Research Article

Modeling and Optimization of Stochastic Joint Replenishment and Delivery Scheduling Problem with Uncertain Costs

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The stochastic joint replenishment and delivery scheduling (JRD) problem is a key issue in supply chain management and is a major concern for companies. So far, all of the work on stochastic JRDs is under explicit environment. However, the decision makers often have to face vague operational conditions. We develop a practical JRD model with stochastic demand under fuzzy backlogging cost, fuzzy minor ordering cost, and fuzzy inventory holding cost. The problem is to determine procedures for inventory management and vehicle routing simultaneously so that the warehouse may satisfy demand at a minimum long-run average cost. Subsequently, the fuzzy total cost is defuzzified by the graded mean integration representation and centroid approaches to rank fuzzy numbers. To find optimal coordinated decisions, a modified adaptive differential evolution algorithm (MADE) is utilized to find the minimum long-run average total cost. Results of numerical examples indicate that the proposed JRD model can be used to simulate fuzzy environment efficiently, and the MADE outperforms genetic algorithm with a lower total cost and higher convergence rate. The proposed methods can be applied to many industries and can help obtaining optimal decisions under uncertain environment.

1. Introduction

The joint replenishment problem (JRP) has been heavily researched since the early work of Shu [1]. The JRP means to group items into the same order from a supplier to achieve the purpose of sharing the main preparation costs and saving the procurement costs (Goyal [2]). The existing research of JRPs can be classified into two categories: (1) the classic JRPs under constant demand, (2) JRPs under stochastic or dynamic demand. Corresponding review is available in Khouja and Goyal [3] and Robinson et al. [4].

Supply chain management has received much attention and global sourcing has been widespread in recent years. Since cost savings can be achieved by using joint replenishment policy, managers have realized that jointly considering JRP and delivery scheduling (JRD) can obtain a scale effect of replenishment and transportation simultaneously, thus further reducing total cost (Qu et al. [5], Wang et al. [6]). Unfortunately, the literature of JRD is limited and most

of scholars studied JRDs with deterministic demand. Sindhuchao et al. [7] developed the inventory and transportation policy for the central warehouse and decentralized suppliers. They considered the transportation capacity and transportation frequency constraints. Chan et al. [8] discussed delivery scheduling after the JRP has been solved and, respectively, designed four different objectives. Cha et al. [9] developed a joint replenishment and transportation model in a one-warehouse, n-retailer system based on improved well-known heuristic named RAND and Genetic Algorithm (GA). They also extended their model with constraints and showed the flexibility of GA. Moon et al. [10] proposed two algorithms to determine the joint replenishment and delivery policies from the perspective of a third party warehouse. Unlike the studies above that consider deterministic demand, Qu et al. [5] discussed a multiitem JRD with modified periodic-review policy under stochastic demand. They considered nonlinear transportation cost and proposed an efficient heuristic algorithm to solve the problem. However, all the existing JRD

models mentioned above assume explicit vital factors, which is too harsh in practice.

In reality, managers often have to make decisions under imprecise operational conditions (Zeng et al. [11]). Uncertainties have been restrained in a variety of ways and fuzzy set theory has a long history for managing inventories. Chen and Chang [12] proposed a fuzzy economic production quantity model with defective productions that cannot be repaired. Wang et al. [13] studied continuous review inventory models in which a fraction of demand is backordered and the remaining fraction is lost during the stock-out period under fuzzy demands. Dey and Chakraborty [14] discussed a single period inventory model under the mixed fuzzy random environment and provided a methodology to handle the model. On the other hand, defuzzification has been a favorite approach in many inventory studies for its simplicity. Defuzzification can easily transfer fuzziness to be explicit without complex analysis (Lin [15]). So, we will also utilize two defuzzification approaches to handle the fuzziness. However, to the best of our knowledge, only a few papers discussed JRD under fuzzy environment (Wang et al. [16]). However, they assume that demand rates are known and constant. It is too ideal and results in the relatively low practical value of the proposed model.

Moreover, it is very difficult to find optimal solutions for the defuzzified model effectively. The JRP has been proven to be an NP-hard problem (Arkin et al., [17]). While optimal solutions are very hard to be obtained, many acceptable solutions are designed in practice. Current approaches for solving JRPs include an iterative algorithm (Goyal [18]), power-of-two (PoT) policy (Lee and Yao [19]), RAND algorithm (Kaspi and Rosenblatt [20]), evolutionary algorithm (Olsen [21]), branch-and-bound (Robinson and Gao [22]), and simulated annealing metaheuristic (Robinson et al. [23]). For the JRD model, there are more decision variables which will influence each other. Unfortunately, defuzzification also increases the complexity of the problem further. It is more difficult to solve the fuzzy JRD model in generally acceptable time and CPU memory requirements. Several heuristics may give acceptable solutions when the scale of optimization is small. However, it is not easy to find a proper heuristic with a robust performance.

On the other hand, metaheuristics have grown quickly in order to attain better solutions. Among these algorithms, the evolutionary algorithms (EAs) and especially genetic algorithm (GA) have been proved to be effective algorithms for the JRD. For example, Cha et al. [9] designed two heuristics and a hybrid GA. Results showed the GA outperformed the best solution of each problem in their experiment (the maximum error found is 0.3660% and the average error found is only 0.0268%). However, the GA displays inherent difficulties in performing local search for some numerical applications and the optimal solution cannot be obtained. So, it is necessary to find a novel algorithm to deal with the fuzzy JRD more effectively.

The aim of this paper is to model and optimize the practical JRD policy with stochastic demands under uncertainty. This topic is interesting because of the widely adoption of the JRD policy in many industries and the operability and rationality of the method to handle uncertain costs. This study makes the work of Qu et al. [5] and Wang et al. [6] become more practical by considering the fuzziness of minor ordering cost, holding cost, and backlogging cost. This study also extends the work of Wang et al. [16] which studied fuzzy JRD under determined environment. The introduction of stochastic demands in the JRD results in rather complex mathematical properties of the proposal model. So, a new approach should be designed to solve this NP-hard problem better. Therefore, we propose a simple and effective modified adaptive differential evolution algorithm (MADE) for the fuzzy JRD model firstly. Results of an example show that an MADE performs better than GA which is also regarded as an effective algorithm for this problem. Then, the MADE is utilized to solve the fuzzy JRD and the results of fuzzy JRD are compared with the JRD. Numerical examples show that the ranges of fuzzy parameters will influence the decision of this problem.

The rest of this paper is organized as follows. In Section 2, we propose a fuzzy JRD model and give the results by two defuzzification approaches. Section 3 proposes an MADE to solve the defuzzified JRD model. Section 4 presents numerical studies and analysis. Section 5 contains the conclusion and provides directions for future research. Preliminaries are presented in the appendix.

2. Fuzzy JRD Model and Analysis

2.1. Stochastic JRD Model with Periodic Review Policy under Crisp

2.1.1. Problem Description and Notations. We consider the similar situation of a central warehouse (where all stocks are kept) and several geographically dispersed suppliers asserted by Qu et al. [5] and Wang et al. [6]. The central warehouse replenishes its stock by dispatching vehicles to collect the goods from vendors, on routes which begin at the warehouse and end there. When all goods are jointly ordered, one main difficulty faced by the central warehouse is to decide the delivery scheduling in each basic replenishment cycle so that the long-run distribution cost is as low as possible. Blumenfeld et al. [24] described a similar supply network used by General Motors (GMs). Similar issues are also frequently faced by an integrated wholesaler/retailer in the supply of grocery products. The notions used are as follows:

i: the index of items ($i = 1, 2, \ldots, n$);

p: the index of suppliers ($p = 1, 2, \ldots, P$);

D_i: annual demand of item I;

S: major ordering cost associated with each order;

s_i: minor ordering cost of item I;

h_i: annual inventory holding cost of item I;

L_i: the lead time of item i;

R_i: the maximal inventory level of item i in its replenishment interval;

z_i: the tercile of the item i on standard normal distribution (decision variable);

δ_i: the variance of demand fluctuations per unit time of item i;

W: the basic interval after all items replenished jointly, means the lowest common multiple of $(k1, k2, \ldots, k_n)$;

τ_i: the shortage cost of item i;

c: the unit distance cost of distribution;

F_p: the stopover cost in supplier p;

j: the index of replenishment period ($j = 1, 2, \ldots, W$);

$d(j)$: the shortest distribution path in the jth replenishment period;

TC: the total annual cost;

T: the basic replenishment cycle per year (decision variable);

k_i: the order interval of the item i (decision variable).

Sometimes, the stock-out factor z_i is predetermined by decision maker. However, in order to obtain the optimal value of z_i or service level, Eynan and Kropp [25] regarded z_i as a decision variable, which is the same with the study of Qu et al. [5]. In this study, we also regard z_i as a decision variable.

2.1.2. Formulation of JRD.

The total cost consists of the following parts: (1) the inventory cost, which includes ordering, holding, and backlog cost; (2) the transportation cost, which includes dispatching and stopover and routing costs. The details of different costs will be discussed as follows.

(1) Inventory Cost. The ordering cost consists of the major ordering cost which is independent of the number of different items in the order and the minor ordering cost which depends on the number of different items in the order. Hence, the annual ordering cost is

$$C_O = \frac{S}{T} + \sum_{i=1}^{n} \frac{s_i}{k_i T}. \tag{1}$$

We suppose the demand of each item follows the normal distribution and the lead time is L_i. As described by Eynan and Kropp [25] and Qu et al. [5], when decision variables k_i and T are determined, the demand of item i will be normally distributed in each replenishment interval ($k_i T + L_i$), and the expectation E is equal to $D_i (k_i T + L_i)$; the variance Var is equal to $\delta_i (k_i T + L_i)$. Take into the lead time, the maximum inventory R_i in each replenishment interval of item i includes the expected demand during the time and the safety stock expressed as a multiple of the standard deviation of demand forecast errors during period ($k_i T + L_i$), that is to say $R_i = D_i(k_i T + L_i) + z_i \sqrt{\delta_i(k_i T + L_i)}$. The average net inventory level of item i during the interval is approximately $R_i - D_i(L_i + k_i T/2) = (1/2)D_i k_i T + z_i \sqrt{\delta_i(L + k_i T)}$; thus, the average inventory holding cost for periodic policy is

$$C_H = \sum_{i=1}^{n} h_i \left(\frac{1}{2} D_i k_i T + z_i \sqrt{\delta_i \left(k_i T + L_i \right)} \right). \tag{2}$$

A stockout occurs following an order placed at time t if the cumulative demand x_i (between t and $t + L_i + k_i T$) exceeds R_i. Then, there will be a penalty cost τ_i for each unit backlogged. If demand has a density function $f(x_i, L_i + k_i T)$ over the interval of length $L_i + k_i T$, the mean backlogging cost in the long run is

$$C_S = \sum_{i=1}^{n} \left(\frac{\tau_i}{k_i T} \right) \left(\int_{R_i}^{\infty} (x_i - R_i) f(x_i, k_i T + L_i) \, dx_i \right). \tag{3}$$

(2) Transportation Cost. Define W as the lowest common multiple of k_i. All items will repeat the same replenishment and distribution policy in every other regeneration cycle WT. So we only need to consider the replenishment and distribution policy in a regeneration cycle WT.

For each replenishment period j, the transportation cost includes the costs of stopover (at those plants visited) plus the travelling costs. Over a regeneration cycle, the average transportation cost is

$$C_T = \frac{\sum_{j=1}^{W} \left(\sum_{p=1}^{P} x_{jp} F_p + cd(j) \right)}{WT}, \tag{4}$$

where

$$x_{jp} = \begin{cases} 1, & \text{if the items are kept in the supplier } p \\ & \text{in the } j\text{th replenishment period} \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

(3) The Objective. Then, the objective is to minimize the total long-run expected cost (TC);

$$\text{Min } TC = C_O + C_H + C_S + C_T$$

$$= \frac{S}{T} + \sum_{i=1}^{n} \frac{s_i}{k_i T}$$

$$+ \sum_{i=1}^{n} h_i \left(\frac{1}{2} D_i k_i T + z_i \sqrt{\delta_i \left(k_i T + L_i \right)} \right)$$

$$+ \sum_{i=1}^{n} \frac{\tau_i}{k_i T} \left(\int_{R_i}^{\infty} (x_i - R_i) f(x_i, k_i T + L_i) \, dx_i \right)$$

$$+ \frac{\sum_{j=1}^{W} \left(\sum_{p=1}^{P} x_{jp} F_p + cd(j) \right)}{WT}, \tag{6}$$

where

$$R_i = D_i \left(k_i T + L_i \right) + z_i \sqrt{\delta_i \left(k_i T + L_i \right)},$$

$$f(x_i, k_i T + L_i) = \frac{1}{\sqrt{2\pi \cdot \delta_i \left(k_i T + L_i \right)}} \tag{7}$$

$$\cdot e^{-[x_i - D_i(k_i T + L_i)]^2 / 2\delta_i(k_i T + L_i)}.$$

2.2. Fuzzy JRD Involving Fuzzy Minor Ordering Cost, Holding Cost, and Backlogging Cost

2.2.1. Formulation. As reported in Vujošević et al. [26], an interesting real problem is when human originated data like holding cost and ordering cost are not precisely known but subjectively estimated or linguistically expressed because of the lack of the accurate history data. Suppose that the minor ordering cost s_i and holding cost h_i are not precisely known. For example, let the linguistic estimates of s_i and h_i be as follows.

"The minor ordering cost is about 8 dollars per order, but not less than 6 dollars per order nor more than 15 dollars per order."

"The holding cost is between 15% and 20% of the average inventory value, but not less than 12% nor more than 25%."

These imprecise definements in linguistics are always described by fuzzy numbers. So, fuzzy variables are also utilized to handle the JRD problem under uncertainty. Two popular kinds of fuzzy numbers characterized by triangular and trapezoidal membership functions (MFs) are presented in the appendix. They were widely used to solve uncertainty problem because of their intuitive appeal and their perceived computational efficacy (Pramanik and Biswas [27]; Giachetti and Young [28]). In addition, trapezoidal fuzzy numbers can be obtained by extending triangular fuzzy numbers (TFNs). That is to say, TFNs can be considered as a particular kind of trapezoidal fuzzy numbers (Vujošević et al. [26]; Giachetti and Young [28]). Thus, we treat the minor ordering cost s_i, holding cost h_i, and backlogging cost τ_i as trapezoidal fuzzy numbers in this study; that is,

$$
\begin{aligned}
\widetilde{s}_i &= [s_{i1}, s_{i2}, s_{i3}, s_{i4}], \\
\widetilde{h}_i &= [h_{i1}, h_{i2}, h_{i3}, h_{i4}], \\
\widetilde{\tau}_i &= [\tau_{i1}, \tau_{i2}, \tau_{i3}, \tau_{i4}].
\end{aligned}
\tag{8}
$$

According to Definition A.4 in the appendix, the MFs of $\widetilde{s}_i$, $\widetilde{h}_i$, and $\widetilde{\tau}_i$ are as follows:

$$
\mu_{\widetilde{s}_i}(x) = \begin{cases}
\dfrac{x - s_{i1}}{s_{i2} - s_{i1}}, & s_{i1} < x \le s_{i2} \\
1, & s_{i2} < x \le s_{i3} \\
\dfrac{s_{i4} - x}{s_{i4} - s_{i3}}, & s_{i3} < x \le s_{i4} \\
0, & x \le s_{i1} \text{ or } x > s_{i4}
\end{cases}
$$

$$
\mu_{\widetilde{h}_i}(x) = \begin{cases}
\dfrac{x - h_{i1}}{h_{i2} - h_{i1}}, & h_{i1} < x \le h_{i2} \\
1, & h_{i2} < x \le h_{i3} \\
\dfrac{h_{i4} - x}{h_{i4} - h_{i3}}, & h_{i3} < x \le h_{i4} \\
0, & x \le h_{i1} \text{ or } x > h_{i4}
\end{cases}
$$

$$
\mu_{\widetilde{\tau}_i}(x) = \begin{cases}
\dfrac{x - \tau_{i1}}{\tau_{i2} - \tau_{i1}}, & \tau_{i1} < x \le \tau_{i2} \\
1, & \tau_{i2} < x \le \tau_{i3} \\
\dfrac{\tau_{i4} - x}{\tau_{i4} - \tau_{i3}}, & \tau_{i3} < x \le \tau_{i4} \\
0, & x \le \tau_{i1} \text{ or } x > \tau_{i4}.
\end{cases}
\tag{9}
$$

Take each TFN into (6) to get the fuzzy number of TC. For the simplify, we denote that

$$
\begin{aligned}
A_i &= \left(\frac{1}{2} D_i k_i T + z_i \sqrt{\delta_i (k_i T + L_i)} \right); \\
B_i &= \frac{1}{k_i T} \int_{R_i}^{\infty} (x_i - R_i) f(x_i, k_i T + L_i) dx_i; \\
C &= \frac{S}{T} + \frac{\sum_{j=1}^{W} \left(\sum_{p=1}^{P} x_{jp} F_p + cd(j) \right)}{WT}.
\end{aligned}
\tag{10}
$$

Hence,

$$
\begin{aligned}
\widetilde{TC} &= C + \sum_{i=1}^{n} \frac{\widetilde{s}_i}{k_i T} + \sum_{i=1}^{n} \widetilde{h}_i \cdot A_i + \sum_{i=1}^{n} \widetilde{\tau}_i \cdot B_i \\
&= \left[C + \sum_{i=1}^{n} \frac{s_{i1}}{k_i T} + \sum_{i=1}^{n} h_{i1} \cdot A_i + \sum_{i=1}^{n} \tau_{i1} \cdot B_i, \right. \\
&\qquad C + \sum_{i=1}^{n} \frac{s_{i2}}{k_i T} + \sum_{i=1}^{n} h_{i2} \cdot A_i + \sum_{i=1}^{n} \tau_{i2} \cdot B_i, \\
&\qquad C + \sum_{i=1}^{n} \frac{s_{i3}}{k_i T} + \sum_{i=1}^{n} h_{i3} \cdot A_i + \sum_{i=1}^{n} \tau_{i3} \cdot B_i, \\
&\qquad \left. C + \sum_{i=1}^{n} \frac{s_{i4}}{k_i T} + \sum_{i=1}^{n} h_{i4} \cdot A_i + \sum_{i=1}^{n} \tau_{i4} \cdot B_i \right] \\
&= [TC_1, TC_2, TC_3, TC_4].
\end{aligned}
\tag{11}
$$

2.2.2. Defuzzified TC. Defuzzification has been a favorite approach in many inventory studies for its simplicity. Defuzzification can easily transfer fuzziness to be explicit without complex analysis. In this study, two approaches will be utilized to defuzzify the fuzzy total cost.

(1) Graded Mean Integration Representation (GMIR) Approach. Chen and Hsieh [29] introduced the GMIR method based on the integral value of the graded mean h-level of the generalized fuzzy number for defuzzifying a generalized fuzzy number. This method is reasonable because it adopts a grade as the important degree of each point of the support set of a fuzzy number for representing the fuzzy number (G. C. Mahata and P. Mahata [30]). So, we

also use the GMIR approach to defuzzify the fuzzy number. If $\widetilde{A} = [a_1, a_2, a_3, a_4]$, then the defuzzified function of $\widetilde{A}$ is

$$P\left(\widetilde{A}\right) = \frac{\int_0^1 h\left[(1/2)\left(a_1 + a_4 + (a_2 - a_1 - a_4 + a_3)h\right)\right]dh}{\int_0^1 h\,dh}$$

$$= \frac{1}{6}\left(a_1 + 2a_2 + 2a_3 + a_4\right), \tag{12}$$

where h is an arbitrary value in the interval of the membership function. According to (12) and (11), we can obtain the defuzzified total cost

$$P\left(\widetilde{TC}\right) = \frac{1}{6}\left[TC_1 + 2TC_2 + 2TC_3 + TC_4\right]. \tag{13}$$

So, the simplified objective is

$$\text{Min } P\left(\widetilde{TC}\right) = \frac{1}{6}\left[6C + \sum_{i=1}^{n}\frac{s_{i1} + 2s_{i2} + 2s_{i3} + s_{i4}}{k_i T}\right.$$

$$+ \sum_{i=1}^{n}\left(h_{i1} + 2h_{i2} + 2h_{i3} + h_{i4}\right) \cdot A_i \tag{14}$$

$$\left. + \sum_{i=1}^{n}\left(\tau_{i1} + 2\tau_{i2} + 2\tau_{i3} + \tau_{i4}\right) \cdot B_i\right].$$

(2) Centroid. Centroid is a widely used approach for ranking fuzzy numbers by distance method based on calculating the centroid point, where distance means from original point to the centroid point (Wang et al. [31]). If $\widetilde{A} = [a_1, a_2, a_3, a_4]$, then the defuzzified function of $\widetilde{A}$ is

$$C\left(\widetilde{A}\right) = \frac{1}{3}\left[a_1 + a_2 + a_3 + a_4\right.$$

$$\left. - \frac{a_4 a_3 - a_1 a_2}{(a_4 + a_3) - (a_1 + a_2)}\right]. \tag{15}$$

Hence, the simplified objective is expressed as

$$\text{Min } C\left(\widetilde{TC}\right) = \frac{1}{3}\left[TC_1 + TC_2 + TC_3 + TC_4\right.$$

$$\left. - \frac{TC_4 TC_3 - TC_1 TC_2}{(TC_4 + TC_3) - (TC_1 + TC_2)}\right]. \tag{16}$$

3. MADE Algorithm for Fuzzy JRD Model

3.1. Rationales of Using Differential Evolution. Differential evolution algorithm (DE) was proposed by Storn and Price [32] for complex continuous nonlinear, nondifferentiable, and multimodal optimization problem. This technique combines simple arithmetic operators with the classical events of crossover, mutation, and selection to evolve from a randomly generated starting population to a final solution. Due to its simple structure, easy implementation, quick convergence,

and robustness, DE has been turned out to be one of the best evolution algorithms in a variety of fields (Wang et al. [33]; Wee et al. [34]; Zheng and Yamashiro [35]).

However, the mutation factor F in a typical DE is a constant, which is actually difficult to determine. The main role of F is to perturb the individual in order to avoid being trapped in a local optimum. So, if F is too small, the degree of population diversity cannot reach a certain value and it is easy to be premature. Vice versa, if F is too large, the chance to find optimal solution will decrease, and the effectiveness of the algorithm will be very low in large decision space. More description about the value of F effects on evolution algorithm can be found in Storn and Price [32] and Salman et al. [36]. In this study, we use a modified adaptive DE to improve the quality of solutions of the defuzzified JRD. According to our rule, in the early stage of search, F is large enough to keep the population diversity and avoid premature. Then, F is decreased gradually, and in the latter stage F is small enough to remain the excellent individual and to increase the chance to converge to the optimal solution.

3.2. MADE-Based Procedures for Fuzzy JRD. The model aforementioned in Section 2.2.2 can be decomposed into a JRP and a transportation problem. We use MADE to determine the optimal combination of k_i, z_i, and T, and then we can obtain a satisfied solution of the annual total cost. The detailed steps are as follows:

Step 1. Representation and initialization. Let N_P represent the population size of a target population, and let $x_{t,G}$ denote the tth target individual at iteration G.

Because k_i is an integer, we set the lower bound $k_i^{\text{LB}} = 1$, which are clearly the lower bounds of item i. For the upper bound k_i^{UB}, we use values of 100 to ensure that the optimal solution will not escape according to the related experience of Cha et al. [9]. According to the properties of the standard normal distribution, we have $P(X \leq z_i) = 1 - P(X > z_i) = 0.995$ when $z_i = 2.575$. So, the range of z_i is set as $[Z^{\text{LB}}, Z^{\text{UB}}] = [0, 4]$ which will cover more than 99.99% of the demand.

According to the related experience of Khouja and Goyal [3] and Qu et al. [5], T is ranged from 0 to 1. It is a fact that the annual demand of each item D_i is larger than 600 in the following examples; so it is reasonable to assume that at least one item will be replenished per year when k_i is not less than 1. That is to say, T should be less than 1.

By combining k_i, z_i, and T, we can obtain the tth individual:

$$x_{t,G} = \left[k_i, z_i, T\right] = \left(k_1, k_2, k_3, k_4, z_1, z_2, z_3, z_4, T\right). \tag{17}$$

Then, create the initial population randomly.

Step 2. If the maximum number of iterations is reached, the algorithm will be stopped and output the optimal solution; otherwise, go to the next step.

Step 3 (mutation). For each target individual $x_{t,G}$ ($i = 1, 2, \ldots, N_P$), a mutant individual $v_{t,G+1}$ is generated according to

$$v_{t,G+1} = x_{r1,G} + F \cdot \left(x_{r2,G} - x_{r3,G} \right), \quad r1 \neq r2 \neq r3 \neq t. \quad (18)$$

With randomly chosen integer indexes $r_1, r_2, r_3 \in \{1, 2, \ldots, N_P\}$. Note that indexes have to be different from each other and from the running index. F is called the mutation factor which controls the amplification of the differential variations $(x_{r2,G}, x_{r3,G})$ and F can be obtained by

$$F = F_{\min} + \left(F_{\max} - F_{\min} \right) \cdot e^{1-(\text{Gen}M/(\text{Gen}M-G+1))}, \quad (19)$$

where $F_{\min}$ is the minimum of F, and $F_{\max}$ is the maximum of F. The way to confirm these parameters will be discussed in Section 4.1.

Step 4 (crossover). The crossover operator implements a discrete recombination of the trial individual $v_{t,G+1}$ and the parent individual $x_{t,G}$ to produce offspring $u_{t,G+1}$. The crossover is implemented as follows:

$$u_{tl,G+1} = \begin{cases} v_{tl,G+1} & \text{if } (\text{rand}(l) \leq \text{CR}) \text{ or } l = \text{rnb}(t) \\ x_{tl,G} & \text{otherwise} \end{cases} \quad (20)$$

$$l = 1, 2, \ldots, D,$$

where $x_{tl,G}$ refers to the lth element of the individual $x_{t,G}$. $u_{tl,G+1}$ and $v_{tl,G+1}$ are similarly defined. rand(l) is the lth evaluation of a uniform random number generator between $[0, 1]$. rnb(t) is a randomly chosen index from $1, 2, \ldots, D$ which ensures that $u_{t,G+1}$ gets at least one parameter from $v_{t,G+1}$. Otherwise, no new parent individual would be produced and the population would not alter. CR is the crossover rate between $[0, 1]$ which must be determined by the user.

Step 5 (selection). The evaluation function of an offspring is one-to-one competition in the MADE. It means that the resulting trial individual will only replace the original if it has a lower objective function value. Otherwise, the parent will remain in the next generation. The rule is as follows:

$$x_{t,G+1} = \begin{cases} u_{t,G+1}, & \text{if } g\left(u_{t,G+1} \right) \geq g\left(x_{t,G} \right) \\ x_{t,G}, & \text{if } g\left(u_{t,G+1} \right) < g\left(x_{t,G} \right), \end{cases} \quad (21)$$

where $g(x)$ means (14) for the GMIR approach or (16) for the centroid approach. This is the same for all variants of the MADE. Then, the best individual of the next generation will be at least as fit as the best individual of the current generation.

Then, the procedure will return to **Step 2** after setting $G = G + 1$.

Step 6. Output the optimal total cost TC^* and the corresponding replenishment cycle T^*, the order interval $(k_1, k_2, \ldots, k_n)$, and the tercile $(z_1, z_2, \ldots, z_n)$.

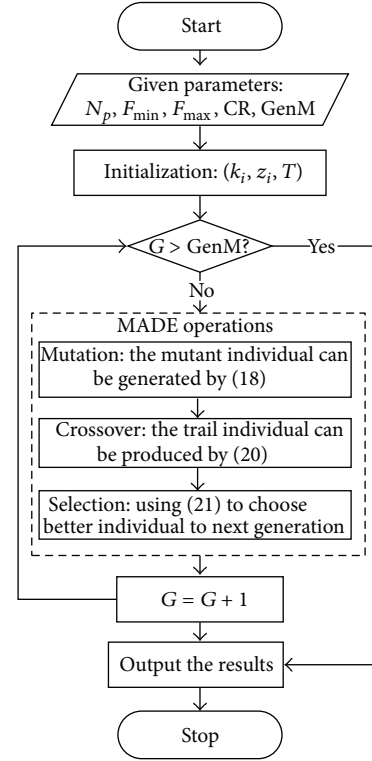The steps can be described by the flow chart as shown in Figure 1.

Figure 1: Flow chart of the MADE.

3.3. An Example of the Implementation of Operations of MADE. In this section, a simple example of four-product and three-supplier problem is presented in order to better understand the implementation of MADE.

(1) Representation. The sequences of each individual t, that is, $x_{t,G} = [k_i, z_i, T]$, can be represented as

$$2 \quad 4 \quad 5 \quad 3 \quad 1.98 \quad 2.05 \quad 1.59 \quad 1.83 \quad 0.109.$$

(2) Mutation (when $F = 0.6$). For the tth target vector, randomly generate 3 numbers r_1, r_2, and r_3, which are different from each other and different from i.

Sequences of $x_{r1,G}$

$$3 \quad 1 \quad 1 \quad 2 \quad 2.09 \quad 1.49 \quad 1.08 \quad 2.39 \quad 0.098.$$

Sequences of $x_{r2,G}$

$$4 \quad 2 \quad 3 \quad 3 \quad 2.56 \quad 1.79 \quad 2.41 \quad 1.38 \quad 0.376.$$

Sequences of $x_{r3,G}$

$$2 \quad 1 \quad 3 \quad 2 \quad 0.68 \quad 1.09 \quad 1.89 \quad 1.76 \quad 0.289.$$

The mutant vector $v_{t,G+1}$ is created by (18):

$$4.2 \quad 1.6 \quad 1 \quad 2.6 \quad 3.22 \quad 1.91 \quad 1.39 \quad 2.16 \quad 0.1502.$$

TABLE 1: The basic data under deterministic situation.

	Item 1	Item 2	Item 3	Item 4
s_i	25	14	20	30
D_i	600	900	1200	1000
δ_i	800	600	700	500
L_i	0.02	0.02	0.02	0.02
h_i	5.6	21	42	15
τ_i	28	35	40	30

TABLE 2: The items supplied by suppliers.

	Supplier 1	Supplier 2	Supplier 3
Item 1	1	0	0
Item 2	0	1	0
Item 3	0	0	1
Item 4	0	0	1
F_p	40	50	60

TABLE 3: The distance between the warehouse and suppliers.

	Warehouse	Supplier 1	Supplier 2	Supplier 3
Warehouse	0	11	9	7
Supplier 1	11	0	5	8
Supplier 2	9	5	0	10
Supplier 3	7	8	10	0

(3) Crossover (when CR = 0.3). For each dimension, the randomly generated number rand (j) is

$$0.25 \quad 0.76 \quad 0.18 \quad 0.39 \quad 0.29 \quad 0.48 \quad 0.21 \quad 0.39 \quad 0.47.$$

And rnb (t) = 6 for this target vector. Therefore, the trial vector $u_{t,G+1}$ is

$$\textbf{4.2} \quad 4 \quad 1 \quad 3 \quad \textbf{3.22} \quad 1.91 \quad 1.39 \quad 1.83 \quad 0.109.$$

Since "4.2" is not an integer, we round it to the nearest integer; that is, it will be changed to "4." Meanwhile, "3.22" exceeds the range of variety; it is substituted by a number randomly generated from [0, 3]. After that, the sequences of adjusted $u_{t,G+1}$ are:

$$4 \quad 4 \quad 1 \quad 3 \quad \textbf{1.87} \quad 1.91 \quad 1.39 \quad 1.83 \quad 0.109.$$

(4) Selection. Since $g(u_{t,G+1})$ = 25110, $g(x_{t,G})$ = 14103, according to (21), the offspring which will remain to the next generation is:

$$4 \quad 4 \quad 1 \quad 3 \quad \textbf{1.87} \quad 1.91 \quad 1.39 \quad 1.83 \quad 0.109$$

4. Numerical Examples and Analysis

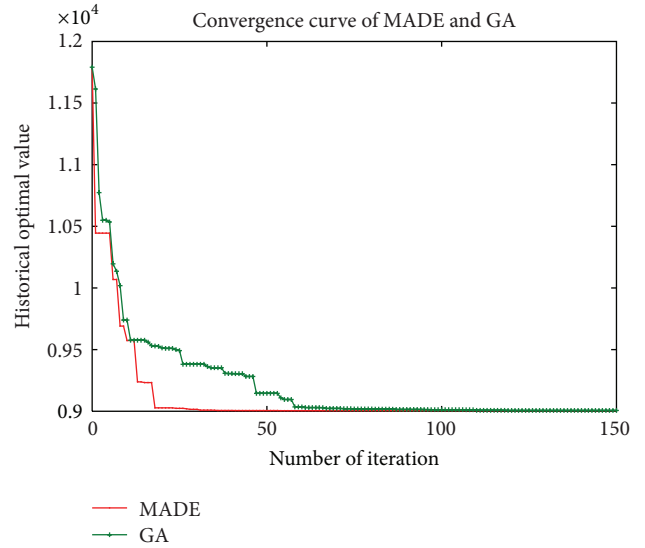In Section 4.1, we will test the performance of MADE with results of GA which was tested to be one of the effective

FIGURE 2: Convergence curve of MADE and GA.

algorithms by a stochastic JRD. Then, the MADE will be used to handle the fuzzy JRD.

4.1. An Example of Stochastic JRD

(1) Basic Data. The data of the example is the same as Qu et al. [5]. We suppose that the central warehouse is in charge of the replenishment of 4 items and the inventory management. There are 3 suppliers and the third supplier produces the third and fourth items. The major ordering cost S is 100, the unit distance cost c is 0.5, and other data are shown in Tables 1, 2, and 3, respectively.

(2) Parameters Setting and Results. Storn and Price [32] suggested that F = 0.5, CR = 0.1 is usually a good initial choice and N_P is appropriate for ten times the dimensionality of the problem. Some scholars also provided useful suggestions. According to the recommendation of Neri and Tirronen [37] and Liu and Lampinen [38], we set the relevant parameters of the MADE as follows: N_P = 100, $F_{\min}$ = 0.2, $F_{\max}$ = 1.2, CR = 0.1, and Gen M = 150. Table 4 shows the result of Qu et al. [5] in comparison to our result solved by MADE and GA. We programmed the approach of [5], MADE, and GA with the language of Matlab 6.5. Comparison of the CPU time in a computer (memory: 4G; CPU: Intel Core i5-2450 M 2.50 GHz) by running the example 50 times is also shown in Table 4.

Both GA and MADE are intelligence EAs. For a perspicuous and direct understanding, we give two convergence curves as shown in Figure 2.

Results listed in Table 4 indicate that the total cost gained by MADE is the smallest and the total cost gotten by GA is the worst. We can also conclude the following: (1) the holding cost of item 1 is 5.6 which is the lowest among all items so that the solved optimal value of k is 2 and the replenishment cycle is the longest; (2) the demand, inventory cost, and shortage cost

TABLE 4: Results comparison of three approaches.

	k_i	z_i	T	TC, min	Convergence rate	CPU time (seconds)
Approach of [5]	3, 1, 1, 1	1.7035, 1.7125, 1.4060, 1.7907	0.0759	9021.2	/	/
MADE	2, 1, 1, 1	1.8537, 1.6576, 1.3765, 1.7441	0.0811	9005.9	100%	4.3734
GA	2, 1, 1, 1	2.2442, 2.1071, 1.9193, 2.2863	0.0775	9213.2	66%	4.4655

of item 3 are the highest among all items so that we should replenish it in every replenishment cycle.

Obviously, MADE is better than GA and the algorithm of Qu et al. [5]. So, the MADE will be used to solve the fuzzy JRD.

4.2. Three Scenarios of Fuzzy JRD Model and Analysis. In this section, we will design three different scenarios to compare the results of JRD and fuzzy JRD.

4.2.1. Scenarios 1. We set the range of the fuzzy set to $[d_1X, d_2X, d_3X, d_4X] = [0.7\,X, 0.95\,X, 1.05\,X, 1.1\,X]$, where X is the value under certainty. The related data is listed in Table 5.

The major ordering cost S was set to 100, 300, 500, and 700, respectively. The results computed by the MADE under uncertainty and determinacy are shown in Table 6. Here, the average CPU time by running each example 20 times in the abovementioned computer is also reported in Table 6. Statistical test of TC is not provided because MADE is quite effective to find the best TC every time for this problem (maximum error is less than 0.002% and is regarded as an optimal solution). In the following Scenarios 2 and 3, the CPU time is obtained by the same method and the statistic of TC is also not given for the same reason.

Table 6 shows the following: (1) results of TC under determinacy are larger than TC under uncertainty; (2) z_i and T obtained under uncertainty are larger than the corresponding values under determinacy; (3) the best results of TC using GMIR are larger than the corresponding values using centroid; (4) results of z_i and T obtained by centroid are larger than GMIR; (5) the CPU times of JRD and fuzzy JRD with GMIR are very close, but the CPU times of fuzzy JRD with centroid are much larger because of the relatively complex computation process.

4.2.2. Scenarios 2 (Symmetrical Fuzzy MFs). Set the range of the fuzzy set $[d_1X, d_2X, d_3X, d_4X] = [0.9\,X, 0.95\,X, 1.05\,X, 1.1\,X]$. Table 7 shows the data of the ordering cost $\widetilde{s}_i$, holding cost $\widetilde{h}_i$, and shortage cost $\widetilde{\tau}_i$ under uncertainty.

When S was set to 100, 300, 500, and 700, respectively, the results computed by the MADE under uncertainty and determinacy are shown in Table 8.

Results shown in Table 8 indicate that the results of the best TC and T under uncertainty and determinacy are the same for this situation. However, in reality, we are always not able to acquire the accurate value of the parameters or we only can acquire it with very high cost. So, we can use fuzzy theory to solve it reasonably.

4.2.3. Scenarios 3. We extend the range of the fuzzy set to $[d_1X, d_2X, d_3X, d_4X] = [0.9\,X, 0.95\,X, 1.05\,X, 1.3X]$ and the related data is shown in Table 9.

We set the major ordering cost S to 100, 300, 500, and 700, respectively, and solve the problem in the same way. Table 10 shows the comparison of results under uncertainty and determinacy after extension.

Table 10 indicates that the results of the best TC under uncertainty are larger than the corresponding results of determinacy situation. In comparison with results of T under determinacy, the basic replenishment cycle is smaller.

From Tables 6, 8, and 10, we can reasonably come to the following conclusions:

(1) the ranges of fuzzy parameters will inevitably influence the decision of the JRD. So, it is important to utilize every useful information and to correct judgment to confirm the ranges of fuzzy numbers scientifically and reasonably;

(2) MADE is quite effective for this fuzzy JRD problem because it can find the best minimum long-run average total cost with 100% convergence rate. The CPU time is also accepted for decision makers.

5. Conclusion and Future Research

This paper is an interdisciplinary research of the fuzzy inventory model and intelligent optimization algorithm. A fuzzy JRD model for the one-warehouse, n-supplier system is studied while assuming warehouse's minor ordering cost, warehouse's inventory holding cost, and warehouse's storage cost to be fuzzy numbers and a new DE is proposed. The main contributions are as follows:

(1) Due to the nonavailability of sufficient and precise input data, accurate predicted values of the minor ordering cost, inventory holding cost, and backlogging cost cannot be obtained easily; while fuzzy numbers can efficiently model the imprecise values. Obviously, it is more reasonable to handle imprecise values using the fuzzy theory in the JRD model. It is the first time to introduce fuzziness into the stochastic JRD, which will widen the application field of fuzzy theory and will make the JRD become more practical.

(2) Since the stochastic JRD policy is widely used in many industries, such as manufacturing, wholesale, maintenance, repair, and operating (MRO) supplies, the proposed fuzzy JRD models with two defuzzification methods can also be applied in these industries.

TABLE 5: The basic data after extension.

Item	$\widetilde{s}_i$	$\widetilde{h}_i$	$\widetilde{\tau}_i$
1	[17.5, 23.75, 26.25, 27.5]	[3.92, 5.32, 5.88, 6.16]	[19.6, 26.6, 29.4, 30.8]
2	[9.8, 13.3, 14.7, 15.4]	[14.7, 19.95, 22.05, 23.1]	[24.5, 33.25, 36.75, 38.5]
3	[14, 19, 21, 22]	[29.4, 39.9, 44.1, 46.2]	[28, 38, 42, 44]
4	[21, 28.5, 31.5, 33]	[10.5, 14.25, 15.75, 16.5]	[21, 28.5, 31.5, 33]

TABLE 6: Results comparison under different situations (scenarios 1).

S	Situation	k_i	z_i	T	TC, min	CPU time (seconds)
100	Certainty	(2, 1, 1, 1)	(1.8459, 1.6578, 1.3710, 1.7442)	0.0811	9005.90	4.3686
	Uncertainty, GMIR	(2, 1, 1, 1)	(1.8399, 1.6514, 1.3637, 1.7379)	0.0822	8805.18	4.4798
	Uncertainty, centroid	(2, 1, 1, 1)	(1.8349, 1.6459, 1.3575, 1.7327)	0.0832	8643.61	20.1834
300	Certainty	(2, 1, 1, 1)	(1.7319, 1.5343, 1.2301, 1.6252)	0.1041	11165.68	4.3143
	Uncertainty, GMIR	(2, 1, 1, 1)	(1.7249, 1.5268, 1.2214, 1.6179)	0.1057	10934.50	4.4220
	Uncertainty, centroid	(2, 1, 1, 1)	(1.7191, 1.5205, 1.2142, 1.6118)	0.1070	10747.98	20.258
500	Certainty	(2, 1, 1, 1)	(1.6525, 1.4478, 1.1302, 1.5421)	0.1231	12926.66	4.2566
	Uncertainty, GMIR	(2, 1, 1, 1)	(1.6449, 1.4396, 1.1206, 1.5342)	0.1250	12668.79	4.3772
	Uncertainty, centroid	(2, 1, 1, 1)	(1.6387, 1.4328, 1.1127, 1.5277)	0.1266	12460.48	19.8587
700	Certainty	(1, 1, 1, 1)	(1.8962, 1.3601, 1.0276, 1.4581)	0.1448	14415.60	4.1407
	Uncertainty, GMIR	(1, 1, 1, 1)	(1.8893, 1.3514, 1.0174, 1.4497)	0.1471	14132.48	4.2276
	Uncertainty, centroid	(1, 1, 1, 1)	(1.8836, 1.3442, 1.0089, 1.4428)	0.1491	13903.65	19.9156

TABLE 7: The basic data under uncertainty.

Item	$\widetilde{s}_i$	$\widetilde{h}_i$	$\widetilde{\tau}_i$
1	[22.5, 23.75, 26.25, 27.5]	[5.04, 5.32, 5.88, 6.16]	[25.2, 26.6, 29.4, 30.8]
2	[12.6, 13.3, 14.7, 15.4]	[18.9, 19.95, 22.05, 23.1]	[31.5, 33.25, 36.75, 38.5]
3	[18, 19, 21, 22]	[37.8, 39.9, 44.1, 46.2]	[36, 38, 42, 44]
4	[27, 28.5, 31.5, 33]	[13.5, 14.25, 15.75, 16.5]	[27, 28.5, 31.5, 33]

TABLE 8: Results comparison under different situations (scenarios 2).

S	Situation	k_i	z_i	T	TC, min	CPU time (seconds)
100	Certainty	(2, 1, 1, 1)	(1.8459, 1.6578, 1.3710, 1.7442)	0.0811	9005.90	4.4341
	Uncertainty, GMIR	(2, 1, 1, 1)	(1.8459, 1.6578, 1.3710, 1.7442)	0.0811	9005.90	4.6048
	Uncertainty, centroid	(2, 1, 1, 1)	(1.8459, 1.6578, 1.3710, 1.7442)	0.0811	9005.90	21.0726
300	Certainty	(2, 1, 1, 1)	(1.7319, 1.5343, 1.2301, 1.6252)	0.1041	11165.68	4.3173
	Uncertainty, GMIR	(2, 1, 1, 1)	(1.7319, 1.5343, 1.2301, 1.6252)	0.1041	11165.68	4.4136
	Uncertainty, centroid	(2, 1, 1, 1)	(1.7319, 1.5343, 1.2301, 1.6252)	0.1041	11165.68	20.3023
500	Certainty	(2, 1, 1, 1)	(1.6525, 1.4478, 1.1302, 1.5421)	0.1231	12926.66	4.2955
	Uncertainty, GMIR	(2, 1, 1, 1)	(1.6525, 1.4478, 1.1302, 1.5421)	0.1231	12926.66	4.37988
	Uncertainty, centroid	(2, 1, 1, 1)	(1.6525, 1.4478, 1.1302, 1.5421)	0.1231	12926.66	19.8634
700	Certainty	(2, 1, 1, 1)	(1.8962, 1.3601, 1.0276, 1.4581)	0.1448	14415.60	4.1261
	Uncertainty, GMIR	(2, 1, 1, 1)	(1.8962, 1.3601, 1.0276, 1.4581)	0.1448	14415.60	4.2493
	Uncertainty, centroid	(2, 1, 1, 1)	(1.8962, 1.3601, 1.0276, 1.4581)	0.1448	14415.60	19.8194

TABLE 9: The basic data after extension.

item	$\widetilde{s}_i$	$\widetilde{h}_i$	$\widetilde{\tau}_i$
1	[22.5, 23.75, 26.25, 32.5]	[5.04, 5.32, 5.88, 7.28]	[25.2, 26.6, 29.4, 36.4]
2	[12.6, 13.3, 14.7, 18.2]	[18.9, 19.95, 22.05, 27.3]	[31.5, 33.25, 36.75, 45.5]
3	[18, 19, 21, 26]	[37.8, 39.9, 44.1, 54.6]	[36, 38, 42, 52]
4	[27, 28.5, 31.5, 39]	[13.5, 14.25, 15.75, 19.5]	[27, 28.5, 31.5, 39]

Table 10: Results comparison under different situations after extension (scenarios 3).

S	Situation	k_i	z_i	T	TC, min	CPU time (seconds)
100	Certainty	(2, 1, 1, 1)	(1.8459, 1.6578, 1.3710, 1.7442)	0.0811	9005.90	4.3782
	Uncertainty, GMIR	(2, 1, 1, 1)	(1.8516, 1.664, 1.378, 1.7501)	0.0801	9205.31	4.5183
	Uncertainty, centroid	(2, 1, 1, 1)	(1.856, 1.6687, 1.3833, 1.7547)	0.0793	9363.95	20.6277
300	Certainty	(2, 1, 1, 1)	(1.7319, 1.5343, 1.2301, 1.6252)	0.1041	11165.68	4.2908
	Uncertainty, GMIR	(2, 1, 1, 1)	(1.7386, 1.5416, 1.2385, 1.6322)	0.1026	11394.79	4.4135
	Uncertainty, centroid	(2, 1, 1, 1)	(1.7437, 1.5472, 1.2449, 1.6376)	0.1015	11576.66	20.0418
500	Certainty	(2, 1, 1, 1)	(1.6525, 1.4478, 1.1302, 1.5421)	0.1231	12926.66	4.2934
	Uncertainty, GMIR	(2, 1, 1, 1)	(1.6597, 1.4558, 1.1393, 1.5497)	0.1212	13181.85	4.3672
	Uncertainty, centroid	(2, 1, 1, 1)	(1.665, 1.4616, 1.1461, 1.5553)	0.1199	13384.18	20.1400
700	Certainty	(1, 1, 1, 1)	(1.8962, 1.3601, 1.0276, 1.4581)	0.1448	14415.60	4.1279
	Uncertainty, GMIR	(1, 1, 1, 1)	(1.9029, 1.3685, 1.0374, 1.466)	0.1426	14695.63	4.2845
	Uncertainty, centroid	(1, 1, 1, 1)	(1.908, 1.3749, 1.045, 1.4722)	0.1410	14917.54	19.8705

(3) The proposed simple and effective MADE-based approach can find the optimal cycle time and safety factor of each item for the defuzzified JRD effectively. Moreover, the convergence rate of MADE outperforms another popular GA-based approach. Therefore, managers can always know the corresponding minimum total cost under fuzzy environment, just like the case of precise data. This study expands the application field of the DE.

In the future, a dependent-chance programming model or a chance-constrained programming model can be designed for the JRD model of the one-warehouse, n-retailer system in fuzzy environment. The DE still can provide good solutions to these problems.

Appendix

In order to solve the fuzzy JRD model, we need to use the following definitions and properties.

Definition A.1. A fuzzy set $\widetilde{a}$ on $R = (-\infty, +\infty)$ is called a fuzzy point if its MF is

$$\mu_{\widetilde{a}}(x) = \begin{cases} 1, & x = a \\ 0, & x \neq a, \end{cases} \tag{.22}$$

where point a is called its support.

Let the family of all fuzzy points be $F_p = \{\widetilde{a} \mid \forall a \in R\}$. The real number $a \in R$ and the fuzzy point $\widetilde{a} \in F_p$ are in one-to-one and onto mapping.

Definition A.2. A fuzzy set $[a_x, b_x]$ on R, $0 \leq a \leq 1$, $a < b$, is called a level α fuzzy interval if its MF is

$$\mu_{[a_x, b_x]}(x) = \begin{cases} \alpha, & a \leq x \leq b \\ 0, & \text{otherwise}. \end{cases} \tag{.23}$$

Definition A.3. A fuzzy set $\widetilde{A} = [a, b, c]$ on R, $a < b < c$, is called a triangular fuzzy number if its MF is

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{x - a}{b - a}, & a < x \leq b \\ \dfrac{c - x}{c - b}, & b < x \leq c \\ 0, & x \leq a, \text{ or } x > c. \end{cases} \tag{.24}$$

When $a = b = c$, we have fuzzy point $(c, c, c) = \widetilde{c}$.

Definition A.4. A fuzzy set $\widetilde{A} = [a, b, c, d]$ on R, $a < b < c < d$, is called a trapezoidal fuzzy number if its MF is

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{x - a}{b - a}, & a < x \leq b \\ 1, & b < x \leq c \\ \dfrac{d - x}{d - c}, & c < x \leq d \\ 0, & x \leq a \text{ or } x > d. \end{cases} \tag{.25}$$

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