \end{pmatrix}, \\ E_{C_2} &= \begin{pmatrix} 0.3 & 0 \\ 0 & 0.5 \end{pmatrix}, \\ \Pi_1 &= \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix}, \\ \Pi_2 &= \begin{pmatrix} 0.1 & 0 \\ 0 & 0.1 \end{pmatrix}, \\ H_{A_i} = H_{B_i} = H_{C_i} &= \begin{pmatrix} 3 & 0 \\ 0 & 5 \end{pmatrix}, \\ G_{A_i}(t) = G_{B_i}(t) = G_{C_i}(t) &= \begin{pmatrix} \sin t & 0 \\ 0 & \cos t \end{pmatrix}. \end{aligned} \tag{75}$$

Just from assumptions (H1) and (H2), we can obtain $\underline{\alpha} = 2$, $\overline{\alpha} = 4$, $b_i = 1.5$, $\delta_i^- = 0$, $\delta_i^+ = 1$, $\tau_m = 0.1$, $\tau_M = 0.6$, $i = 1, 2$, $\mu = 0.5$, $\iota = 1$, $\underline{K}(1) = \operatorname{diag}(4, 1/2)$.

Therefore, for $\beta = 2$, by solving LMIS (56) and (57), we get

$$\begin{aligned} P_1 &= 1 \times 10^{-3} \begin{pmatrix} 0.1551 & 0 \\ 0 & 0.1551 \end{pmatrix}, \\ Q_{11} &= \begin{pmatrix} 1.3273 & -0.0001 \\ -0.0001 & 1.3550 \end{pmatrix}, \\ Q_{21} &= \begin{pmatrix} 17.8857 & -0.0011 \\ -0.0011 & 18.0732 \end{pmatrix}, \\ Q_{31} &= \begin{pmatrix} 1.4294 & 0.0001 \\ 0.0001 & 1.3633 \end{pmatrix}, \\ Q_{41} &= \begin{pmatrix} 33.2092 & 0.0020 \\ 0.0020 & 39.9381 \end{pmatrix}, \\ S_1 &= \begin{pmatrix} 3.7674 & -0.0002 \\ -0.0002 & 3.7960 \end{pmatrix}, \end{aligned}$$

$$
\begin{aligned}
R_1 &= \begin{pmatrix} 3.6154 & -0.0002 \\ -0.0002 & 3.6527 \end{pmatrix}, \\
U_1 &= \begin{pmatrix} 42.8137 & -0.0140 \\ -0.0140 & 43.1853 \end{pmatrix}, \\
Z_1 &= \begin{pmatrix} 11.3563 & -0.0003 \\ -0.0003 & 11.3904 \end{pmatrix}, \\
\widetilde{Z}_1 &= \begin{pmatrix} 11.4414 & -0.0003 \\ -0.0003 & 11.4748 \end{pmatrix}, \\
L_1 &= \begin{pmatrix} 6.3948 & 0 \\ 0 & 6.3948 \end{pmatrix}, \\
P_2 &= 1 \times 10^{-3} \begin{pmatrix} 0.1885 & 0 \\ 0 & 0.1885 \end{pmatrix}, \\
Q_{12} &= \begin{pmatrix} 1.5297 & -0.0001 \\ -0.0001 & 1.6248 \end{pmatrix}, \\
Q_{22} &= \begin{pmatrix} 20.2951 & -0.0004 \\ -0.0004 & 20.5209 \end{pmatrix}, \\
Q_{32} &= \begin{pmatrix} 1.7322 & 0.0002 \\ 0.0002 & 1.6554 \end{pmatrix}, \\
Q_{42} &= \begin{pmatrix} 40.2433 & 0.0049 \\ 0.0049 & 47.6254 \end{pmatrix}, \\
S_2 &= \begin{pmatrix} 4.2168 & -0.0001 \\ -0.0001 & 4.2518 \end{pmatrix}, \\
R_2 &= \begin{pmatrix} 4.0988 & -0.0001 \\ -0.0001 & 4.1437 \end{pmatrix}, \\
U_2 &= \begin{pmatrix} 47.0352 & -0.0358 \\ -0.0358 & 47.1122 \end{pmatrix}, \\
Z_2 &= \begin{pmatrix} 12.4873 & 0 \\ 0 & 12.5300 \end{pmatrix}, \\
\widetilde{Z}_2 &= \begin{pmatrix} 12.5778 & 0 \\ 0 & 12.6198 \end{pmatrix}, \\
L_2 &= \begin{pmatrix} 6.9869 & 0 \\ 0 & 6.9869 \end{pmatrix}, \\
\lambda_1 &= 1.78, \\
\lambda_2 &= 2.18.
\end{aligned}
\tag{76}
$$

Taking $F_j(0) = 0$, $J = 0$ and using (59), we can obtain the average dwell time $\tau_a > \tau_a^* = \max\{6.6718, 6.8237\}$. Therefore, one can choose $\tau_a = 7$. The simulations of arbitrary switching signal with the average dwell time $\tau_a = 7$ can be shown in Figure 1.

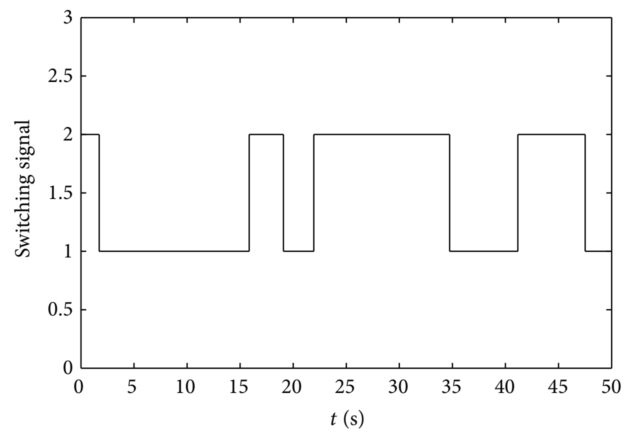

Figure 1: Arbitrary switching signal with the average dwell time $\tau_a = 7$.

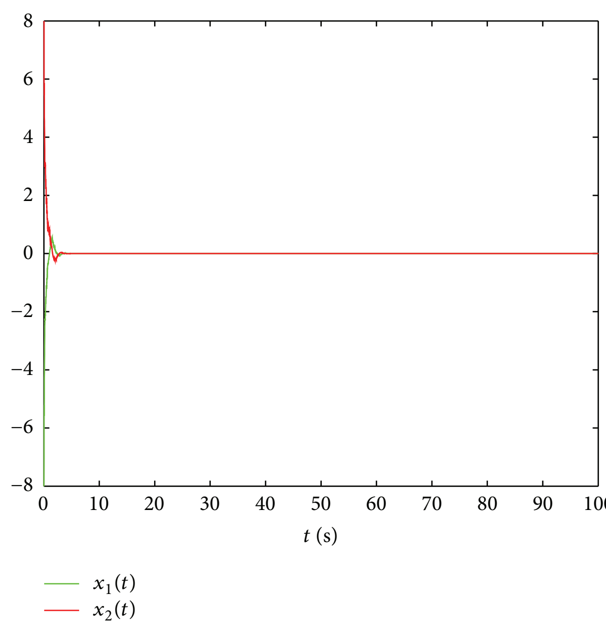

Figure 2: The mean-square exponential stability with the initial value as $x_0 = [-8, 8]$.

The mean-square exponential stability of system (55) with the initial value as $x_0 = [-8, 8]$ can be shown in Figure 2. With the help of MATLAB, the time evolutions of state variables of the system (55) can be shown in Figure 3. In the above conditions, phase portraits of simulations under initial condition of the system (55) are shown in Figure 4.

5. Conclusion

This paper has studied the problem of boundedness for a class of switched stochastic Cohen-Grossberg neural networks with both average dwell time and norm-bounded parameter uncertainties. By employing multiple Lyapunov-Krasovskii functionals (25) and (60), we formulate a method that derives

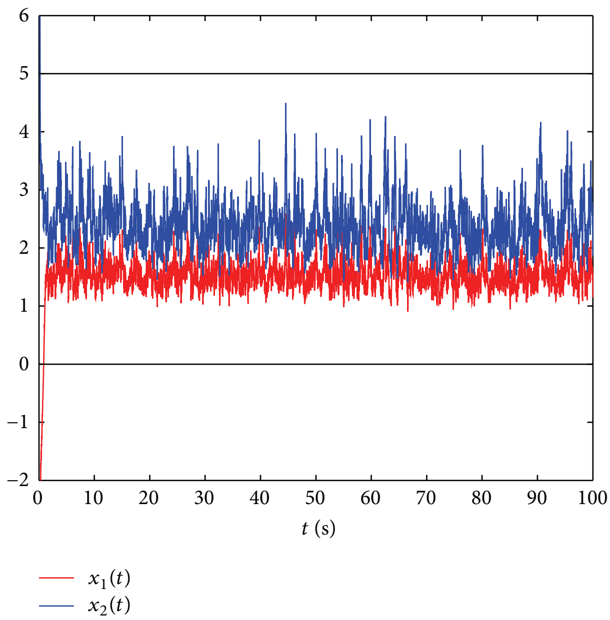

FIGURE 3: The mean-square uniformly ultimate boundedness.

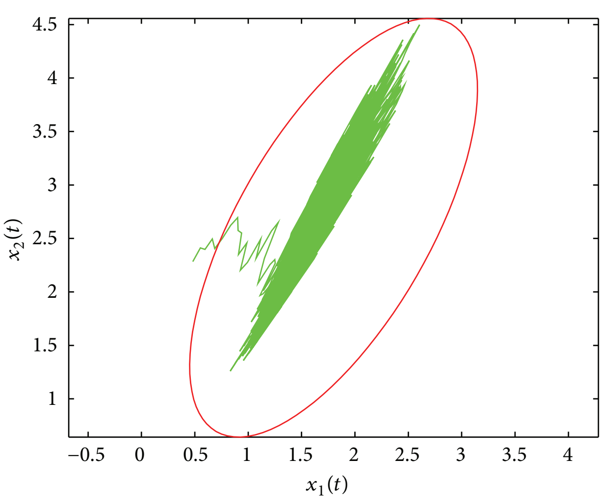

FIGURE 4: The attractor set $\mathbb{A}$.

new sufficient conditions guaranteeing the mean-square uniformly ultimate boundedness, the existence of an attractor, and the mean-square exponential stability. A numerical example has been presented to demonstrate the effectiveness and the merits of the proposed method. It is expected that the approach presented in this paper can be easily extended to analyze other neural networks.

Competing Interests

The authors declare that they have no competing interests.

Acknowledgments

The work is partially supported by the National Natural Science Foundation of China (numbers 11101053 and 71471020), China Postdoctoral Science Foundation (numbers 2014M550097 and 2015T80144), Hunan Provincial Natural Science Foundation (number 16JJ1015), Scientific Research Fund of Hunan Provincial Education Department (number 15A003), and Hunan Provincial Innovation Foundation for Postgraduate (number CX2015B373).

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